Title	Location and Evaluation of Maximum Dynamic Effects on a Simply Supported Beam due to a Quarter-Car Model
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Publication date	2008-12
Conference details	Bridge and Infrastructure Research In Ireland (BRI 2008), Galway, Ireland, December, 2008
Link to online version	http://www.bcri.ie/
Item record/more information	http://hdl.handle.net/10197/6190

LOCATION AND EVALUATION OF MAXIMUM DYNAMIC EFFECTS ON A SIMPLY SUPPORTED BEAM DUE TO A QUARTER-CAR MODEL

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Abstract

Most current research on dynamic effects due to traffic load on simply supported bridges focuses on the mid-span section of the bridge, since this location corresponds to the worst static bending moment. However, the maximum total moment may be located relatively far apart from the mid-span location and differ considerably from the maximum mid-span moment. This paper uses a quarter-car vehicle model travelling over an Euler-Bernoulli beam to analyse this phenomena. The vehicle parameters are varied using Monte-Carlo simulations. The influence of road profile roughness and bridge length on the magnitude of the differences between mid-span and the worst possible section are also investigated.

Keywords: Dynamic Amplification Factor, bridge, vehicle-bridge interaction, critical section

1. Introduction

Abundant work has been carried out in the field of bridge dynamics, showing that impact factors recommended by existing codes may result over-conservative when assessing the dynamics effects of moving loads on highway bridges (DIVINE 1998, SAMARIS 2006). This conservatism is due to the uncertainty of many of the parameters involved in the vehicle bridge dynamic interaction. This uncertainty can be reduced using on-site data and experimentally validated mathematical models to derive more realistic dynamic amplification factors.

On the other hand, while most of the research (González 2002, Sethilvasan et al 2002, Savin 2001, NCHRP 19998) focus on the study of stresses developed at a selected number of sections, mainly mid-span, there is not much research on the stresses developed through the whole bridge length. However, the maximum bending moment due to a vehicle crossing does not always take place at mid-span and it can be of significant higher magnitude. This paper evaluates the magnitude and the area of influence of these critical sections with maximum stresses when using a quarter-car model over a simply supported beam model. First, the models employed to simulate the crossing of a vehicle over a beam are introduced. Secondly, the results of the simulations are discussed.

2. Vehicle-Bridge Interaction Model

The vehicle model is a quarter-car (Q-car) travelling at constant velocity, *c*, moving from left to right. The tyre is modelled like a mass, m_t, linked to the road by a spring of stiffness K_t. On the other hand, the main mass of the vehicle, m_s, is linked to the tyre by a spring of stiffness K_s in parallel with a passive viscous damper of value C_s as seen in Figure 1. The vehicles parameters are taken from the values proposed by Cebon (1999) and are listed in Table 1.

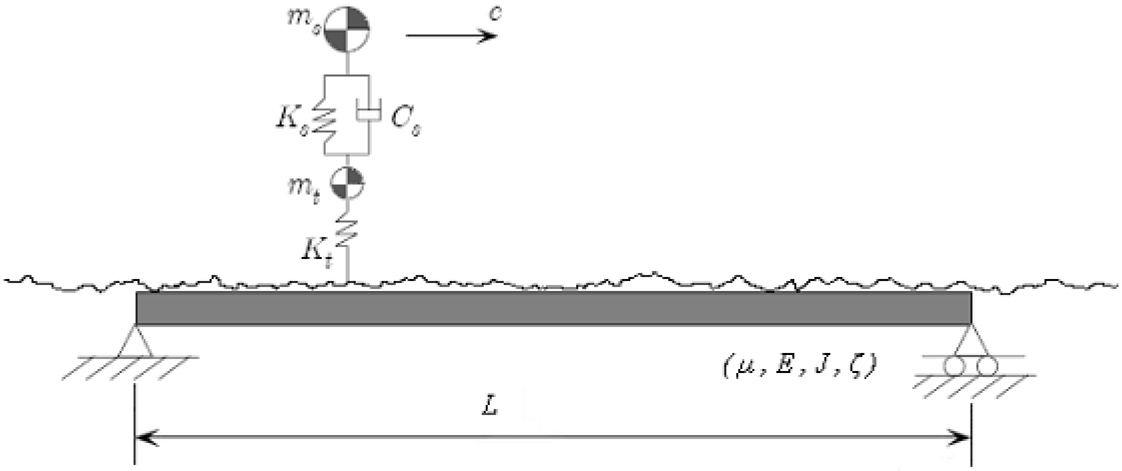

Figure 1 – Sketch of Vehicle and Beam Models

The beam model is a simply supported Euler-Bernoulli beam of length L with modulus of elasticity E, second moment of area J, constant mass per unit length μ and structural damping ζ, as pictured in Figure 1.

Table 1 – Vehicle parameters variability

	Mean value	**Standard deviation**	**Minimum**	**Maximum**	**Unit**
Speed (*c*)	20	10	10	35	m/s
Gross Vehicle Weigh (m_s+m_t)	$10\cdot10^3$	$5\cdot10^3$	$5\cdot10^3$	$15\cdot10^3$	Kg
Unsprung mass (m_t)	750	250	500	1000	Kg
Suspension Stiffness (k_s)	$750\cdot10^3$	$250\cdot10^3$	$500\cdot10^3$	$1\cdot10^6$	N/m
Suspension Damping (c_s)	$10\cdot10^3$	$2.5\cdot10^3$	$5\cdot10^3$	$15\cdot10^3$	N·s/m
Tyre Stiffness (k_t)	$3.5\cdot10^6$	$1\cdot10^6$	$2\cdot10^6$	$5\cdot10^6$	N/m

The crossing of the Q-car over the beam is simulated based on the approach proposed by Frýba (1972). The vertical displacements of both, beam and vehicle, are described by a system of coupled differential equations that can be solved using standard numerical techniques. In this paper Wilson-θ integration scheme (Bathe et al 1976) was adopted that, compared to Rung-Kutta, provides similar accuracy (Xie 1996). The Wilson-θ method is essentially an extension of the linear acceleration method, in which a linear variation of acceleration from time '*t*' to time '*t* + Δ*t*' is assumed (Tan et al 1998).

3. Magnitude and Location of Maximum Bending Moment

The Vehicle-bridge interaction (VBI) model described in the previous section is used to determine the response of a 25m simply supported bridge to a Q-car travelling at 25m/s (90km/h) on a perfectly smooth road profile. The resulting total bending moment is normalised by dividing by the maximum static moment at mid-span. This Normalised Bending Moment (NBM) at mid-span is illustrated in Figure 2 in which the maximum corresponds to the Dynamic Amplification Factor (DAF) that in this case is found to be 0.9959.

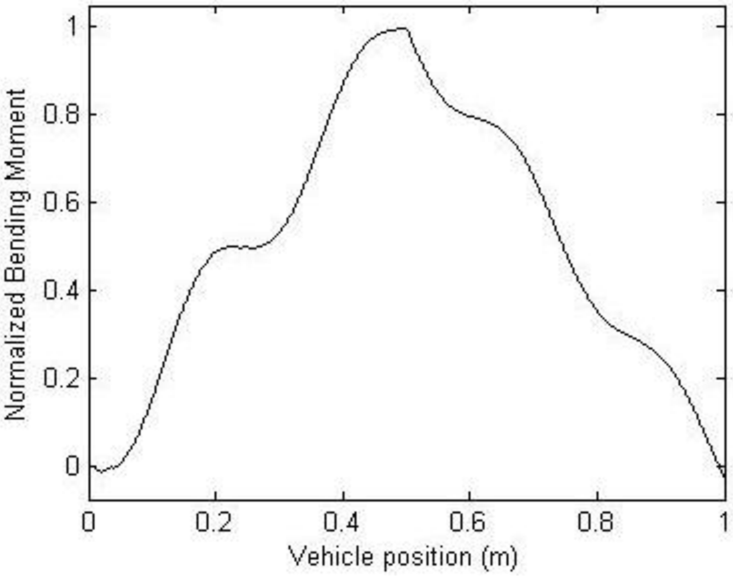

Figure 2 – Mid-span Normalized Bending Moment

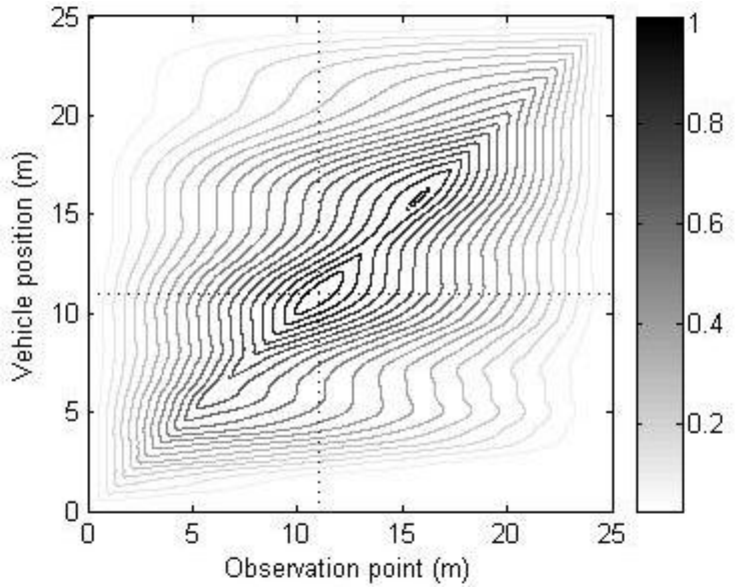

Figure 3 – Contour plot of Normalized Bending Moment at all Possible Observation Points. Dashed Lines show the Location of Maximum.

However, as the total response due to VBI is a complex problem, the maximum total bending moment value is commonly not located at exactly mid-span. Therefore, for the same case study, the *NBM* is obtained, in Figure 3, for a number of observation points along the beam, and not only mid-span. In this paper the overall maximum of the NBM is referred as FDAF, abbreviation derived from Full bridge length DAF (Cantero 2008). This new dynamic amplification factor takes into account the whole bridge extension rather that just mid-span, and has a value of 1.0648 in this particular case.

Therefore, the constant γ is defined to evaluate the difference between mid-span and full length maximum values (Equation (1)). In this particular case, γ has a value of 0.0692, showing that the DAF should be increased in 6.92% to account for the bridge full length.

$$FDAF = (1 + \gamma) \cdot DAF \qquad (1)$$

Also the Critical Observation Point (COP) is defined here as the observation point on the bridge where the maximum bending moment takes place. In the case of Figure 3, the COP is located at 11.00m from the start of the bridge. This means that for this particular case, the absolute maximum occurs at a considerable distance of 1.50m away from the mid-span location.

4. Monte Carlo Simulation

The values of FDAF and the location of the COP will depend on the velocity of the load, profile roughness, the bridge length, its boundary conditions and in a lesser extent the rest of vehicle and beam parameters. That is why a Monte Carlo simulation for 16000 events was performed to statistically evaluate the COP location and γ values. The vehicle parameters were randomly generated following a normal distribution and maintaining the values within a certain threshold (Table 1). Table 2 lists the range of bridge lengths and structural parameters considered in the simulations.

Table 2 – Beam model parameters ($E = 3.5 \cdot 10^{10} N/m^2$ and $\zeta = 3\%$)

L (m)	μ (kg/m)	J (m^4)	1st natural frequency (Hz)
17	15002	0.4911	5.8179
19	15741	0.6660	5.2950
21	16530	0.8722	4.8405
23	17419	1.1133	4.4411
25	18358	1.3901	4.0915
27	19372	1.7055	3.7824
29	20486	2.0634	3.5069
31	21650	2.4651	3.2630

As the condition of the road profile is a major factor influencing the response of the bridge to a passing vehicle (DIVINE 1998), simulations have been carried out to analyse the influence of different profiles generated using ISO recommendations (ISO8608 1996). This road generation is a random process described by a power spectral density function that vary depending on the road class from A (‘very good’) to E (‘very poor’).

Figure 4 shows the distribution of COP location expressed in percentage of bridge length, for all simulated events. It can be seen that the variation of COP location is within a range of 30% to 70%. Note these results show the response for all 8 bridge spans under consideration, obtaining similar figures if each beam length is analyzed separately. For instance, for a 25m long bridge the maximum bending moment will most likely occur within 7.5m and 17.5m range and not just at mid-span. This has implications at the design stage of the beam, where it might be necessary to increase the length of the central bending moment reinforcement.

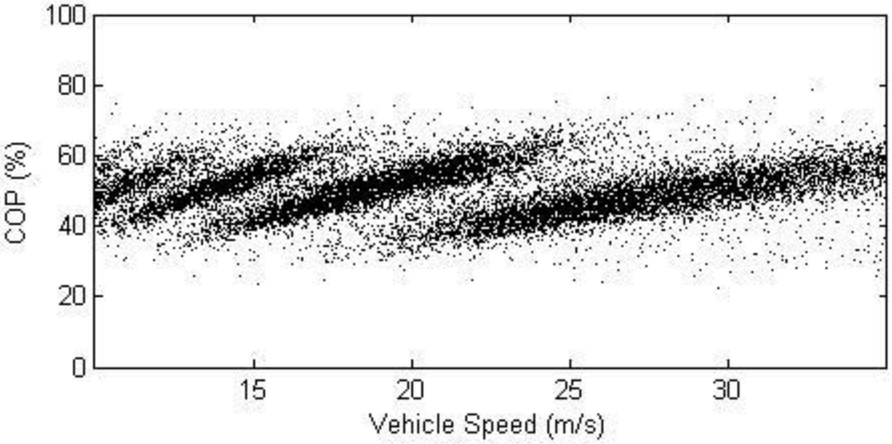

Figure 4 – Critical Observation Points for all Monte Carlo Simulation Events

Furthermore, it is evident from Figure 4 that the distribution of COP follows a pattern where the maximum bending moment is most likely to occur for a given speed. Note that the distribution showed remains similar for any bridge length. This pattern is the distorted version of the constant load case, and the observed dispersion is due to the vehicle-bridge interaction, variability in road profiles and vehicles mechanical properties.

Figure 5 illustrates the average γ value of the studied Monte Carlo events for each beam length and road profile class. While the influence of the profile unevenness is evident in the values of γ, there is no clear tendency in relation to the bridge length.

Therefore stresses measured and calculated at mid-span should be increased by a certain amount γ that in the case of the Q-car discussed in this paper reaches average values above 5% for a wide range of bridge lengths and road profiles.

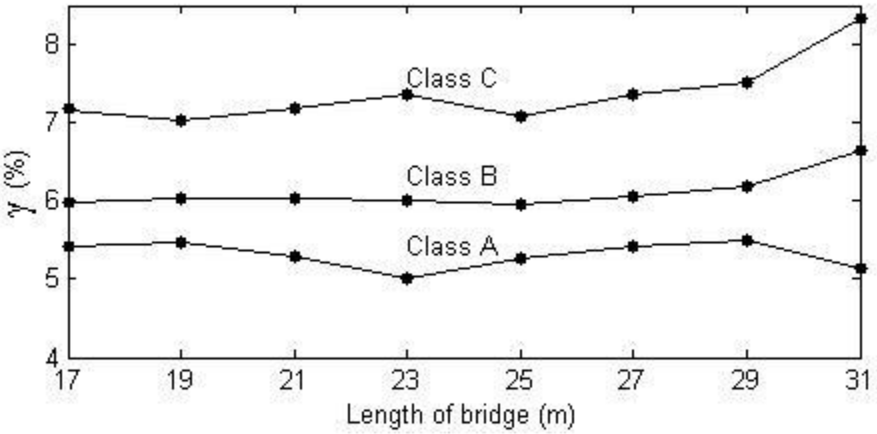

Figure 5 – Mean γ Values of Simulated Events

Moreover, it is a known fact that the increase of Gross Vehicle Weight leads to smaller DAF values (SAMARIS 2006), due to a higher vehicle-bridge mass ratio that reduces the dynamic response of the bridge. This has been confirmed as well with the results of the described Monte Carlo simulation. On the other hand, the characteristic load of a highway bridge is due to the heaviest truck within a certain return period. Thus, small DAF values should be expected for this extreme load. But it is precisely for small DAF values that γ becomes bigger, as seen in Figure 6. Therefore, special care should be taken when assessing a highway bridge using recorded strains from mid-span, or at the design stage when the characteristic load is calculated.

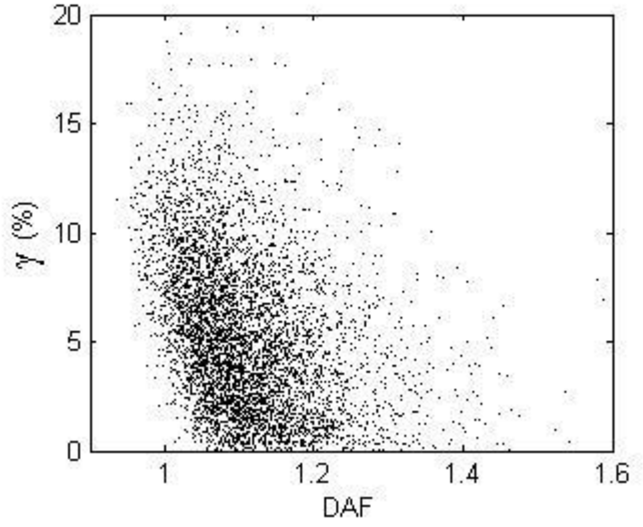

Figure 6 – γ for DAF Values (Class A Profiles)

Finally, in Figure 7 γ values are presented according to its corresponding event speed showing different peaks and valleys. The valleys relate to events whose COP is very close to mid-span, whereas for the peaks COP occurs at a significant distance from the centre of the beam, as showed in Figure 4. Therefore, from Figure 7 critical speeds can be defined (approximately 12, 16 and 23m/s) for the studied velocity range, and have the property to be roughly the same for all profiles, bridge spans and vehicle parameters.

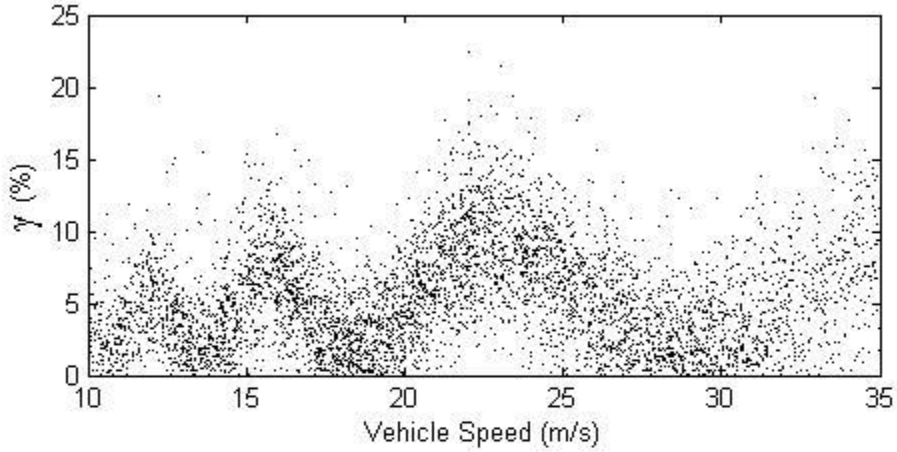

Figure 7 – γ for Vehicle Speed Values (Class A Profiles)

5. Conclusion

A Q-car model travelling on a simple supported beam has been used to study the location of the maximum bending moment and compare it to the one at mid-span. Results show that the difference is significant, typically above 5%. The absolute maximum stress occurs in a location between 30% and 70% of the bridge length.

Vehicle, bridge and road profile properties were varied to search for critical scenarios. As the road roughness increased, the difference between worst total moment and total moment at mid-span clearly became more important. Therefore it is shown that the consideration of all observation points across the full beam length is necessary when dealing with dynamics effects.

FDAF is defined as a new dynamic amplification factor that considers the full bridge length, and is related to DAF by the studied γ values. In addition, the location on the beam of the overall maximum is described as COP, and found to be strongly correlated with γ values, and therefore with FDAF.

At the moment, the authors are extending the study to more complex vehicle and bridge models. The elaboration of a multiple axle vehicle model over a planar finite element slab is at an early stage, and so far shows similar results to the ones explained in this paper.

Acknowledgments

The authors would like to express their gratitude for the financial support received from the 6th European Framework Project ARCHES (Assessment and Rehabilitation of Central European Highway Structures) towards this investigation.

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