



Title	Estimation of voltage sensitivities in low voltage feeders with photovoltaics
Authors(s)	Rigoni, Valentin, Keane, Andrew
Publication date	2018-10-25
Publication information	Rigoni, Valentin, and Andrew Keane. "Estimation of Voltage Sensitivities in Low Voltage Feeders with Photovoltaics." IEEE, October 25, 2018. https://doi.org/10.1109/ISGTEurope.2018.8571580 .
Conference details	The 8th IEEE Power & Energy Systems (PES) Innovative Smart Grid Technologies Europe, Sarajevo, Bosnia and Herzegovina, 21-25 October 2018
Publisher	IEEE
Item record/more information	http://hdl.handle.net/10197/26344
Publisher's statement	© 2018 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.
Publisher's version (DOI)	10.1109/ISGTEurope.2018.8571580

Downloaded 2026-05-01 23:33:25

The UCD community has made this article openly available. Please share how this access benefits you. Your story matters! (@ucd_oa)



© Some rights reserved. For more information

Estimation of voltage sensitivities in low voltage feeders with photovoltaics

Valentin Rigoni, Student Member, IEEE, and Andrew Keane, Senior Member, IEEE
School of Electrical & Electronic Engineering
University College Dublin
Dublin, Ireland
valentin.rigoni@ucdconnect.ie

Abstract— With increasing penetration levels of photovoltaic panels (PVs) in low voltage (LV) networks, voltage statutory limits are likely to be violated. To avoid this, most strategies rely on the capacity of inverters to modify power injections as needed. In cases for which the required set points are not obtained from a constrained formulation of the power flow problem, voltage sensitivities to power injections are required. Therefore, recent publications have proposed methods to estimate these sensitivities in residential LV feeders. However, as they rely on extensive monitoring and communication infrastructure, they may result impractical for operators to implement. This paper introduces a novel methodology that allows every PV to autonomously estimate voltage sensitivities based merely on the real-time local voltage at the customer point of connection. A robust accuracy assessment under multiple system states is performed and benchmarked against a more traditional method.

Index Terms— Distributed power generation, Estimations, Smart grids, Solar panels, Reactive power, Voltage sensitivities.

I. INTRODUCTION

High penetration levels of residential-scale photovoltaic panels (PVs) can rise voltage levels over their statutory limit on distribution low voltage (LV) networks [1]. Therefore, to guarantee their secure integration, Distribution Network Operators (DNOs) need to move from passive operation to active network management schemes. For the general case of Distributed Generation (DG) units, multiple strategies have been proposed [2]–[8]. These methods are based on the innate capacity of most inverters to curtail active power generation and consume reactive power as needed. These algorithms, which can be extended to PVs, are built around the availability of network sensitivities to power injections. This is because the DG units' set points do not result from a constrained power flow formulation (e.g., centralized optimal power flow).

There exists a series of traditional methods used for obtaining voltage sensitivities in medium voltage (MV) distribution systems with DG. A common one is the perturb-and-observe approach where network variables are compared to those obtained from a power flow calculation after small

modifications [2], [3]. This requires previous knowledge of the system state and of all nodal power injections. Sensitivity data is also accessible from the inverse of the Jacobian matrix J from the Newton Raphson power flow method [4], [5]. As to calculate J knowledge of the state variables is again needed, simplifications are made when this information is unavailable (e.g. uniform constant load). Furthermore, as the Newton Raphson method linearizes non-linear equations around fixed state variables, accuracy decreases if power variations are not small. Finally, sensitivities have been determined from direct analytical expressions derived from classical power flow equations describing flows and voltage drops [6]–[8]. In all cases, the real system state is ignored and either marginal voltage fluctuations, balanced networks, no remote power injections and/or negligible power losses are presumed.

Regardless all mentioned methods being common for MV networks, their use on LV feeders poses a series of non-addressed research questions. For instance, methods that rely on full network observability (e.g. perturb-and-observe) require the prior application of State Estimation (SE) [9]. While traditional SE is common in transmission systems, its use at the LV level is still a challenge for DNOs. Not only monitoring equipment is generally inexistent, but also the complex network modelling and the coordination of such a widespread communication infrastructure may prove it to be incompatible with real-time applications [10]. This practical limitation could be overcome by the analytical equations [6]–[8]; where the system state is ignored. However, no studies have assessed the errors that may take place due to LV feeders having different characteristic to those of the upstream network.

Focusing on LV feeders, recent publications have proposed new ways of estimating voltage sensitivities during on-line operation; reckoning their dependence on the system state [11], [12]. In the first, sensitivities are obtained from historical smart meter data (required to be available prior to implementation). In the latter, power electronic devices apply small on-line perturbations while assessing the corresponding effects via a communication network. While these methodologies do not need topological data, they both depend on a full monitoring

This publication has emanated from research conducted with the financial support of Science Foundation Ireland under the SFI Strategic Partnership Programme Grant Number SFI/15/SPP/E3125. This work was conducted in UCD Energy Institute.

and communication infrastructure. This prompts again practical limitations by representing not only a costly investment but resulting in complex operation. For example, signal delays and lack of synchronism and noise due to residential load stochastic variations could result in very inaccurate sensitivities.

Based on the above, we introduce a decentralized method able to estimate, in real-time, voltage sensitivities for unbalanced LV feeders with no need of communication between units nor measurement of residential loads. The proposed approach combines off-line Monte Carlo (MC) simulations and regression analysis. Crucially, the only measurement needed for on-line application is the local single-phase voltage from each PV inverter. The paper is organized as follows: Section II describes the proposed methodology. In Section III, the obtained sensitivities are tested on a real LV distribution feeder and benchmarked against a more traditional method. Finally, discussion and conclusions are drawn in Sections IV and IV.

II. METHODOLOGY

In the proposed approach, polynomial equations are used to describe voltages sensitivities to active and reactive power injections. These equations result from an off-line Monte Carlo methodology that counts for multiple system scenarios. The method only requires monitoring of the local voltage at the customer point of connection (CPOC) for on-line application.

A. Off-line stage - Monte Carlo power flow simulations and regression analysis

Fig. 1 summarizes the overall Monte Carlo based approach that will be used to characterize voltage sensitivities under multiple demand and generation scenarios with simple polynomial functions. First, we will concentrate on the inner loop enclosed by the dashed lines; where the system scenario, from the perspective of a local PV, is fixed (i.e. set points for all remote PVs and demand of every house already defined). This loop starts by setting either reactive power Q_g or active power P_g for the local unit to their lower operation bound. This selection depends on whether we intend to obtain sensitivities to reactive or active power respectively. Next, this set point is modified in regular steps until reaching the unit's maximum reactive power capability or active power rating as appropriate. At each $Q_g|P_g$ step, an unbalanced AC power flow simulation is solved and the voltage magnitude at every CPOC stored.

When all the steps of the inner loop have been explored, the final compilation of calculated voltages feeds a first-stage of polynomial fitting that characterizes their variation as a linear function of the local injected active or reactive power at every step. This requires two coefficients a_{ki} to be determined for each k customer voltage V_k :

$$V_k = a_{k1} + a_{k2}Q_g \quad \text{or} \quad V_k = a_{k1} + a_{k2}P_g \quad (1)$$

This determination is performed by polynomial regression analysis. The procedure consists of a least squares estimation that minimizes the sum of square of residuals between the data and the evaluation of the fitted polynomials. This is solved by an optimization algorithm that determines the polynomial coefficients beginning from an initial guess and converging

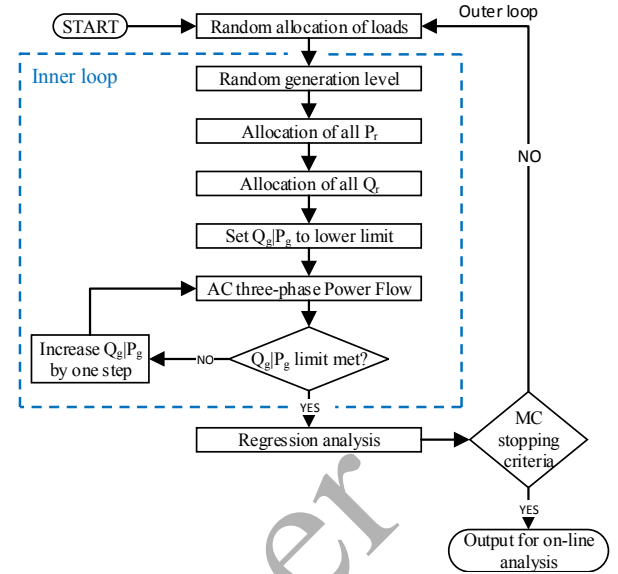


Fig. 1. Off-line procedure – Power flow simulations

toward a specified tolerance [13]. From (1), it can be inferred that the slope of this equation (i.e., a_{k2}) is the sensitivity of the voltage at the k th customer to the variation of Q_g or P_g (whether the method is obtaining sensitivities to reactive or active power). This can be observed from the definite integrals (2) and (3) of the derivatives of (1) evaluated for either reactive or active power injection variations ΔQ_g or ΔP_g :

$$\int_{Q_g^i}^{Q_g^f} \frac{dV_k}{dQ_g} dQ_g = a_{k2} \Delta Q_g \quad \text{where: } \Delta Q_g = Q_g^f - Q_g^i \quad (2)$$

$$\int_{P_g^i}^{P_g^f} \frac{dV_k}{dP_g} dP_g = a_{k2} \Delta P_g \quad \text{where: } \Delta P_g = P_g^f - P_g^i \quad (3)$$

All calculations from the inner loop are associated to a specific system scenario, identified by pre-defined power injections at each remote node. This means that, despite maintaining the same $Q_g|P_g$ steps, the set of calculated voltages will change if remote power injections differ (i.e. leading to new a_{k2} values). Therefore, an outer framework is designed to generate individual MC scenarios from the perspective of the local unit under analysis. At each MC iteration, load profiles are randomly allocated at every house according to a statistical load model. Next, the network generation level is randomly selected from zero to maximum active power output. This value, in absence of active power curtailment, equals the system rating divided by the active power generation of each unit. Later, the active power P_r and reactive power Q_r set points of all remote PVs are selected from their lower to upper bound according to how these units are expected to operate. Consequently, each execution of the outer-loop will result in a new set of a_{k2} coefficients for each local PV.

In real-time operation, each local unit will need to estimate which set of voltage sensitivities corresponds the best to the remote nodal power injections. The problem is that, due to the limited observability, that information is unknown. However, we propose that every a_{k2} can be estimated based on local accessible measurements. Measurements that we have limited to the single-phase CPOC voltage of the local PV (index l) and

the network generation level. Hence, a second stage of regression analysis fits the sensitivity coefficients from all MC iterations with a second-degree equation of those variables:

$$a_{k2} = b_{k1}xV_l^* + b_{k2}xG_{level} + b_{k3}xG_{level}^2 + b_{k4} \quad (4)$$

where a_{k2} is the voltage sensitivity coefficient in (1), G_{level} is the network generation level and V_l^* the voltage at the local CPOC in the first inner loop power flow calculation (i.e., Q_g or P_g at their lower bound). The last parameter is specified in this manner as a voltage in a common operation point is needed to compare MC scenarios. This second stage then requires four shaping coefficients b_{ki} to be determined for each possible V_k . To finalize the overall procedure, at the end of each outer-loop the variation of all considered b_{ki} with respect to the precedent MC run are used as decision variable for a stopping criterion. The algorithm stops when these variations remain within 0.1% for 100 consecutive scenarios (i.e. to account for outliers).

The order of the fitted polynomials in the two previous stages was defined based on the observation of the training data obtained from the MC methodology. Each equation corresponds to the highest order polynomial that does not result on an ill-conditioned regression model [13] (i.e. best possible capture of the data trend).

B. On-line estimation based on real-time measurements

Up to this point, everything is performed off-line using computer-based simulations aimed at exploring the space of possible system states and the associated sensitivity coefficients. Once the results from the off-line analysis are available, every unit can estimate during real-time operation the sensitivities described by the slope a_{k2} in (1) based on a limited series of steps. First, a value for the reference voltage V_l^* in (4) needs to retrieve. As mentioned, V_l^* is the local voltage calculated in each inner loop with either Q_g or P_g at their lower bound. Nevertheless, in on-line circumstances the current set point can be taking any other value. Therefore, V_l^* from (4) must be obtained based on the current voltage at the local CPOC using the local voltage sensitivities for the unit under analysis which, according to (4), can be expressed as:

$$\frac{\partial V_l}{\partial Q_g|P_g} = b_{11}xV_l^* + b_{12}xG_{level} + b_{13}xG_{level}^2 + b_{14} \quad (5)$$

where V_l is the measured single-phase voltage and its derivate the sensitivity a_{12} . Simultaneously, V_l^* can be written as:

$$V_l^* = V_l - Q_g|P_g x \frac{\partial V_l}{\partial Q_g|P_g} \quad (6)$$

which states that V_l^* is a function of itself, the generation level and the local voltage. Consequently, equations (5) and (6) can be merged into (7) to obtain V_l^* :

$$V_l^* = \frac{V_l + Q_g|P_g x (b_{12}xG_{level} + b_{13}xG_{level}^2 + b_{14})}{(1 + Q_g|P_g x b_{11})} \quad (7)$$

Once V_l^* is obtained, the appropriate coefficients in (4) are used to estimate the corresponding values for a_{k2} based on its value and the generation level. Finally, if all a_{k2} are estimated, the integrals (2) or (3) provide the changes in any voltage V_k with respect to both type of power variations.

These on-line estimations consist on solving a few direct parametric equations; posing negligible computational burden

and no risk of non-convergence (there is no non-convex problem during on-line estimations). This, together with no need of a widespread monitoring is a clear practical benefit for DNOs during real-time operation.

III. TEST CASE

This analytical approach is tested on a real unbalanced LV feeder under a multi-scenario simulation. This section introduces the residential demand model used during the off-line analysis and describes the study case network. It also contains an accuracy assessment of the estimated sensitivities.

A. Domestic residential load model

The model used for the random load allocation was developed in [14] and is based on a 1-minute resolution changing load composition for each house. As many others, it represents statistical behaviour and it is therefore not limited to an individual network. In this one, load composition is modelled in terms of constant impedance, constant current and constant power, i.e. ZIP model, for active and reactive power.

The load model used for this test is not inherent to the methodology. While advance models are expected to be a better representation of customers behaviour, DNOs can use the data that is available to them. In addition, the complexity of the regression analysis stages and on-line operation are independent of the load characterization. This can be accounted as a benefit when compared to other methods. For instance, load voltage dependency was neglected in previous analytical expressions due to the considered assumptions. Furthermore, in methods where previous State Estimation is performed, its solution would become more complex as a ZIP model would make power injections a function of the state variables.

It needs to be mentioned that domestic residential demand is time dependent and presents seasonal behaviour. Consequently, the methodology proposed in Section I.A is repeated for any time condition that is to be taken into consideration. This poses no limitation as the off-line part of the methodology is performed before real-time application at a pre-operation stage (i.e. far from being highly time constrained). The selected time resolution for the off-line stage will depend on the variability, over time, of the selected demand model. As a consequence, this must be considered during on-line operation; where the most appropriate polynomial must be used according to the time resolution of the regulation itself.

B. Test Network

The analysis is implemented on a real LV feeder selected from a set of UK networks [15]. The results obtained in [1] group this feeder within the most common residential feeders that are expected to present technical problems due to high penetrations of PVs. It consists of 83 single-phase houses and 1,684 metres of 4-wire cable (3 phases and neutral) and is depicted in Fig. 2 where customers are identified by dots and the head of the feeder represented by a black triangle.

Every house is considered to have a PV (i.e., worst case condition of 100% penetration), modelled as a constant PQ generator. Statistics from [16] are used for a realistic sizing of those units with installed capacities that can go from 1 to 4 kW.

Finally, a nominal voltage of 0.4kV is adopted.

C. Power injections bounds and allocation for remote units

All PVs' AC/DC inverters are presumed to be capable of consuming reactive power Q ; upper and lower bounded by:

$$0 \leq |Q| \leq \min(\sqrt{S^2 - P^2}, P \times \tan(\cos^{-1}(0.95))) \quad (8)$$

$$\text{with } S = 1.1 \times P_{max}$$

where P is the generated active power and S is the inverter's apparent power rating; with a 10% oversize with respect to the generator's active power rating P_{max} . A minimum power factor of 0.95 is chosen accordingly to Ireland's distribution grid code [17]. Additionally, active power is allowed to take any value from zero to the maximum PV installed capacity P_{max} .

In Section I.A, it was specified that the MC methodology summarized in Fig. 1 must be implemented for sensitivities to reactive and active power individually. This is because the allocation of the operation points for all units is expected to be different for each case. For instance, in all the previously mentioned articles, reactive power is always preferred as the control variable instead of active power. Otherwise stated, active power curtailment takes place when all inverters' maximum reactive power capability has been reached. For the case of reactive power sensitivities, once the network generation level is defined, the active power injection of all PV units in the system is obtained as the latter multiplied by each unit's P_{max} (i.e., no power curtailment). Here, the generation level is the same for all units as solar irradiance is considered uniform for the whole network. Then, reactive power Q_r of every remote unit is randomly selected from lower to upper bound using a uniform distribution. For the case of active power sensitivities, we follow a similar approach as the one for reactive power. Once the generation level for the current MC iteration is known, active power P_r for each unit is randomly selected from zero to maximum possible value (i.e., $P_{max} \times G_{level}$). Then, all units' reactive power Q_r is obtained from the upper bound defined by (8). This presumes that active power curtailment takes place only when all units' reactive power capability has been reached.

While not accounted for in this test case, different orientations (e.g. east/west) can occur for residential PVs. Therefore, instead of a uniform generation level, an expected fraction of it could be allocated for each PV based on its orientation, the operation time and the season.

D. Validation and results

1) First stage of the off-line polynomial regression

Once loads and generators' characteristics are defined, it is possible to run the inner data gathering loop for each Monte Carlo execution. In order to do that, at every scenario, Q_g or P_g are incremented from zero to the upper bound in steps of 25%. To have a fair comparison between all houses, the local unit is always considered to have a P_{max} equal to 4 kW. At each step, an AC power flow is solved and the voltages of interest stored for the regression analysis stage. The step granularity is not necessarily limited to 25% but it needs to be considered that unnecessarily small steps will result on extra needless calculations and higher computational times.

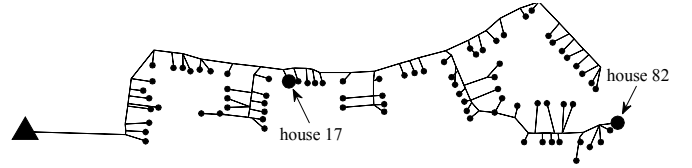


Fig. 2. Real residential LV feeder

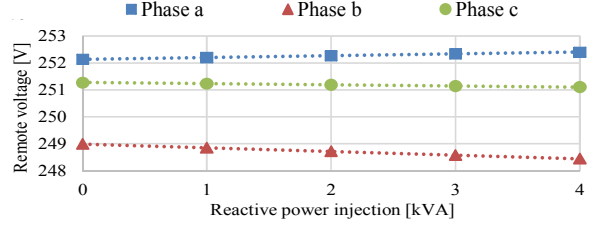


Fig. 3. Trend of remote voltages at multiple reactive power injection set points

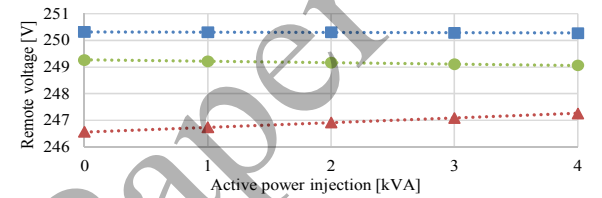


Fig. 4. Trend of remote voltages at multiple active power injection set points

Fig. 3 and Fig. 4 show remote voltage values at different steps for an individual scenario. Local power injections are those on phase b , i.e., CPOC at house 17 (see Fig. 2). Here and in future results, remote voltages always refer to a hypothetical three-phase node at house 82. House 82's node is critical as it presents the highest impedance path to the head of the feeder. This exposes it to voltage violations. It can be observed how the polynomial proposed in (1) results in a good fit for the voltages at each power flow calculation (dots). It is interesting to notice how a linear approximation is a good representation for voltage variations. As customers are single-phase, contributions to voltage rise or drop and reactive or active power flow variations differ from phase to phase. Both figures show that only phase b presents, as it would be expected on a balanced system, positive contribution from active power generation and negative contribution from reactive power consumption to voltage variation. Finally, the slopes of the obtained lines (i.e., a_{k2}) are always much higher for phase b (i.e. self-contribution) compared to phases a and c .

2) Second stage of the off-line polynomial regression

The overall off-line methodology was performed for each of the existing 83 PVs considering a typical summer week-day. The hypothesis formulated in Section II stating that the network generation level and the reference local voltage could be used to track the sensitivities obtained from the previous stage was verified. For instance, Fig. 5 shows the polynomial surface (4) for phase b remote voltage sensitivities to reactive power injections from the PV in house 17. Every dot represents the a_{k2} sensitivity coefficient obtained at each scenario (i.e. outer-loop iteration). The surface, which fits the trend of all possible values of sensitivities, basically expresses them as a function of the parameters that will be locally accessible during on-line estimations. The square of Pearson's coefficient R (0.75) is included. This value can be interpreted as the amount of a_{k2} variation explained by the selected independent variables.

Traditionally, values over 0.5 and 0.7 correspond to moderate and strong positive correlations respectively [1].

Presuming now that house 82 was connected to phase *c* instead of *b* and the voltage at house 17 (phase *b*) was still used for the estimation, a reduction on R^2 to 0.22 would be noticed. This correlation reduction is not a limitation of the methodology but of the local monitoring. For instance, the plot in Fig. 6 shows a correlation of $R^2=0.61$ if for the same hypothetical case a local phase *c* voltage was used for the estimation. This consideration is expected to extend to any methodology with equal monitoring limitations as the uncertainty in all phases cannot be captured with single phase measurements. However, as self-contributions are higher than variations on other phases it might not worth for DNOs to extend local observability.

3) Sensitivity analysis error

To rigorously assess the accuracy of the method, multiple scenarios are generated following again a MC approach. Nonetheless, the network generation level is not randomized but comes from a profile that varies from zero to 100% and is depicted in Fig. 7. As it can be observed, this profile is very smooth (i.e., negligible clouds). This will help to visually identify how the generation level affects the trend of the estimations' uncertainty.

We define the sensitivity analysis error δ as the percent error between real voltage variations and the ones estimated:

$$\delta = 100\% \left(\frac{\Delta V_{real} - \Delta V_{estimated}}{\Delta V_{real}} \right) \quad (9)$$

where $\Delta V_{estimated}$ comes from the integrals in (2) or (3). In both cases, the highest possible local power injection variations are considered (i.e. from lower to upper bound). Results are shown from Fig. 8 to Fig. 10. Recall that remote voltage sensitivities refer to a hypothetic three-phase node at house 82 (results correspond to phase self-contributions). Furthermore, we will account only for the time frame in which the generation level is different from zero (e.g., 5am to 7pm).

Fig. 8 includes the error for remote voltage sensitivities with respect to reactive power injections at house 17. Results are presented in 10-minute time-steps as boxplots where medium horizontal bars reveal median values, rectangles 50% of results distribution, slim black lines 99.3% and small crosses outliers. Errors show normal distributions quasi-centred on zero with results that are approximately within a 1.5% and 3% for 50% and 99.3% of the sample. It can be noticed that the slight deviation of the mean follows a pattern that resembles the generation level profile in Fig. 7; with more error at higher solar irradiances. Fig. 9 shows similar results for the case of remote voltage sensitivities to active power injections.

Fig. 10 shows the error for remote voltage sensitivities with respect to all houses reactive power injections at 12:30pm (i.e. generation level peak). Even if the behaviour of a higher number of loads and PVs must be englobed from local to remote point as we move away from house 82, error proves not to largely increase. This means that, independently of the distance between remote and local nodes and on them being placed in different radial ramifications, accuracy does not significantly diverge. Results are approximately within a 2% for 50% and within 3.5% for the 99.3% of all measurements.

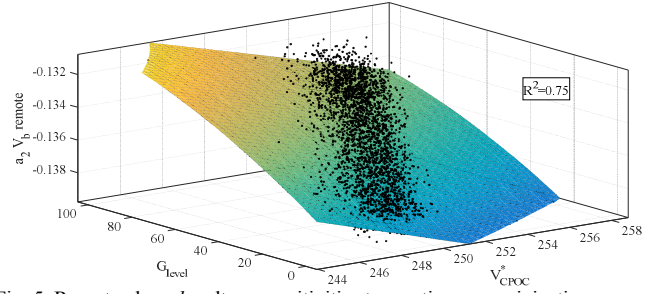


Fig. 5. Remote phase *b* voltage sensitivities to reactive power injections vs V_{CPOC}^* and G_{level}

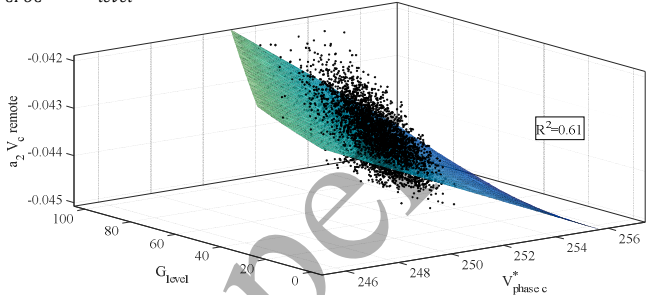


Fig. 6. Remote phase *c* voltage sensitivities to reactive power injections vs $V_{phase\ c}^*$ and G_{level}

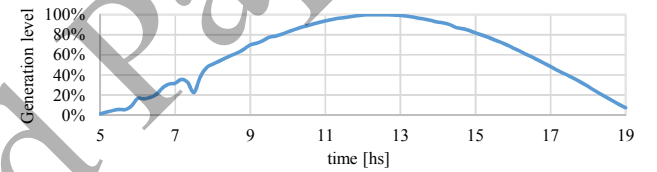


Fig. 7. Generation level profile

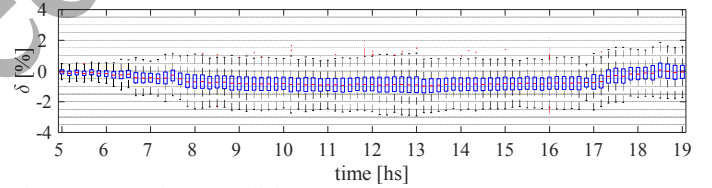


Fig. 8. Remote voltage sensitivity to Q_g error

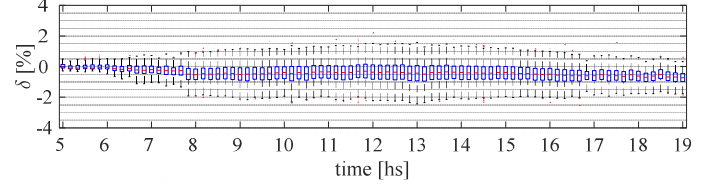


Fig. 9. Remote voltage sensitivity to P_g error

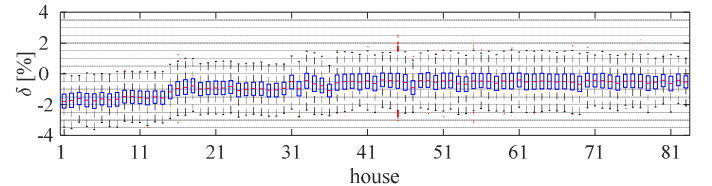


Fig. 10. Remote voltage sensitivity to Q_g error – All units at 12:30pm

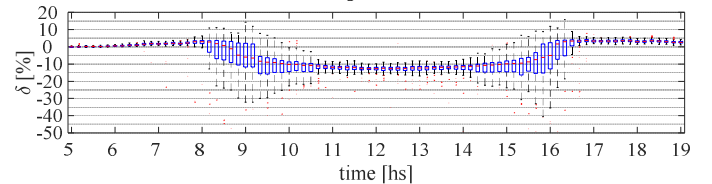


Fig. 11. Benchmark case considering constant sensitivities - Remote voltage sensitivity to Q_g error

As mentioned in Section I, the direct analytical equations [6]–[8] used in MV networks are not expected to hold with the same exactness at the LV level. However, none of the previous publications proposing widespread monitoring have benchmarked their results with these more simplistic methods to assess how big the accuracy gap may be. The values of δ in Fig. 11 correspond to sensitivities calculated using a single-phase line equivalent connecting the customer with the slack bus and neglecting all but local power injections (resulting in constant sensitivities). It can be seen how sensitivities are underestimated and show a relative error of up to 40% for 99.3% of the sample. This difference comes mainly from disregarding mutual impedances and the effect of voltage variations on nodal power injections. It is very interesting how the error is related to nodal power injections composition; with maximum deviation during demand peaks and mean value distancing from zero with the increment of solar irradiance.

IV. DISCUSSION

One of the key characteristics of this method is the avoidance of most computational hardness during real-time application (on-line estimations in Section III took only an average computation time of 0.4ms). Thanks to the use of the off-line precomputed polynomial models, there are no-iterative calculations (i.e., no risk of non-convergence). Most importantly, this is achieved without compromising the access to accurate sensitivities.

During the off-line simulations, a Monte Carlo approach is used to cope with the system uncertainties. While this non-trivial procedure is performed in anticipation of on-line regulations, its computational times can be still reduced if desired. For instance, as no multi-period simulations are solved, parallel processing tools can be utilized to significantly improve time efficiency [18]. Here, an Intel Xeon E5-2699 running 20 parallel instances (only 30% of CPU usage) allowed obtaining all sensitivities from Section III in a total of 16.7 hours (with an average of 3827 iterations needed for each MC to converge). In addition, other techniques such as data clustering [1] could be used to simplify calculations even more.

It must be mentioned that for a good performance, validated network data is required. While access to this information has traditionally been unusual within DNOs, data gathering campaigns prove that this reality is changing [15]. Additionally, regarding scalability, variations on the network characteristics (e.g. new customers or PV, reconnections, etc.) will start affecting the method accuracy until recalculating the polynomials (same as with any analytical method). Fortunately, this is infrequent within residential LV feeders.

Distribution grid codes may require a three-phase PV connection if the rated power exceeds a certain limit. In that case, that would need to be accounted for in the network model but will not affect the off-line algorithm from Fig. 1. During on-line operation, local and remote voltage sensitivities will still be estimated based on the local voltage at each phase; as described in Section II.B. In such cases, the inherent access to local three-phase monitoring can allow for accurate sensitivities independently of the connection phase of remote customers.

V. CONCLUSIONS

This paper has introduced a methodology for decentralized-autonomous estimation of voltage sensitivities in LV feeders with PVs. This novel procedure uses an off-line MC approach and regression analysis to generate polynomials describing the evolution of multiple parameters under different operation points. In line with the current lack of observability on LV feeders, the methodology relies only on local single-phase voltage measurements at each local CPOC. Crucially, no load monitoring or communication between units is required.

Results from a thorough accuracy assessment places the methodology in a very encouraging position, especially when considering the context in which sensitivities are estimated: no knowledge of the system state, stochastic load behaviour and real-time voltage data limited to the local CPOC (already available from the PV inverters). This, together with very reduced on-line computation times and the lack of convexity issues (key features during real-time application) give the method high practical value from DNOs' perspective.

REFERENCES

- [1] V. Rigoni, L. F. Ochoa, G. Chicco, A. Navarro-Espinosa, and T. Gozel, "Representative residential LV feeders: A case study for the North West of England," *IEEE Trans. Power Syst.*, vol. 31, no. 1, 2016.
- [2] F. Tamp and P. Ciufo, "A sensitivity analysis toolkit for the simplification of MV distribution network voltage management," *IEEE Trans. Smart Grid*, vol. 5, no. 2, pp. 559–568, 2014.
- [3] A. Keane, L. F. Ochoa, E. Vittal, C. J. Dent, and G. P. Harrison, "Enhanced utilization of voltage control resources with distributed generation," *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 252–260, 2011.
- [4] R. Aghatehrani and R. Kavasseri, "Sensitivity-Analysis-Based Sliding Mode Control for Voltage Regulation in Microgrids," *Sustain. Energy, IEEE Trans.*, vol. 4, no. 1, pp. 50–57, 2013.
- [5] C. Murphy and A. Keane, "Local and Remote Estimations using Fitted Polynomials in Distribution Systems," *IEEE Transactions on Power Systems*, vol. PP, no. 99, p. 1, 2016.
- [6] M. Brenna et al., "Automatic distributed voltage control algorithm in smart grids applications," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 877–885, 2013.
- [7] R. Yan and T. K. Saha, "Voltage variation sensitivity analysis for unbalanced distribution networks due to photovoltaic power fluctuations," *IEEE Trans. Power Syst.*, vol. 27, pp. 1078–1089, 2012.
- [8] K. Youssef, "A New Method for Online Sensitivity-Based Distributed Voltage Control and Short Circuit Analysis of Unbalanced Distribution Feeders," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1253–1260, 2015.
- [9] A. Abur and A. Gomez Exposito, *Power System State Estimation: Theory and Implementation*. 2004.
- [10] M. C. De Almeida and L. F. Ochoa, "An Improved Three-Phase AMB Distribution System State Estimator," *IEEE Trans. Power Syst.*, vol. 32, no. 2, pp. 1463–1473, 2017.
- [11] S. Weckx, R. D'Hulst, and J. Driesen, "Voltage Sensitivity Analysis of a Laboratory Distribution Grid With Incomplete Data," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1271–1280, 2015.
- [12] G. De Carne, M. Liserre, and C. Vournas, "On-line load sensitivity identification in LV distribution grids," *IEEE Transactions on Power Systems*, vol. PP, no. 99, p. 1, 2016.
- [13] S. C. Chapra and R. P. Canale, *Numerical methods for engineers.*
- [14] K. McKenna and A. Keane, "Residential Load Modeling of Price-Based Demand Response for Network Impact Studies," *IEEE Transactions on Smart Grid*, vol. PP, no. 99, p. 1, 2015.
- [15] ENWL, "LV network models," *Low Voltage Network Solutions*, 2014.
- [16] Energy Saving Trust, "Solar Energy Calculator Sizing Guide."
- [17] *ESB Networks Limited, "Distribution Code," 2016.*
- [18] A. T. Procopiou, J. Quiros-Tortos, and L. F. Ochoa, "HPC-Based Probabilistic Analysis of LV Networks with EVs: Impacts and Control," *IEEE Trans. Smart Grid*, vol. 8, no. 3, pp. 1479–1487, 2017.