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Examining the benefits of load shedding strategies using a rolling-horizon stochastic mixed complementarity equilibrium model

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Abstract

As a result of government policies increasing the amount of electricity generated from fluctuating renewable sources in many countries, the requirement for flexibility in the corresponding electricity systems increases. On the demand side, load shedding is one demand response mechanism contributing to an increased flexibility. Traditionally, load shedding was based on rather static or rotational strategies, whereby the system operator chooses the consumers for load shedding. However, ongoing technological developments provide the basis for smarter and more efficient load shedding strategies. We therefore examine the costs and strategies associated with such mechanisms by modelling an electricity market with different types of generators and consumers. Some consumers provide flexibility through load shedding only while others additionally have the ability to generate their own electricity. Focussing on the impacts of how and to whom consumers with own generation ability can supply electricity, the presence of market power and generator uncertainty, we propose a rolling horizon stochastic mixed complementarity equilibrium model, where the individual optimisation problems of each player are solved simultaneously and in equilibrium. We find that a non-static strategy reduces consumer costs while allowing consumers to provide own generation to the whole market results in minimal benefits. The presence of market power was found to increase costs to consumers.

Keywords: OR in Energy, Stochastic Programming, Load shedding, Stochastic mixed complementarity, Rolling horizon

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1. Introduction

Many governments have adopted policies for expanding the use of renewable energy sources (RES) aimed at reducing greenhouse gas emissions. As a result of an increasing use particularly of fluctuating RES, such as wind and solar energy, the volatility of the system residual load will increase strongly leading to growing flexibility requirements Bertsch et al. (2016). In order to meet these requirements, electricity markets need to become more flexible. While traditionally, flexibility has been mainly provided by the supply side, demand side flexibility has gained increasing interest over the last decade and is expected to become increasingly important in the future Palensky & Dietrich (2011); De Jonghe et al. (2012). Kirby & Hirst (1999) as well as Chen et al. (2010), for instance, describe the system benefits (mainly efficiency gains and cost reductions) of an increased demand side flexibility. In this context, Palensky & Dietrich (2011) distinguish between four categories of demand side management: energy efficiency, time of use, demand response and spinning reserve. Aimed at exploring market-based solutions to meet short-term flexibility requirements, we focus on the demand response category in this paper. Within the demand response category, Albadi & El-Saadany (2008) distinguish between load reduction/shedding, load shifting and customer owned distributed self-generation, whereas Bayer (2014) distinguishes between reduction/shedding, shifting and increase of load. The majority of existing research concentrates on load shifting. However, empirical research findings suggest that consumers respond to higher prices by reducing electricity consumption during peak periods Faruqui & Sergici (2010), but that they do not necessarily shift their consumption to off-peak periods Allcott (2011); Di Cosmo et al. (2014). We therefore focus on the examination of benefits of load shedding strategies in this paper. Consequently, we study the temporary short-term reduction of load in situations where the demand for electricity exceeds the supply capacity or where there is inadequate transmission capacity available to deliver sufficient electricity to the areas and consumers where it is needed.

Traditionally, load shedding involved strategies where the system operator chooses the consumers that must shed their load - mostly following a rather static or a rotational scheme. Under a static scheme, the system operator can shed load of specific consumers according to predefined conditions (e.g., sheddable capacity and corresponding price) laid down in a contract or according to predefined priorities of consumers Calderaro et al. (2010). Under a rotational scheme, the system operator can shed load in a specific part of the electricity network at a time, where the affected areas and consumers will change over time in order to ensure a fair burden sharing. While being a common event in many developing countries, load shedding, particularly the rotational scheme is rather a measure of last resort in developed countries today, used by the system operator to avoid a total blackout of the power system.

However, the increasing digitisation driven by ongoing developments in information and communication technology (ICT) enables the transformation of electricity distribution grids towards active distribution grids Woo et al. (2014); Ruppert et al. (2015) and provides the basis for smarter and more efficient (non-static) load shedding strategies. For instance, shedding load of a particular consumer would not need to result in a complete blackout for this consumer but could simply imply a partial load reduction ("brownout"). Such interruptible and curtailable electricity load programmes have also been reported and explored by others Ströhle & Flath (2016); Albadi & El-Saadany (2008); Woo et al. (2014); Associates (2005); of Energy (2006); Faruqui et al. (2010). In essence, while implying curtailments for some consumers, such approaches, are aimed at avoiding blackouts and therefore at increasing energy security on a system level. The European Energy Security Strategy EC (2014) and the European Directive on Security of Network and Information Systems EC (2016) both acknowledge the need for increasing energy system security and underline the relevance of such approaches, while at the same time highlighting the need for addressing these challenges in a competitive market environment.

Our focus in this paper is therefore to examine the potential costs and benefits of different strategies for load shedding as one set of instruments within the field of demand response. For this purpose, we assume a competitive electricity market with multiple generators and different types of consumers which can be distinguished according to their load shedding ability and costs. We also assume that some consumers provide flexibility to the market through load shedding only while others additionally have the ability to generate their own electricity by auxiliary power generation units (APUs). Moreover, we consider uncertain generator availability. In such an environment, we are particularly interested in exploring the following research questions:

1. What are the benefits of allowing consumers with own generation to provide generation to the whole market?
2. How do 'smart' (non-static) load shedding strategies compare with static and rotational load shedding schemes and how do they differ in terms of costs to consumers?
3. How does the presence/absence of market power affect costs to consumers?
4. What are the benefits of a stochastic planning approach in the light of the uncertainties?

Demand response has been studied intensely in literature (see e.g. reviews by Albadi & El-Saadany (2008); Hornby et al. (2011); Economics & First (2012); Boßmann & Eser (2016); Haider et al. (2016); Esther & Kumar (2016)). Load shedding, in particular, has been investigated using heuristic techniques Laghari et al. (2013) as well as linear or nonlinear programming techniques Subramanian (1971). Wang and Billinton Wang & Billinton (2000), for instance, consider time-dependent, linear load shedding cost functions of different consumer types in an optimal load shedding approach. However, in order to explore the research questions set out above, most existing approaches are limited with respect to at least one of the following two characteristics:

- The load shedding cost functions are assumed to be linear.
- Load shedding is optimised from a central planning perspective using a single optimisation problem.

In relation to the first limitation, we wish to note that the costs associated with load shedding should not be assumed to increase linearly when the amount of load shedding is increased. Low amounts of lost load, for instance, may only lead to low-cost effects (e.g., reduced illumination) whereas higher amounts of lost load may induce much higher losses across different consumer types Ruppert et al. (2015). In relation to the second limitation, a central planning optimisation does not take into account individual optimisation targets of different players. Hence, methods are needed that allow for the simultaneous consideration of multiple, individual optimisation problems (such as complementarity problems) and for the incorporation of consumer-specific, nonlinear load shedding cost functions. Moreover, with a view to our research questions, the methods should be able to consider market power, electricity generation by consumers and stochastic supply. Chen et al. (2010), for instance, use a game-theoretic equilibrium model with a quadratic load shedding cost function. The model by De Jonghe et al. (2012) is very similar. However, both don't take into account market power, APU generation or stochastic supply.

We therefore propose using a game-theoretic equilibrium model, namely a stochastic mixed complementarity problem (MCP) with quadratic load shedding cost functions, to analyse interactions of different players in a competitive electricity market. MCPs have been used to model various types of energy markets Devine et al. (2016); Lynch & Devine (2017); Egging (2013); Hobbs (2001); Gabriel et al. (2009); Huppmann (2013). They allow the optimisation problems of multiple individual players to be solved simultaneously and in equilibrium by combining the Karush-Khun-Tucker (KKT) conditions for optimality of each of the players and connecting them via market clearing conditions. In addition, MCPs allow both primal variables (eg., power generation) and dual variables (eg., prices) to be constrained together Gabriel et al. (2012) while also allowing players with constrained optimisation problems to be modelled as either price-takers or price-makers, hence, incorporating market power into such models Gabriel et al.

(2005); Lee (2016). Traditionally, price-makers have been modelled using simple linear demand curves ($Demand = A - B \times Price$). However, in this work, we model price-makers in a novel manner by combining a supply-demand equation with the KKT conditions of the consumers.

We apply the proposed stochastic MCP in the context of a case study based on data for Ireland. The players that we consider in the case study include different types of generators and consumers. The generators produce electricity to maximise their profits and may be price-takers or price-makers as described above. The consumers in our model choose how much of their load to shed in order to meet their demand at minimum costs and may differ in terms of their electricity demand profiles, their load shedding potential and cost functions and their ability to generate their own electricity. We consider consumers with the ability to generate electricity as *active* load shedding consumers and consumers without this ability as *passive* load shedding consumers. Note that, in reality, neither active nor passive consumers would usually decide themselves whether or not and when to shed any load. Rather, we assume that there will be an aggregator who acts on behalf of the consumers Good et al. (2017); Ceseña et al. (2015); Burger et al. (2016), with the objective of minimising their energy supply costs. For the model, however, this does not make a difference. Moreover, the market we model is one with a significant presence of smart meters, such as the Irish electricity market in future Comission for Energy Regulation (2014). These smart meters will allow consumers or aggregators on their behalf to react quickly to short-term changes in the wholesale market. As load shedding is of greatest significance in times when demand is high and supply is low, we initially assume that RES generation is not available during the entire time period. In addition, we assume that one of the conventional generators is unavailable for generation and the time when it will return online is the model's main source of uncertainty. While our model considers a conventional generator returning online, one can similarly think of this uncertainty as representing the time when RES generation becomes available again.

In order to take this uncertainty into account, we use a rolling horizon stochastic MCP in this work, which involves solving multiple MCPs along multiple sequences paths where the results from the previous problem are used to update the parameters for the subsequent problem, i.e. for each problem solved, only a subset of all time steps are considered. The alternative approach to solving optimisation/equilibrium models would be the perfect foresight approach which involves solving the problem once over all time steps. However, rolling horizon models have been used to solve many optimisation/equilibrium problems in energy markets Tuohy et al. (2009); Devine et al. (2014); Guigues et al. (2014) as they have been shown to model such markets more realistically than perfect foresight approaches and in a computationally efficient manner Devine et al. (2016, 2014). Details of the rolling horizon solution approach used in this work can be found in section 2

The contribution of our paper can be summarised as follows: We examine the benefits of load shedding strategies using a rolling horizon stochastic MCP with quadratic load shedding cost functions to allow the optimisation problems of multiple individual players to be solved simultaneously. Beyond the existing work by Chen et al. (2010) and De Jonghe et al. (2012), we consider market power, own generation by consumers and stochastic generator availability. Note that the proposed model is a short-term model, i.e. we do not consider aspects related to investment planning in this paper.

The remainder of this paper is structured as follows: In section 2, we introduce the mathematical model. In section 3, we present the data of our case study followed by the results. In section 4, we discuss and interpret the results before concluding the paper in section 5. In Appendix A, we provide additional data related to the case study.

2. Model formulation

In this section we describe the formulation of the rolling-horizon stochastic mixed complementarity approach. This solution approach involves solving a sequence of stochastic MCPs Devine et al. (2016). Each MCP corresponds to a roll of the rolling horizon and consists of solving the problem over a subset of the model timesteps. At each roll r the model considers H timesteps: the first timestep is $t = r$ while the last timestep is $t = r + H - 1$. The set of timesteps for roll r is $T(r) = \{r, r + 1, .., H + r - 1\}$. For example, if $H = 48$, then the time set for the first roll would be $T(1) = \{1, 2, ..., 48\}$ while for the second roll it would be $T(2) = \{2, ..., 48, 49\}$, and so on. Once the MCP for a given roll is solved, the model steps forward to the next MCP problem where the demand and storage parameters, in addition to parameters describing whether generators are on/offline, are updated. In particular, storage parameters are updated from the storage decision variables in the previous roll. In this way, the model updates itself with decisions made in the previous roll; see Section 2.3.1 for details on how the stochastic MCP in this work updates between rolls. The rolling horizon approach should be contrasted with a (perfect foresight) approach where all the future time periods are considered at once in a single optimisation Devine et al. (2016).

The stochastic MCP at each roll r models an electricity market with $|I|$ conventional generators, $|J|$ passive load shedding consumers and $|K|$ active load shedding consumers. The conventional generators produce electricity in order to maximise their profits (subject to capacity constraints) and may be modelled as either price-takers or price-makers. Thus, the model can incorporate market power. The two types of consumers choose how much of their load to shed, subject to constraints, in order to meet their demand at minimum costs. In addition, active load shedding consumers also have the ability to generate their own electricity, known as auxiliary generation, to help meet their demand (again, subject to capacity and storage constraints).

Each of these three types of players has separate optimisation problems that are connected through market clearing conditions. The stochastic MCP is made up of these market clearing conditions along with the Karush-Kuhn-Tucker (KKT) conditions for optimality from each of the players. Thus, the MCP at roll r solves the optimisation problem of each player simultaneously and in equilibrium.

For the MCP at roll r, each player has first-stage decisions that represent 'here and now' decisions or actual decisions for timestep $t = r$. These decisions are scenario-independent. Furthermore, each player also has second-stage decisions that represent 'wait and see' or hypothetical decisions for all other hourly timesteps (i.e., $r < t < H + r$). These decisions are scenario-dependent. One of the conventional generators is assumed to be unreliable, i.e., this generator is available/unavailable for generation at different timesteps. Each MCP is solved over $|S|$ scenarios with each scenario s representing different outage lengths for the unreliable generator (see Section 2.3). Once the unreliable generator returns online it is assumed it does not go offline again. We believe this assumption is reasonable, as the model is solved over a relatively short timescale (no more than 48 hours). In addition, to include scenarios where the unreliable generator could return online and then go offline again would require an excess amount of scenarios, which would make the model computationally intractable. All other generators are assumed to be fully available for generation for each hourly timestep and scenario.

In this work, we consider $|L|$ different rolling horizon sequence paths. There are $|R|$ rolls in each sequence path l (described in detail in Section 2.3) with $|L|$ paths in total. The sequence paths differ by when the unreliable generator returns online. In the l^{th} sequence path, the unreliable generator returns after the l^{th} roll, i.e., the unreliable generator is offline for the l^{th} roll and before but available again for the $(l+1)^{\text{th}}$ roll. In total, the MCP is solved $|L|\times|R|$ times. When the last MCP of a given rolling horizon sequence is solved (i.e., the MCP for roll $r = |R|$), demand and storage parameters for auxiliary generation are reset to their original levels and the model moves forward to the first MCP of the next sequence path. The set of hourly timesteps, for roll r, remains the same for each sequence path l, i.e., $T(r) = \{r, r+1, .., H+r-1\}\ \forall l \in L$.

Including multiple rolling horizon sequence paths is in contrast to the analysis in Devine et al. (2016) where only one sequence paths is considered.

Tables 1 - 4 describe the sets, variables and parameters used in the model. The following conventions are used: lower-case Roman letters indicate indices or variables, upper-case Roman letters represent parameters (i.e., data, functions), while Greek letters indicate endogenous or exogenous prices. The variables in parentheses alongside each constraint in this section are the Lagrange multipliers associated with those constraints.

Table 1: Sets for MCP solved at roll r.

$r \in R$	Rolls. For each roll an MCP is solved.
$l \in L$	Path of rolling horizon sequences.
$i \in I$	Conventional generators.
$j \in J$	Passive load shedding consumers.
$k \in K$	Active load shedding consumers.
$t \in T(r) = \{r, .., r + H - 1\}$	Hourly timeteps for roll r where H is the time horizon.

Table 2: Primal variables for MCP solved at roll r. All units are MW.

$g_{i,t,s}$	Electricity generated by generator i in timestep t and scenario s.
$g^{\mathrm{FS}}_{i,t=r}$	Electricity generated by generator i for scenario-independent first timestep $(t = r)$.
$\Delta g^{\mathrm{P}}_{j,t,s}$	Load shed by passive load shedding consumer j in timestep t and scenario s.
$\Delta g^{\mathrm{P,FS}}_{k,t=r}$	Load shed by passive load shedding consumer j for scenario-independent first timestep $(t = r)$.
$\Delta g^{\mathrm{A}}_{k,t,s}$	Load shed by active load shedding consumer k in timestep t and scenario s.
$\Delta g^{\mathrm{A,FS}}_{j,t=r}$	Load shed by active load shedding consumer k for scenario-independent first timestep $(t = r)$.
$g^{\mathrm{APU}}_{k,t,s}$	Electricity generated by active load shedding consumer k in timestep t and scenario s.
$g^{\mathrm{APU,FS}}_{k,t=r}$	Electricity generated by active load shedding consumer k for scenario-independent first timestep $(t = r)$.

2.1. Stochastic mixed complementarity problem for roll r

We now describe details of the optimisation problems and KKT conditions for the generators, passive load shedding consumers and the active load shedding consumers. In addition, we provide the equations that connect the different optimisation problems, i.e., the market clearing conditions.

Table 3: Dual variables for MCP solved at roll r.

$\gamma_{t,s}$	System electricity price for timestep t and scenario s (€/MW h).
$\gamma_{t=r}$	System electricity price for scenario-independent first timestep ($t = r$) (€/MW h).
$\lambda^{\cdot}_{G\#}$	Lagrange multipliers associated with the constraints in generators' problems (unit depends on constraint).
$\lambda^{\cdot}_{P\#}$	Lagrange multipliers associated with the constraints in passive load shedding consumers' problems (unit depends on constraint).
$\lambda^{\cdot}_{A\#}$	Lagrange multipliers associated with the constraints in active load shedding consumers' problems (unit depends on constraint).

Table 4: Parameters for MCP solved at roll r.

PR_s	Probability associated with scenario s for MCP solved at roll r.
PR_l	Probability associated with path l.
H	Time horizon, i.e., total number of hourly timesteps for each MCP.
F_i	Marginal costs for generator i (€/MW h).
$G_i^{\max}$	Generator i's maximum capacity for each timestep (MW).
M_i^{G}	Binary parameter indicating whether generator i is a price-maker ($M_i^G = 1$) or price-taker ($M_i^{\mathrm{G}} = 0$).
$D_{j,t}^{\mathrm{P}}$	Reference demand for passive load shedding consumer j at timestep t (MW).
$\Delta g_j^{\mathrm{P,max}}$	Maximum amount of electricity passive load shedding consumer j can shed at timestep t (MW).
$C_{j,t}^{\mathrm{P}}(.)$	Passive load shedding consumer j's marginal cost function for time t (€/MW h).
$E_{j,t}^{\mathrm{P}}$	Intercept for passive load shedding consumer j's marginal cost function for time t (€/MW h).
$B_{j,t}^{\mathrm{P}}$	Slope for passive load shedding consumer j's marginal cost function for time t.
$D_{k,t}^{\mathrm{A}}$	Reference demand for active load shedding consumer k at timestep t (MW).
$\Delta g_k^{\mathrm{A,max}}$	Maximum amount of electricity active load shedding consumer k can shed at timestep t (MW).
$C_{k,t}^{\mathrm{A}}(.)$	Active load shedding consumer k's marginal cost function for time t (€/MW h).
$E_{k,t}^{\mathrm{A}}$	Intercept for active load shedding consumer k's marginal cost function for time t (€/MW h).
$B_{k,t}^{\mathrm{A}}$	Slope for active load shedding consumer k's marginal cost function for time t.
F_k^{APU}	Marginal costs of auxiliary generation for active load shedding consumer k (€/MW h).
$g_k^{\mathrm{APU,max}}$	Maximum amount of electricity active load shedding consumer k can generate at each timestep (MW).
V_k	Maximum capacity of electrical energy active load shedding consumer k can store [a] (MW h).

[a] In reality, while the energy in storage would be some sort of fuel (e.g., diesel or gas), V_k represents the total amount of electrical energy the stored fuel can generate.

2.1.1. Generator i's problem

Generator i maximises its expected profits (revenues less costs) by choosing the amount of electricity to generate. For timestep $t = r$, the generator makes scenario-independent first-stage decisions ($g^{\mathrm{FS}}_{i,t=r}$), representing actual decisions. For timesteps $t > r$ it makes second-stage hypothetical decisions, for each scenario s ($g_{i,t,s}$). The marginal price generators receive is $\gamma_{t=r}$ for timestep $t = r$ and $\gamma_{t,s}$ for timesteps $t > r$, $\forall s \in S$. The marginal costs of generation for generator i are F_i. Generator i's problem at roll r is

$$\max_{g^{\mathrm{FS}}_{i,t=r}, g_{i,t,s}} \quad (\gamma_{t=r} - F_i) g^{\mathrm{FS}}_{i,t=r} + \sum_{s \in S} PR_s \sum_{t>r}^{r+H-1} (\gamma_{t,s} - F_i) g_{i,t,s}, \tag{1a}$$

subject to

$$g^{\mathrm{FS}}_{i,t=r} \quad \leq \quad G^{\max}_i, \;\; (\lambda^{\mathrm{FS}}_{G1_{i,t=r}}) \tag{1b}$$

$$g_{i,t,s} \quad \leq \quad G^{\max}_i, \;\; \forall s, t > r, \;\; (\lambda_{G1_{i,t,s}}). \tag{1c}$$

Constraints (1b) and (1c) ensure that the amount of electricity generator i produces in each hourly timestep and scenario is capped. Generator i's primal decision variables are also constrained to be non-negative.

If generator i is assumed to be a price-taker, then its decision variables cannot affect the prices $\gamma_{t=r}$ and $\gamma_{t,s}$. Hence, these price variables are assumed exogenous to generator i's problem whilst still being variables of the overall MCP. If generator i is assumed to be a price-maker, then its decision variables can affect price. As a result we derive the following relationship between prices and generation for price-makers:

$$\begin{aligned} \gamma_{t=r} = & -\frac{1}{2}\Bigg(\frac{(\sum_{j\in J} B^{\mathrm{P}}_{j,t=r})(\sum_{k\in K} B^{\mathrm{A}}_{k,t=r})}{(\sum_{j\in J} B^{\mathrm{P}}_{j,t=r}) + (\sum_{k\in K} B^{\mathrm{A}}_{k,t=r})}\Bigg)\Bigg(\sum_{i\in I} g^{\mathrm{FS}}_{i,t=r} + \sum_{k\in K} g^{\mathrm{APU,FS}}_{k,t=r} \\ & - \sum_{j\in J}\big(D^{\mathrm{P}}_{j,t=r} + \frac{1}{2\sum_{j\in J} B^{\mathrm{P}}_{j,t=r}}(E^{\mathrm{P}}_{j,t=r} + \lambda^{\mathrm{FS}}_{P1_{j,t=r}} - \lambda^{\mathrm{FS}}_{P2_{j,t=r}})\big) \\ & - \sum_{k\in K}\big(D^{\mathrm{A}}_{k,t=r} + \frac{1}{2\sum_{k\in K} B^{\mathrm{A}}_{k,t=r}}(E^{\mathrm{A}}_{k,t=r} + \lambda^{\mathrm{FS}}_{A1_{k,t=r}} + \lambda^{\mathrm{FS}}_{A2_{k,t=r}} - \lambda^{\mathrm{FS}}_{A5_{k,t=r}})\big)\Bigg), \end{aligned} \tag{2}$$

and

$$\begin{aligned} \gamma_{t,s} = & -\frac{1}{2}\Bigg(\frac{(\sum_{j\in J} B^{\mathrm{P}}_{j,t})(\sum_{k\in K} B^{\mathrm{A}}_{k,t})}{(\sum_{j\in J} B^{\mathrm{P}}_{j,t}) + (\sum_{k\in K} B^{\mathrm{A}}_{k,t})}\Bigg)\Bigg(\sum_{i\in I} g_{i,t,s} + \sum_{k\in K} g^{\mathrm{APU}}_{k,t,s} \\ & - \sum_{j\in J}\big(D^{\mathrm{P}}_{j,t} + \frac{PR_s}{2\sum_{j\in J} B^{\mathrm{P}}_{j,t}}(\lambda_{P1_{j,t,s}} - \lambda_{P2_{j,t,s}} + \frac{E^{\mathrm{P}}_{j,t}}{PR_s})\big) \\ & - \sum_{k\in K}\big(D^{\mathrm{A}}_{k,t} + \frac{PR_s}{2\sum_{k\in K} B^{\mathrm{A}}_{k,t}}(\lambda_{A1_{k,t,s}} + \lambda_{A2_{k,t,s}} - \lambda_{A5_{k,t,s}} + \frac{E^{\mathrm{A}}_{k,t}}{PR_s})\big)\Bigg), \;\; \forall s, t > r. \end{aligned} \tag{3}$$

These relationships are then substituted into the objective function (1a). Equation (2) is determined by combining the market clearing condition (equation (16a) in section 2.1.7 below), with the KKT conditions that determine how passive and active load shedding change with the system price, see equations (9a) and (13a) resepectively. Similarly, equation (3) is determined by combining equations (16b), (9b) and (13b).

2.1.2. Analysis of generator i's KKT conditions

Generator i has four KKT conditions[1]:

$$0 \leq g^{\mathrm{FS}}_{i,t=r} \quad \perp \quad -\gamma_{t=r} - M^{\mathrm{G}}_i g^{\mathrm{FS}}_{i,t=r} \frac{\partial \gamma_{t=r}}{\partial g^{\mathrm{FS}}_{i,t=r}} + F_i + \lambda^{\mathrm{FS}}_{G1_{i,t=r}} \geq 0, \tag{4a}$$

$$0 \leq g_{i,t,s} \quad \perp \quad PR_s\big(- \gamma_{t,s} - M^G_i g_{i,t,s} \frac{\partial \gamma_{t,s}}{\partial g_{i,t,s}} + F_i\big) + \lambda_{G1_{i,t,s}} \geq 0, \ \ \forall s, t > r, \tag{4b}$$

$$0 \leq \lambda^{\mathrm{FS}}_{G1_{i,t=r}} \quad \perp \quad -g^{\mathrm{FS}}_{i,t=r} + G^{\max}_i \geq 0, \tag{4c}$$

$$0 \leq \lambda_{G1_{i,t,s}} \quad \perp \quad -g_{i,t,s} + G^{\max}_i \geq 0, \ \ \forall s, t > r, \tag{4d}$$

where M^G_i is a binary parameter indicating whether generator i is a price-maker ($M^G_i = 1$) or price-taker ($M^{\mathrm{G}}_i = 0$) and

$$\frac{\partial \gamma_{t=r}}{\partial g^{\mathrm{FS}}_{i,t=r}} = \frac{\partial \gamma_{t,s}}{\partial g_{i,t,s}} = -\frac{1}{2}\bigg(\frac{(\sum_{j \in J} B^{\mathrm{P}}_{j,t})(\sum_{k \in K} B^{\mathrm{A}}_{k,t})}{(\sum_{j \in J} B^{\mathrm{P}}_{j,t}) + (\sum_{k \in K} B^{\mathrm{A}}_{k,t})} \bigg), \ \ \forall s, t. \tag{5}$$

Equation (5) is obtained from equations (2) and (3). Conditions (4a) and (4b) denote the stationarity associated with generator i's generation decisions while conditions (4c) and (4d) describe its generation constraints. If generator i is a price-taker, then their problem is linear and hence convex. If generator i is a price-maker, then their problem is strictly convex, as $B^{\mathrm{P}}_{j,t} > 0$, $\forall j \in J, t \in T$ and $B^{\mathrm{A}}_{k,t} > 0$, $\forall k \in K, t \in T$.

If generator i is a price-taker, then its decision to generate in the scenario-independent timestep $t = r$ depends on the price $\gamma_{t=r}$:

1. If $\gamma_{t=r} < F_i$, then KKT condition (4a) can only be satisfied if, and only if, $g^{\mathrm{FS}}_{i,t=r} = 0$ as $\lambda^{\mathrm{FS}}_{G1_{i,t=r}}$ must be non-negative from condition (4c).
2. Likewise if $\gamma_{t=r} > F_i$, then KKT condition (4a) can only be satisfied if, and only if, $\lambda^{\mathrm{FS}}_{G1_{i,t=r}} > 0$, which forces $g^{\mathrm{FS}}_{i,t=r} = G^{\max}_i$ in order to satisfy condition (4c).
3. If $0 < g^{\mathrm{FS}}_{i,t=r} < G^{\max}_i$, then conditions (4a) and (4c) are only satisfied if $\gamma_{t=r} = F_i$. In this situation, generators with higher marginal costs will not generate and, consequently,

[1]The 'perb' notation $0 \leq a \perp b \geq 0$ is equivalent to $a \geq 0$, $b \geq 0$ and $a.b = 0$.

generator i is the marginal generator. The exact level of generation is determined by the supply/demand market clearing condition; see (16a) in Section 2.1.7.

Similar relationships can also be derived for $g_{i,t,s}$ and $\gamma_{t,s}$ using conditions (4b) and (4d). If generator i is a price-maker, then its decision to generate in the scenario-independent timestep $t = r$ also depends on the price $\gamma_{t=r}$:

1. If $\gamma_{t=r} < F_i$, then KKT conditions (4a) and (4c) can only be satisfied if, and only if, $g^{\text{FS}}_{i,t=r} = 0$.
2. If $\gamma_{t=r} \geq F_i$, then the same two conditions determine that generator i's optimal generation decision is

$$g^{\text{FS}}_{i,t=r} = \min\left[-\frac{\partial \gamma_{t=r}}{\partial g^{\text{FS}}_{i,t=r}}(\gamma_{t=r} - F_i), G^{\max}_i \right]. \tag{6}$$

   In contrast to price-taking generators, a price-making generator will not necessarily generate to its maximum capacity if $\gamma_{t=r} > F_i$.

As before, similar relationships can also be derived for $g_{i,t,s}$ and $\gamma_{t,s}$ using conditions (4b) and (4d). Note: Equation (5) shows that $\frac{\partial \gamma_{t=r}}{\partial g^{\text{FS}}_{i,t=r}} < 0$. It also tells us that a price-making generator can increase the marginal price, and hence their profits, by reducing their generation level. Further details on this result, and how it affects consumers, are described in Section 2.2.2.

The analysis in this section also shows, assuming without loss of generality that $F_i < F_{i+1}$ $\forall\ i \in I$, that generator i will only generate if, and only if, $g^{\text{FS}}_{\hat{\imath},t=r} = G^{\max}_{\hat{\imath}}$, $\forall\ \hat{\imath} \in \{I \mid \hat{\imath} < i \text{ and } M^G_{\hat{\imath}} = 0\}$. In Section 3, we assume that there is only one price-making generator and it is the generator with the highest marginal costs. Consequently, this analysis shows that the price-making generator in our case study will only generate if all other generators are generating at maximum capacity.

2.1.3. Passive load shedding consumer j's problem

Passive load shedding consumer j seeks to minimise the expected costs of meeting their demand plus the expected costs of any load shedding. Their demand consists of a reference demand less any load shedding. Reference demand ($D^{\text{P}}_{j,t}$) is the demand consumer group j would have at time t if they did not shed any of their load. For timestep $t = r$, passive load shedding consumers make scenario-independent first-stage decisions ($\Delta g^{\text{P,FS}}_{j,t=r}$), representing actual decisions. For timesteps $t > r$ they make second-stage hypothetical decisions, for each scenario s ($\Delta g^{\text{P}}_{j,t,s}$). Passive load shedding consumer j's problem at roll r is:

$$\begin{aligned}\min_{\Delta g^{\text{P,FS}}_{j,t=r},\Delta g^{\text{P}}_{j,t,s}} \quad & \gamma_{t=r}(D^{\text{P}}_{j,t=r} - \Delta g^{\text{P, FS}}_{j,t=r}) + \Delta g^{\text{P,FS}}_{j,t=r} C_{j,t=r}(\Delta g^{\text{P, FS}}_{j,t=r}) + \\ & \sum_{s\in S} PR_s \sum_{t>r}^{r+H-1} \left(\gamma_{t,s}(D^{\text{P}}_{j,t} - \Delta g^{\text{P}}_{j,t,s}) + \Delta g^{\text{P}}_{j,t,s} C^{\text{P}}_{j,t}(\Delta g^{\text{P}}_{j,t,s})\right),\end{aligned} \tag{7a}$$

subject to

$$\Delta g^{\text{P, FS}}_{j,t=r} \leq \Delta g^{\text{P,max}}_{j}, \;\; (\lambda^{\text{FS}}_{P1_{j,t=r}}), \tag{7b}$$

$$\Delta g^{\text{P}}_{j,t,s} \leq \Delta g^{\text{P,max}}_{j}, \;\; \forall s, t > r, \;\; (\lambda_{P1_{j,t,s}}), \tag{7c}$$

$$\Delta g^{\text{P,FS}}_{j,t=r} \geq 0, \;\; (\lambda^{\text{FS}}_{P2_{j,t=r}}), \tag{7d}$$

$$\Delta g^{\text{P}}_{j,t,s} \geq 0 \;\; \forall s, t > r, \;\; (\lambda_{P2_{j,t,s}}). \tag{7e}$$

Equations (7b) and (7c) constrain the amount of the load passive consumer's can shed for each hourly timestep and scenario while constraints (7d) and (7e) ensure that load-shedding is non-negative. We assume the marginal costs of load shedding for passive consumers j at time $t, \forall s \in S$ are linear:

$$C^{\text{P}}_{j,t}(x) = E^{\text{P}}_{j,t} + B^{\text{P}}_{j,t} x, \tag{8}$$

which means their total costs of load shedding are quadratic. As $B^{\text{P}}_{j,t} > 0$, $\forall t \in T$, consumer j's problem is strictly convex.

2.1.4. Analysis of passive load shedding consumer j's KKT conditions

Passive load shedding consumer j's KKT conditions are

$$-\gamma_{t=r} + E^{\text{P}}_{j,t=r} + 2B^{\text{P}}_{j,t=r}\Delta g^{\text{P,FS}}_{j,t=r} + \lambda^{\text{FS}}_{P1_{j,t=r}} - \lambda^{\text{FS}}_{P2_{j,t=r}} = 0, \;\; \forall \tag{9a}$$

$$PR_s\left(-\gamma_{t,s} + E^{\text{P}}_{j,t} + 2B^{\text{P}}_{j,t}\Delta g^{\text{P}}_{j,t,s}\right) + \lambda_{P1_{j,t,s}} - \lambda_{P2_{j,t,s}} = 0, \;\; \forall s, t > r, \tag{9b}$$

$$0 \leq \lambda^{\text{FS}}_{P1_{j,t=r}} \perp -\Delta g^{\text{P,FS}}_{j,t=r} + \Delta g^{\text{P,max}}_{j} \geq 0, \tag{9c}$$

$$0 \leq \lambda_{P1_{j,t,s}} \perp -\Delta g^{\text{P}}_{j,t,s} + \Delta g^{\text{P,max}}_{j} \geq 0, \;\; \forall s, t > r, \tag{9d}$$

$$0 \leq \lambda^{\text{FS}}_{P2_{j,t=r}} \perp \Delta g^{\text{P,FS}}_{j,t=r} \geq 0, \tag{9e}$$

$$0 \leq \lambda_{P2_{j,t,s}} \perp \Delta g^{\text{P}}_{j,t,s} \geq 0, \;\; \forall s, t > r. \tag{9f}$$

Conditions (9a) and (9b) denote the stationarity associated with consumer j's load shedding decisions while conditions (9c) - (9f) describe their load shedding constraints.

In a similar manner to generators passive load shedding consumer j's decision to shed their load in the scenario-independent timestep $t = r$ depends on the price $\gamma_{t=r}$:

1. If $\gamma_{t=r} < E^{\mathrm{P}}_{j,t=r}$, then KKT condition (9a) can only be satisfied if, and only if, $\lambda^{\mathrm{FS}}_{P2_{j,t=r}} > 0$, as $\lambda^{\mathrm{FS}}_{P1_{j,t=r}}$ must be non-negative from condition (9c). Consequently, in order to satisfy condition (9e), consumer j will not shed any load in this situation, i.e., $\Delta g^{\mathrm{P,FS}}_{j,t=r} = 0$.
2. If $\gamma_{t=r} \geq E^{\mathrm{P}}_{j,t=r}$, then the same KKT conditions determine that $\lambda^{\mathrm{FS}}_{P2_{j,t=r}} = 0$ and consumer j's optimal load shedding decision is

$$\Delta g^{\mathrm{P,FS}}_{j,t=r} = \min\left[\frac{\gamma_{t=r} - E^{\mathrm{P}}_{j,t=r}}{2B^{\mathrm{P}}_{j,t=r}}, \Delta g^{\mathrm{P,max}}_{j}\right]. \tag{10}$$

From equation (10) it is clear to see that $\frac{\partial \Delta g^{\mathrm{P,FS}}_{j,t=r}}{\partial \gamma_{t=r}} \geq 0$, telling us that as the price in the scenario-independent timestep $t = r$ increases so does the amount of electricity consumer j sheds. Similar relationships can also be derived for $\Delta g^{\mathrm{P,FS}}_{j,t=r}$ and $\gamma_{t,s}$ using conditions (9b), (9d) and (9f).

2.1.5. Active load shedding consumer k's problem

Active load shedding consumer k seeks to minimise the expected costs of meeting their demand plus the expected costs of any load shedding or auxiliary generation. Their demand consists of a reference demand less any load shedding. Reference demand ($D^{\mathrm{A}}_{k,t}$) is the demand consumer group k would have at time t if they did not shed any of their load or produce any auxiliary generation. For timestep $t = r$, active load shedding consumers make scenario-independent first-stage decisions on the amount of their load to shed ($\Delta g^{\mathrm{A,FS}}_{k,t=r}$) and on the level of auxiliary generation ($g^{\mathrm{APU,FS}}_{k,t=r}$), both representing actual decisions. For timesteps $t > r$ they make second-stage hypothetical decisions, for each scenario s ($\Delta g^{\mathrm{A}}_{k,t,s}$ and $g^{\mathrm{APU}}_{k,t,s}$). Active load shedding consumer k's problem at roll r is:

$$\begin{aligned}\min_{\substack{\Delta g^{\mathrm{A,FS}}_{k,t=r},\\ g^{\mathrm{APU,FS}}_{k,t=r},\\ \Delta g^{\mathrm{A}}_{k,t,s},\\ g^{\mathrm{APU}}_{k,t,s}}} \; & \gamma_{t=r}(D^{\mathrm{A}}_{k,t=r} - \Delta g^{\mathrm{A,FS}}_{k,t=r} - g^{\mathrm{APU,FS}}_{k,t=r}) + \Delta g^{\mathrm{A,FS}}_{k,t=r} C^{\mathrm{A}}_{k,t=r}(\Delta g^{\mathrm{A,FS}}_{k,t=r}) + g^{\mathrm{APU,\ FS}}_{k,t=r} F^{\mathrm{APU}}_{k} \\ & + \sum_{s\in S} PR_s \sum_{t>r}^{r+H-1} \left(\gamma_{t,s}(D^{\mathrm{A}}_{k,t} - \Delta g^{\mathrm{A}}_{k,t,s} - g^{\mathrm{APU}}_{k,t,s}) + \Delta g^{\mathrm{A}}_{k,t,s} C_{k,t}(\Delta g^{\mathrm{A}}_{k,t,s}) + g^{\mathrm{APU}}_{k,t,s} F^{\mathrm{APU}}_{k}\right),\end{aligned} \tag{11a}$$

subject to

$$\begin{aligned}
\Delta g^{\mathrm{A,FS}}_{k,t=r} &\le \Delta g^{\mathrm{A,max}}_{k}, \ (\lambda^{\mathrm{FS}}_{A1_{k,t=r}}), && (11b)\\
\Delta g^{\mathrm{A}}_{k,t,s} &\le \Delta g^{\mathrm{A,max}}_{k}, \ \forall s, t>r, \ (\lambda_{A1_{k,t,s}}), && (11c)\\
\Delta g^{\mathrm{A,FS}}_{k,t=r} + g^{\mathrm{APU,FS}}_{k,t=r} &\le D^{\mathrm{A}}_{k,t=r}, \ (\lambda^{\mathrm{FS}}_{A2_{k,t=r}}), && (11d)\\
\Delta g^{\mathrm{A}}_{k,t,s} + g^{\mathrm{APU}}_{k,t,s} &\le D^{\mathrm{A}}_{k,t}, \ \forall s, t>r, \ (\lambda_{A2_{k,t,s}}), && (11e)\\
g^{\mathrm{APU,FS}}_{k,t=r} &\le g^{\mathrm{APU,max}}_{k}, \ (\lambda^{FS}_{A3_{k,t=r}}), && (11f)\\
g^{\mathrm{APU}}_{k,t,s} &\le g^{\mathrm{APU,max}}_{k} \ \forall s, t>r, \ (\lambda_{A3_{k,t,s}}), && (11g)\\
g^{\mathrm{APU,FS}}_{k,t=r} + \sum_{t>r}^{r+H-1} g^{\mathrm{APU}}_{k,t,s} &\le V_k, \ \forall s, \ (\lambda_{A4_{k,s}}), && (11h)\\
\Delta g^{\mathrm{A,FS}}_{k,t=r} &\ge 0, \ (\lambda^{\mathrm{FS}}_{A5_{k,t=r}}), && (11i)\\
\Delta g^{\mathrm{A}}_{k,t,s} &\ge 0, \ \forall s, t>r, \ (\lambda_{A5_{k,t,s}}). && (11j)
\end{aligned}$$

Equations (11b) and (11c) constrain the amount of the load active consumer's can shed for each hourly timestep and scenario while equations (11d) and (11e) ensure their auxiliary generation is capped at their demand, i.e., they cannot provide auxiliary generation to meet demand other than their own. Equations (11f) and (11g) constrains the amount of auxiliary generation they can produce at each timestep and scenario while equation (11h) constrains the total amount of auxiliary generation they can produce for scenario s over the entire time horizon of roll r. Note: the variable $g^{\mathrm{APU,FS}}_{k,t=r}$ appears in every one of the $|S|$ instances of constraint (11h) reflecting how this variable represents an actual decision that must hold for each possible future scenario. The parameter V_k represents the maximum capacity of electrical energy that active load shedding consumer k can store (e.g., through diesel tanks) and is updated after the MCP at each roll r is solved; see Section 2.3.1. Constraints (11i) and (11j) ensure active load shedding consumer k's load shedding decisions are non-negative.

Active load shedding consumer k's marginal costs for auxiliary generation are F^{APU}_{k} while we assume their marginal costs for load shedding at time t are linear:

$$C_{k,t}(x) = E^{\mathrm{A}}_{k,t} + B^{\mathrm{A}}_{k,t}x, \tag{12}$$

which means their total costs of load shedding are quadratic. As $B^{\mathrm{A}}_{k,t} > 0$, $\forall t \in T$, consumer k's problem is strictly convex.

2.1.6. Analysis of active load shedding consumer k's KKT conditions

Active load shedding consumer k's KKT conditions are

$$
\begin{aligned}
-\gamma_{t=r} + E^{\mathrm{A}}_{k,t=r} + 2B^{\mathrm{A}}_{k,t=r}\Delta g^{\mathrm{A,FS}}_{k,t=r} + \lambda^{\mathrm{FS}}_{A1_{k,t=r}} + \lambda^{\mathrm{FS}}_{A2_{k,t=r}} - \lambda^{\mathrm{FS}}_{A5_{k,t=r}} &= 0, && \text{(13a)}\\
PR_s\big(-\gamma_{t,s} + E^{\mathrm{A}}_{k} + 2B^{\mathrm{A}}_{k}\Delta g^{\mathrm{A}}_{k,t,s}\big) + \lambda_{A1_{k,t,s}} + \lambda_{A2_{k,t,s}} - \lambda_{A5_{k,t,s}} &= 0, \ \forall s, t > r, && \text{(13b)}\\
0 \le g^{\mathrm{APU,FS}}_{k,t=r} \perp -\gamma_{t=r} + F^{\mathrm{APU}}_{k} + \lambda^{\mathrm{FS}}_{A2_{k,t=r}} + \lambda^{\mathrm{FS}}_{A3_{k,t=r}} + \sum_{s\in S}\lambda_{A4_{k,s}} &\ge 0, && \text{(13c)}\\
0 \le g^{\mathrm{APU}}_{k,t,s} \perp -PR_s\big(\gamma_{t,s} - F^{\mathrm{APU}}_{k}\big) + \lambda_{A2_{k,t,s}} + \lambda_{A3_{k,t,s}} + \lambda_{A4_{k,s}} &\ge 0, \ \forall s, t > r, && \text{(13d)}\\
0 \le \lambda^{\mathrm{FS}}_{A1_{k,t=r}} \perp -\Delta g^{\mathrm{A,FS}}_{k,t=r} + \Delta g^{\mathrm{A,max}}_{k} &\ge 0, && \text{(13e)}\\
0 \le \lambda_{A1_{k,t,s}} \perp -\Delta g^{\mathrm{A}}_{k,t,s} + \Delta g^{\mathrm{A,max}}_{k} &\ge 0, \ \forall s, t > r, && \text{(13f)}\\
0 \le \lambda^{\mathrm{FS}}_{A2_{k,t=r}} \perp -\Delta g^{\mathrm{A}}_{k,t=r} - g^{\mathrm{APU,FS}}_{k,t=r} + D^{\mathrm{A}}_{k,t=r} &\ge 0, && \text{(13g)}\\
0 \le \lambda_{A2_{k,t,s}} \perp -\Delta g^{\mathrm{A}}_{k,t,s} - g^{\mathrm{APU}}_{k,t,s} + D^{\mathrm{A}}_{k,t} &\ge 0, \ \forall s, t > r, && \text{(13h)}\\
0 \le \lambda^{\mathrm{FS}}_{A3_{k,t=r}} \perp -g^{\mathrm{APU,FS}}_{k,t=r} + g^{\mathrm{APU,max}}_{k} &\ge 0, && \text{(13i)}\\
0 \le \lambda_{A3_{k,t,s}} \perp -g^{\mathrm{APU}}_{k,t,s} + g^{\mathrm{APU,max}}_{k} &\ge 0, \ \forall s, t > r, && \text{(13j)}\\
0 \le \lambda_{A4_{k,s}} \perp -g^{\mathrm{APU,FS}}_{k,t=r} - \sum_{t>r}^{r+H-1} g^{\mathrm{APU}}_{k,t,s} + V_k &\ge 0, \ \forall s, && \text{(13k)}\\
0 \le \lambda^{\mathrm{FS}}_{A5_{k,t=r}} \perp \Delta g^{\mathrm{A,FS}}_{k,t=r} &\ge 0, && \text{(13l)}\\
0 \le \lambda_{A5_{k,t,s}} \perp \Delta g^{\mathrm{A}}_{k,t,s} &\ge 0, \ \forall s, t > r && \text{(13m)}
\end{aligned}
$$

Conditions (13a) and (13b) denote the stationarity associated with consumer k's load shedding decisions while conditions (13c) and (13d) denote the stationarity associated their auxiliary generation decisions. Conditions (13e) and (13f) provide constraints for the amount of load they can shed while conditions (13g) - (13k) describe the constraints associated with their auxiliary power unit. Conditions (13l) and (13m) ensure consumer k's load shedding decision must be non-negative.

Active load shedding consumer k's decision to shed their load in the scenario-independent timestep $t = r$ is similar to those of passive consumers and depends on the price $\gamma_{t=r}$:

1. If $\gamma_{t=r} < E^{\mathrm{A}}_{k,t=r}$, then KKT condition (13a) can only be satisfied if, and only if, $\lambda^{\mathrm{FS}}_{A5_{k,t=r}} > 0$ as both $\lambda^{\mathrm{FS}}_{A1_{k,t=r}}$ and $\lambda^{\mathrm{FS}}_{A2_{k,t=r}}$ must be non-negative from conditions (13e) and (13g). Consequently, in order to satisfy condition (13l), consumer k will not shed any load in this situation, i.e., $\Delta g^{\mathrm{A,FS}}_{k,t=r} = 0$.
2. If $\gamma_{t=r} \ge E^{\mathrm{A}}_{k,t=r}$, KKT condition (13a) can only be satisfied if, and only if,

$$
\Delta g^{\mathrm{A,FS}}_{k,t=r} = \frac{\gamma_{t=r} - E^{\mathrm{A}}_{k,t=r}}{2B^{\mathrm{A}}_{k,t=r}}, \tag{14}
$$

or if either $\lambda^{\text{FS}}_{A1_{k,t=r}} > 0$ or $\lambda^{\text{FS}}_{A2_{k,t=r}} > 0$ which require $\Delta g^{\text{A,FS}}_{k,t=r} = \Delta g^{\text{A,max}}_{k}$ or $\Delta g^{\text{A,FS}}_{k,t=r} = D^{\text{A}} - g^{\text{APU}}_{k,t,s}$ in order to satisfy condition (13e) or (13g) respectively. In each of these, $\Delta g^{\text{A,FS}}_{k,t=r} > 0$ which requires $\lambda^{\text{FS}}_{A5_{k,t=r}} = 0$ to satisfy condition (13l). Consequently, consumer k's optimal load shedding decision is

$$\Delta g^{\text{A,FS}}_{k,t=r} = \min\left[\frac{\gamma_{t=r} - E^{\text{A}}_{k,t=r}}{2B^{\text{A}}_{k,t=r}}, \Delta g^{\text{A,max}}_{k}, D^{\text{A}}_{k,t} - g^{\text{APU}}_{k,t,s}\right]. \tag{15}$$

Equation (15) shows that $\frac{\partial \Delta g^{\text{A,FS}}_{k,t=r}}{\partial \gamma_{t=r}} \geq 0$, telling us that as the price in the scenario-independent timestep $t = r$ increases so does the amount of electricity consumer k sheds, up until either condition (13e) or (13g) become binding. Similar relationships can also be derived for $\Delta g^{\text{A}}_{k,t,s}$ and $\gamma_{t,s}$ using conditions (13b) (13f), (13h) and (13m).

Active load shedding consumer k's decision to use their auxiliary generation in the scenario-independent timestep $t = r$ also depends on the price $\gamma_{t=r}$:

1. If $\gamma_{t=r} < F^{\text{APU}}_{k}$, then KKT condition (13c) can only be satisfied if, and only if, $g^{\text{APU,FS}}_{k,t=r} = 0$, as both $\lambda^{\text{FS}}_{A2_{k,t=r}}$ and $\lambda^{\text{FS}}_{A3_{k,t=r}}$ in addition to $\lambda_{A4_{k,s}}$ $\forall$ $s \in S$ must be non-negative from conditions (13g), (13i) and (13k).
2. $\gamma_{t=r} > F^{\text{APU}}_{k}$, then KKT condition (13c) can only be satisfied if, $\lambda^{\text{FS}}_{A2_{k,t=r}} > 0$, $\lambda^{\text{FS}}_{A2_{k,t=r}} > 0$ or if $\sum_{s \in S} \lambda_{A4_{k,s}} > 0$:
   (a) If $\lambda^{\text{FS}}_{A2_{k,t=r}} > 0$, then condition (13g) requires $g^{\text{APU,FS}}_{k,t=r} = D^{\text{A}}_{k,t} - \Delta g^{\text{A,FS}}_{k,t=r}$.
   (b) If $\lambda^{\text{FS}}_{A3_{k,t=r}} > 0$, then condition (13i) requires $g^{\text{APU,FS}}_{k,t=r} = g^{\text{APU,max}}_{k}$.
   (c) If $\sum_{s \in S} \lambda_{A4_{k,s}} > 0$, then $\exists$ $s' \in S$ such that $\lambda_{A4_{k,s'}} > 0$ which, by condition (13k), requires $V_k = g^{\text{APU,FS}}_{k,t=r} + \sum^{r+H-1}_{t>r} g^{\text{APU}}_{k,t,s'}$. This is tells us that, in scenario s', consumer k uses all the energy in their storage tank. In this case, $g^{\text{APU,FS}}_{k,t=r}$ may equal zero despite the system price being greater than their marginal costs. This is because, in future scenario s', the difference between $\gamma_{t,s'}$ and F^{APU}_{k} may be large, which requires a large value for $\lambda_{A4_{k,s'}}$ for condition (13d) to be satisfied. A value for $\lambda_{A4_{k,s'}}$ larger than $\gamma_{t=r} - F^{\text{APU}}_{k}$ requires $g^{\text{APU,FS}}_{k,t=r} = 0$ for condition (13c) to be satisfied.
3. If $\gamma_{t=r} = F^{\text{APU}}_{k}$, then consumer k is indifferent to using auxiliary generation or not. Their decision will be determined by the difference between $\gamma_{t,s}$ and $g^{\text{APU}}_{k,t,s}$, as explained above, or if auxiliary generation is required to meet the supply/demand balance of the overall system; see equation (16a) in Section 2.1.7.

Both consumer k's load shedding and auxiliary generation decisions in timestep $t = r$ depend on the price $\gamma_{t=r}$. The decision of which of these measures they choose first depends on the respective costs. If $F_k^{\text{APU}} < E_{k,t=r}^{\text{A}}$ then consumer k would choose auxiliary generation before load shedding and would continue to do so until either condition (13g) or (13i) become binding. If $F_k^{\text{APU}} > E_{k,t=r}^{\text{A}}$ then they would choose load shedding first and would continue to do so until $E_{k,t=r}^{\text{A}} + 2B_{k,t=r}^{\text{A}} \Delta g_{k,t=r}^{\text{A,FS}} \geq F_k^{\text{APU}}$ or either condition (13e) or (13g) become binding, at which point consumer k could also begin to use auxiliary generation to meet their demand. Similar relationships can also be derived for timesteps $t > r$ using F_k^{APU}, $E_{k,t}^{\text{A}}$ and conditions (13f), (13h) and (13j).

In Section 3.1.2, a test case where active load shedding consumers are able to provide auxiliary generation greater than their own demand is considered, i.e., they are able to meet the demands of other consumers. In this case, conditions (13g) and (13h) are removed from the KKT conditions. Moreover, $\lambda_{A2_{k,t=r}}^{\text{FS}}$ and $\lambda_{A2_{k,t,s}}$ and are also removed from conditions (13a) - (13d). Removing these constraints and Lagrange multipliers means that both types of consumers may need to shed less of their load in order to satisfy the aforementioned supply/demand market clearing conditions, equations (16a) and (16b). As this section and Section 2.1.4 describe, less load shedding puts downward pressure on prices and hence reduces consumer costs.

2.1.7. Market clearing condition

The $|I| + |J| + |K|$ optimisation problems are connected via the following market clearing conditions:

$$\sum_{i \in I} g_{i,t=r}^{\text{FS}} + \sum_{k \in K} g_{k,t=r}^{\text{APU,FS}} = \sum_{j \in J} \left(D_{j,t=r}^{\text{P}} - \Delta g_{j,t=r}^{\text{P,FS}} \right) + \sum_{k \in K} \left(D_{k,t=r}^{\text{A}} - \Delta g_{k,t=r}^{\text{A,FS}} \right), (\gamma_{t=r}), \tag{16a}$$

$$\sum_{i \in I} g_{i,t,s} + \sum_{k} g_{k,t,s}^{\text{APU}} = \sum_{j \in J} \left(D_{j,t}^{P} - \Delta g_{j,t,s}^{P} \right) + \sum_{k \in K} \left(D_{k,t}^{\text{A}} - \Delta g_{k,t,s}^{\text{A}} \right), \forall s, t > r, (\gamma_{t,s}), \tag{16b}$$

which state that the total amount of electricity produced from conventional generation and auxiliary generation must equal the sum of the passive and active load shedding demand (reference demand less load shedding). The prices $\gamma_{t=r}$ and $\gamma_{t,s}$ are the free Lagrange multiplier associated with conditions (16a) and (16b) respectively.

2.1.8. The complete MCP for roll r

As each of the optimisation problems are convex, the KKT conditions are both necessary and sufficient for optimality for each type of player Gabriel et al. (2012). The MCP for roll r consists of conditions (4), (9), (13) and the market clearing conditions (16).

2.2. *Interactions between the players' optimal decisions*

In this section we examine the interactions between the optimal decisions of each player. Firstly, between price-taking generators and consumers, secondly, between between price-making generators and consumers and finally, between the two different types of consumers. For a discussion of the interactions between different generators, please see section 2.1.2 above.

2.2.1. *Price-taking generators and consumers*

Both price-taking generators' and consumers' optimal decisions depend on their marginal costs:

1. If generators have sufficient capacity to meet all reference demands:
   (a) If $\max_i \{F_i\} < \min\,[E^{\mathrm{P}}_{j,t}, E^{\mathrm{A}}_{k,t}, F^{\mathrm{APU}}_k]$, then it will be too expensive for consumers to shed their load or auxiliary generate and generators will meet the reference demand for both types of consumers. The prices $\gamma_{t=r}$ and $\gamma_{t,s}$ will be set at the marginal costs of the marginal generator; see Section 2.1.2.
   (b) If $\exists\; i' \in I$ such that $F_{i'} > \min\,[E^{\mathrm{P}}_{j,t}, E^{\mathrm{A}}_{k,t}, F^{\mathrm{APU}}_k]$, then generator i' may not generate to their maximum capacity and load shedding and/or auxiliary generation would be utilised to satisfy market clearing conditions (16).
2. If conventional generators' capacity is not sufficient to meet all reference demands, then either load shedding and/or auxiliary generation is required to market clearing conditions (16). This, as Sections 2.1.4 and 2.1.6 describe, requires

$$\gamma_{\cdot} \geq [E^{\mathrm{P}}_{j,t}, E^{\mathrm{A}}_{k,t}, F^{\mathrm{APU}}_k], \tag{17}$$

   with the exact prices dependent on the levels of load shedding and/or auxiliary generation required.

2.2.2. *Price-making generators and consumers*

If there is at least one price-making generator, then that generator will reduce its generation (compared with a situation where it is a price-taker and generating); see Section 2.1.2. As a result, extra load shedding and/or auxiliary generation is required to meet market clearing conditions (16). This, as Sections 2.1.4 and 2.1.6 describe, leads to increased prices. Consequently, the presence of a price-making generator increases consumer costs and generators' profits. This result can be seen numerically in the test cases studied in Section 3.2.1 where test cases with and without a price-making generator are considered.

For timestep $t = r$, the interaction between a price-making generator's optimal generation decision and passive consumer j's optimal load shedding can be determined by combining equations (6) and (10):

$$g^{\mathrm{FS}}_{i,t=r} = \max\left[\min\left[-\frac{\partial \gamma_{t=r}}{\partial g^{\mathrm{FS}}_{i,t=r}}(E^{\mathrm{P}}_{j,t=r} + 2B^{\mathrm{P}}_{j,t=r}\Delta g^{\mathrm{P,FS}}_{j,t=r} - F_i), G^{\max}_i\right], 0\right], \tag{18}$$

while the interaction between a price-making generator's optimal generation decision and active load shedding consumer k's decisions can be determined by combining equations (6) and (15):

$$g^{\mathrm{FS}}_{i,t=r} = \max\left[\min\left[-\frac{\partial \gamma_{t=r}}{\partial g^{\mathrm{FS}}_{i,t=r}}(E^{\mathrm{A}}_{k,t=r} + 2B^{\mathrm{A}}_{j,t=r}\Delta g^{\mathrm{P,FS}}_{k,t=r} - F_i), -\frac{\partial \gamma_{t=r}}{\partial g^{\mathrm{FS}}_{i,t=r}}(F^{\mathrm{APU}}_k - F_i), G^{\max}_i\right], 0\right], \tag{19}$$

where $-\frac{\partial \gamma_{t=r}}{\partial g^{\mathrm{FS}}_{i,t=r}}(F^{\mathrm{APU}}_k - F_i)$ represents a situation where active consumer k utilises their auxiliary generation but does not shed their load. Similar interactions can also be found for timesteps $t > r$.

2.2.3. Interactions between both types of consumers

Sections 2.1.4 and 2.1.6 show that both types of consumers' optimal decisions in timestep $t = r$ depend on the system price $\gamma_{t=r}$. If $E^{\mathrm{P}}_{j,t=r} < E^{\mathrm{A}}_{k,t=r}$, then passive consumer j will shed their load before active consumer k and vice-versa. Similarly, if $E^{\mathrm{P}}_{j,t=r} < F^{\mathrm{APU}}_k$ then passive consumer j will shed their load before active consumer k utilises their auxiliary generation and vice-versa. If one consumer group's load shedding is constrained to be zero in timestep $t = r$ then other consumers may need to increase their load shedding and/or auxiliary generation, to ensure market clearing condition (16a) is satisfied. This, as sections 2.1.4 and 2.1.6 describe, leads to increased values for $\gamma_{t=r}$ and, consequently, increased consumer costs. Similar conclusions can be drawn for timesteps $t > r$. Sections 3.1.2 and 3.2.1 considers different numerical test cases where consumer load shedding is constrained to be zero for some or all timesteps.

In the absence of constraints, equations (10) and (15) show that optimal load shedding decision for consumer group j and k in timestep $t > r$ are

$$\Delta g^{\mathrm{P,FS}}_{j,t=r} = \frac{\gamma_{t=r} - E^{\mathrm{P}}_{j,t=r}}{2B^{\mathrm{P}}_{j,t=r}}, \tag{20}$$

and

$$\Delta g^{\mathrm{A,FS}}_{k,t=r} = \frac{\gamma_{t=r} - E^{\mathrm{A}}_{k,t=r}}{2B^{\mathrm{A}}_{k,t=r}}, \tag{21}$$

respectively. Combining equations (20) and (21) gives

$$\Delta g^{\mathrm{P,FS}}_{j,t=r} = \frac{E^{\mathrm{A}}_{k,t=r} + 2B^{\mathrm{A}}_{k,t=r}\Delta g^{\mathrm{A,FS}}_{k,t=r} - E^{\mathrm{P}}_{j,t=r}}{2B^{\mathrm{P}}_{j,t=r}}, \tag{22}$$

or

$$\Delta g^{\mathrm{A,FS}}_{k,t=r} = \frac{E^{\mathrm{P}}_{j,t=r} + 2B^{\mathrm{P}}_{j,t=r}\Delta g^{\mathrm{P,FS}}_{j,t=r} - E^{\mathrm{A}}_{k,t=r}}{2B^{\mathrm{A}}_{k,t=r}}, \tag{23}$$

and shows how both types of consumers optimal load shedding decisions interact with each other before any load shedding constraints become binding. As before, similar relationship can be found for timesteps $t > r$.

2.3. Scenarios and sequence paths

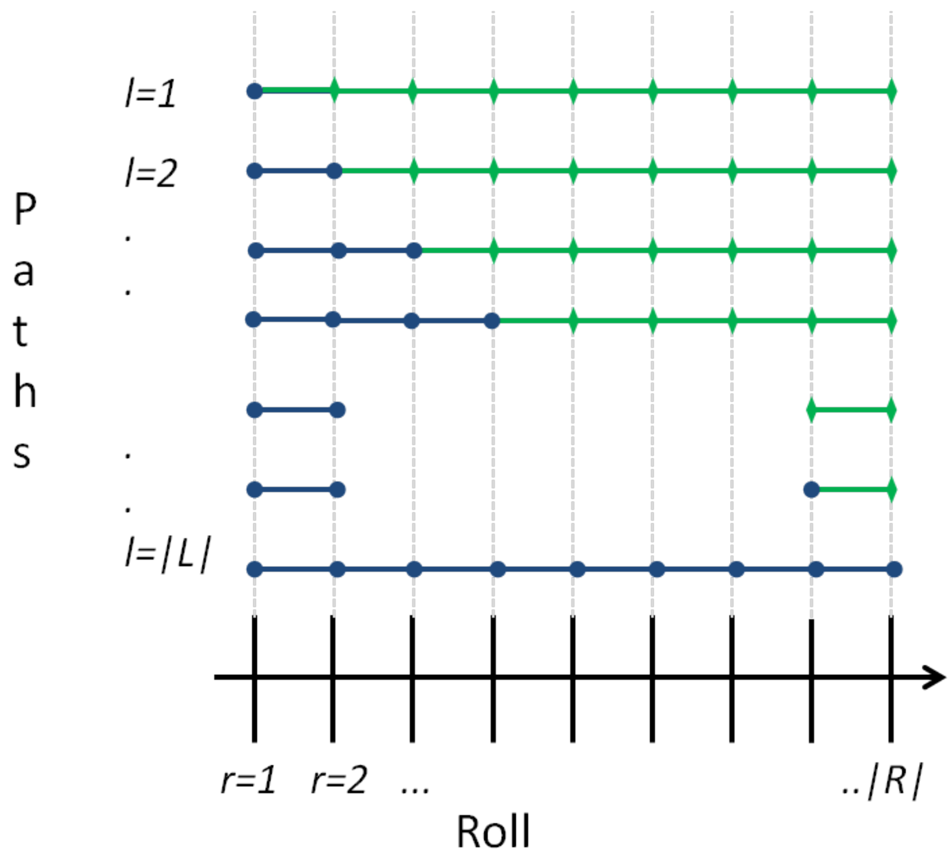


Figure 1: Schematic of rolling horizon sequence paths. For circular nodes, the MCP is solved with multiple scenarios (see Figure 2) while for diamond shaped nodes, the MCP is solved with only one sceanrio.

As mentioned previously, the overall model involves solving the stochastic MCP for $|R|$ rolls along each possible sequence path l. Figure 1 describes these paths and shows that they differ by when the unreliable generator returns online. Each path l represents different sequences of rolls with each node presenting a different instance of when the MCP is solved. Circular nodes in Figure 1 model a situation where the unreliable generator is initially offline and the other players are unsure of when it will return. Consequently, the MCPs solved at these rolls contain multiple scenarios, representing the uncertain information the players have when making their decisions. Diamond shaped nodes reflect a situation where the unreliable generator has returned online and all players assume it will not go on outage again. As a result, MCPs solved at these rolls are deterministic, i.e., $S = \{1\}$ and $PR_{s=1} = 1$. Each path l represents different time points when the unreliable returns online. For path l the unreliable generator is assumed initially offline (with uncertainty about its return) for the MCPs solved at rolls $r \leq l$ while, for rolls $r > l$, the unreliable generator is assumed to have returned online for all timesteps and scenarios.

Figure 2 describes the $|S|$ scenarios for the MCPs associated with circular nodes in Figure 1. It reflects the uncertain information the other players have about the return of the unreliable generator when making decisions at these rolls. Circular nodes in Figure 2 represent timesteps when the unreliable generator is offline while diamond shaped nodes represent timesteps where this generator is online. For scenario s, the unreliable generator is assumed offline for $t-r+1 \leq s$ and online for $t-r+1 > s$. Consequently, these scenarios have the same probabilities as the paths, i.e., $PR_s = PR_l$, $(\{\forall s \in S,\ \forall l \in L\ |s = l\})$. For the first hourly timestep $(t = r)$ for each roll described with circular nodes in Figure 2, the unreliable generator is assumed offline in every scenario. Hence the decisions made at this timestep are scenario-independent for each player.

2.3.1. Update rules

When a MCP is solved, the model moves forward to the next roll on the path and the following parameters are updated before the next roll:

$$
\begin{aligned}
D^{\mathrm{P}}_{j,t} &\rightarrow D^{\mathrm{P}}_{j,t+1}\ \forall j,t, && (24a)\\
D^{\mathrm{A}}_{k,t} &\rightarrow D^{\mathrm{A}}_{k,t+1}\ \forall k,t, && (24b)\\
E^{\mathrm{P}}_{j,t} &\rightarrow E^{\mathrm{P}}_{j,t+1}\ \forall j,t, && (24c)\\
E^{\mathrm{A}}_{k,t} &\rightarrow E^{\mathrm{A}}_{k,t+1}\ \forall k,t, && (24d)\\
B^{\mathrm{P}}_{j,t} &\rightarrow B^{\mathrm{P}}_{j,t+1}\ \forall j,t, && (24e)\\
B^{\mathrm{A}}_{k,t} &\rightarrow B^{\mathrm{A}}_{k,t+1}\ \forall k,t, && (24f)\\
V_k &\rightarrow V_k - g^{\mathrm{APU,FS}}_{k,t=r}\ \forall k. && (24g)
\end{aligned}
$$

Equations (24a) and (24b) reflect what happens in real-world electricity markets: when a new time period begins (i.e., when the model moves to a new roll), those in the market have updated information regarding demand. Similarly, Equations (24c) - (24f) reflect how the costs of load

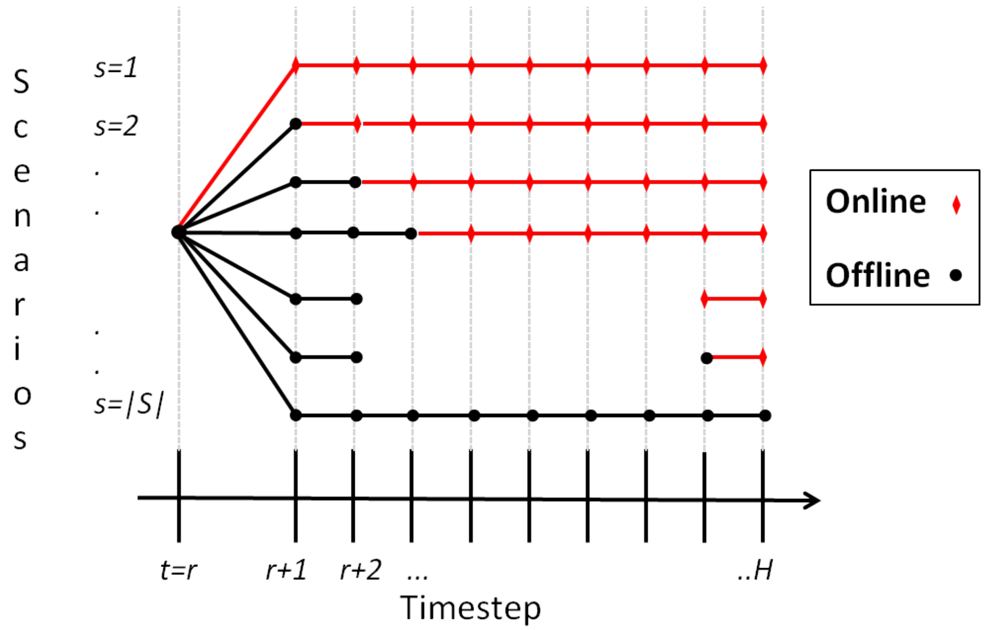


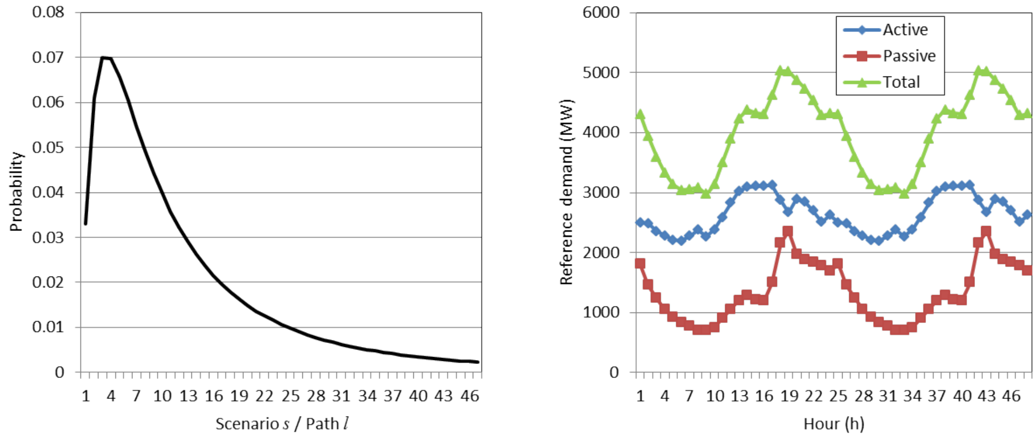
Figure 2: Schematic of scenarios associated with nodes the MCP is solved with the unreliable generator is initially assumed offline and its return is uncertain.

shedding for the consumers changes when load shedding decisions are being made for a new time period. As $g^{\mathrm{APU,FS}}_{k,t=r}$ represents the actual amount of auxiliary generation (in contrast to the hypothetical $g^{\mathrm{APU}}_{k,t,s}$ decisions), equation (24g) reflects how the amount of electrical energy in storage is reduced after it is used for auxiliary generation.

After the last roll of a sequence path is solved, these parameters are reset to their original values (described in Section 3.1) for roll $r = 1$ of the next sequence path.

3. Case Study and Results

In this section, we present the case study assumptions (section 3.1), followed by the results for these assumptions (section 3.2). The assumptions concerning the load shedding costs constitute a crucial part of the input data for the analysis in this paper. With respect to this aspect, our analysis builds on Leahy & Tol (2011) who estimate the value of lost load (VOLL) for different types of consumers in Ireland (see section 3.1.1 below for further details on how their results are used for our analysis).



(a) Probabilities associated with each path l and scenario s (only for MCPs at nodes in Figure 1 where the unreliable generator is assumed offline).

(b) Reference demand for passive ($D^{\mathrm{P}}_{j=1,t}$) and active ($D^{\mathrm{A}}_{k=1,t}$) consumers including total ($D^{\mathrm{P}}_{j=1,t} + D^{\mathrm{A}}_{k=1,t}$).

Figure 3: Input parameters.

3.1. Model input assumptions for the case study

3.1.1. Generation and demand data

In this section, numerical examples are presented. We firstly describe the parameters for the main policy test case, the *Base Case*. This policy assumes that the regulator/system operator maintains a non-static load-shedding strategy which means that, at each hourly timestep, the ability of both consumers to shed their load is not constrained nor is their demand prioritised over the other player's demand. In addition, the *Base Case* policy assumes that active consumers have the ability to use their auxiliary generation to meet their demand. The *Base Case* is formulated with $|I| = 5$ generators, $|J| = 1$ passive load shedding consumers, $|K| = 1$ active load shedding consumers and $|L| = 48$ rolling horizon sequence paths. Each sequence path has $|R| = 48$ rolls. For the MCPs solved when the unreliable generator is initially assumed offline, there are $|S| = 48$ scenarios with the probability associated with these (PR_s) described in Figure 3a and Table A.7. These probabilities are normalised such that $\sum_s PR_s = 1$ and are initially taken form a log-normal distribution with values of 2.233 and 1 for the location and scale parameters respectively. These parameters are chosen such that each player assumes that the expected outage length of the unreliable generator is 12 hours. Note: these probabilities are also the same as those associated with the $|L| = 48$ rolling horizon sequence paths (PR_l).

This expectation holds for each MCP where the unreliable generator is offline. Consequently, if the first timestep of roll r represents 01:00, then each player would expect the unreliable generator to return online at 13:00. Furthermore, in this instance, the first timestep of roll $r+1$ would represent 02:00 and each player would expect the unreliable generator to return online at 14:00. In this sense, the expected return time of the unreliable generator updates from roll-to-roll, despite the probabilities of the scenarios remaining constant. In addition, a sensitivity analysis with log-normal distributions that gave expected outage lengths of 6 and 24 hours were also conducted. These sensitivities did not qualitatively affect the results or conclusions to be discussed later.

For MCPs solved once the unreliable generator has returned online there is only $|S| = 1$ scenario for all timesteps. For each MCP solved there are $H = 24$ hourly timesteps, i.e., each player has a look-ahead foresight of 24 hours. Hence, the set of hourly timesteps are $t = \{1, 2, ..., 24\}$ for rolls $r = 1$, $t = \{2, 3, ..., 25\}$ for rolls $r = 2$ and $t = \{48, 49, ..., 72\}$ for rolls $r = |R| = 48$.

The five different thermal generators represent different technologies with marginal costs and maximum capacity values described in Table 5. The baseload generator ($i = 1$) is assumed to be a modern hard coal power plant with high efficiency. Midload generators are assumed to be combined cycle gas turbines (CCGTs) with high ($i = 2$) and moderate efficiency ($i = 3$) respectively. Peak generator $i = 4$ is assumed to be an open cycle gas turbine (OCGT) whereas peak generator $i = 5$ is assumed to be oil-fired. The maximum capacity values by technology type described in Table 5 are based on EirGrid (2016). Note, however, that we focus on the dispatchable, thermal capacities only here. The marginal costs in Table 5 were calculated using fuel and carbon prices of the corresponding futures markets for 2017 as obtained from the European Energy Exchange (www.eex.com). For this purpose, we used the average market results of the futures markets for 2017 as traded during 2016.

The unreliable generator, that is assumed initially offline for some rolls, is generator $i = 4$ while all other generators are assumed online and available for all rolls and timesteps. All generators are assumed to be price-takers except for generator $i = 5$, i.e., $M_i^{\mathrm{G}} = 0$, for $i = 1, 2, 3, 4$ and $M_i^{\mathrm{G}} = 1$, for $i = 5$. Generator $i = 5$ represents a peaking unit with the largest marginal costs. Therefore, following the KKT analysis in Section 2.1.2, this generator only generates when all other generators are generating at their maximum capacity and their electricity is also needed to meet demand. Consequently, in this situation, generator $i = 5$ is able to force consumers to shed their load and/or auxiliary generate by holding back on their own generation. This leads to higher marginal prices and hence higher profits for generator $i = 5$ and indeed all other generators. Generators $i = 1, .., 4$ are not able to manipulate prices in this manner as there is never a situation where they are the last/only generator needed/available to meet demand.

Table 5: Values for generator marginal costs and maximum hourly capacity.

Generator	F_i	$G_i^{\max}$
$i = 1$	38	1200
$i = 2$	34	1700
$i = 3$	41	1000
$i = 4$	50	600
$i = 5$	133	700

Table 6: Parameter values for passive and active load shedding consumer.

Parameter	Value
$\Delta g_{j=1}^{\mathrm{P,max}}$	500
$E_{j=1,t}^{\mathrm{P}}$	200 ($\forall t$)
$\Delta g_{k=1}^{\mathrm{A,max}}$	500
$E_{k=1,t}^{\mathrm{A}}$	150 ($\forall t$)
$F_{k=1}^{\mathrm{APU}}$	176
$g_{k=1}^{\mathrm{APU,max}}$	200
$V_{k=1}$	100

Table 6 displays the maximum amount of electricity both types of consumers can shed in each hourly timestep as well as the intercepts of their load shedding marginal cost functions, which we assume do not vary in time. In addition, Table 6 also displays the marginal costs, maximum hourly capacity and total storage capacity for the active load shedding consumer's auxiliary generation. The total storage capacity is equal to five hours worth of generation. Figure 3b displays reference demand for both types of consumers (as well as their sum) while Table A.8 gives the values for these reference demands in addition to the slopes associated with both types of consumers' marginal cost functions. The values for both the parameters in the marginal load shedding cost functions as well as the reference demands are obtained on the basis of Leahy & Tol (2011). While they distinguish between industrial, commercial and residential consumers, we focus on passive and active load shedding consumers. We assume residential consumers to be passive while we assume industrial, in addition to commercial consumers, to be active, i.e. we aggregate industrial and commercial consumers for illustrative purposes. The passive reference demand values $D^{\mathrm{P}}_{j,t}$ in Table A.8 are therefore identical with the demand values in Leahy & Tol (2011) while the active reference demand values $D^{\mathrm{A}}_{k,t}$ represent the sum of the industrial and commercial demand values in Leahy & Tol (2011).

The slopes $B^{\mathrm{P}}_{j,t}$ and $B^{\mathrm{A}}_{k,t}$ of the marginal load shedding cost functions are calculated on the basis of the time-dependent VOLL values, according to equations (25a) and (25b). As for the reference demand, the values of lost load for the passive consumers ($VOLL^{\mathrm{P}}_{j,t}$) are identical to the residential consumer VOLLs in Leahy & Tol (2011). The values of lost load of the active consumers ($VOLL^{\mathrm{A}}_{k,t}$) are calculated as the demand-weighted average of the industrial and commercial VOLLs. The calculation according to equations (25a) and (25b) is based on the idea that low amounts of lost or shed load lead to rather low-cost effects and that the marginal load shedding costs increase with increasing amounts of load shedding Ruppert et al. (2015). Equations (25a) and (25b) assume the marginal load shedding costs of the last MWh of demand that can be shed to be equal to the VOLL for each considered type of consumer. Since we are aware of the uncertainty related to this assumption, we shall consider different marginal load shedding cost function slopes aimed at exploring the sensitivity of this assumption (see section 4).

$$B^{\mathrm{P}}_{j,t} = \frac{VOLL^{\mathrm{P}}_{j,t}}{D^{\mathrm{P}}_{j,t}} \tag{25a}$$

$$B^{\mathrm{A}}_{k,t} = \frac{VOLL^{\mathrm{A}}_{k,t}}{D^{\mathrm{A}}_{k,t}} \tag{25b}$$

Note: the parameters $D^{\mathrm{P}}_{j,t,s}, D^{\mathrm{A}}_{k,t,s}, B^{\mathrm{P}}_{j,t,s}, B^{\mathrm{A}}_{k,t,s}$ and V_k update after each roll[2], thus the values described here for these parameters are the values for rolls $r = 1$ only with the values for all other rolls being obtained as described in Section 2.3.1. All other parameters remain the same for each path solved.

3.1.2. Considered load shedding policies

The above describes the values for the parameters used in the *Base Case* which represents a non-static load shedding policy. We now proceed and describe the other policies, in particular the difference between them and the *Base Case*:

[2]As $E^{\mathrm{P}}_{j,t,s}$ and $E^{\mathrm{A}}_{k,t,s}$ are assumed to be constant through time, there is no need to update these parameters after each roll.

1. *APU-to-Market Case*:

   This case focusses on the active load shedding consumer (see objective function 11a in section 2.1.5) and assumes a modified set of constraints. In the *Base Case*, constraints (11d) and (11e) ensure that an active load shedding consumer can only provide auxiliary generation to meet their own demand. In contrast, these constraints are removed from the MCPs solved in the *APU-to-Market Case*. Thus, the active load shedding consumer can provide auxiliary generation to the full market. As with the *Base Case*, this test represents a non-static load shedding policy. The analysis in Section 2.1.6 suggests that this test case may reduce system prices, and hence total consumer costs, when compared with the *Base Case*.

2. Static (Priority/Rotational) Policies: The next three cases focus on the interaction between the active (see objective function 11a in section 2.1.5) and passive (see objective function 7a in section 2.1.3) load shedding consumers as discussed theoretically in section 2.2.3. For each of the cases, a certain parameter will be changed/fixed when compared to the *Base Case* as outlined below.

   (a) *Priority (No passive load shedding) Case*:

   In this case, only the active load shedding consumer's demand is shed. As such, the passive load shedding consumer's demand is prioritised by setting the maximum amount of passive load shedding to zero, i.e., $\Delta g_j^{\mathrm{P,max}} = 0$.

   (b) *Priority (No active load shedding & no APU) Case*:

   In this case, only the passive load shedding consumer's demand is shed. As such, the active load shedding consumer's demand is prioritised by setting the maximum amount of active load shedding to zero, i.e., $\Delta g_k^{\mathrm{A,max}} = 0$. In addition, the active load shedding consumer cannot provide any auxiliary generation, i.e., $g_k^{\mathrm{APU,max}} = 0$.

   (c) *Rotational load shedding Case*:

   In this case, load shedding switches between the two types of consumers. For the first 24-hour period, only the passive load shedding consumer's demand can be shed (case 2b above) while for the second 24-hour period, only the active load shedding consumer's demand can be shed (case 2a above).

   The analysis in Section 2.2.3 suggests that constraining some consumers' load shedding to be zero may lead to increased system prices, and hence total consumer costs, when compared with the *Base Case*.

3. *No Market Power Case*:

This case focusses on generator $i = 5$ (see objective function 1a in section 2.1.1) and the interaction between price-making generators and consumers (see section 2.2.2). In the *Base Case*, generator $i = 5$ is assumed to be a price-maker (see equations 2 and 3 for the endogenous relationship between the price-maker's generation and prices). In the *No Market Power Case*, generator $i = 5$ is assumed to be a price-taker along with all other generators, i.e. instead of equations 2 and 3, the price variables $\gamma_{t=r}$ and $\gamma_{t,s}$ are assumed to be exogenous to the optimisation problem of generator $i = 5$. As with the *Base Case*, this test represents a non-static load shedding policy. The analysis in Section 2.2.2 shows that the absence of a price-making generator reduces the amount of load that is shed and system prices and, hence, total consumer costs.

3.2. Results

In the following, we present the results of our study for the input data described above. We start by presenting and comparing the results for different load shedding policies, followed by describing the effect of market power. Subsequently, we present our findings concerning the value of stochastic solution as well as the expected value of perfect information.

3.2.1. Load shedding policies

In this section the results from the different load shedding policies are compared. Figure 4 displays the Total Expected Consumer Costs (TECC) for the different test cases. These costs are calculated by taking the optimal decisions from the scenario-independent timesteps $t = r$ from each MCP solved:

$$\text{TECC} = \sum_{l} PR_l \text{TECC}_l, \tag{26}$$

where

$$\begin{aligned}\text{TECC}_l = \sum_{r} \Big(& \gamma_{t=r}(D^{\text{P}}_{j,t=r} + D^{\text{A}}_{k,t=r} - \Delta g^{\text{P,FS}}_{j,t=r} - \Delta g^{\text{A,FS}}_{k,t=r} - g^{\text{APU,FS}}_{k,t=r}) \\ & + \Delta g^{\text{P,FS}}_{j,t=r} C_{j,t=r}(\Delta g^{\text{P, FS}}_{j,t=r}) + \Delta g^{\text{A,FS}}_{k,t=r} C^{\text{A}}_{k,t=r}(\Delta g^{\text{A,FS}}_{k,t=r}) + g^{\text{APU, FS}}_{k,t=r} F^{\text{APU}}_{k} \Big).\end{aligned} \tag{27}$$

These decisions represent the actual decisions the players would make assuming scenario path l and hence these costs are calculated as a weighted sum of the costs associated with each path.

Figure 4 shows that the *Base Case* and *APU-to-Market* case have the lowest TECC. In fact, for the data considered in this work, these two cases give the same optimal decision values for each MCP solved. This is despite the analysis in Section 2.1.6 suggesting that allowing auxiliary generation to meet reference demands of consumers other than themselves can decrease system

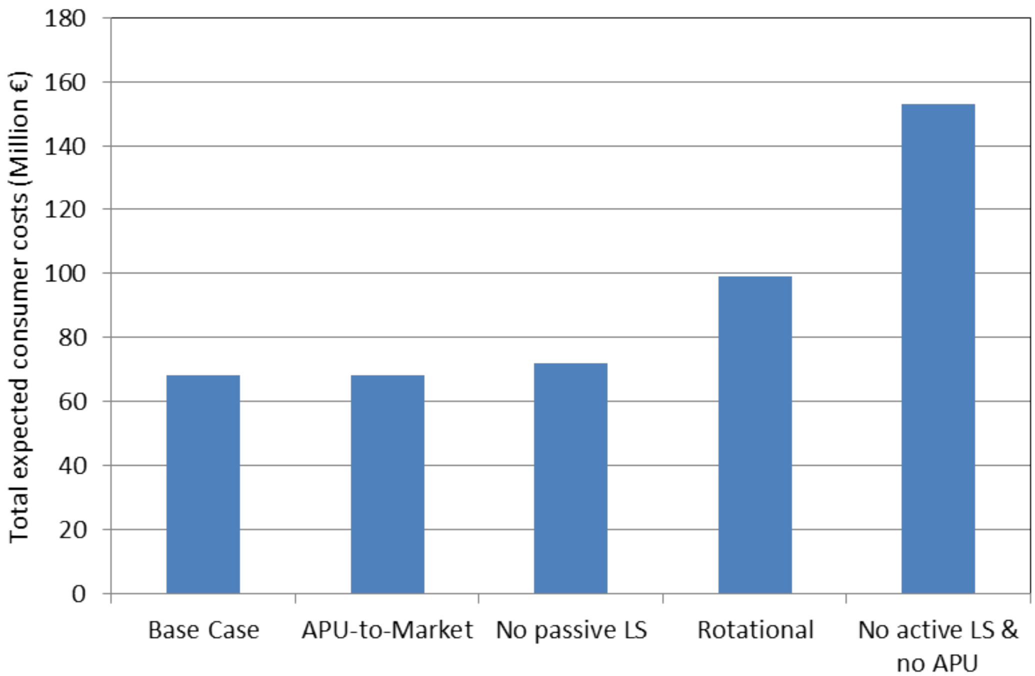


Figure 4: Total expected consumer costs associated with different load shedding policies.

prices. The difference between the two cases is that constraints (11d) and (11e) are excluded from the *APU-to-Market* case. However, in the *Base Case*, these two constraints are never binding. This is because APU generation is constrained (through both hourly and storage capacity constraints)[3] on the amount of electricity it can provide. Thus, APU generation is never large enough to be able to meet demand from active consumers, never mind demand from both types of consumers. In order to examine a case where APU generation is large enough to meet at least some of both types of demand, we also, as a sensitivity check, examined both of these cases with extremely low (and unrealistic) reference demand for active consumers. However, only a minor reduction in TECC was found by allowing auxiliary generation be available to the full market. Henceforth, the *Base Case* and *APU-to-Market* are considered equivalent.

Figure 4 also shows that, overall, the *Base Case* has the lowest TECC. For each of the other cases, load shedding and/or the APU are restricted in some way. Excluding the *Base Case*, the lowest TECC is seen when the passive consumers' load is prioritised (i.e., there is load shedding

[3]Given that the model is solved over a relatively short period of time (48) hours, we assume the active consumer is unable to refuel the storage tank associated with APU generation.

for active consumers only) while the highest TECC is found when the active consumers' load is prioritised. Consequently, when load shedding rotates between active and passive consumers, there are intermediate levels of TECC. This suggests that if a regulator had to prioritise between the two types of consumers, then active consumers should be chosen for load shedding while rotational policies should be chosen ahead of prioritising active consumers only. This result concurs with the analysis in Section 2.2.3, which details how system prices may increase as load shedding is constrained to be zero.

In a similar manner to equations (26) and (27), expected total amount of load shedding and expected prices are also calculated; see Figures 5 and 6. These results match with those seen in Figure 4 with decreased potential for load shedding leading to lower amounts of load shedding, increased expected prices and hence increased TECC.

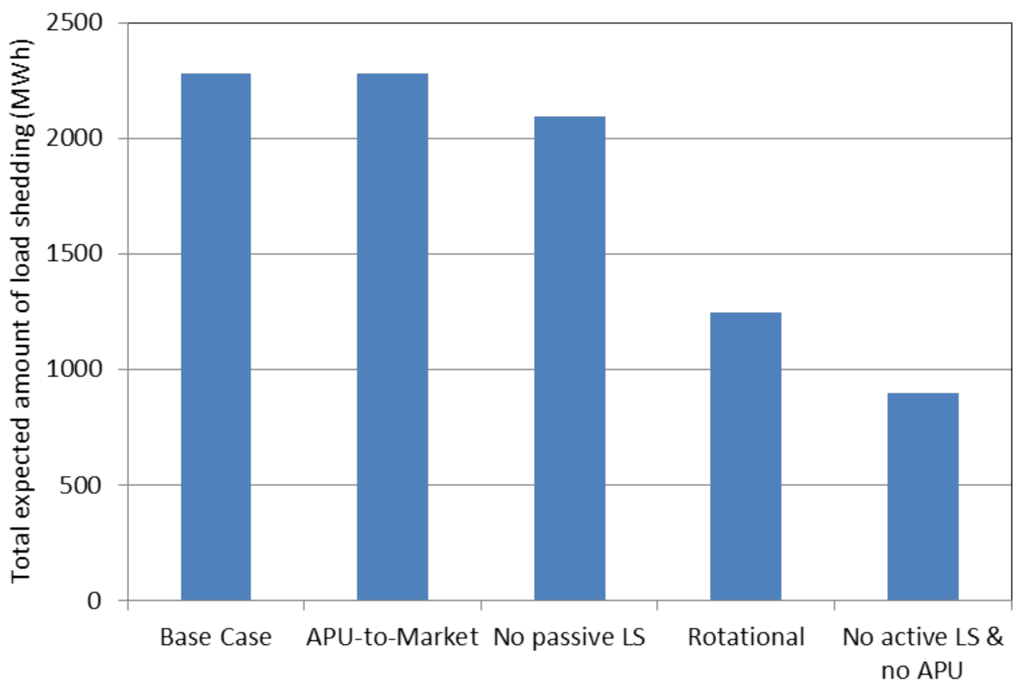


Figure 5: Total expected amount of load shed for different load shedding policies.

3.2.2. Effect of market power

Figure 7 displays the TECC and total expected amount of load shedding for the *Base Case* and the case where no market power is present. Unsurprisingly both TECC and load shedding are lower when there is no market power present. In the *Base Case* the price-maker (generator

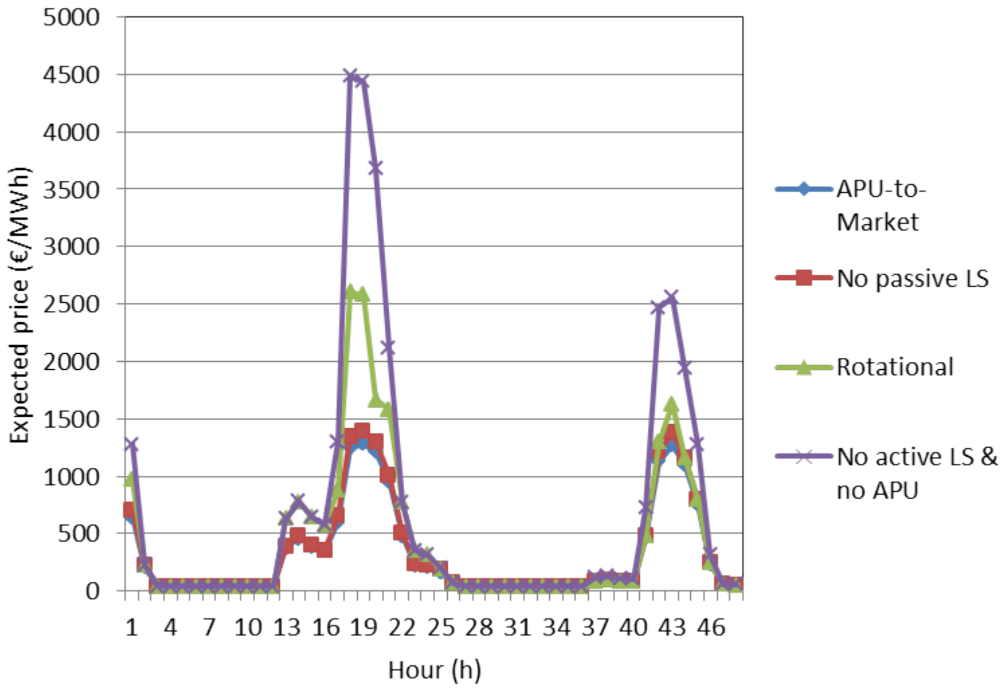
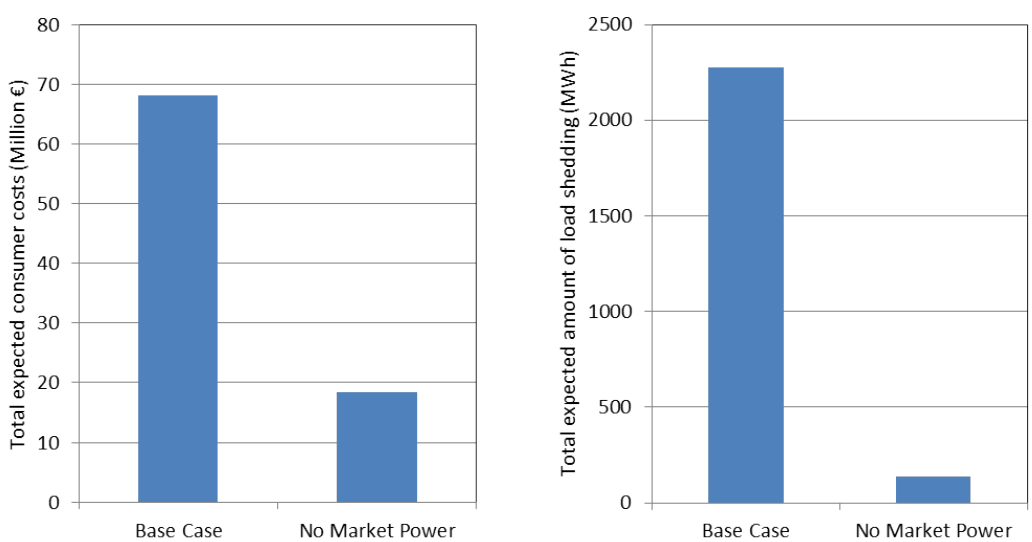


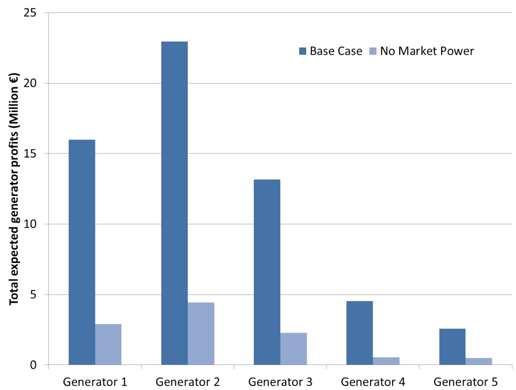
Figure 6: Expected price time series for different load shedding policies.



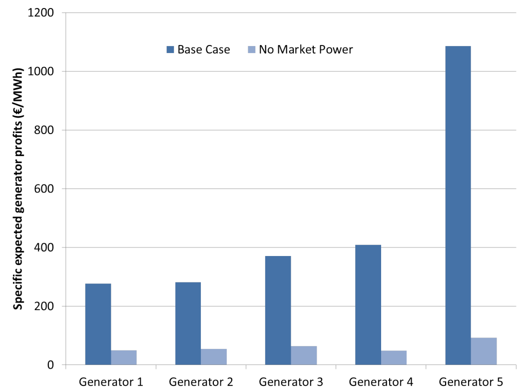
(a) Total expected consumer costs.

(b) Total expected amount of load shed.

Figure 7: Results with and without market power.



(a) Total expected generator profits with and without market power present.



(b) Specific expected generator profits with and without market power present.

Figure 8: Profits with and without market power.

$i = 5$) holds back on the amount of electricity it can provide. It is able to do this at times of high demand and low supply, i.e., when all other generators are supplying electricity at maximum capacity and are hence unable to prevent the price-maker's strategic behaviour. Consequently, load shedding, prices and the price-maker's profits all increase. This result concurs with the analysis in Section 2.2.2, which details how the presence of a price-making generator increases load shedding and hence system prices.

Figure 8a presents the generator profits for the *Base Case* and the *No Market Power* case and shows that each of the generators substantially increase its profits when market power is present. This is despite only generator $i = 5$ being modelled as a price-maker. As the price-maker strategically chooses its production to increase the system price and hence its profit, all other generators benefit too. Ironically, generator $i = 5$ has the smallest absolute and second smallest percentage increase (427%) in profit between the *Base Case* and the *No Market Power* case. This is because this generator is modelled as a peaking unit and is generating the least amount in comparison to the other units. However, Figure 8b shows that generator $i = 5$ has the highest increase in specific profits (€/MWh) as expected.

3.2.3. Value of stochastic solution and expected value of perfect information

To justify the use of stochastics and rolling-horizons in the modelling approach employed, we also calculate well known metrics in stochastic programming: the Value of Stochastic Solution (VSS) and the Expected Value of Perfect Information (EVPI) Birge & Louveaux (2011). The EVPI measures the increase in costs as a result of having uncertainty in the model, i.e., it measures the costs of not having perfect information. In this work, it is calculated using the following three steps:

1. For each MCP, taking the difference in expected consumer costs (sum of objective functions (7a) and (11a)) between the *Base Case* and when, for each MCP, the scenarios described in Figure 2 are replaced with a deterministic process whereby each player knows exactly when the unreliable generator will return.
2. For each path, averaging these values over the total number of stochastic MCPs in that path[4] (circular shaped nodes in Figure 1)).
3. Calculating a weighted average over all paths, using the probabilities associated with each path.

The EVPI's value was found to be €12,676,231 which is 28.6% of the corresponding costs associated with the *Base Case*.

The VSS measures the costs of considering expected values instead of stochastic solutions. In this work, it is calculated in a similar way to the EVPI described above but, for each MCP, the scenarios described in Figure 2 are replaced with a deterministic process whereby each player assumes, with probability one, that the unreliable generator will return in 12 hours (the expected outage length), regardless of which path or scenario they are on. It's value was found to be €2,753,283 which is 6.2% of the corresponding costs associated with the *Base Case*.

4. Discussion

The four main findings of our research can be summarised as follows. While the first three are particularly relevant for system operators or regulators, the fourth provides insights particularly valuable for the players in the market - consumers as well as producers. The fourth finding is related to the methodology.

First, allowing consumers with auxiliary generation ability to supply their generation to the entire market only results in marginal benefits. Again, this finding needs to be interpreted in the context of our assumptions but even the sensitivity check with an unrealistically low reference demand for active consumers did not show a noteworthy benefit for this policy.

Second, a load shedding policy which does not discriminate or prioritise between different consumer groups leads to the lowest consumer costs: amongst the considered policies, smart load

[4]There is no EVPI or VSS associated with deterministic MCPs (diamond-shaped nodes in Figure 1)

shedding policies (considered as *Base Case* in section 3) lead to the lowest costs to consumers, followed by shedding load of active consumers only and a rotational load shedding scheme (alternating between shedding active and passive consumers). Shedding load of passive consumers only leads to the highest costs to consumers. At the same time, the amount of load shedding (in MWh) is highest for smart load shedding and lowest for shedding passive consumers only. Since high amounts of load shedding represent a high demand side flexibility, this finding suggests that increasing demand side flexibility can strongly reduce consumer costs. Obviously, these results are driven by our assumptions to a large extent, particularly concerning the load shedding cost functions for the two different consumer groups. Leahy & Tol (2011), on the basis of whose results we derive the cost functions for our case study, find the VOLL to be highest for the residential sector. This assumption, together with the fact that we did not consider households with auxiliary generation ability, is important for interpreting the results. However, the generalised result that a load shedding policy which does not discriminate or prioritise between different consumer groups leads to the lowest consumer costs holds regardless of these assumptions. In addition, we ran the model with different slopes for the load shedding cost functions as a sensitivity check. While the total costs to consumers changed strongly with the slopes, the ranking of the policies remained unchanged. Finally, our results suggest that the importance of a deeper understanding of the load shedding costs and the value of lost load for different consumer groups will increase in future.

Third, preventing market power will significantly reduce consumer costs as well as the amount of load shedding. While this finding is not surprising, it is especially relevant for markets with dominant players on the generation side. Moreover, it is interesting to note that costs and load shedding between absence and presence of market power differ by a factor of six approximately. Costs and load shedding between the *Base Case* and *Priority (No active load shedding & no APU)* policy differ by a factor of approximately two only.

Fourth, the calculated EVPI and VSS provide proof that it is advisable to use a rolling-horizon stochastic programming approach in this context. Incorporating stochastics into the model more realistically replicates the uncertain information those in the market have when making decisions. Additionally, and in contrast to a perfect foresight approach, the rolling-horizon model allows for the capture of the actual decisions players would make for each timestep, as well as the hypothetical decision they would consider making. Consequently, the results obtained replicate real-world markets more convincingly.

Critically reflecting our approach, we wish to acknowledge its short-term nature. This means that we do neither consider refueling strategies for the limited auxiliary generation storage nor do we consider interactions between load shedding and investment decisions. Moreover, we did not analyse the impact of load shifting of the consumers or of own generation ability or decentralised storage availability for residential consumers within the scope of this paper. Concerning the data, while being based on actual values for Ireland, the case study has an illustrative character. In particular, there is uncertainty around the VOLL which, at the same time, drives the results. Furthermore, we wish acknowledge that players may assume that the expected (remaining) outage length of the unreliable generator decreases over time. However, due the lack of information/data on how that expectation may do so, the model's players always expect the unreliable generator to return online in 12 hours, regardless of how long it has already been offline for. Each of these limitations will be subject to future research activities.

5. Conclusions

In this paper, we examine the costs of strategies associated with load shedding as one demand response mechanism contributing to an increased demand side flexibility. For this purpose, we model an electricity market with different types of generators and consumers. More specifically, we propose a rolling horizon stochastic mixed complementarity equilibrium model.

Overall, we found that leveraging demand side flexibility can reduce costs to consumers. When comparing different policies, a smart load shedding policy, which does not discriminate between consumer groups, leads to the lowest costs. Furthermore, we found that allowing consumers with generation ability to supply their generation to the entire market only provides marginal benefits, whereas the prevention of market power is highly important in terms of ensuring low costs to consumers. In the light of these findings and bearing the smart meter rollout in mind, which is imminent particularly in many European countries, it is therefore now important that the specification of the technical requirements allows realisation of such policies - ideally without or with limited loss of comfort. This will ensure that full use of the smart infrastructure to be installed can be made in future.

Future research in this area should explore the impact of decentralised storage and own generation ability of residential consumers as well as interdependencies between the introduction of load shedding policies and investment decisions into generation capacity. Moreover, a deeper understanding of load shedding cost functions should be sought for different consumer groups.

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Appendix A. Supplementary data

Table A.7: Probabilities associated with each path l and scenario s (only for nodes where unreliable generator is offline).

s/l	1	2	3	4	5	6	7	8	9	10	11	12
PR_s/PR_l	0.035	0.064	0.074	0.073	0.069	0.063	0.058	0.052	0.047	0.042	0.038	0.034
s/l	13	14	15	16	17	18	19	20	21	22	23	24
PR_s/PR_l	0.031	0.028	0.025	0.023	0.021	0.019	0.017	0.016	0.014	0.013	0.012	0.011
s/l	25	26	27	28	29	30	31	32	33	34	35	36
PR_s/PR_l	0.010	0.010	0.009	0.008	0.008	0.007	0.007	0.006	0.006	0.005	0.005	0.005
s/l	37	38	39	40	41	42	43	44	45	46	47	48
PR_s/PR_l	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002

Table A.8: Values for reference demands and slopes of marginal load shedding cost functions.

t	1	2	3	4	5	6	7	8	9	10	11	12
$D^{\mathrm{P}}_{j=1,t}$	1808	1459	1239	1052	926	842	780	705	703	750	905	1058
$D^{\mathrm{A}}_{k=1,t}$	2501	2485	2354	2285	2207	2195	2276	2376	2268	2385	2591	2831
$B^{\mathrm{P}}_{j=1,t}$	4.7	2.4	1.2	1.7	1.2	2.2	15.0	68.2	102.0	85.7	58.2	40.4
$B^{\mathrm{A}}_{k=1,t}$	4.5	4.6	4.8	5.0	5.1	5.2	5.0	4.8	5.0	4.8	4.4	4.0
t	13	14	15	16	17	18	19	20	21	22	23	24
$D^{\mathrm{P}}_{j=1,t}$	1207	1293	1216	1198	1500	2155	2354	1979	1889	1835	1777	1690
$D^{\mathrm{A}}_{k=1,t}$	3027	3091	3107	3111	3121	2872	2670	2896	2843	2696	2507	2625
$B^{\mathrm{P}}_{j=1,t}$	32.6	28.4	31.6	33.5	23.6	13.8	14.2	20.7	23.4	25.6	25.0	16.1
$B^{\mathrm{A}}_{k=1,t}$	3.7	3.7	3.7	3.6	3.6	4.0	4.3	3.9	4.0	4.2	4.5	4.3
t	25	26	27	28	29	30	31	32	33	34	35	36
$D^{\mathrm{P}}_{j=1,t}$	1808	1459	1239	1052	926	842	780	705	703	750	905	1058
$D^{\mathrm{A}}_{k=1,t}$	2501	2485	2354	2285	2207	2195	2276	2376	2268	2385	2591	2831
$B^{\mathrm{P}}_{j=1,t}$	4.7	2.4	1.2	1.7	1.2	2.2	15.0	68.2	102.0	85.7	58.2	40.4
$B^{\mathrm{A}}_{k=1,t}$	4.5	4.6	4.8	5.0	5.1	5.2	5.0	4.8	5.0	4.8	4.4	4.0
t	37	38	39	40	41	42	43	44	45	46	47	48
$D^{\mathrm{P}}_{j=1,t}$	1207	1293	1216	1198	1500	2155	2354	1979	1889	1835	1777	1690
$D^{\mathrm{A}}_{k=1,t}$	3027	3091	3107	3111	3121	2872	2670	2896	2843	2696	2507	2625
$B^{\mathrm{P}}_{j=1,t}$	32.6	28.4	31.6	33.5	23.6	13.8	14.2	20.7	23.4	25.6	25.0	16.1
$B^{\mathrm{A}}_{k=1,t}$	3.7	3.7	3.7	3.6	3.6	4.0	4.3	3.9	4.0	4.2	4.5	4.3

References

Albadi, M. H., & El-Saadany, E. (2008). A summary of demand response in electricity markets. *Electric Power Systems Research*, *78*, 1989–1996.

Allcott, H. (2011). Rethinking real-time electricity pricing. *Resource and energy economics*, *33*, 820–842.

Associates, C. R. (2005). Primer on demand-side management with an emphasis on price-responsive programs. *Report prepared for The World Bank, Washington, DC, CRA No. D06090, available online: http://www.worldbank.org (2005)*, .

Bayer, B. (2014). Demand response–is the usa a role model for Germany? analysis of the integration of demand response into the american capacity and balancing markets. *IASS Working Paper*, .

Bertsch, J., Growitsch, C., Lorenczik, S., & Nagl, S. (2016). Flexibility in europe's power sectoran additional requirement or an automatic complement? *Energy Economics*, *53*, 118–131.

Birge, J. R., & Louveaux, F. (2011). *Introduction to stochastic programming*. Springer Science & Business Media.

Boßmann, T., & Eser, E. J. (2016). Model-based assessment of demand-response measuresa comprehensive literature review. *Renewable and Sustainable Energy Reviews*, *57*, 1637–1656.

Burger, S., Chaves-Ávila, J. P., Batlle, C., & Pérez-Arriaga, I. J. (2016). The value of aggregators in electricity systems. *MIT working paper*, .

Calderaro, V., Galdi, V., Lattarulo, V., & Siano, P. (2010). A new algorithm for steady state load-shedding strategy. In *12th International Conference on Optimization of Electrical and Electronic Equipment (OPTIM)* (pp. 48–53). IEEE.

Ceseña, E. A. M., Good, N., & Mancarella, P. (2015). Electrical network capacity support from demand side response: Techno-economic assessment of potential business cases for small commercial and residential end-users. *Energy Policy*, *82*, 222–232.

Chen, L., Li, N., Low, S. H., & Doyle, J. C. (2010). Two market models for demand response in power networks. *IEEE SmartGridComm*, *10*, 397–402.

Comission for Energy Regulation (2014). CER National Smart Metering Programme Smart Metering High Level Design. Available online: http://www.cer.ie/docs/000699/CER14046%20High%20Level%20Design.pdf.

De Jonghe, C., Hobbs, B. F., & Belmans, R. (2012). Optimal generation mix with short-term demand response and wind penetration. *IEEE Transactions on Power Systems*, *27*, 830–839.

Devine, M. T., Gabriel, S. A., & Moryadee, S. (2016). A rolling horizon approach for stochastic mixed complementarity problems with endogenous learning: Application to natural gas markets. *Computers & Operations Research*, *68*, 1–15.

Devine, M. T., Gleeson, J. P., Kinsella, J., & Ramsey, D. M. (2014). A rolling optimisation model of the uk natural gas market. *Networks and Spatial Economics*, *14*, 209–244.

Di Cosmo, V., Lyons, S., & Nolan, A. (2014). Estimating the impact of time-of-use pricing on irish electricity demand. *The Energy Journal*, *35*, 117–136.

EC (2014). European energy security strategy.

EC (2016). Directive (eu) 2016/1148 of the european parliament and of the council of 6 july 2016 concerning measures for a high common level of security of network and information systems across the union.

Economics, F., & First, S. (2012). Demand side response in the domestic sector - a literature review of major trials. *Report prepared for the Department of Energy & Climate Change (DECC), London, UK (2012)*, .

Egging, R. (2013). Benders decomposition for multi-stage stochastic mixed complementarity problems–applied to a global natural gas market model. *European Journal of Operational Research*, *226*, 341–353.

EirGrid (2016). All-island generation capacity statement 2016-2025.

of Energy, U. D. (2006). Benefits of demand response in electricity markets and recommendations for achieving them. *Report to the United States Congress, February 2006, available online: http://eetd.lbl.gov (2006)*, .

Esther, B. P., & Kumar, K. S. (2016). A survey on residential demand side management architecture, approaches, optimization models and methods. *Renewable and Sustainable Energy Reviews*, *59*, 342–351.

Faruqui, A., Hledik, R., & Sergici, S. (2010). Rethinking prices: The changing architecture of demand response in america. *Public Utilities Fortnightly*, *148*, 30–39.

Faruqui, A., & Sergici, S. (2010). Household response to dynamic pricing of electricity: a survey of 15 experiments. *Journal of Regulatory Economics*, *38*, 193–225.

Gabriel, S. A., Conejo, A. J., Fuller, J. D., Hobbs, B. F., & Ruiz, C. (2012). *Complementarity modeling in energy markets* volume 180. Springer Science & Business Media.

Gabriel, S. A., Kiet, S., & Zhuang, J. (2005). A mixed complementarity-based equilibrium model of natural gas markets. *Operations Research*, *53*, 799–818.

Gabriel, S. A., Zhuang, J., & Egging, R. (2009). Solving stochastic complementarity problems in energy market modeling using scenario reduction. *European Journal of Operational Research*, *197*, 1028–1040.

Good, N., Ellis, K. A., & Mancarella, P. (2017). Review and classification of barriers and enablers of demand response in the smart grid. *Renewable and Sustainable Energy Reviews*, *72*, 57–72.

Guigues, V., Sagastizábal, C., & Zubelli, J. P. (2014). Robust management and pricing of liquefied natural gas contracts with cancelation options. *Journal of Optimization Theory and Applications*, *161*, 179–198.

Haider, H. T., See, O. H., & Elmenreich, W. (2016). A review of residential demand response of smart grid. *Renewable and Sustainable Energy Reviews*, *59*, 166–178.

Hobbs, B. (2001). Linear complementarity models of nash-cournot competition in bilateral and poolco power markets. *IEEE Transactions on Power Systems*, *16*, 194–202.

Hornby, R., Hurley, D., & Knight, P. (2011). A review of demand response potential in the United States. *Synapse Energy Economics, Inc., Report prepared for the US Environmental Protection Agency (2011)*, .

Huppmann, D. (2013). Endogenous production capacity investment in natural gas market equilibrium models. *European Journal of Operational Research*, *231*, 503–506.

Kirby, B., & Hirst, E. (1999). Load as a resource in providing ancillary services. *Technical report, Oak Ridge National Laboratory. Oak Ridge, TN*, .

Laghari, J., Mokhlis, H., Bakar, A., & Mohamad, H. (2013). Application of computational intelligence techniques for load shedding in power systems: a review. *Energy Conversion and Management*, *75*, 130–140.

Leahy, E., & Tol, R. S. (2011). An estimate of the value of lost load for Ireland. *Energy Policy*, *39*, 1514–1520.

Lee, C.-Y. (2016). Nash-profit efficiency: A measure of changes in market structures. *European Journal of Operational Research*, *255*, 659–663.

Lynch, M. Á., & Devine, M. T. (2017). Investment vs. refurbishment: Examining capacity payment mechanisms using mixed complementarity problems with endogenous probability. *The Energy Journal*, *38*, 27–51.

Palensky, P., & Dietrich, D. (2011). Demand side management: Demand response, intelligent energy systems, and smart loads. *IEEE Transactions on Industrial Informatics*, *7*, 381–388.

Ruppert, M., Bertsch, V., & Fichtner, W. (2015). Optimal load shedding in distributed networks with sigmoid cost functions. In *2015 International Symposium on Smart Electric Distribution Systems and Technologies (EDST)* (pp. 159–164). IEEE.

Ströhle, P., & Flath, C. M. (2016). Local matching of flexible load in smart grids. *European Journal of Operational Research*, *253*, 811–824.

Subramanian, D. (1971). Optimum load shedding through programming techniques. *IEEE Transactions on Power Apparatus and Systems*, *1*, 89–95.

Tuohy, A., Meibom, P., Denny, E., & O'Malley, M. (2009). Unit commitment for systems with significant wind penetration. *IEEE Transactions on Power Systems*, *24*, 592–601.

Wang, P., & Billinton, R. (2000). Optimum load-shedding technique to reduce the total customer interruption cost in a distribution system. *IEE Proceedings-Generation, Transmission and Distribution*, *147*, 51–56.

Woo, C.-K., Sreedharan, P., Hargreaves, J., Kahrl, F., Wang, J., & Horowitz, I. (2014). A review of electricity product differentiation. *Applied Energy*, *114*, 262–272.