A Robust Skewness-Kurtosis Descriptor for Damping Calibration from Frequency Response

Vesna Jaksic and Vikram Pakrashi

Abstract

This paper attempts to define simple, consistent and robust statistical descriptors for the calibration of damping in linear and non-linear systems through frequency response in the presence of variability and uncertainties due to noise and sampling intervals. The work employs the frequency response of a linear system and a Duffing Oscillator simulating hardening and softening springs. The Skewness – Kurtosis descriptor was observed to be efficiently calibrating the nature of the system and the extent of damping with robustness against measurement noise and sampling effects. The descriptors allow rapid computation and can be applied to experimental data without the requirement of assuming a specific underlying model. The findings are general and applicable to a very broad spectrum of linear and non-linear systems and applications.

Keywords: Damping, Non-linear Systems, Frequency Response, Skewness, Kurtosis, Aerospace Engineering

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INTRODUCTION

The identification of damping in a structural system is often a very important, but difficult problem. A main reason behind this difficulty is the lack of information regarding the energy dissipation method of the system. Even when the dissipation mechanism can be acceptably modeled as an equivalent viscous damping ratio for practical applications, the identification of the extent of damping can be problematic due to the lack of information on the linearity or the non-linearity and condition of the system. For linear structures, a significant number of classical and new approaches are available to establish damping including the use of logarithmic decrement (Lamarque, et al., 2000), energy loss per cycle, frequency response function (Clough and Penzien, 1993) and the analyses of vibration response in time and frequency (Prandina, et al., 2009) or time-frequency (Yin, et al., 2004) domain. The description or calibration of damping, even under the assumption of an equivalent viscous damping ratio is scarce for non-linear systems. In fact, within the domain of definition of equivalent viscous damping ratios, sub-critical damping of low magnitude (0-10%) tends to govern the dynamics of an extremely wide range of elastomechanical systems. Of all the approaches of characterizing damping in a system, the use of frequency response functions (Phani and Woodhouse, 2007, Yin, 2010) is beneficial. The frequency response functions, when available, tend to characterize the linear or the non-linear system they belong to. The shapes of the functions are affected by damping and this opens up a possibility of exploring simple, robust and consistent descriptors to calibrate damping ratios employing the entire curve. Additionally, the frequency response functions can be directly related to the efficiency and capacity of a number of energy harvesting devices (Ali, et al., 2010, Challa, et al., 2009, Mann and Sims, 2009). Consequently, an appropriate descriptor for damping calibration can be related to the performance of energy harvesters as well. Duffing like systems also have a potential to act as vibration absorbers by tuning their dynamic characteristics. Filtering based method for parameter estimation for linear and non-linear SDOF system that acts as Duffing oscillator is discussed by (Khalil, et al., 2009). In Multi degree of freedom (MDOF) systems it is usually not found that all the segments are non-linear, but rather one segment is extremely non-linear and the others are nearly linear in behaviour. For example, structural systems such as damaged beam (two DOF system) can be modeled as bilinear damped mechanical system of SDOF (Peng, et al., 2007).

This paper explores and recommends a simple, consistent and robust statistical descriptor to calibrate damping ratios in linear and non-linear systems where the non-linear system is modeled as a Duffing oscillator in the form of hardening and softening springs. The validation of the descriptors on these systems automatically opens up possibilities of application on a vast number of very disparate and interesting fields (De and Aluru, 2006, Michon, et al., 2008). The general approach and the findings are immediately applicable under model free conditions for frequency responses that typically contain a single significant global extremum within the analysis window. With slight modification on windowing, it is also readily applicable for responses with multiple significant extrema. Noise stress tests and effects of sampling rate variability have been investigated to establish the robustness, efficiency and the consistency of the descriptors.

FREQUENCY REPONSE OF LINEAR AND DUFFING SYSTEMS

A non-linear single degree of freedom (SDOF) system in the form of a Duffing Oscillator is considered in this paper in order to establish and illustrate proposed approach. The linear SDOF system is simply a special case of the nonlinear equation. Although a great number of studies have looked into the computational or phenomenological aspects (Cross and Worden, 2009, Ravindra and Mallik, 1994), a practical description of such a system through frequency response does not seem to have been approached.

The non-linear SDOF system is a Duffing Oscillator governed by the non-dimensional equation

$$\ddot{x} + 2\xi\dot{x} + x + \alpha x^3 = \mathrm{Cos}(\Omega\tau) \tag{1}$$

where *x* is the non-dimensional displacement for the non-linear single degree of freedom Duffing Oscillator, ξ is the equivalent viscous damping ratio, α is a constant proportional to the cubic non-linearity of the hardening or softening dynamical system, *F* is the non-dimensional amplitude of the harmonic force with non-dimensional frequency Ω impressed upon the system and τ is the non-dimensional time parameter. These parameters are defined as $\xi = \frac{c}{2m\omega_n}; \omega_n = \sqrt{\frac{k_1}{m}}; \alpha = \frac{k_3 x_0^2}{k_1}; \Omega = \frac{\omega}{\omega_n}; \tau = \omega_n t$ where k_1 forms the linear part of the stiffness of the system and k_3, the cubic non-linear part. Consequently, the term α represents the ratio of the non-linear and the linear stiffness, since $x_0 = \left.\frac{F}{k_1}\right|_{k_3=0,\omega=0}$ and ω is the frequency of the harmonic excitation and F is the amplitude of the harmonic excitation. The term ω_n is not the natural frequency but a characteristic frequency of the linearised system. The term t represents time. The overdots in equation 1 represent differentiation with respect to the non-dimensional time parameter. A range of non-dimensional frequency ratio between 0 and 2 is considered throughout the paper. The frequency response of this non-linear system can be given as the roots of a quadratic function in the form:

$$\Omega_1 = \sqrt{(1+\frac{3}{4}\alpha x^2 - 2\xi^2) - \frac{1}{x}\sqrt{1-4\xi^2 x^2(1-\xi^2+\frac{3}{4}\alpha x^2)}} \qquad (2)$$

and

$$\Omega_2 = \sqrt{(1+\frac{3}{4}\alpha x^2 - 2\xi^2) + \frac{1}{x}\sqrt{1-4\xi^2 x^2(1-\xi^2+\frac{3}{4}\alpha x^2)}} \qquad (3)$$

where the subscripts of Ω represent the roots.

The Duffing Oscillator, with α=0 behaves as a linear system. Figure 1 presents the response amplitude against the non-dimensional frequency for a linear case, and for situations related to non-linear coefficient value, i.e. maximum (α_{max}), critical (α_{crit}) and limit (α_{lim}) respectively. The value $|\alpha|_{max} = \frac{4}{3}\xi^2$ limits the value α can assume for a jump phenomenon to take place, $|\alpha|_{crit} = \frac{2^8}{3^{(5/2)}}\xi^3$ provides the limit of jump avoidance and $|\alpha|_{lim} = 2\frac{16}{3}\frac{\xi^2(1-\xi^2)}{\xi^4}$ is the situation where the jump down frequency is equal to the natural frequency. These values of α can be readily computed and interpreted following existing work (Carrella, 2008). It is clearly observed that the shapes of the curves are governed by the type of non-linearity (hardening for cases where α>0 or softening where α<0), the degree of non-linearity and the damping ratio. Also, the hardening and the softening p arts are generally not symmetric about the linear response.

Existing works (Peng, et al., 2007; Yin, 2010) also represent similar frequency reponse conditions that can be associated to a change of shape of frequency response to characterise the system parameters. This change of shape is the motivation behind this work. The next section presents the appropriate descriptors of damping ratio.

SKEWNESS-KURTOSIS DESCRIPTORS FOR DAMPING CALIBRATION

Figure 1, in addition to their change of shape, illustrated that the frequency response curves tend to have a single significant global maximum within the domain of definition. The global maximum is being referred to here since a number of local maxima can form when the ideal curve is corrupted by measurement noise. Under these circumstances, in terms of the description of shape properties, the resemblance of the frequency response curves with probability distributions are exploited. Consequently, the statistical moments of the discretely sampled curves forming the frequency response describe the shape of the curves. Of the various simple descriptors (Pakrashi et al., 2009[a]) available in this regard, the skewness and the kurtosis are chosen since the nature of the system and the damping ratio is observed to significantly affect the peakedness and the symmetry of the frequency response. The uses of these statistical descriptors have become popular in the field of structural health monitoring in recent years (Hadjileontiadis and Douka, 2007; Hadjileontiadis, et al., 2005; Pakrashi, et al., 2007; Pakrashi et al.,2009[b]). In a sense, it is attempted to relate the nature of the system and the damping ratio through a relative deviation from the statistical moments obtained from a Gaussian function. The skewness and the kurtosis of a discretely sampled curve is computed as

$$\lambda = \frac{\frac{1}{N}\sum_{i=1}^{N}(f_i - \mu)^3}{\left(\sqrt{\frac{1}{N}\sum_{i=1}^{N}(f_i-\mu)^2}\right)^3} \quad (4)$$

and

$$\kappa = \frac{\frac{1}{N}\sum_{i=1}^{N}(f_i - \mu)^4}{\left(\frac{1}{N}\sum_{i=1}^{N}(f_i-\mu)^2\right)^2} \quad (5)$$

where λ is the skewness, κ is the kurtosis, f is a discretely sampled function, N is the number of points at which the function f is discretely sampled and μ is the mean of the function f in this regard.

Figure 2 presents calibration of a range of damping ratio employing skewness and kurtosis descriptors when system have linear (α_{lin}) or non-linear (α_{max}, α_{crit} and α_{lim}) behavior in the case of softening (α_{maxsof}, $\alpha_{critsof}$ and α_{limsof}) and hardening(α_{maxhar}, $\alpha_{crithar}$ and α_{limhar}). The values of skewness and kurtosis for observed damping ratios are given in Table1. The variation of calibration is relatively high for equivalent viscous damping ratios up to 2%. The variation may be explained by considering the frequency response functions presented in Figure 1 where very low damping ratios form high and thin frequency peaks. Consequently, the variations associated with very low damping ratios are relatively higher. A kurtosis based calibration of damping ratio is observed to be monotonic, consistent and significantly independent of the system non-linearity. The level of calibration values can be easily related to the system damping, while the relative change in the region of damping under consideration allows obtaining a practical and appropriate resolution. Unless the system becomes exceptionally non-linear, these observations hold true. Consequently, a kurtosis based calibration is extremely useful. A skewness based calibration provides a consistent and monotonic calibration against damping as well. However, it is significantly dependent on the type and the level of non-linearity present in the system. The calibration curves are distinctly bunched according to whether the system is significantly soft, hard or more or less linear. This observation leads to the proposition of a combined skewness-kurtosis descriptor based calibration. Given the kurtosis, the damping ratio can be calibrated independent of the system. The value of skewness at that level of damping estimates the type and the degree of non-linearity of the system. At a practical level it is helpful to establish an estimate of the equivalent viscous damping ratio first and then establish the type of nonlinearity by at a skewness value corresponding to this estimated damping ratio. Skewness also independently provides and estimate of damping and consequently this operation may be carried out iteratively. However, for many practical purposes such iterations may not be required. The method is immediately transferable to multi-degree-of-freedom (MDOF) systems when the frequency response functions of each mode are available or when the frequency response peaks corresponding to the degrees of freedom are appropriately separated. For relatively closely spaced peaks, calculations similar to what have been presented here may be carried out.

Figure 3 and Figure 4 computes and graphically demonstrates best-fit power law curves for the calibration through kurtosis and skewness respectively corresponding to the different system behavior, i.e. $\alpha=0$ (linear); $\alpha>0$ (hardening) or $\alpha<0$ (softening). Additionally, a calibration table (Table 1) is provided to establish a one to one relationship between the equivalent viscous damping ratio with the skews and kurtosis respectively. The robustness of these calibrations under noise effects and variable rates of sampling will establish their usefulness as descriptors for practical purposes and for a potentially wide range of applications.

EFFECTS OF NOISE AND SAMPLING RATE

To establish the effect of noise, the situation where the discretely measured data is corrupted by significantly high levels of (up to as high as 100%) Gaussian white noise has been considered. The percentage noise in this case has been defined as the percentage ratio of the integrals of the squared signal and the noise amplitudes respectively. The nature of the noise is not very important here, as the resistance of the calibrations against broadband noise is to be demonstrated. We have considered a wide range of noise levels for the various calibrations and have established the distribution of the calibration values about the calibration values obtained from pure data. The descriptors were observed to be defined within tight standard deviation bands of calibration. The deviations of calibrations employing kurtosis is presented in Figure 5 while that corresponding to skewness based calibration is presented in Figure 6 for all of the cases considered. Kurtosis calibrations tend to form a tightly defined master curve for each case even in the presence of significant noise. It is also observed that the standard deviation bounds are smaller for a relatively higher damping ratio. This is related to how close the uniformly distributed discrete samples are in the kurtosis axis. For low damping, the separations are high even for relatively closely spaced non-dimensional frequency ratios and a measurement noise can significantly affect the shape of the curve near the global maxima. The effect reduces for higher damping as the separation in the kurtosis axis becomes relatively lesser. The variation about the mean calibration value is higher for skewness.

Consequently, it is recommended to use kurtosis values for a more stable estimate of damping ratio. In both cases, however, the calibration was observed to be fairly stable in the presence of high noise for all different cases considered and the skewness-kurtosis descriptor pair may be considered to be robust against noise.

The effects of variations in sampling rate of the non-dimensional frequency ratio are presented in Figure 7. Both kurtosis and skewness calibrations are considered in this regard where α_{lim} (hard) describes the non-linear system. It is observed, that barring extremely low damping ratios (as low as 1%), the calibration values are relatively independent of the discrete sampling rate covering wide logarithmic ranges of sampling rates. The calibrations presented in this paper are thus extremely consistent against sampling rates as well. The tests against noise and sampling rate establish a high degree of confidence on the proposed dual descriptors.

CONCLUSIONS

A skewness-kurtosis descriptor has been presented for the calibration of damping ratios in linear and non-linear systems. The validation of the proposed descriptor has been carried out on the frequency response of a Duffing Oscillator. The calibrations are observed to be fast, computationally simple, consistent and robust against measurement noise and sampling rates. The kurtosis measure tends to characterize the damping while the skewness measure is more also important for characterizing the type and the degree of non-linearity in the system. The calibrations can be applied to discretely sampled experimental data and can be used without assuming specific dynamical models. The findings of this paper are general and the application can be in a very wide range of applications including system identification, energy harvesting and adaptive control of dynamical systems.

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REFERENCES

Ali, S.F., Friswell, M.I. and Adhikari, S.(2010) "Piezoelectric Energy Harvesting with Parametric Uncertainty". *Smart Materials and Structures*, 18, pp. 105010 (1-9).

Carrella, A.(2008) "Passive Vibration Isolators With High-Static-Low-Dynamic-Stiffness". *Faculty of Engineering, Science and Mathematics; Institute of Sound and Vibration Research*, Doctor of Philosophy(51276), pp. 226.

Challa, V.R., Prasad, M.G. and Fisher, F.T.(2009) "A Coupled Piezoelectric-Electromagnetic Energy Harvesting Technique for Achieving Increased Power Output through Damping Matching ". *Smart Materials and Structures*, 18(9), pp. 095029 (1-11).

Clough, R.W. and Penzien, J.(1993) *Dynamics of Structures*. McGraw-Hill Book Co., Singapore.

Cross, E.J. and Worden, K.(2009) "Approximation of the Duffing Oscillator Frequency Response Function using the FPK Equation,". *Journal of Physics: Conference Series*, 181(1), pp. 012085 (1-9).

De, S.K. and Aluru, N.R.(2006) "Complex Non-linear Oscillations in Electrostatically Actuated Microstructures". *Journal of Microelectromechanical Systems*, 15(2), pp. 355-369.

Hadjileontiadis, L.J. and Douka, E.(2007) "Kurtosis Analysis for Crack Detection in Thin Isotropic Rectangular Plates". *Engineering Structures*, 29(9), pp. 2353 – 2364.

Hadjileontiadis, L.J., Douka, E. and Trochidis, A.(2005) "Crack Detection in Beams using Kurtosis". *Computers and Structures*, 83, pp. 909-919.

Khalil, M., Sarkar, A. and Adhikari, S.(2009) "Non-linear filters for chaotic oscillatory systems". *Non-linear Dynamics*, 55(1-2), pp. 113-137.

Lamarque, C.H., Pernot, S. and Cuer, A.(2000) "Damping Identification in Multi-Degree-of-Freedom Systems via a Wavelet-Logarithmic Decrement – Part 1:Theory". *Journal of Sound and Vibration*, 235(3), pp. 361-374.

Mann, B.P. and Sims, N.D.(2009) "Energy harvesting from the Non-linear Oscillations of Magnetic Levitation". *Journal of Sound and Vibration*, 319(1-2), pp. 515–530.

Michon, G., Manin, L., Parker, R.G. and Dufour, R.(2008) "Duffing Oscillator with Prametric Excitation: Analytical and Experimental Investigation on a Belt-Pulley System". *ASME Journal of Computational and Non-linear Dynamics*, 3, pp. 031001(1-6).

Pakrashi, V., Basu, B. and O' Connor, A.(2007) "Structural damage detection and calibration using a wavelet-kurtosis technique". *Engineering Structures*, 29(9), pp. 2097-2108

Pakrashi, V., Basu, B. and O' Connor A.(2009)."Non-Detection, False Alarm and Calibration Insensitivity in Kurtosis and Pseudofractal based Singularity Detection". *ASCE Journal of Aerospace Engineering, 22(4), 466-470.*

Pakrashi, V., Basu, B. and O' Connor A.(2009)."A Statistical Measure for Wavelet based Singularity Detection". *ASME Journal of Vibrations and Acoustics, 131(4), 041015-1-041015-6.*

Peng, Z.K., Lang, Z.Q., Billings, S.A. and Lu, Y.(2007) "Analysis of bilinear oscillators under harmonic loading using non-linear output frequency response functions". *International Journal of Mechanical Sciences*, 49(2007), pp. 1213–1225.

Phani, S.A. and Woodhouse, J.(2007) "Viscous Damping Identification in Linear Vibration". *Journal of Sound and Vibration*, 303(3-5), pp. 475-500.

Prandina, M., Mottershead, J.E. and Bonisoli, E.(2009) "An Assessment of Damping Identification Methods". *Journal of Sound and Vibration*, 323(3-5), pp. 662-676.

Ravindra, B. and Mallik, A.K.(1994) "Role of Non-linear Dissipation in Soft Duffing Oscillators". *Physical Review E*, 49(6), pp. 4950-4954.

Yin, H.P.(2010) "An Average Inverse Power Ratio Method for the Damping Estimation from a Frequency Response Function ". *Mechanical Systems and Signal Processing*, 24(3), pp. 617-622.

Yin, H.P., Duhamel, D. and Argoul, P.(2004) "Natural Frequencies and Damping Estimation using Wavelet Transform of Frequency Response Function ". *Journal of Sound and Vibration*, 271(3-5), pp. 999-1014.

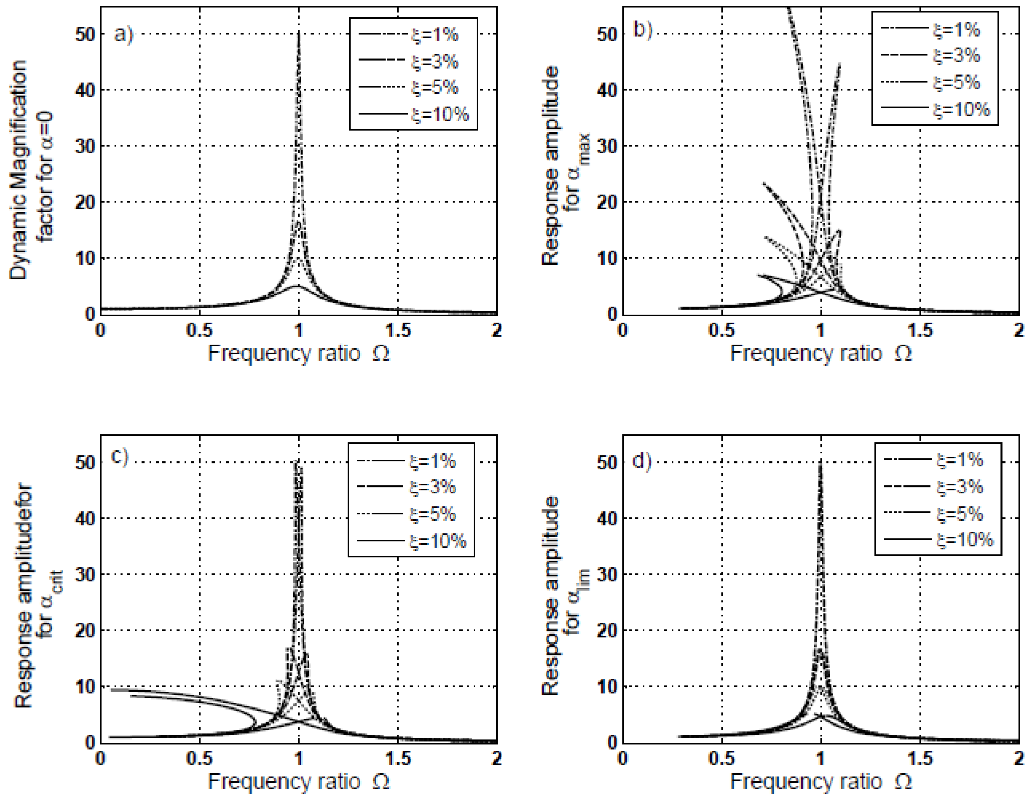

Figure 1

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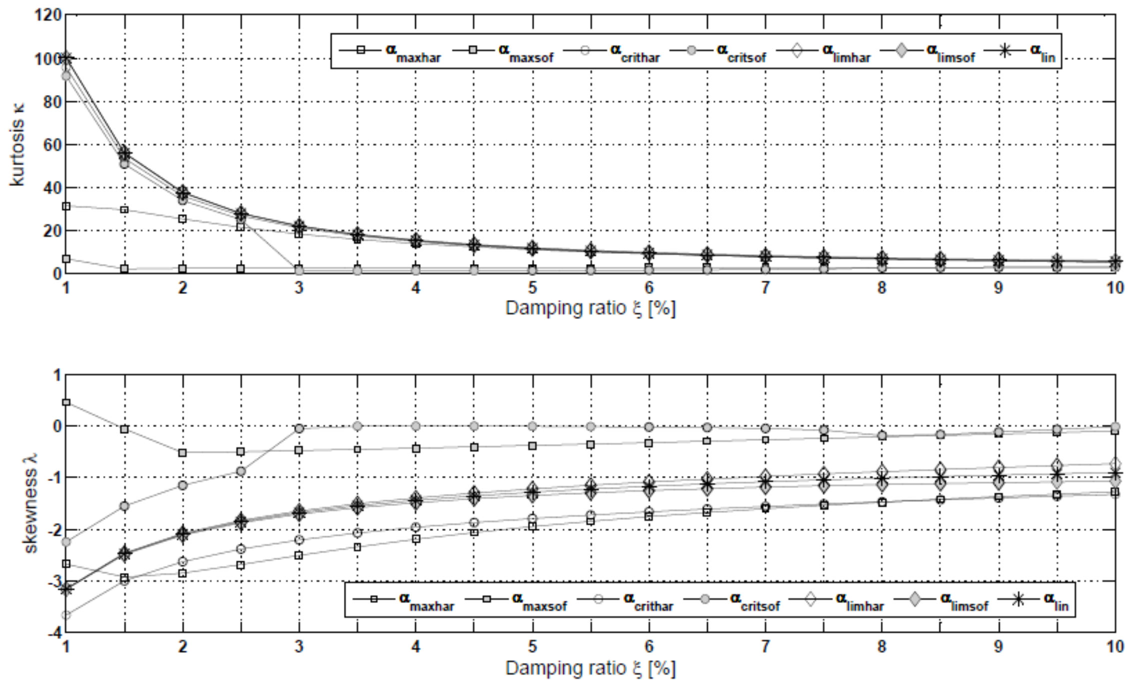

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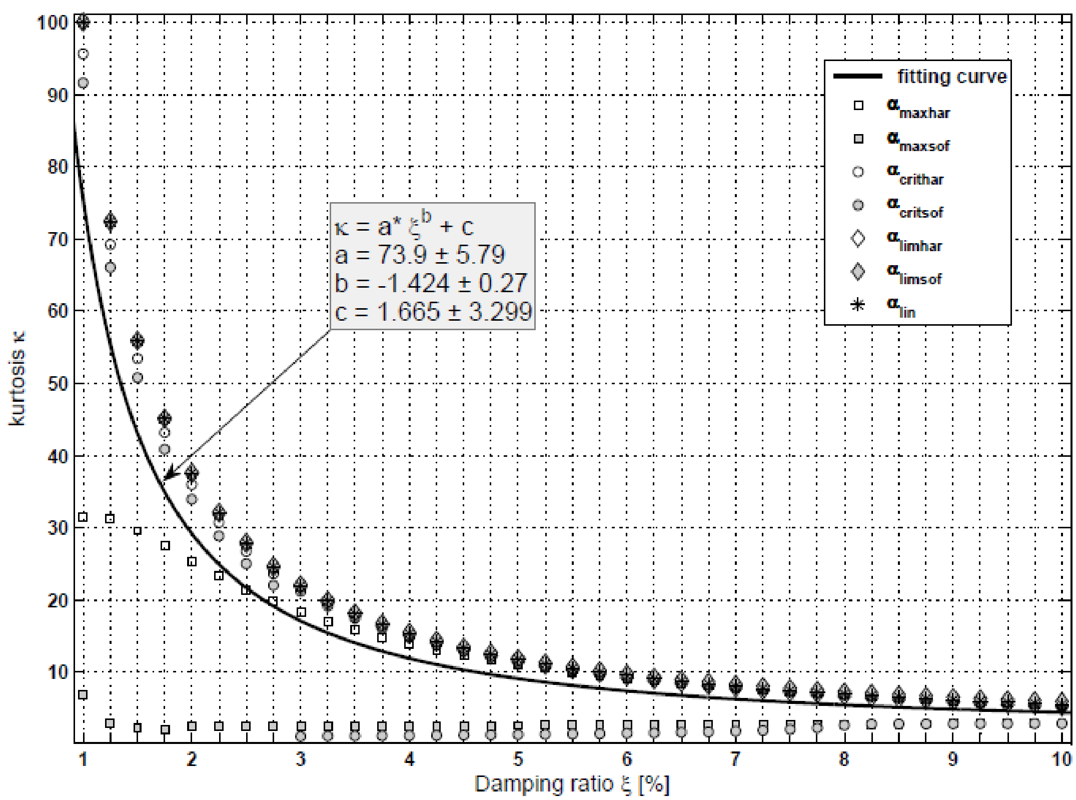

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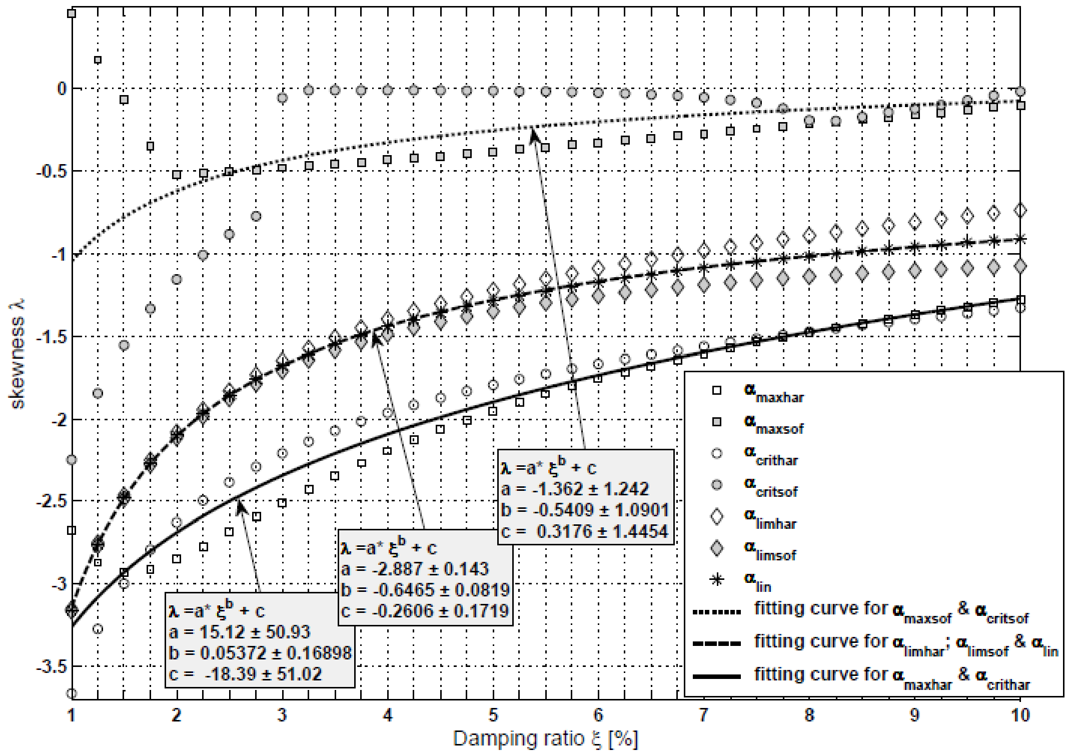
Figure 3



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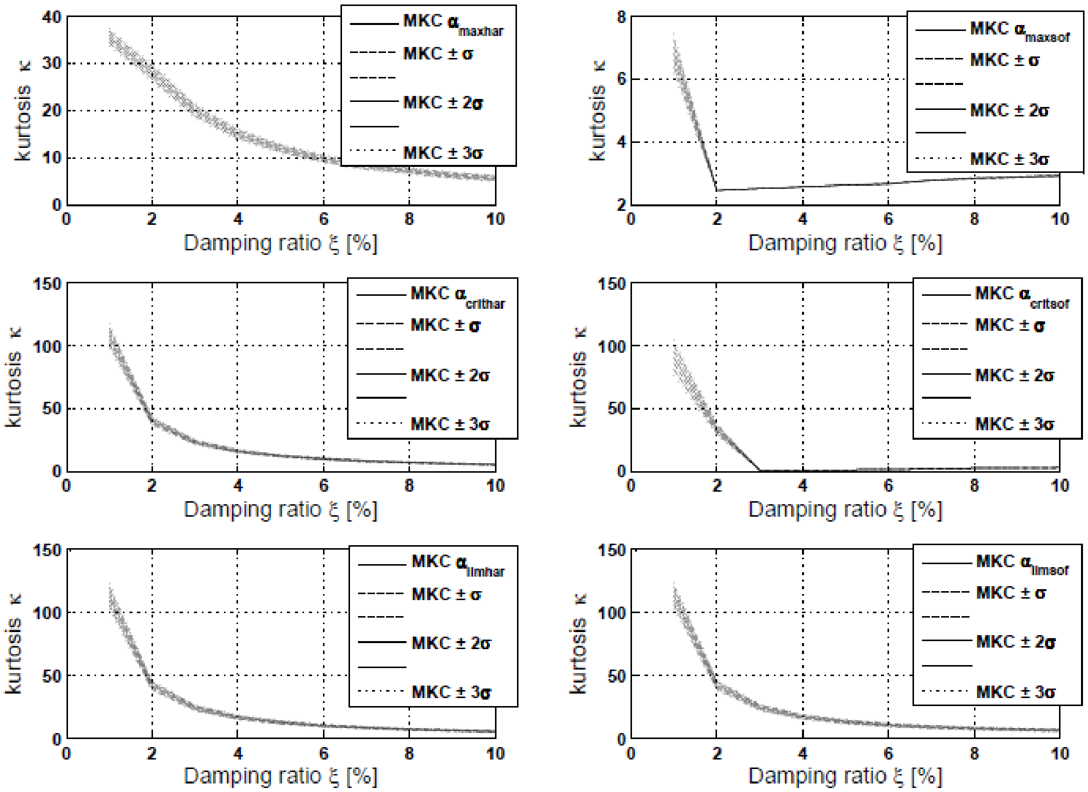
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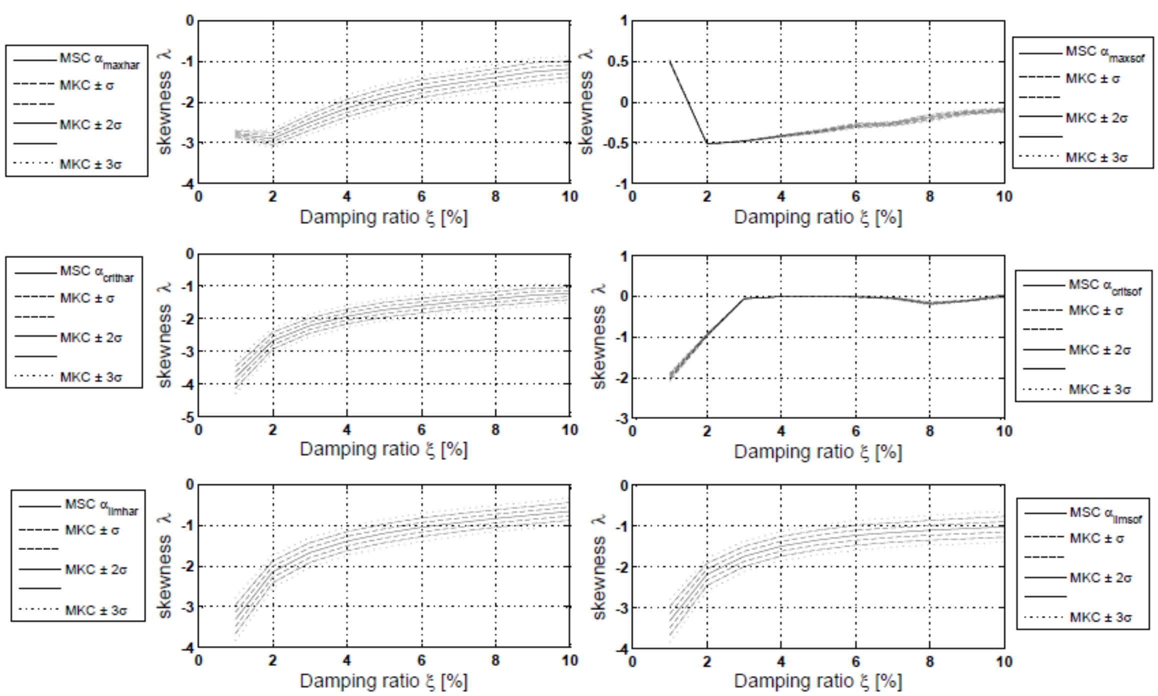
Damping ratio ξ [%]

Figure 5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Damping ratio ξ [%]

Figure 6

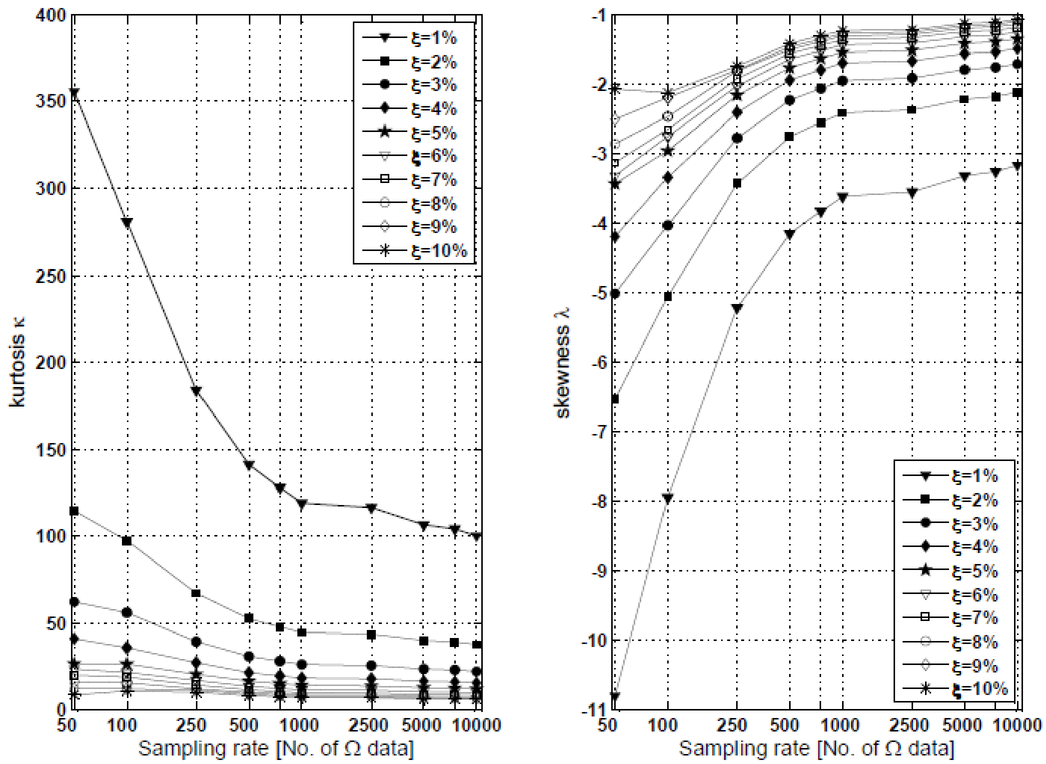

Figure 7