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Title: Numerical and finite element analysis of heat transfer in a closed loop geothermal system

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Abstract

Analysis of the thermal regime created by a geothermal borehole heat exchanger is performed using a closed form radial heat flow equation, a geothermal borehole heat exchanger design tool and a finite element model. Climatic, heat exchanger construction and building load data are entered into the heat exchanger design tool in order to create a theoretical model along with thermal parameters from a number of geological formations. Output data from the design tool model are used in conjunction with the closed form radial heat flow equation to calculate the predicted temperature with respect to time and distance from the heat exchanger for the modelled ground formations. The output data from the design tool is also used to create a number of finite element method models against which the predictions calculated using the closed form radial heat flow equation can be compared. A good correlation between the temperatures predicted by the finite element models and the closed form equation calculations is observed. However when used within its recommended limiting conditions, the closed form equation is shown to slightly underestimate the temperature of the ground when compared to the finite element model predictions. The limiting conditions associated with the closed form equation are discussed in the context of the output from the finite element method models.

Keywords: Geothermal energy, ground source energy, finite element analysis, heat conduction, renewable energy

Nomenclature:

c	volumetric heat capacity of soil (J/m^3/°C)
D	uppermost borehole section length (m)
F	unit heat flux (W/m^2)
h	capacity for heat storage (J/m^3/°C)
H	lowermost borehole section length (m)
L	latent heat of water (kJ/kg)
Q	applied boundary flux (W/m^3)
q	specific heat extraction rate (W/m)

r	distance from borehole centre (m)
r_b	borehole radius (m)
T	temperature at distance r and at time t (ºC)
t	time from simulation commencement (sec)
T_a	annual average air temperature (ºC)
T_b	borehole wall temperature (ºC)
T_c	steady state temperature of heat carrier fluid (°C)
T_o	initial ‘at rest’ ground temperature (ºC)
t_s	time at which ‘steady state’ heat extraction begins (s)
x	distance (m)

Greek letters:

α	geothermal gradient (K/m)
λ	thermal conductivity (W/mK)
λ_x	thermal conductivity in the x-direction (W/mK)
λ_y	thermal conductivity in the y-direction (W/mK)
Θ_u	total unfrozen volumetric water content

1. Introduction

Geothermal systems for heating and / or cooling of structures may be broadly categorised into two groups, closed loop and open loop. Closed loop geothermal systems are dealt with in this paper, they involve installing loops of piping into some sub-surface media (e.g. boreholes, piles, tunnel walls and anchors, lakes). A heat carrier fluid is circulated around the closed loop and used to exchange heat with the media surrounding the heat exchanger piping.

Design of ground source energy systems is currently dominated by a small number of specialists, and the vast majority of well-established consultants and contractors in many countries do not currently have the required expertise to design these systems. A need for the development of expertise and appropriate design and analysis tools is therefore required in order to allow design and analysis of ground source energy systems by a wider group of professionals (Preene and Powrie, 2009). A pre-text to the development of any design / analysis software or tool is a thorough understanding of the thermal interaction between a borehole heat exchanger and the surrounding ground, which is essential for accurate design of geothermal systems. Finite element analysis techniques offer the opportunity to model more complex thermal problems relative to numerical techniques, which tend to be quite complicated and not user friendly when used to analyse complex problems. Modelling the flow of heat through soil with a numerical solution can be very complex. Temperature in natural soil deposits are generally highly dependent on boundary conditions and in a time dependent analysis, boundary conditions often change with time and cannot always be defined with certainty at the beginning of analysis. In fact, the correct boundary condition can sometimes be part of the solution, as is the case for geothermal systems, where the amount of heat flow in the ground around the heated or chilled pipe in borehole depends on the difference between the pipe fluid temperature and the ground temperature itself. In this case, it is the temperature that is the primary unknown in soil elements and needs to be determined. Because of this, an iterative numerical technique is required to match the computed temperature and the material property, making the solution highly non-linear. These complexities make it necessary to use some form of numerical analysis to analyse thermal problems. A common thermal modelling approach which is not currently routinely used in geothermal industry practice is to use Finite Element Method formulations. TEMP/W (GeoStudio, 2007) is an example of a FEM software tool which can be used to perform thermal modelling.

Several models have been constructed using TEMP/W and other software in order to consider the thermal effects of sub surface conductive heat flow (Cha et al., 2007, Darrow, 2011, Jantzer, 2005, Kokelj et al., 2010, Kristensen et al., 2008, Mobley and Barlow, 2003, Mottaghy and Rath, 2006, Weaver and Kulas, 2003, Lee and Jeong, 2008). However, most of these models were constructed in order to understand the mechanics of permafrost action on sub-surface thermal regimes (for example, Jantzer (2005) modelled the temporal temperature changes in an embankment located in northern Sweden using TEMP/W in order to examine if and how the material was influenced by the freezing and thawing processes and concluded that the 'freezing plane' advanced significantly into the core of the dam, thereby negatively influencing the functions of the central core and filter zones of the embankment) and very few examples of the use of finite element methods exist in the ground source energy industry.

This paper utilises a geothermal borehole heat exchanger design tool called Energy Earth Designer (EED), the closed form radial heat flow equation developed by Claesson and Eskilson (Eskilson, 1987, Claesson and Eskilson, 1988) and finite element method analysis developed using TEMP/W in order to investigate the temperature predictions with respect to time and distance resulting from analysis using the various methods. Development of FEM methods for the analysis of geothermal problems is important so that more complex problems can be analysed and in order to provide more powerful modelling capability. Although the closed form solution (which is described later) is applicable in modelling heat transfer and propagation in soil, it is very simplified and has several limitations. Also analysis using closed form equation is typically more time consuming and less flexible than numerical analysis, particularly when input parameters or dimensions of a problem are changed. This is because once the heat transfer problem has been defined in an FEM code, subsequent analyses with, for example different input parameters or providing temperature values at varying distances from the borehole heat exchanger, can be performed rapidly, while subsequent analyses using the closed form equation would require separate calculation for each time step or each distance from the borehole.

For example, the closed form equation does not have the ability to consider the backfill material specifications and therefore results of temperature predictions from the closed form equation are independent of the backfill materials thermal properties. In addition the closed form equation has several limitations with regard to both radial distance from the borehole heat exchanger and maximum and minimum analysis time – these constraints significantly curtail the use of the closed form equation for practical applications and do not apply to numerical analyses. These closed form equation limitations are summarised later in this paper and described in detail by Eskilson (1987).

The theory behind the development of the radial heat flow equation, inherent assumptions and limiting conditions of the radial heat flow equation are presented, and the resulting consequences of these limiting conditions are discussed.

2. Claesson & Eskilson's Radial Heat Flow Equation

The fundamental equations governing radial heat flow between a line source and a surrounding material were developed by a number of authors such as Carslaw and Jaeger (1946) and Ingersoll et. al (1948). This section outlines the theory behind the subsequent equation developed by Claesson and Eskilson (Claesson and Eskilson, 1988, Eskilson, 1987) which describes radial heat transfer in the vicinity of a closed loop borehole heat exchanger extracting or injecting heat from / to the sub-surface.

A number of assumptions and simplifications were made by Claesson & Eskilson during the development of the following theory. These are: (i) the seasonal fluctuation of ground temperatures in the top few meters of soil caused by fluctuating seasonal air temperatures and other environmental factors (e.g. freezing, snow, air to ground thermal resistance) are neglected; (ii) the ground is assumed to be homogenous in all directions, therefore, in cases where for example a ground formation dominated by limestone but overlain by 10 meters or less of natural soils, the thermal effects of the overlying natural soils are ignored; (iii) the uppermost part of the borehole (denoted D in Figure 1) is required in order to take account of the effects of either the standing groundwater level or an insulating casing surrounding the uppermost part of the borehole; (iv) variations in temperature between the upward and downward channels of the borehole heat exchanger and thermal interaction between the upward and downward channels are ignored, the thermal effects from the borehole wall outwards are thus considered, and assume that the temperature at the borehole wall (denoted T_b in Figure 1) is constant along the length of the borehole at any particular point in time and (v) the initial temperature along the length of the borehole heat exchanger (i.e. prior to any exchange of heat) may be considered to be equal to the sum of the annual average air temperature (T_a) and the vertical geothermal gradient (α) times the length of the borehole heat exchanger ($D+H$) divided by two, resulting in a constant initial temperature over the length of the borehole heat exchanger (which is denoted T_0 in subsequent equations) – the resulting expression is shown in Equation 1 for clarity.

$$T_0 = T_a + \alpha\left(\frac{D+H}{2}\right) \qquad (1)$$

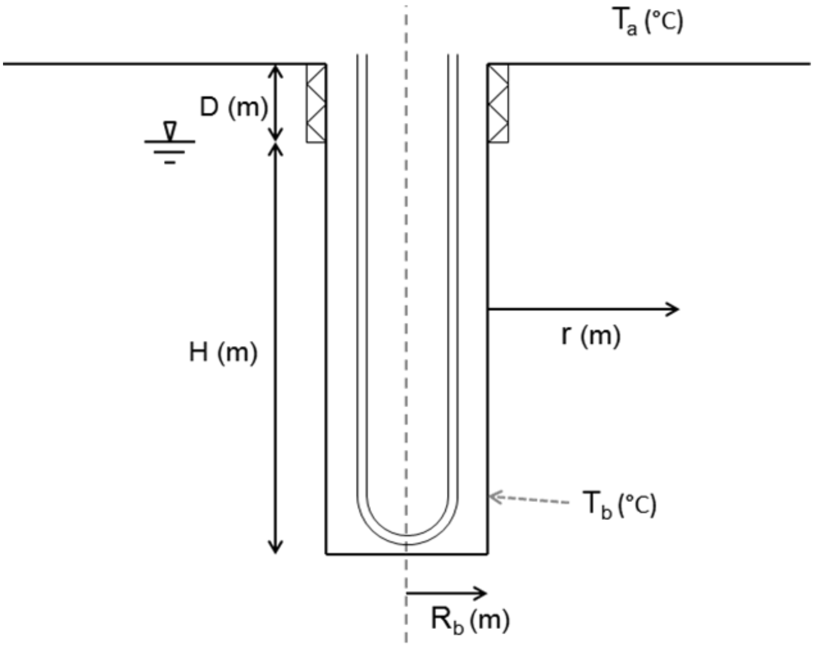

Fig. 1. Borehole heat exchanger

Cognisant of the foregoing assumptions, Eskilson (1987) states that ground temperature satisfies the heat conduction equation in Cartesian co-ordinates (Equation 2), where λ and h are the thermal conductivity (W/mK) and specific heat capacity (J/m^{3}K) of the rock / soil being analysed.

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{h}{\lambda}\frac{\partial T}{\partial t} \qquad (2)$$

Banks (2008) states that with the boundary conditions at $t=0$, $T=T_0$ for all values of r and z; and as $r\rightarrow\infty$, $T=T_0$ for all values of t, Equation 2 can be approximated with the above boundary conditions so that we can write Equation 3.

$$T_0-T_b=\frac{q}{4\pi\lambda}E(u)=\frac{q}{4\pi\lambda}\left[-\gamma-\ln(u)-\sum_{n=1}^{\infty}(-1)^n\frac{u^n}{n.n!}\right] \qquad (3)$$

where $u=(r_b^2h)/(4\lambda t)$, T_b is the average temperature of the carried fluid for any given time (t) and γ is Euler's constant = 0.5772. The previously stated simplification (number iv), is now considered, meaning that the temperature of the heat carrier fluid in the borehole heat exchanger piping can be assumed to be equal to the temperature at the borehole wall (i.e. at a distance r_b from the borehole centre). This allows simplification of

Equation 3 to form Equation 4 which is valid during the early stages of heat exchange as described by Claesson and Eskilson (1988).

$$T_0 - T_b \approx -\frac{q}{4\pi\lambda}\left[\ln\left(\frac{4\lambda t}{r_b^2 h}\right) - 0.5772\right] \quad (4)$$

Substituting T_b for T and r_b for r, we may know rewrite Equation 4 as Equation 5 such that the temperature (T) at distance (r) from the borehole at any time (t) may be calculated.

$$T \approx T_0 + \frac{q}{4\pi\lambda}\left[\ln\left(\frac{4\lambda t}{r^2 h}\right) - 0.5772\right] \quad (5)$$

Equation 5 is valid only within the range $5r_b^2h / \lambda < t < t_s / 10$ (Banks, 2008), where t_s is the time at which 'steady state' heat exchange begins i.e. the point in time at which three-dimensional heat flow effects become important, and therefore the radial heat flow equation (5) is no longer valid. The lower time constraint ensures that the heat capacity of the volume of the borehole can be neglected in the line source approximation (Eskilson and Hellstrom, 1987), and therefore if the value of t is lower than this constraint, the mathematics of Equation 5 become invalid, and if the value of t is higher than the upper constraint, the equation is also invalid as the system begins to induce heat flow from the ground surface and begins to approach steady state conditions (Banks, 2008). The point in time at which three-dimensional heat flow effects become important is described by Eskilson (1987) as $t_s = e^{\gamma}(D+H)^2h / 18\lambda$. At times greater than this, Equation 6 can be used to approximate the steady state temperature of the heat carrier fluid. Equation 6 is only valid where $H >> r_b$.

$$T_c = \frac{q}{2\pi\lambda}\ln\left(\frac{H}{r_b\sqrt{4.5}}\right) \quad (6)$$

3. Generation of Model Data

Energy Earth Designer (EED, 2010) is a borehole heat exchanger software package which was developed through collaboration between the Institute of Applied Geosciences of Justus-Liebig University, Germany and the Department of Mathematical Physics in the University of Lund, Sweden (Hellström and Sanner, 1994, Hellström et al., 1997) and is currently one of the most commonly used geothermal heat exchanger design tools employed by the industry in Europe. Building energy loads, geological data, borehole thermal parameters and heat carrier fluid information are entered into the EED borehole heat exchanger design program in order to generate a model. The output data from this simulation (i.e. the calculated borehole heat exchanger fluid temperatures), along with the appropriate building energy load and geological data are used as input parameters for the finite element model and Claesson & Eskilson's radial heat flow equation (Equation 5). The following sections provide an overview of the governing EED input parameters and summarize the output from the simulation.

3.1 Heat Exchanger Design Tool Input Parameters

The building energy loads, borehole heat exchanger length and other borehole heat exchanger details are selected so that ground freezing will not occur during the simulation – this is because ground freezing typically does not occur below a depth of one to two meters under Irish climatic conditions.. One of the inputs for the radial heat flow equation is a value for specific heat extraction rate (W/m). In order to provide a single value for the specific heat extraction rate for use in the equation, the users have the option to (i) specify a variable monthly heating / cooling load in line with what would be expected due to the seasonal fluctuation in heating / cooling requirements or (ii) specify a constant year-averaged heating / cooling load for use in the simulation. If the first option were selected as the input for the EED simulation, a long-term averaged specific heat extraction rate would have to be calculated for use in the radial heat flow equation, thereby introducing an anomaly between the EED simulation and subsequent radial heat flow equation / finite element TEMP/W simulation. The latter option was therefore selected resulting in a constant building heating base load of 2.5 MWh per month is assumed. Peak loads and variations in the base load above the previous value are assumed to be covered by an auxiliary system (e.g. a gas boiler).

Ground thermal parameters have been selected based on a geological formation assumed to be dominated by saturated sand / limestone. Values for thermal conductivity and specific heat capacity of 2.4 W/(mK) and 2.5 MJ/(m^3K) respectively have been selected. An average ground surface temperature of 9.5 ºC and a geothermal heat flux value of 0.07 W/m^2 have been selected – these values are commensurate of what would be expected under Irish climatic and geological conditions. A sketch of the borehole and borehole heat exchanger piping layout is provided in Figure 3. The borehole has been assigned a diameter of 140 mm. The borehole heat exchanger has the following dimensions: 40 mm outer pipe diameter; 3 mm pipe wall thickness and shank spacing (the distance between the centres of the down-flow and up flow pipes) of 80 mm. Polyethylene has been selected for the borehole heat exchanger piping and a thermal conductivity value of 0.42 W/(mK) has been assigned. The backfill material between the borehole heat exchanger pipe and the borehole wall is assumed to be a thermally enhanced grout with low permeability (thereby preventing the development of thermosiphon along the borehole heat exchanger length – thermosiphon is a phenomenon where an un-grouted borehole heat exchanger is intersected by one or more fractures which provide a path for ground water flow between the borehole and surrounding rock. In such boreholes an enhanced heat transfer may occur due to the induced convective water flow, driven by the volumetric expansion of the heated water within the borehole. This results in warm water leaving through fractures in the upper part of the borehole while groundwater of ambient temperature enters the borehole through fractures located at greater depths (Gehlin et al., 2003)) and a thermal conductivity of 1.47 W/(mK).

Monoethyleneglycol, which has a freezing point of approximately - 14 ºC, is selected as the heat carrier fluid circulating within the borehole heat exchanger piping in order to ensure freezing is avoided. A flow rate of 0.4 l/s is selected so that a turbulent flow regime is developed and maintained within the borehole heat exchanger piping, thereby maximizing heat transfer efficiency. A conservative seasonal performance factor (SPF, the ratio of heat produced over the course of a season to the electrical demand required to drive the heat pump over the same period) of three is selected for use in the simulation.

3.2 Heat Exchanger Design Tool Outputs

The design tool simulation indicates that a borehole heat exchanger length of 108 m is required in order to prevent ground freezing over a simulation period of 25 years. This results in a specific heat extraction rate of 21.15 W/m. The average BHE (borehole heat exchanger) fluid temperature profile (i.e. the average of the down-flow and up-flow BHE pipe temperatures) over the first 24 months of the simulation is shown on Figure 2. The temperature profile observed is commensurate of that which would be expected for a heating-dominated geothermal system (i.e. a sharp reduction in ground temperature in the first month, followed by a gradual reduction in ground temperature due to the extraction of heat from the ground).

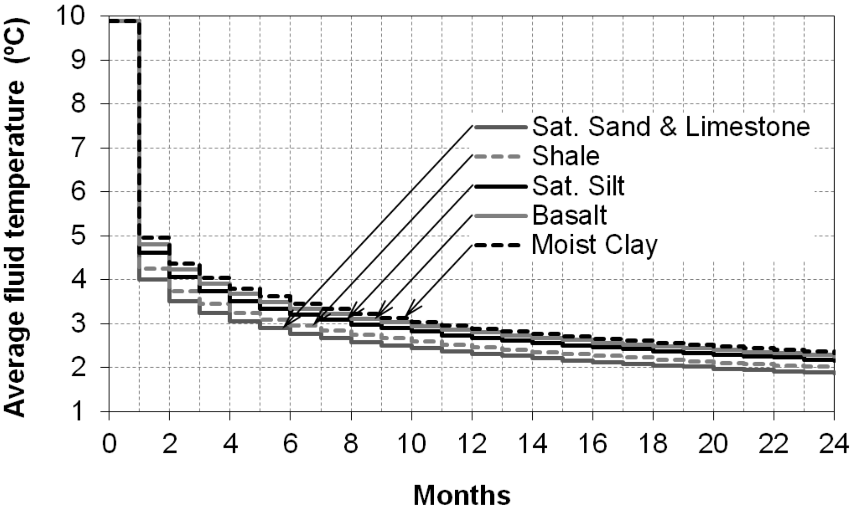

Fig. 2. BHE carrier fluid temperature development profile

4. Radial Heat Flow Analysis

4.1. Analysis using Closed Form Equation

Analysis of radial heat flow is performed using Claesson & Eskilson's radial heat flow equation (Equation 5) in conjunction with the input parameters used in the original EED simulation. This allows calculation of the temporal evolution of ground temperatures with respect to distance from the borehole heat exchanger. The initial ground temperature (T_0) is set to 9.9 °C, the specific heat extraction rate (q) is set to 21.15 W/m, the thermal conductivity (λ) is set to 2.4 W/mK and the specific heat capacity (h) is set to 2,500,000 J/m^{3}K as was for the case for the EED design tool simulation. The time from simulation commencement (t) and distance from the borehole centre (r) are then varied in order to calculate the temperature at a number of distance variations for each month of the simulation (T) and thereby create Table 1. The previously stated limiting conditions for Equation 5 (i.e. $5r_b^2h / \lambda < t < t_s / 10$) mean that the temperature predictions shown in Table 1 for month one, months one to seven and months one to seventeen are not valid at distances one meter, two meters and three meters respectively from the BHE. However, these temperature predictions have been considered in the presented research work to check the possible proficiency of FEM analysis in overcoming the limitations of the closed form equation.

Table. 1. Temperature with respect to distance r and time t

Time (months)	Temperature at distance r from the BHE				
	r = 0.25 m	r = 0.5 m	r = 1 m	r = 2 m	r = 3 m
1	6.91	7.88	8.86	9.83	10.40
2	6.42	7.40	8.37	9.34	9.91
3	6.14	7.11	8.08	9.06	9.63
4	5.94	6.91	7.88	8.86	9.42
5	5.78	6.75	7.73	8.70	9.27
6	5.65	6.63	7.60	8.57	9.14
7	5.54	6.52	7.49	8.46	9.03
8	5.45	6.42	7.40	8.37	8.94
9	5.37	6.34	7.31	8.29	8.86
10	5.29	6.27	7.24	8.21	8.78
11	5.23	6.20	7.17	8.15	8.71
12	5.17	6.14	7.11	8.08	8.65
13	5.11	6.08	7.06	8.03	8.60
14	5.06	6.03	7.00	7.98	8.55
15	5.01	5.98	6.96	7.93	8.50
16	4.96	5.94	6.91	7.88	8.45
17	4.92	5.89	6.87	7.84	8.41
18	4.88	5.85	6.83	7.80	8.37
19	4.84	5.82	6.79	7.76	8.33
20	4.81	5.78	6.75	7.73	8.30
21	4.77	5.75	6.72	7.69	8.26
22	4.74	5.71	6.69	7.66	8.23
23	4.71	5.68	6.66	7.63	8.20
24	4.68	5.65	6.63	7.60	8.17

5. Finite Element Method (FEM) Analysis

Various multi-purpose commercial finite element packages such as ABAQUS 6.10 (2010), ANSYS CFX 12 (2009) and FEFLOW 6 (2003) have the capability to model heat transfer in porous material. There are also several commercially available FEM packages such as TEMP/W (2010), FEHT (2006), and SVHEAT (2009) which have been developed specifically for finite element analysis of heat transfer in porous materials.

In this research paper, TEMP/W is employed for FEM analysis of heat transfer to model the radial heat flow around a closed loop borehole heat exchanger. TEMP/W is a finite element software product that can be used to model the thermal changes in the ground due to environmental changes, or due to the construction of facilities such as buildings or pipelines. The comprehensive formulation of the software makes it possible to analyse both simple and highly complex geothermal problems (GeoStudio, 2007). However the authors are not aware of any previous work involving the modelling of geothermal borehole heat exchangers using TEMP/W. TEMP/W is typically used for thermal design for roads and airstrips, ground freezing for soil stabilization, insulation design for shallow buried piping, thawing or freezing beneath heated or chilled structures, freezing around chilled pipelines and convective cooling of surfaces. Its ability to model full thermal models, simplified thermal models and coupled convective thermal models in both steady state and time dependent transient conditions makes this software a useful tool for modelling various geothermal problems.

Although TEMP/W has been used by a number of researchers to investigate ground thermal regimes (Darrow, 2011, Jantzer, 2005, Kristensen et al., 2008, Mottaghy and Rath, 2006, Mobley and Barlow, 2003, Weaver and Kulas, 2003, Cha et al., 2007, Kokelj et al., 2010), all of this previous research was conducted primarily to understand the interaction between permafrost action and thermal regimes. For example, Jantzer (2005) modelled the temporal temperature changes in an embankment located in northern Sweden using TEMP/W in order to examine if and how the embankment material was influenced by the freezing and thawing processes. The author concluded that the 'freezing plane' advanced significantly into the core of the dam, thereby negatively influencing the functions of the central core and filter zones of the embankment. The model, which was created using TEMP/W, allowed the authors to understand temperature changes in the embankment resulting from conductive heat transfer.

TEMP/W assumes that heat flux due to conduction is governed by Equation 7, where *F* is the heat flux, λ is the thermal conductivity, *T* is the temperature and *x* is distance.

$$F = -\lambda \frac{\partial T}{\partial x} \qquad (7)$$

This equation shows that heat flow due to conduction is directly dependent on the thermal conductivity of the soil medium and temperature gradient. The negative sign indicates that the temperature decreases in the direction of increasing *x*; that is, the heat flows in the direction from high temperature to low temperature (Krahn, 2004).

The differential equation used in TEMP/W to model the heat flow in unfrozen soil is reproduced as Equation 8. In this equation *T* is temperature, λ_x is thermal conductivity in the x-direction, λ_y is thermal conductivity in the y-direction (note that in this case, the thermal conductivity is assumed to be equal in all radial directions and therefore $\lambda_x = \lambda_y$), *Q* is applied boundary flux, *h* is capacity for heat storage and *t* is time.

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right) + Q = h\frac{\partial T}{\partial t} \qquad (8)$$

Equation 8 states that the difference between the heat flow entering and leaving an unfrozen elemental volume of soil (assuming unit depth in the z-direction) at a point in time is equal to the change in the stored heat energy of that soil volume. However, in non-time dependent steady-state conditions, the flux entering and leaving an element is the same all time, so the right side of the Equation 8 disappears and therefore Equation 9 may be written.

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right)+Q=0 \qquad (9)$$

In TEMP/W formulation, under time dependent transient conditions, the capacity to store heat is made-up of two parts. The first part is the volumetric heat capacity of the soil (either frozen or unfrozen) and the second part is the latent heat associated with the phase change (see Equation 10).

$$h=c+L\frac{\partial \Theta_u}{\partial T} \qquad (10)$$

Where, c is volumetric heat capacity of soil, L is latent heat of water, Θ_u is total unfrozen volumetric water content, T is temperature, and h is capacity for heat storage. Although in the case of presented analysis, ground freezing does not occur and no phase change analysis is required, the second part of Equation 10 cannot be eliminated since TEMP/W is not an open source code. However, the second part of Equation 10 is equal to zero at all stages and therefore the capacity for heat storage is directly equal to the volumetric heat capacity of the soil.

The geometry of the FEM model is defined in plan view. A soil cluster of dimensions 20 m × 20 m is chosen for the analysis. The borehole is placed in centre of soil geometry with 10 m distance from boundaries as shown in Figure 3. The model includes 6,700 quad and triangular elements while the integration order is 4 and 3 for quad and triangular elements respectively. Although the borehole is located 10 m from the boundaries, ideally, the boundary should be as far as possible away from the borehole so as to not influence the results around the borehole which is the main area of interest. For the purposes of this analysis, rectangular grids of infinite elements are used on the boundaries in order to idealise the boundary conditions (see Figure 3). TEMP/W code uses a formulation developed by Bettess (1992)to model infinite elements. To use this formulation, the relationship between local and global coordinate systems must be described by a shape function. TEMP/W uses the Serendipity family of mapping functions presented by Bettess (1992) to make the relationship between the local and global coordinate systems (GeoStudio, 2007). This function is related to the placement of infinite element in the model geometry and can be written in five different forms for 1-D infinite elements and three different forms for 2-D infinite elements. These infinite elements are a convenient way of extending the far field of a problem and in general three points are required to describe the shape function. Each element needs to be an 8-noded quadrilateral, so secondary nodes are required to form the decay function. For more information please refer to Bettess (1992, pp. 53-85) and GeoStudio (2007, pp. 234-240).

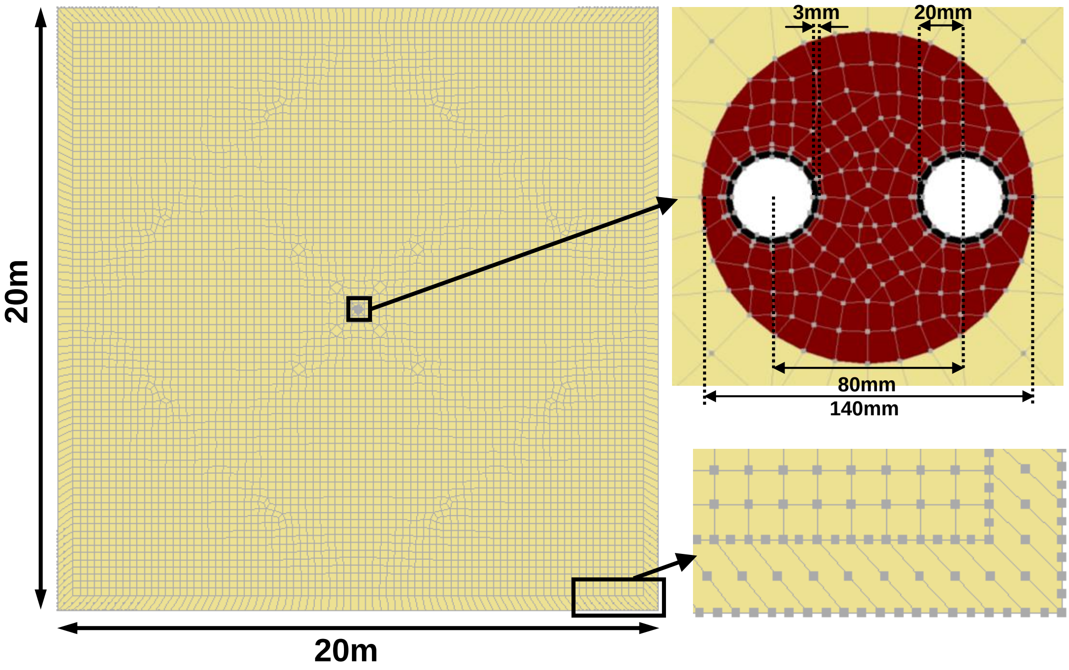

Fig. 3. FEM dimensions and layout

Material specifications were given separately to the pipe, borehole and ground region as presented in Table 2. The analysis includes one steady state and one transient thermal stage. The reason for this two-stage analysis is that TEMP/W requires definition of the initial steady state temperature at all nodes and elements, prior to starting the time dependent transient analysis. In the initial steady state analysis, the value of T_0 = 9.9 ºC was given to all nodes (which results from the EED simulation and was used for the closed form equation analysis) so that a uniform temperature distribution was developed in all elements. In the transient stage, the average BHE fluid temperature was given to the nodes on the inside pipe surface based on the temperature profile shown in Figure 2 which was developed using the EED borehole heat exchanger design tool. The transient analysis consisted of 24 one-month time steps. The parallel direct equation solver was used instead of direct equation solver due to the large number of elements in the simulation. The parallel solver saves the matrices in a compressed format to eliminate zero’s and it has many advanced schemes to solve large systems of equations more efficiently. It also offers the ability to make use of multiple processors on the computer which is 2.2 GHz dual core processor.

Table. 2. Material Specifications

Material	Thermal Conductivity (W/mK)	Volumetric Heat Capacity (MJ/m³/°C)
Saturated sand / limestone	2.4	2.5
Shale	2.1	2.3
Saturated silt	1.8	2.2
Basalt	1.7	2.4
Moist Clay	1.6	2.4
Backfill material	1.47	1.8
Pipe	0.42	1.9

Total CPU processing time for solving both first and second stages was measured to be 50 seconds. This represents an excellent analysis time for a transient time dependent analysis with a geometry made up of 6,700 elements. Three grid independence tests consisting of 38,900, 6,700 and 2,002 elements were carried out alongside a time step independence test consisting of 48, 24 and 1 time step(s) in order to validate the numerical accuracy of the model. The dependency test results revealed that the model outputs are independent of the considered grid and time steps. The radial heat flow profile after one month is shown in Figure 4. The difference between each iso-counter line is 0.5 ºC in all figures. The numbers shown on Figures 4 and 5 represent the temperature in degrees centigrade of each iso-counter line. Figure 5 shows the heat flow profiles after 4, 8, 12, 16, 20 and 24 months for the saturated sand / limestone soil formation. By comparing Figures 4 and 5, it is obvious that soil in the close vicinity of the borehole is affected significantly during the first month of heat extraction and that rate of heat propagation caused by extraction of heat by the borehole decreases as the distance from the borehole increases. The temperature of the soil element located a distance 0.57 m from the borehole decreases from 9.9 ºC to 8.5 ºC during the course of the first month of the analysis (see Figure 4). This means that the average temperature variation rate at this point is 1.4 ºC/month. By comparison, it takes 24 months for the soil element at a distance 3.91 m from the borehole to reach the same temperature level, resulting in an average temperature variation rate of 0.058 ºC/month.

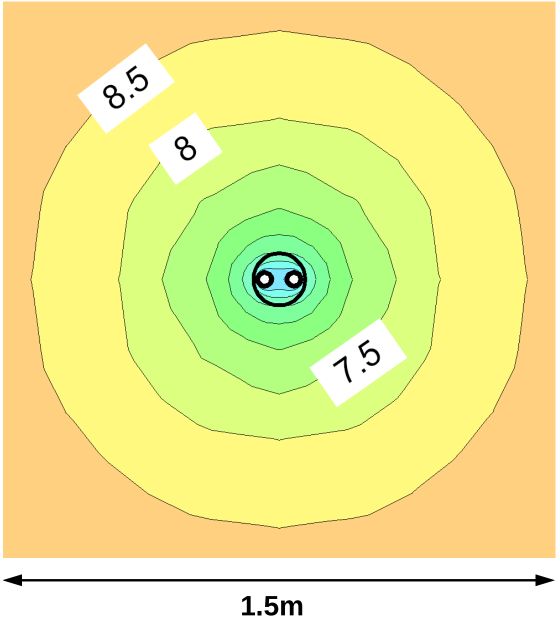

Fig. 4. Heat flow profile after one month (Saturated Sand / Limestone)

Fig. 5. Heat flow profiles at various time steps

A plot of ‘time required to decrease the ground temperature by 1 °C’ versus ‘distance from the borehole’ and ‘area around the borehole’ (Figure 6) is created in order to investigate the rate of heat propagation and its relationship with distance from the borehole. Although the change in temperature value has a power relationship with affected distance from the borehole, the relationship between affected area and change in temperature value is perfectly linear. This explains why the iso-counter lines are not located at uniform distances in Figures 4 and 5. The TEMP/W model presented is analysed in plan view and therefore considers the entire model to have unit thickness (or unit depth), therefore area and volume are directly linked. The linear relationship presented in

Figure 6 for the change in temperature with respect to area is therefore explained by the constant volumetric heat capacity is applied in the analysis.

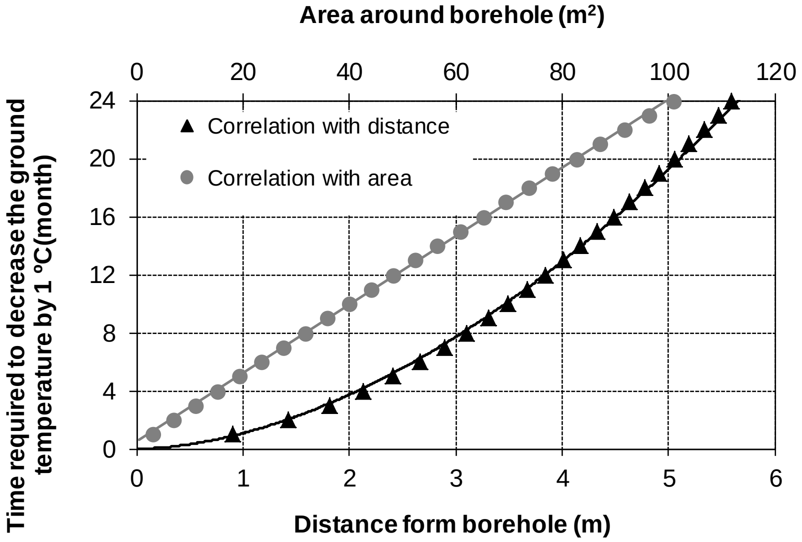

Fig. 6. Time taken to decrease ground temperature by 1 ºC

5.1. Comparison between predictions by FEM and Closed form equation

Prediction of temperature at various distances between 0.25 m to 3.0 m from the borehole for the saturated sand / limestone ground formation is presented in Figure 7 (a-e). Both the FEM and the closed form equation (Equation 5) analyses show similar shapes of curvature of heat change profile during the considered time. When the closed form equation is used within the previously described application limits (see Section 2 of this paper) it tends to slightly underestimate the temperature of the ground when compared to the FEM model predictions. However in instances where the closed form equation is outside of the lower bound application limit, it tends to overestimate the predicted temperatures when compared to the FEM model predictions. The overestimated value is even more than the initial ground temperature which is 9.9 ºC in the soil element at a distance of 3.0 m from the borehole (see Figure 7(e)).

Figure 8 shows that the range of prediction difference decreases with increasing distance from the borehole for each of the soil formations analysed. This is because the lower time-limiting condition of the closed-form equation is governed by the square of the distance from the borehole wall and also because the closed form equation cannot take account of the thermal specifications of the backfill material between the heat exchanger pipes and the borehole wall, while the finite element model does have the ability to take account of the thermal influence of the backfill material. It is also evident from Figure 8 that the difference between the temperatures predicted by the TEMP/W and closed form equation methods increases with decreasing ground formation thermal conductivity. Decreasing the thermal conductivity has a similar effect (in terms of modelled predicted temperature) to increasing the time of the analysis and therefore because the closed form equation tends to underestimate the predicted temperature relative to the TEMP/W model, and this underestimation increases with increasing analysis time, therefore decreasing the thermal conductivity of the modelled ground formation has the effect of accentuating the difference between the predicted values by the two methods.

The maximum difference in predicted values in the case of the saturated sand / limestone ground formation is 6 % and occurs in the soil element placed at a distance of 0.25 m from the borehole. The minimum difference in predicted values is 0 % and occurs in the soil element at a distance of 3.0 m from the borehole for the time period considered in the analysis. It is clear from Figure 8 that for a fixed distance from the borehole, the difference in predicted temperature values increases with increasing time. In general the estimated temperatures predicted using both the FEM and closed form equation methods are in good agreement.

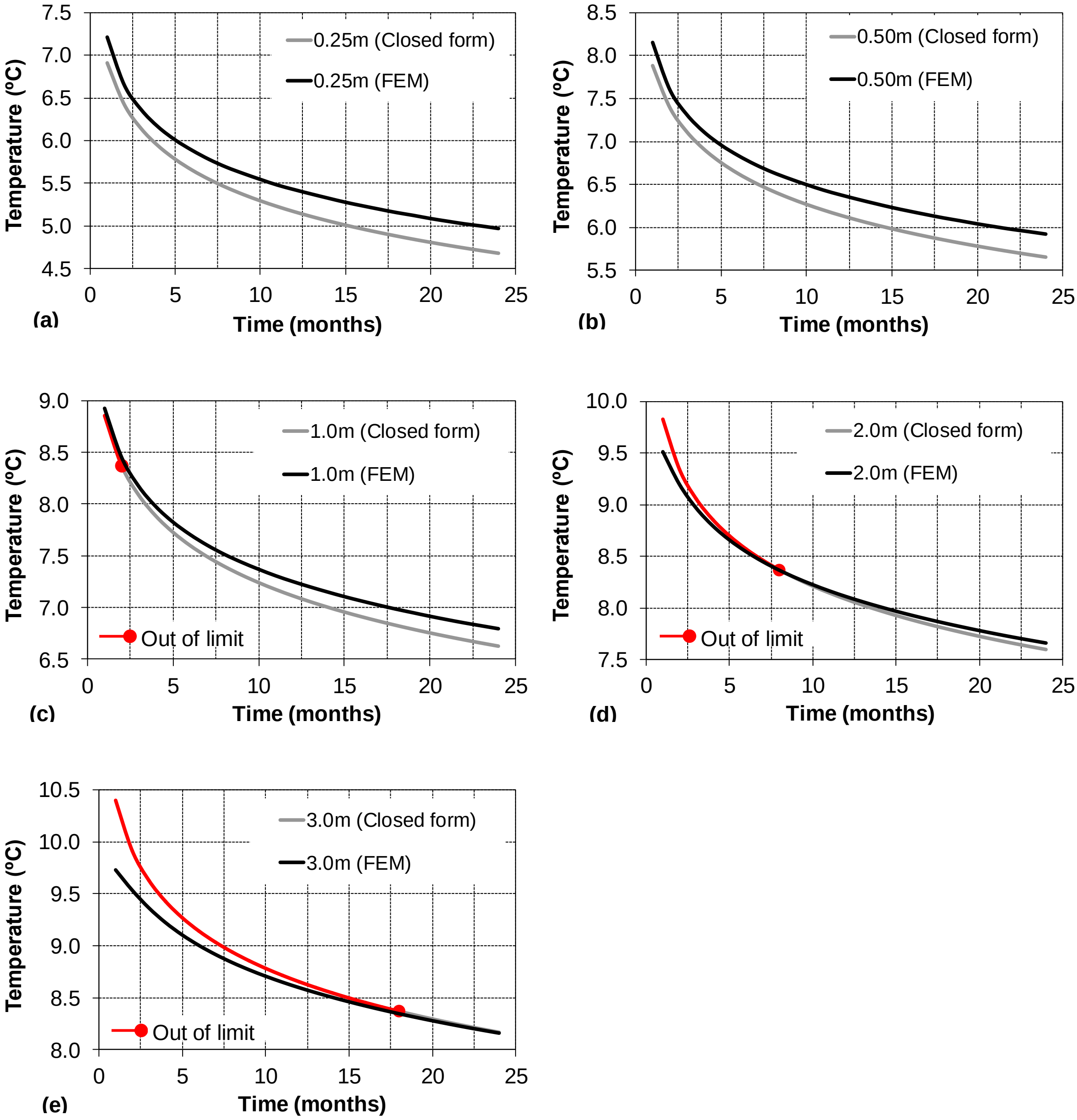

Fig. 7 (a) - (e). Results from FEM analyses and closed form calculations (saturated sand / limestone)

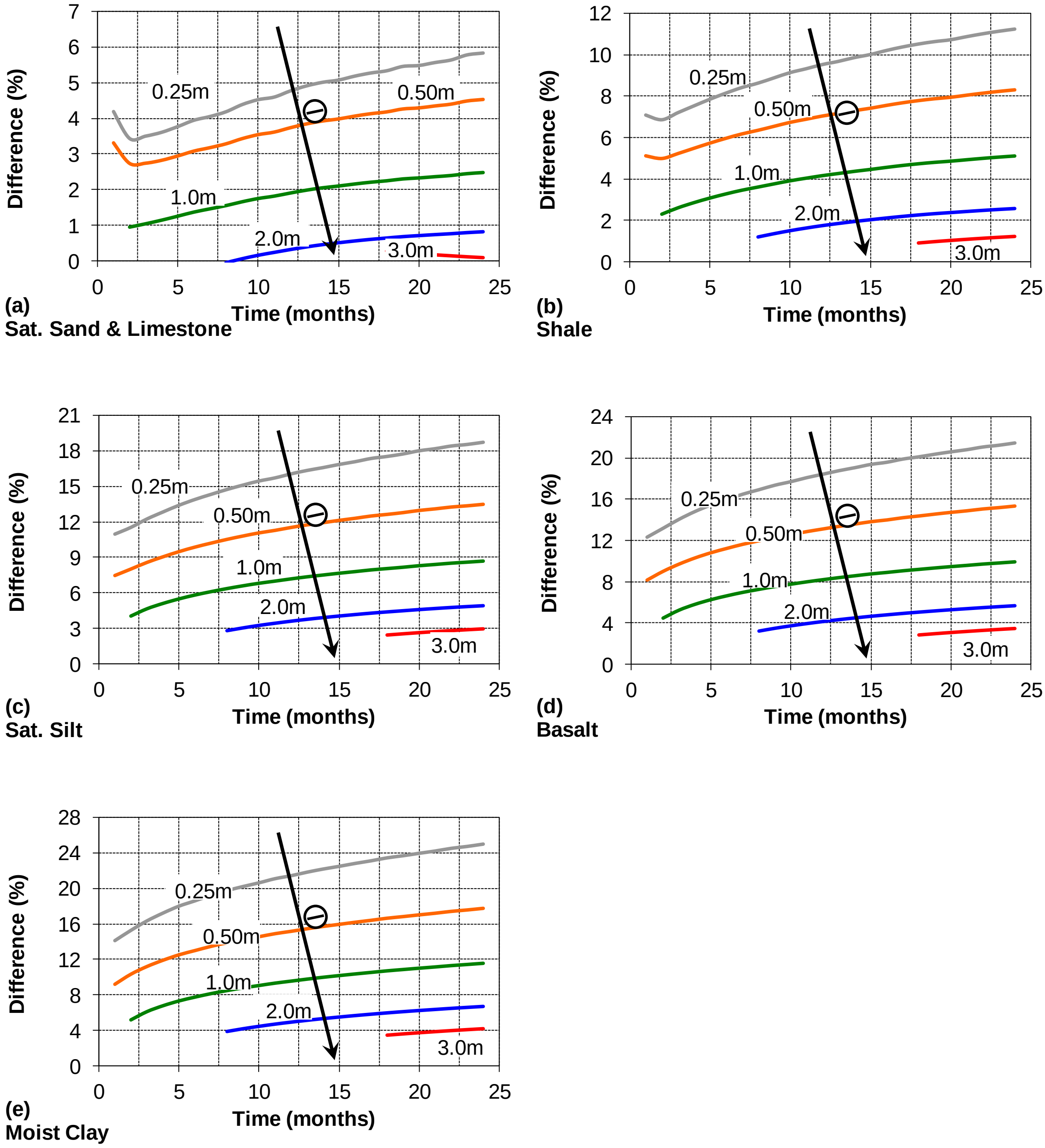

Fig. 8 (a) - (e). Comparison between results of FEM and closed form solution for different soil

6. Discussion and Conclusions

Analysis of the thermal regime created by a geothermal borehole heat exchanger is performed using a closed form radial heat flow equation, a geothermal borehole heat exchanger design tool and a finite element model. Geological, climatic, heat exchanger construction and building load data are entered into the heat exchanger design tool (EED) in order to create a theoretical model. Output data from the design tool model are used in conjunction with the closed form radial heat flow equation to calculate the predicted temperature with respect to time and distance from the heat exchanger. The output data from the design tool is also used to create a finite element method model using FEM package TEMP/W against which the predictions calculated using the closed form radial heat flow equation can be compared.

Although TEMP/W can be used to model thermal problems, the authors are not aware of any previous work involving the modelling of geothermal borehole heat exchangers using TEMP/W. It is typically employed to model the interaction between permafrost action and ground thermal regimes – a number of citations relating to this type of research have been provided. TEMP/W is employed in this research paper in order to model the radial heat flow around a closed loop borehole heat exchanger which is extracting energy at a constant rate.

A good correlation is shown between the temperatures predicted by the FEM and closed form equation analyses; however the closed form equation is shown to slightly underestimate the ground temperature relative to the FEM model when used within its recommended limiting conditions. It is evident from Figure 8 that the difference between the temperatures predicted by the TEMP/W and closed form equation methods increases with decreasing ground formation thermal conductivity Decreasing the thermal conductivity has a similar effect (in terms of modelled predicted temperature) to increasing the time of the analysis and therefore because the closed form equation tends to underestimate the predicted temperature relative to the TEMP/W model, and this underestimation increases with increasing analysis time, therefore decreasing the thermal conductivity of the modelled ground formation has the effect of accentuating the difference between the predicted values by the two methods.

The finite element model presented is capable of providing a comprehensive analysis of the ground thermal regime created by a borehole heat exchanger for the period outside of the recommended range of the closed form equation. An additional advantage of the FEM analysis method is that it provides an improved graphical output relative to other analysis methods.

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