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OPTIMAL PLASTIC DESIGN OF PITCHED ROOF FRAMES FOR MULTIPLE LOADING

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Abstract—The paper describes the use of algebraic linear programming for the minimum weight design of steel portal frames subject to the constraints of the Kinematic Theorem of plastic collapse. Minimum weight design is a classic linear programming problem which can be solved algebraically for classes of frames with arbitrary geometric dimensions and arbitrary load magnitudes. In a recent paper, the process of algebraic linear programming was reduced to the repeated application of a number of vector formulas and a computer program was developed for the derivation of the solution charts for specific classes of frames. In this paper the method is extended to the problem of frames subjected to multiple load cases. It is shown that simple problems whose solution can normally be displayed in the form of two-dimensional charts now require three-dimensional charts or a number of two-dimensional charts. © 1997 Civil-Comp Ltd and Elsevier Science Ltd.

1. INTRODUCTION

The theory of plasticity has been well established over the years [1, 2] and is widely used today in the design of single-storey steel frames to resist ultimate loads [3]. Minimum weight design using plastic methods of analysis was established [4] as early as 1951 and can be carried out using standard linear programming techniques [5]. Although design for minimum weight, subject solely to the constraints imposed by plastic theory, does not necessarily result in frames of minimum weight overall, such methods are useful at the preliminary stages of design for frames with known geometry and loading. In the 1980's, Brousse [6] and others [7, 8] used the concept of *algebraic* linear programming to determine the closed form solution for a number of minimum weight design problems. More recently, the authors have written a computer program, ALP, with the capability of extending algebraic linear programming techniques to new classes of frames [9, 10]. In this context, a class of frames is defined as all frames conforming to a specified geometric shape and subject to a specified number of loads in specified directions and locations. Subject to these restrictions, the frame dimensions and load magnitudes are arbitrary over certain ranges.

In this paper, the application of algebraic linear programming to classes of frames subject to multiple load cases is considered. The effect of the added constraints on the optimisation problem is shown and the feasibility of solving the resulting algebraic linear programming problem is discussed.

2. ALGEBRAIC LINEAR PROGRAMMING FOR MULTIPLE LOADS

The minimum weight design for the simple class of frames illustrated in Fig. 1 is derived here for combinations of dead, wind and imposed loads (a more detailed exposition is given in Ref. 11). For this class of frames, there exist three possible collapse mechanisms, illustrated in Fig. 2. Corresponding to these mechanisms, there are three inequalities, derived using the Kinematic Theorem, which provide the constraints to the optimisation problem. However, the inequality for the beam mechanism is less stringent than that for the frame mechanism and so it can be ignored.

Also, a collapse mechanism resulting from dead load alone will always be less critical than the corresponding mechanism in which combined dead and imposed load is present. Therefore, only three combinations of load need to be considered: dead load acting with imposed load, dead load acting with wind load, and dead load acting with imposed and wind loads. The optimisation problem involves three constraints for each of the sway and frame mechanisms, one for each of the three load combinations. However, five of these constraints can be discarded as being less stringent than others or as leading to trivial solutions. Using the load combination factors recommended by BS 5950 [12], the remaining inequalities are as follows for sway mechanism:

$$\text{dead + wind:} \quad 2\min(x_1, x_2) \geqslant 1.4\alpha \qquad (1)$$

and for frame mechanisms:

dead + imposed:

$$(2\rho + 2)\min(x_1, x_2) + 2x_1 \geqslant 1.4 + 1.6\beta \quad (2)$$

dead + wind:

$$(2\rho + 2)\min(x_1, x_2) + 2x_1 \geqslant 1.4 + 1.4\alpha \quad (3)$$

dead + wind + imposed:

$$(2\rho + 2)\min(x_1, x_2) + 2x_1 \geqslant 1.2 + 1.2\beta + 1.2\alpha \quad (4)$$

where $x_1 = X_1/(P_d l)$, $x_2 = X_2/(P_d l)$, $\alpha = Q_w h/(P_d l)$, $\beta = P_i/P_d$ and where P_d and P_i are the (unfactored) dead and imposed components, respectively, of the central vertical load, Q_w is the horizontal wind load at the eaves and X_1 and X_2 are the plastic moments of resistance of the rafters and columns, respectively. The optimisation problem is that of minimising $x_2 + S^0 x_1$ subject to Inequalities (1)–(4), where $S^0 = l_r/h$.

The algebraic solution to this problem consists of two stages:

(i) identification of all possible vertices of the simplex and determination of the conditions under which each vertex is feasible;
(ii) determination of the conditions under which each vertex is optimal

Consideration of the multiple load combinations has the effect of introducing two loading parameters, α and β, into the linear programming problem. With only one loading parameter, α, it was possible to identify intervals of α for which each vertex was valid. These intervals in α were then ranked in increasing order and the vertices of each interval considered in turn. For the multiple loadings problem, it is only possible to rank the limits of α for specified intervals of β. For example, the vertex associated with Inequalities (1) and (3) in the region of design space, $x_2 < x_1$ is:

$$(x_1, x_2) = (0.7(1 - \rho\alpha), 0.7\alpha)$$

This vertex satisfies Inequality (2) for $\alpha \geqslant 8\beta/7$ and satisfies Inequality (4) for $\alpha \geqslant 6\beta - 1$. The first of these limits on α is the more stringent for $\beta \leqslant 7/34$; the second for $\beta > 7/34$.

For all intervals of α and β, the coordinates for each of the valid vertices and the corresponding ranges in α and β are given in Table 1. The ranges for all vertices are illustrated graphically in Fig. 3. It can be seen clearly in the figure that the vertex denoted V3 is bounded on the left side by the line, $\alpha = 8\beta/7$, while $\beta < 7/34$ and that the line, $\alpha = 6\beta - 1$, becomes the lower limit when $\beta > 7/34$.

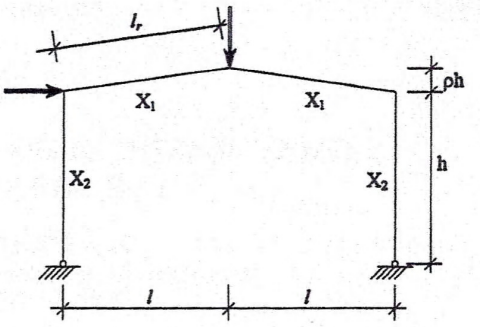

Fig. 1. Dimensions and loading for a class of pitched-roofed frames. Plastic moment of resistance for each rafter is X_1 and for each column is X_2.

To determine the minimum weight solution, each region in $\alpha - \beta$ space is considered in turn. For each region, the conditions under which particular vertices are optimal are considered through a comparison of slopes with that of the objective function. For this simple example, there are no more than two vertices in any region. It has been found using ALP that for $S^0 < 1/(1 + \rho)$, the first of the pair listed in Fig. 3 is optimal for all ranges (i.e. V1, V3, V4 and V6). The second of the pair is optimal in all cases when $S^0 > 1/(1 + \rho)$. The solution for this optimisation problem is presented in the form of a three-dimensional design chart in Fig. 4.

3. MINIMUM WEIGHT DESIGN OF MORE COMPLEX CLASSES OF FRAMES SUBJECT TO MULTIPLE LOAD COMBINATIONS

For the simple class of frames illustrated in Fig. 1, integration of multiple load case combinations

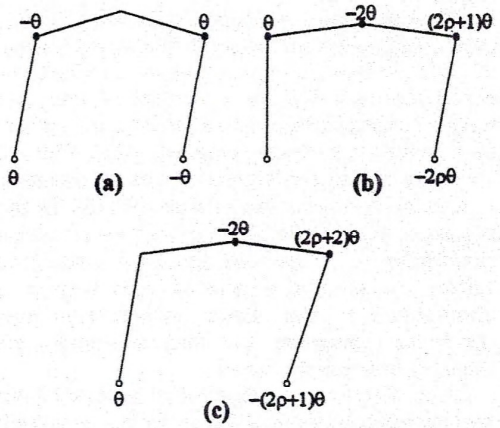

Fig. 2. Possible collapse mechanisms for the class of frames illustrated in Fig. 1, including hinge rotation magnitudes: (a) sway mechanism; (b) beam mechanism; (c) frame mechanism.

Table 1. Valid vertices of the simplex and the range for which each vertex is valid

Vertex and associated inequalities	x_1	x_2	Range of α		Range of β	
			Lower	Upper	Lower	Upper
V1	$\frac{7+8\beta-\alpha(7+7\rho)}{10}$	$\frac{7\alpha}{10}$	0	$\frac{8\beta}{7}$	0	$\frac{7}{34}$
			0	$\frac{2\beta+1}{6}$	$\frac{7}{34}$	None
V2	$\frac{7+8\beta}{20+10\rho}$	$\frac{7+8\beta}{20+10\rho}$	0	$\frac{8\beta}{7}$	0	$\frac{7}{34}$
			0	$\frac{2\beta+1}{6}$	$\frac{7}{34}$	None
V3	$\frac{7-7\rho\alpha}{10}$	$\frac{7\alpha}{10}$	$\frac{8\beta}{7}$	$\frac{1}{(1+\rho)}$	0	$\frac{7}{34}$
			$6\beta-1$	$\frac{1}{(1+\rho)}$	$\frac{7}{34}$	$\frac{(2+\rho)}{6(1+\rho)}$
V4	$\frac{7\alpha}{10}$	$\frac{7\alpha}{10}$	$\frac{1}{(1+\rho)}$	None	0	$\frac{(2+\rho)}{6(1+\rho)}$
			$\frac{(6+6\beta)}{(8+7\rho)}$	None	$\frac{(2+\rho)}{6(1+\rho)}$	None
V5	$\frac{7(1+\alpha)}{20+10\rho}$	$\frac{7(1+\alpha)}{20+10\rho}$	$\frac{8\beta}{7}$	$\frac{1}{(1+\rho)}$	0	$\frac{7}{34}$
			$6\beta-1$	$\frac{1}{(1+\rho)}$	$\frac{7}{34}$	$\frac{(2+\rho)}{6(1+\rho)}$
V6	$\frac{6+6\beta-\alpha(1+7\rho)}{10}$	$\frac{7\alpha}{10}$	$\frac{2\beta+1}{6}$	$6\beta-1$	$\frac{7}{34}$	$\frac{(2+\rho)}{6(1+\rho)}$
			$\frac{2\beta+1}{6}$	$\frac{(6+6\beta)}{(8+7\rho)}$	$\frac{(2+\rho)}{6(1+\rho)}$	None
V7	$\frac{3(1+\alpha+\beta)}{5(2+\rho)}$	$\frac{3(1+\alpha+\beta)}{5(2+\rho)}$	$\frac{2\beta+1}{6}$	$6\beta-1$	$\frac{7}{34}$	$\frac{(2+\rho)}{6(1+\rho)}$
			$\frac{2\beta+1}{6}$	$\frac{(6+6\beta)}{(8+7\rho)}$	$\frac{(2+\rho)}{6(1+\rho)}$	None

into the optimisation procedure increased both the algebraic complexity of the procedure and the complexity of the resulting solution (three vertices for a single load case versus seven vertices for multiple load cases). There are 20 possible collapse mechanisms of the more complex example illustrated in Fig. 5. Consideration of this class for multiple load combinations obviously involves a substantial increase in the complexity of the optimal design solution. Considering the 20 collapse mechanisms for three load combinations results in a total of 60 inequalities which form the constraints of the optimisation problem. In fact, it has been shown [10] that 44 of these constraints can be discarded as being less or equally stringent to others. For the remaining 16 inequalities, however, it has been determined that it is necessary to consider 17 intervals of β (as compared to just three for the previous example) in order to solve the optimisation problem. Due to the large number of collapse mechanisms it would be reasonable to expect that there would be a significant number of alternative optimal solutions for each interval of $\alpha - \beta$ space. This would result in the division of the S^0 axis of the design chart into a significant number of different regions. The implication is that it would not be possible to illustrate the solution in a form similar to Fig. 4.

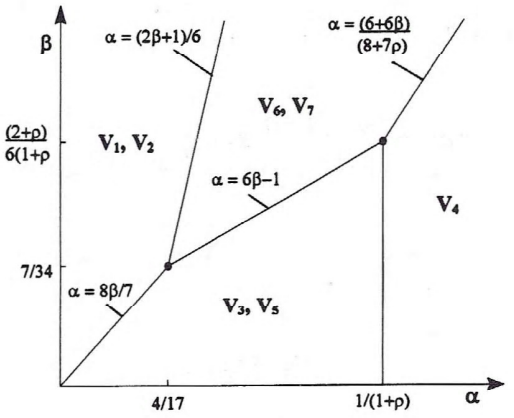

Fig. 3. Ranges of α and β for which vertices are valid.

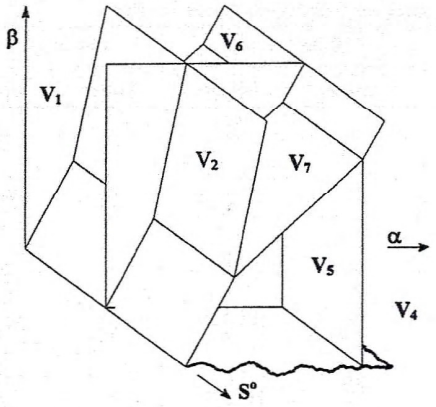

Fig. 4.

As the identification of optimal solution involves a comparison of the slopes of the lines bounding each inequality with the slope of the objective function, the number of distinct regions of S^0 in the solution chart cannot exceed the number of different slopes. For the class of frames illustrated in Fig. 5, there are only seven distinct slopes. Thus, while it may not be possible to illustrate the solution in the form of a three-dimensional chart, it is possible to illustrate it in the form of not more than six two-dimensional charts, one for each interval of S^0.

4. EXAMPLE

A pitched roof portal frame, from the class illustrated in Fig. 1, is to be designed for minimum weight where $l = 10$ m, $h = 6$ m, $\rho = 0.15$, $P_d = 40$ kN, $P_i = 60$ kN and $Q_w = 10$ kN.

Solution: The nondimensional parameters are calculated as $\alpha = 0.15$, $\beta = 1.5$ and $S^0 = 1.67$. As $\alpha < 4/17$ and the point (0.15, 1.5) is above the line, $\alpha = 8\beta/7$, in Fig. 3, only V1 and V2 are valid vertices of the simplex. The slope, S^0, exceeds the value, $1/(1 + \rho)$ given in Fig. 4. Hence the optimal vertex is V2 with coordinates (from Table 1) of:

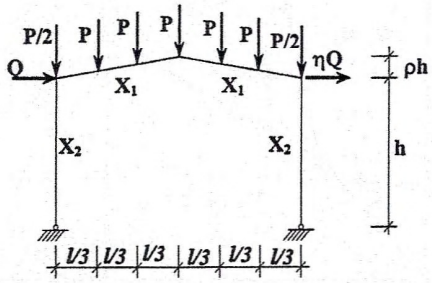

Fig. 5.

$$(x_1, x_2) = \left(\frac{7 + 8\beta}{20 + 10\rho}, \frac{7 + 8\beta}{20 + 10\rho}\right) = (0.884, 0.884)$$

The optimal plastic moments of resistance are hence $(0.884)(40 \times 10) = 353$ kNm for both the rafters and the columns.

5. CONCLUSIONS

Algebraic linear programming has been applied to the minimum weight design of the class of frames of Fig. 1 subject to multiple combinations of dead, imposed and wind loads. It has been shown that it is possible to represent the solution in the form of a three-dimensional design chart. The complete preliminary design of minimum weight for any frame whose geometry and loading conform to this standard can be obtained directly from this chart.

The application of the method to more complex classes of frames subject to multiple load combinations is shown to involve a great quantity of algebraic manipulation. The solutions for such problems can be presented in the form of a series of two-dimensional charts.

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