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Uncertainty quantification and calibration of a modified fracture mechanics model for reliability-based inspection planning

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Abstract: Inspection planning for welded joints is normally based on prediction of crack propagation while neglecting the crack initiation stage. A common problem with existing crack propagation models is that distribution and statistics of the initial flaw size is often hard to obtain due to difficulties in measuring and sampling. In addition, the flaw size in some high-quality joints may be so small that the crack initiation life is not negligible. This paper proposes a modified fracture mechanics (FM) model that predicts the fatigue life allowing for the crack initiation life. Sources of uncertainty associated with the model are quantified in probabilistic terms. Main novelty of the modified FM model is calibration using S-N curves with criteria in reliability and fatigue life. The modified FM model is applied to reliability-based inspection planning for a typical ship structural detail. Optimum inspection plan formulated with the model is compared to those formulated with existing S-N and FM model. Results show to what extent is the optimum inspection plan influenced by the crack initiation life. The modified model can be a reliable tool for both fatigue design and fatigue management with respect to inspection and maintenance planning.

1 Introduction

Initiation and propagation of fatigue cracks is a common problem that needs to be addressed in the life cycle of steel structures. Although in the design stage fatigue check is conducted with S-N method, a structural detail is still likely to fail within its service life due to uncertainties associated with the fatigue process and fatigue analysis method. In addition, there are inevitable gross errors during construction and operation, which can also result in fatigue and fracture failure of a structural detail [1].

To reduce the uncertainties and identify gross errors, inspections are implemented to gather information that is not available in the design stage, and provide additional information with respect to the actual state and performance of a structure. With the additional information, the structure can be re-assessed. Accordingly, one gets more confidence in the fatigue reliability of the structure in case of no crack detection or makes decisions on maintenance in case of crack detection.

However, inspection and maintenance actions are costly, so it makes sense to plan those actions in advance so that limited inspection and maintenance resources can be optimally allocated to the most critical details. A predictive model for crack evolution is of decisive help for such planning. Inspection planning is normally performed based on fatigue analysis with FM method [2-4]. FM method is favorable because it predicts fatigue damage in terms of crack evolution, which is comparable with inspection results.

FM method assumes that there are detectable initial flaws/cracks in any structure, and fatigue degradation begins with crack propagation. However, the problem is that distribution and statistics of the initial flaw/crack for specific application is often hard to obtain. In addition, the dimension of the initial flaw/crack in some structural details may be so small that it takes considerable time to develop a macroscopic crack [5]. In these two cases, FM method cannot be applied directly.

The problem is addressed with a new FM model incorporating the crack initiation time. Uncertainties associated with the crack initiation and propagation process are quantified in probabilistic terms. The crack initiation time is treated as an additional variable to existing crack propagation models. e.g. Paris's law [6]. The distribution of the crack initiation life is calibrated to S-N curves. Methods for such calibration are proposed and evaluated.

2 Crack Propagation Modelling

FM is the most important approach to address crack propagation. From the perspective of FM, fatigue failure is caused by crack evolution: crack initiation, propagation and finally fracture. As the final fracture normally occurs very quickly, the fatigue life is mostly comprised of the crack initiation life and the crack propagation life. FM assumes that there are macroscopic initial flaws/cracks in any structural detail and the crack initiation life is negligible (e.g. see crack evolution 1 in Figure 1). Accordingly, fatigue degradation begins with crack propagation.

Paris's law [6] shows the relationship between crack propagation rate and material property, loads, crack geometry and intensity of stresses around a crack. The formulation of the model is expressed as

$$\frac{da}{dN} = C\Delta K^m \quad (1)$$

where da is increment in crack propagation for dN stress cycles; C and m are material parameters; ΔK is stress intensity factor range, given by

$$\Delta K = \Delta\sigma Y(a)\sqrt{\pi a} \quad (2)$$

where $\Delta\sigma$ is stress range and $Y(a)$ is geometry function.

Paris's model is widely used in inspection planning due to its generality and simplicity. Based on Paris's model, several models are proposed to account for additional influence factors and thus add more complexity, e.g. Elber's model [7], bi-linear model [8], two-directional model [9], etc. These models may be more accurate than Paris' model for some specific applications, but require much more input parameters and computational efforts, which hinder their wide application in probabilistic crack propagation modelling and inspection planning. In this paper, a modified FM is proposed based on Paris's model.

By integration, Equation (1) can be rewritten as

$$N_F = \frac{1}{\pi^{m/2} C \Delta\sigma^m} \int_{a_0}^{a_c} \frac{da}{a^{m/2} Y(a)^m} \quad (3)$$

As can be seen from Equation (3), crack propagation life is determined by material properties C and m , stress range $\Delta\sigma$, geometry function $Y(a)$, initial and critical crack size a_0 and a_c . Among others, previous studies have shown that uncertainties associated with initial crack size a_0 , crack growth rate C and stress range $\Delta\sigma$ are more influential in the predicted crack propagation life [10]. In this paper, C and $\Delta\sigma$ are treated as variables. Their statistical characteristics, e.g. mean values and coefficient of variations (CoV) are quantified. A model is proposed below for cases where statistical information on a_0 is unavailable or a_0 is so small that crack initiation life can't be neglected (e.g. see crack evolution 2 in Figure 1).

3 The Proposed Model

In the case of crack evolution 2 in Figure 1, the actual initial crack size a_0^2 is smaller than the smallest crack size a_d that can be detected reliably by non-destructive testing (NDT) methods. It is thus hard to obtain statistical information on a_0^2 by normal NDT methods. However, it is convenient to substitute the actual initial crack size a_0^2 with a_d and calibrate the number of cycles N_I^2 (spent for the crack to develop from a_0^2 to a_d) to SN curves. The formulation of the proposed model is expressed as

$$N_F = T_I \cdot N_0 + \frac{1}{\pi^{m/2} C \Delta\sigma^m} \int_{a_d}^{a_c} \frac{da}{a^{m/2} Y(a)^m} \quad (4)$$

where N_0 is the number of cycles experienced by the structure every year, T_I is the years needed to develop a crack size of a_d and is defined as crack initiation life here.

T_I can be measured from specimen tests by using special gauges. However, it is expensive to implement such tests [11]. For practical applications in engineering, it is desirable if the parameter T_I can be related to some parameters that is already known. From this point, it is suggested to calibrate the crack initiation life T_I from S-N curves, as S-N curves for common welded joints are well established. The following three calibration criteria are proposed.

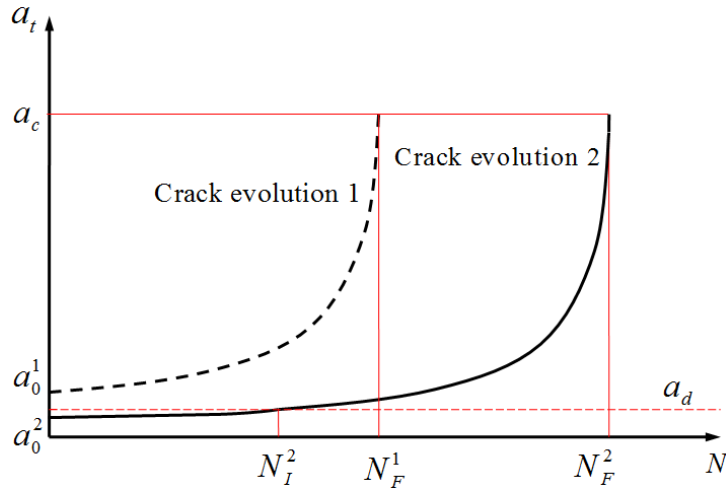


Figure 1: Schematic figure of crack evolution process

3.1 Reliability During the Service Life

The crack initiation life T_I can be calibrated to S-N curves with the criteria that the difference between the reliabilities calculated with S-N and FM method are minimum in the whole service life T_{SL} . The following function is minimised with respect to T_{I1}

$$\min_{T_{I1}} \sum_{t=1}^T [\beta_{FM}(t; T_{I1}) - \beta_{SN}(t)]^2 \quad (5)$$

3.2 Reliability When $\beta_{SN} < \beta_t$

As fatigue design is based on S-N method, it makes sense to determine the time for the first inspection according to the reliability decline curve calculated with S-N method. The first inspection is supposed to be carried out when the reliability index calculated with S-N method β_{SN} is below a target reliability level β_t , i.e. $\beta_{SN} < \beta_t$. At that time, the following function is minimized with respect to T_{I2}

$$\min_{T_{I2}} [\beta_{FM}(t; T_{I2}) - \beta_{SN}(t)]^2 \quad (6)$$

3.3 Mean Fatigue Life

From the perspective of FM, fatigue life of welded joints is comprised of crack initiation life and crack propagation life. The mean of crack initiation life T_I can be obtained from the mean of fatigue life N_c deducted by the mean of crack propagation life N_F according to Equation (7). The mean of fatigue life N_c can be obtained from an S-N curve if the standard deviation (SD) or coefficient of variation (CoV) of the fatigue life is known. The mean of crack propagation life N_F can be calculated from a FM model.

$$T_{I3} = (N_c - N_F)/N_0 \quad (7)$$

4 Limit State Function

4.1 FM Method

A failure criterion is normally defined based on serviceability in FM-based fatigue reliability analysis and inspection planning. It is thought that a structural detail is not serviceable if a through thickness crack exists, so a critical crack size equals to plate thickness can be used as a failure criterion. Using this criterion, a limit-state function can be formulated as

$$M(t) = a_c - a(t) \quad (8)$$

where a_c is the critical crack size, $a(t)$ is the actual crack size at time t under fatigue loading.

The capacity and load can also be expressed in number of cycles, and limit-state function (9) can be used.

$$M(t) = N_F - N(t) \quad (9)$$

where N_F is the capacity of a structural detail against fracture and $N(t)$ is the fatigue loading experienced by the detail till time t .

Investigations on inspection planning generally calculate N_F by integration of a crack propagation model, while in the present paper, the crack initiation life is included in N_F and Equation 1 is used. Uncertainties associated with T_I , C and $\Delta\sigma$ are quantified.

4.2 S-N Method

Fatigue reliability can also be calculated based on S-N curves and Miner's rule. The following limit-state function is used.

$$g = \Delta - D \quad (10)$$

where Δ is a variable signifying the fatigue damage at failure, and D is fatigue damage expected during the service life of a structural detail, given by Miner's rule

$$D = \sum_{i=1}^{n_b} \frac{n_i}{N_{ci}} \quad (11)$$

where n_i is number of load cycles at the i stress range level; N_{ci} is the fatigue capacity under the i stress range level; and n_b is the number of stress range levels.

Fatigue capacity N_{ci} is given by S-N curves. In this paper, two-segments S-N curves [12] are used. The curves can be expressed as

$$\begin{cases} N_c \Delta \sigma^{m_1} = \bar{a}_1 & N_c \leq 10^7 \\ N_c \Delta \sigma^{m_2} = \bar{a}_2 & N_c \geq 10^7 \end{cases} \quad (12)$$

where m_1 and m_2 are the fatigue strength exponents, and \bar{a}_1 and \bar{a}_2 are the fatigue strength coefficients.

Due to inherent uncertainties of the fatigue process, probabilistic methods are used to assess the reliability of a structural detail against fatigue failure. Major uncertainties for fatigue analysis based on S-N method come from: 1) uncertainty on fatigue capacity expressed by S-N curve; 2) uncertainty on modelling of damage accumulation, e.g. Miner's rule; 3) uncertainty on calculation of load effects, i.e. stress ranges. Uncertainties from those three aspects are considered in this paper.

5 Reliability-based Inspection Planning

Based on the proposed modified FM model, the initial reliability curve without inspection can be calculated. Inspections are assigned every time the calculated reliability index is below the target reliability level, and the inspection results are used to update the reliability level. Based on the new reliability curve, the next inspection is planned.

Reliability updating is based on the definition of conditional probability and Bayesian Theorem

$$P(F|I) = \frac{P(F \cap I)}{P(I)} \quad (13)$$

where $P(F|I)$ is the probability the event F occurs given that event I occurs.

In this paper, F is fracture failure occurs, and I are the inspection outcomes. F is given by

$$F(t) = N_F - N(t) \leq 0 \quad (14)$$

The event of no detection is considered, as it is the most common outcome. The event can be expressed by

$$I(t) = a_t - a_d \leq 0 \quad (15)$$

If n inspections are implemented with no detection, then the event can be expressed as

$$I(t_1, t_2, \dots, t_n) = a_{t_1} - a_d \leq 0 \cap a_{t_2} - a_d \leq 0 \cap \dots \cap a_{t_n} - a_d \leq 0 \quad (16)$$

where $a_{t_1}, a_{t_2}, \dots, a_{t_n}$ are the predicted crack size at time t_1, t_2, \dots, t_n . In this paper, it is assumed that each inspection is independent.

Substituting Equation (14) and (16) into Equation (13), one can obtain the probability of fracture failure given that n inspections have been implemented with no detection,

$$P(F|I) = \frac{P(N_F - N(t) \leq 0 \cap a_{t_1} - a_d \leq 0 \cap a_{t_2} - a_d \leq 0 \cap \dots \cap a_{t_n} - a_d \leq 0)}{P(a_{t_1} - a_d \leq 0 \cap a_{t_2} - a_d \leq 0 \cap \dots \cap a_{t_n} - a_d \leq 0)} \quad (17)$$

6 An Illustrative Example

The proposed model is illustrated on a typical T joint subjected to fatigue in a steel ship structure. The critical location is shown in Figure 2. There are thousands of such T joints in a ship structure, in which stiffeners are welded to plates to improve stability of plates. However,

fatigue reliability of such joints is a problem that needs to be addressed during the life cycles of the ship, as surface cracks are highly likely to initiate and propagation along the weld toes of the joints. The structural detail is designed initially for a service life $T_{SL} = 20$ years with S-N method. At the late stage of service life, the operator of the ship want to know if the service life can to extended to $T = 25$ years. Fatigue reliability of the structural detail needs to be analysed to support decision-making under uncertainties. In addition, inspection plan needs to be developed to provide information regarding actual state of the structural detail. FM method is employed for in-service performance assessment and inspection planning, because with the method, all the information of the detail, loads and inspections is utilized. Also, using FM method, crack size can be predicted and compared with inspection results, in which way epistemic uncertainties are reduced. However, the problem is that the initial crack size is unknown, and thus FM method cannot be used directly for fatigue life prediction. The modified FM model proposed in this paper is applied to the structural detail.

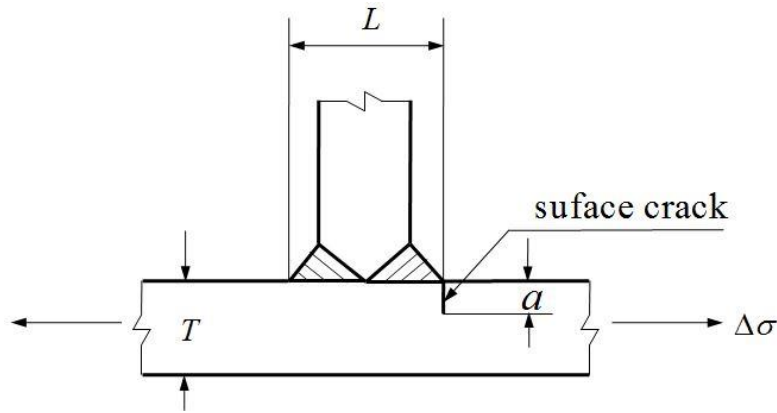


Figure 2: Crack initiation and propagation in a welded T joint

Table 1: Distributions and statistics of parameters

Parameters	Distribution	Unit	Mean	SD	CoV
N_0	Deterministic	cycle	5×10^6	-	-
N_c	Lognormal	cycle	$50 N_0$	-	0.25/0.50
Δ	Lognormal	-	1	-	0.30/0.60
$\log_{10} \bar{a}_1$	Deterministic	$N^4 \cdot \text{mm}^{-6}$	11.855	-	-
$\log_{10} \bar{a}_2$	Deterministic	$N^4 \cdot \text{mm}^{-6}$	15.091	-	-
m_1	Deterministic	-	3	-	-
m_2	Deterministic	-	5	-	-
$\log_{10} C$	Normal	$N^{-4} \cdot \text{mm}^{5.5}$	-12.74	0.11	-
B	Normal	-	1	0.15	-
N_I	Weibull	cycle	fitted	-	0.34
a_d	Deterministic	mm	0.1	-	-
T	Deterministic	mm	25	-	-
T_{SL}	Deterministic	year	20	-	-

6.1 Calibration of the Modified FM Model

The probabilistic model in Table 1 is used for reliability analysis and inspection updating. The parameter B is introduced to take into account the uncertainties associated with load and load effect calculation. Figure 3 and 4 shows the initial fatigue reliability without inspection calcu-

lated with S-N method and modified FM method respectively. Uncertainty in fatigue capacity N_c , fatigue damage at failure Δ , and crack initiation life N_I is quantified, and sensitivity of reliability index to those variables are studied. It can be seen from Figure 3 that the fatigue reliability of the joint is strongly influenced by the uncertainty in N_c and Δ . According to the reliability decline curves provided by Figure 3, the time for the first inspection can be made based on certain target reliability. For example, if target reliability $\beta_{SN} = 3$, the year for the first inspection is listed in Table 2.

Three methods proposed in Section 3 are tested to calibrate the crack initiation life T_I to S-N F curve. T_{I1} and T_{I2} are calibrated based on the criterion of reliability, so the results, which are listed in Table 3 and 4 respectively, are dependent on the uncertainty in N_c and Δ . T_{I3} is, on the other hand, calibrated based on the criterion of mean fatigue life, and is constant $T_{I3} = 34$ years. The reliability curves calculated with the modified FM model are compared with the reliability curve calculated with S-N method in Figure 5, in which FM₁, FM₂ and FM₃ represents modified FM model with crack initiation life T_{I1} , T_{I2} and T_{I3} respectively.

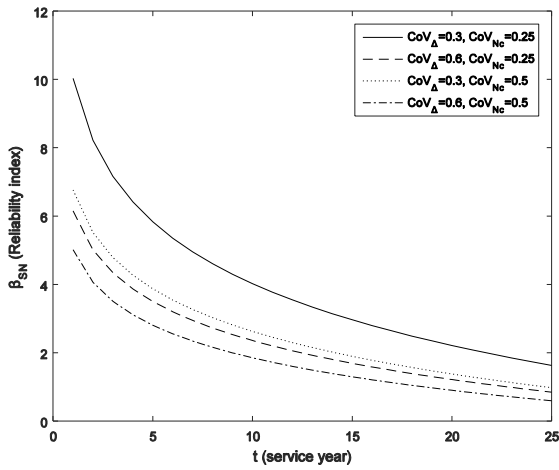


Figure 3: Reliability with S-N method

Table 2: The year for the first inspection based on $\beta_{SN} = 3$

CoV $_{\Delta}$	CoV $_{N_c}$	
	0.25	0.50
0.30	15	9
0.60	7	5

Table 3: Crack initiation life T_{I1} (unit: years)

CoV $_{\Delta}$	CoV $_{N_c}$	
	0.25	0.50
0.30	28	13
0.60	12	9

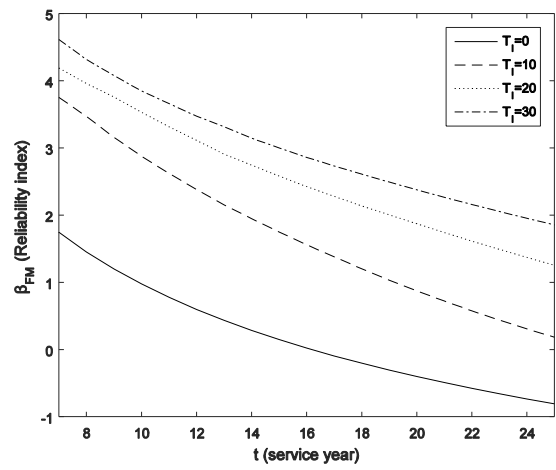


Figure 4: Reliability with modified FM model

Table 4: Crack initiation life T_{I2} (unit: years)

CoV $_{\Delta}$	CoV $_{N_c}$	
	0.25	0.50
0.30	29	7
0.60	6	1

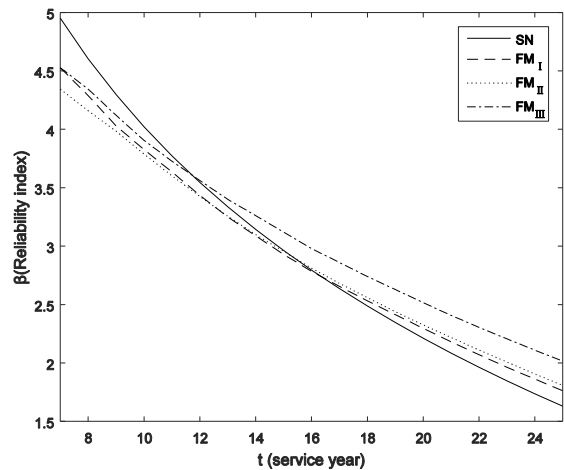


Figure 5: Reliability with calibrated T_I

6.2 Optimum Inspection Plans

The considered inspection method is magnetic particle inspection (MPI) with mean detectable crack size of 0.89 mm [2]. The time for inspection is determined based on prediction of crack evolution. The inspection plans formulated with the modified model FM₁, FM₂ (Figure 8 and 9) and FM₃ (Figure 7) are compared with the inspection plan formulated with traditional FM model (Figure 6). The symbol $\beta_0, \beta_1, \beta_2, \beta_3$ in those figures means reliability without inspection, reliability after one, two, and three inspections respectively. It can be seen from Figure 7 that based on traditional FM model, four inspections are needed to ensure that no crack is detected in the joint within 25 years. The inspection time is the 5th, 10th, 16th, and 22nd service year. Based on the modified model FM₃, only two inspection are needed, which should be carried out on the 16th and 23rd service year. The inspection plans based on modified model FM₁ and FM₂ are dependent on the uncertainty in N_c and Δ . It can be seen from Figure 8 and 9 that inspection plans formulated with model FM₁ and FM₂ are similar and only two inspections are needed if $CoV_{N_c}=0.25, CoV_{\Delta}=0.30$. However, if $CoV_{N_c}=0.50, CoV_{\Delta}=0.60$, the inspection plans formulated with model FM₁ and FM₂ are different in the years for inspections, but the same in the number of inspections. The detailed inspection plans based on S-N model, FM model and modified FM model are compared in Table 5.

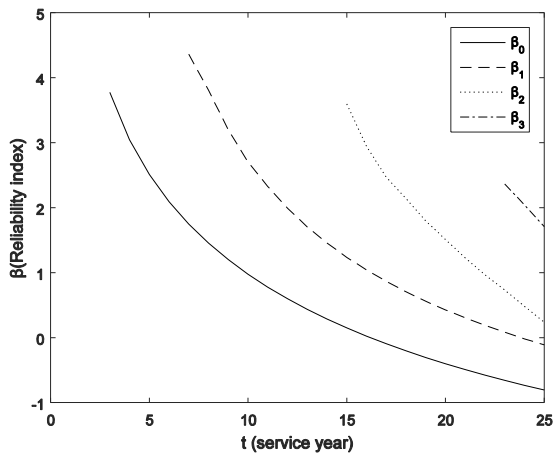


Figure 6: Inspection updating based on FM model

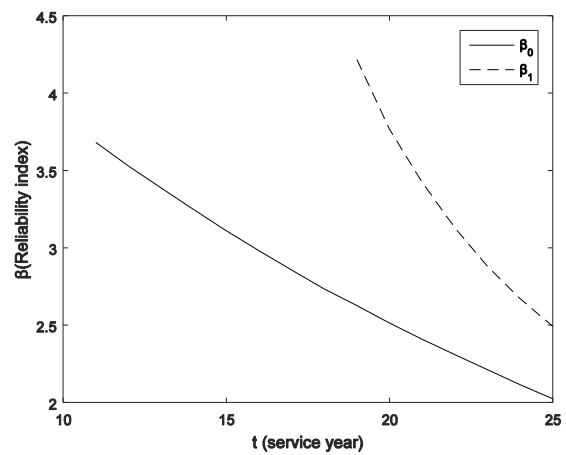


Figure 7: Inspection updating based on FM₃ model

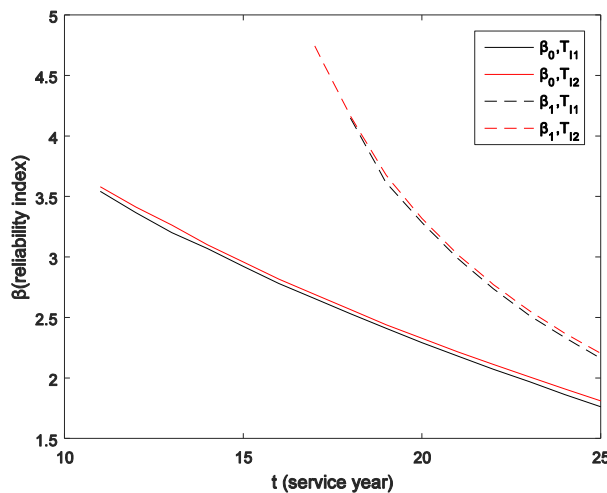


Figure 8: Inspection updating based on FM₁ and FM₂ model ($COV_{N_c}=0.25, COV_{\Delta}=0.30$)

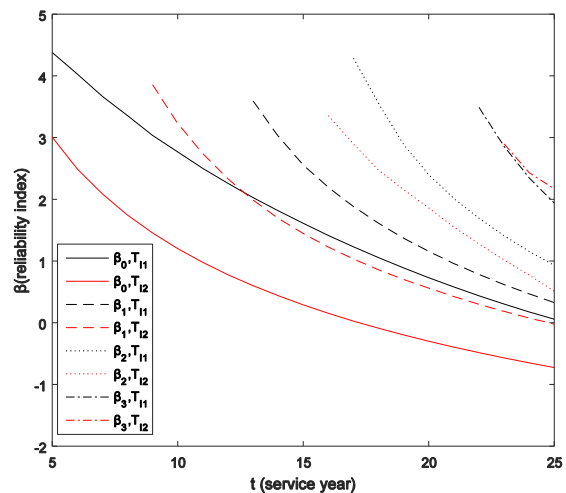


Figure 9: Inspection updating based on FM₁ and FM₂ model ($COV_{N_c}=0.50, COV_{\Delta}=0.60$)

Table 5: Inspection plans (the years for inspections) based on $\beta_t = 3$

Degradation model		The years for inspections	
		CoV _{N_c} =0.25, CoV _Δ =0.30	CoV _{N_c} =0.50, CoV _Δ =0.60
S-N model (only the first inspection can be planned)		15	5
FM model		5/10/16/22	5/10/16/22
Modified FM model	FM ₁	15/21/-	10/15/19/23
	FM ₂	15/22/-	6/11/17/23
	FM ₃	16/23/-	16/23/-

7 Conclusion

Fatigue reliability of welded joints is a critical issue for integrity management of steel structures. Compared with S-N method, FM method is widely used in in-service performance assessment and inspection planning. The major uncertainty in FM-based fatigue life prediction comes from the uncertainty in the initial flaw/crack size, which is one of the most influential parameters, but hard to obtain due to sampling and measuring difficulties.

This paper has addressed the problem with a modified FM model, in which the initial flaw/crack size is substituted with the minimum detectable crack size of a NDT method, and the corresponding crack propagation life is added with a crack initiation life. The crack initiation life is calibrated to S-N curves and three methods for calibration are proposed. Based on comparative studies, the calibration is suggested be done based on probabilistic analysis and with criterion in fatigue reliability accounting for the uncertainties associated with S-N data, fatigue damage accumulation, crack initiation and propagation.

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