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# A Distribution Network Expansion Planning Model considering Distributed Generation Options and Techno-Economical Issues

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## Abstract

This paper presents a dynamic multi-objective model for distribution network expansion, considering the distributed generations as non-wire solutions. The proposed model simultaneously optimizes two objectives namely, total costs and technical constraint satisfaction by finding the optimal schemes of sizing, placement and specially the dynamics (i.e., timing) of investments on DG units and/or network reinforcements over the planning period. An efficient heuristic search method is proposed to find non-dominated solutions of the formulated problem and a fuzzy satisfying method is used to choose the final solution. The effectiveness of the proposed model and search method are assessed and demonstrated by various studies on an actual distribution network.

*Key words:* Distributed generation, Fuzzy satisfying method, Soft constraint handling, Immune algorithm, Multi-objective optimization.

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## List of Symbols

$P_{t,dl}^{grid}$  Active power purchased from grid in year  $t$  and demand level  $dl$

$Y_{ij}^t$  Admittance magnitude between bus  $i$  and  $j$ , in year  $t$

$\theta_{ij}^t$  Admittance angle between bus  $i$  and  $j$ , in year  $t$

$S_{t,dl}^{grid}$  Apparent power imported from grid in year  $t$  and demand level  $dl$

$I_{\ell,t,dl}$  Current magnitude of  $\ell^{th}$  feeder in year  $t$  and demand level  $dl$

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- $Cap_\ell$  Capacity limit of potential feeder  $\ell$
- $Cap_{tr}$  Capacity limit of potential transformer
- $\bar{I}_{\ell,s}^t$  Critical operating limit of feeder  $\ell$  in year t
- $\bar{S}_{tr,c}^t$  Critical operating limit of existing substation feeding the network, in year t
- $\mu^{f_k(X_n)}$  Degree of minimization satisfaction of  $k^{th}$  objective function by solution  $X_n$
- d Discount rate
- $\tau_{dl}$  Duration of demand level  $dl$
- $IC_{dg}$  Investment cost of a DG unit
- $C_\ell$  Investment cost of feeder  $\ell$
- $C_{tr}$  Investment cost of transformer in substation
- $d_\ell$  Length of feeder  $\ell$  in km
- $\bar{P}_{lim}^{dg}$  Maximum operating limit of a DG unit
- $\varsigma^{\max}$  Maximum mutation probability
- $\varsigma_m$  Mutation probability in  $m^{th}$  cloning process.
- $N_b$  Number of buses in the network
- $N_p$  Number of population
- $N_\ell$  Number of feeders in the network
- $N_O$  Number of objective functions
- $N_{dl}$  Number of considered demand levels
- $OC_{dg}$  Operation cost of a DG unit
- T Planning horizon

## 1. Introduction

Distributed Generations (DGs) are defined as the small power resources, located closely to the load points [1]. The role of DG units is increased in the last decade by providing different benefits like cost reduction, reliability of supply, ancillary services, emission reduction, postponement of the transmission and distribution expansion for DG-owner, Distribution Network Operators (DNO) and socio-political acceptance [2, 3]. The traditional method for improving the technical and economical performance of a distribution network, is investing in network components. In some market models, the DNO is authorized to install DG units in his territory [4] along with network reinforcement [4, 2]. However, in some power markets, the DNOs are unbundled from DG ownership while it is done by non-DNO entities [5]. Different models have been proposed in the literature addressing the integration of DG units in distribution networks which consider different objectives, including technical (voltage profile[6], voltage stability improvement [7]), economical (network investment deferral [8, 9], active loss reduction [10, 11, 12, 13, 14]) and environmental (emission reduction [15]) issues. One way of treating with multi-objective problems is converting them into a single objective model [16, 17, 13, 6]. This may deprive the planner of having a set of solutions to do tradeoff analysis. The Pareto optimality concept [18] is used in some models [10, 19, 12, 14, 20] to overcome this problem. These models have some benefits such as: it is not necessary to resolve the problem if the priorities of objectives are changed and they can easily deal with incommensurable objective functions [18]. However, there are some shortcomings associated to the reported multi-objective models of DG-owned DNO, such as: first, they are static and all investments are designed to be done at the beginning of the planning horizon to meet the load at the end of planning period [12]. Considering the time value of money confirms that this assumption makes the planning procedure unrealistic because the solution is proposed to satisfy the load at the end of the planning horizon but implemented at the beginning of it. The second problem is that they do not simultaneously consider the network and DG investment and just use one of their planning options namely, DG units [12, 19, 20] or reinforcement of the distribution network [14].

In this paper, a planning model for distribution system is formulated which is multi-

objective, dynamic and also considers DG units as a planning option along with network reinforcement for DNO. A two-stage algorithm is proposed to solve the problem. In the first stage, the set of Pareto optimal solutions is found using a hybrid multi-objective Immune Genetic Algorithm (IGA), and in the second stage, the best solution is chosen using a fuzzy satisfying technique. The proposed model aims to provide a comprehensive multi-objective model which covers at all three aspects of placement, sizing and timing of DG and network investments simultaneously. The main contributions of this paper are:

1. A dynamic multi-objective dynamic integrated DG and distribution network planning model is proposed.
2. An efficient hybrid heuristic search method is proposed for solving the proposed model

This paper is set out as follows: section 2 presents the problem formulation, section 3 sets out the proposed solution method for solving the problem. The simulation results of the proposed model and solution method are presented in section 4 and finally, section 5 states the findings of this work.

## 2. Problem Formulation

The proposed planning model is formulated and presented in this section. The decision variables are defined as the number of DG units from each specific technology, to be installed in bus  $i$ , in year  $t$ , i.e.,  $\xi_{i,t}^{dg}$ ; reinforcement decision in feeder  $\ell$ , in year  $t$ , i.e.  $\gamma_t^\ell$ , which can be 0 or 1, and finally the number of new installed transformers in year  $t$ , i.e.  $\psi_t^{tr}$ . The assumptions used in problem formulation, constraints and the objective functions are explained next.

### 2.1. Assumptions

The following assumptions are employed in problem formulation:

- The daily load variation is modeled using a load duration curve which is divided into  $N_{dl}$  discrete demand levels. Assuming a base load in the beginning of the planning horizon, i.e.  $S_{i,base}^D$ , a Demand Level Factor, i.e.  $DLF_{dl}$ , and a demand growth rate,

i.e.  $\alpha$ , the demand in bus  $i$ , in year  $t$  and in demand level  $dl$  is described as:

$$S_{i,t,dl}^D = S_{i,base}^D \times DLF_{dl} \times (1 + \alpha)^t \quad (1)$$

- The DNO purchases the energy from the main grid or/and produce it using DG units to supply the customers. In this paper, the variation of electricity price in each demand level is modeled by multiplication of two factors namely, base price, i.e.  $\rho$ , and a Price level Factor in demand level  $dl$ , i.e.  $PLF_{dl}$  [17].
- The DNO is authorized to invest in DG units and/or network components in an integrated framework.

## 2.2. Constraints

There are two types of constraints which should be satisfied in power system planning problems: soft and hard constraints [21]. The hard constraints should be fully satisfied and no violation of them is accepted. However, there are some constraints called the soft constraints which their violation can be tolerated to some degree, in hope of achieving a better solution if other criteria are considered [21]. It is possible to deal with soft constraints the same as the hard constraints but this may cause narrowing the feasible solution space. For this reason, the constraints considered here are grouped into two separate categories: the hard and soft constraints. Each category is explained as follows:

### 2.2.1. Hard Constraints

The hard constraints considered here are explained next:

*Power Flow Constraints.* The power flow equations that should be satisfied in year  $t$  and demand level  $dl$  are:

$$\begin{aligned} -P_{i,t,dl}^D + \sum_{dg} P_{i,t,dl}^{dg} &= V_{i,t,dl} \sum_{j=1}^{N_b} Y_{ij}^t V_{j,t,dl} \cos(\delta_{i,t,dl} - \delta_{j,t,dl} - \theta_{ij}^t) \\ -Q_{i,t,dl}^D + \sum_{dg} Q_{i,t,dl}^{dg} &= V_{i,t,dl} \sum_{j=1}^{N_b} Y_{ij}^t V_{j,t,dl} \sin(\delta_{i,t,dl} - \delta_{j,t,dl} - \theta_{ij}^t) \end{aligned} \quad (2)$$

Where,  $P_{i,t,dl}^{dg}$  and  $Q_{i,t,dl}^{dg}$  are the active and reactive power produced by DG unit in year  $t$  and demand level  $dl$ , respectively.

*Operating limits of DG units* . The DG units should be operated considering the limits of their primary resources, i.e.:

$$P_{i,t,dl}^{dg} \leq \sum_{t=0}^t \xi_{i,t}^{dg} \times \bar{P}_{lim}^{dg} \quad (3)$$

Where,  $\sum_{t=0}^t \xi_{i,t}^{dg}$  denotes the investments done until year t.

The power factor of DG unit is kept constant [22, 23], as follows:

$$\cos\varphi^{dg} = \frac{P_{i,t,dl}^{dg}}{\sqrt{(P_{i,t,dl}^{dg})^2 + (Q_{i,t,dl}^{dg})^2}} = const. \quad (4)$$

### 2.2.2. Soft Constraints

The satisfaction of a soft constraint, in contrary to the hard constraints, is not described by a binary value (one or zero). Because the hard constraints are either fully satisfied or not, but in soft constraints, the satisfaction is defined as a number varying between zero and one. In [24] and [21], a fuzzy model is proposed to model the satisfaction of soft constraints. Fuzzy modeling is used to quantify the satisfaction of technical constraints of voltages and thermal limits of feeders and substation. This paper extends this concept to be used in the dynamic multi-year distribution network expansion problem with different demand levels as follows:

*Voltage profile.* The voltage magnitude of each bus should be kept between the safe operating limits. These limits are dependent on operating condition of the system under study. There are two ways to handle this constraint namely, considering it as a hard constraint [17, 19, 20] or considering it as a soft constraint [24, 21]. If the planner considers the voltage profile as a hard constraint, the violation of this constraint is not tolerated regardless of its severity and duration. But modeling it as a soft constraint helps him to tradeoff between the violation of them (degrading the system performance) and the associated cost saving.

The membership function of the voltage constraint satisfaction is represented by a trapezoidal fuzzy number [12] as depicted in Fig.1. Observe that a voltage magnitude between the upper and lower safe operating limits of bus i, i.e.  $\underline{V}_{i,s}, \bar{V}_{i,s}$ , has a satisfactory value of 1. As the voltage exceeds these limits, the value of satisfaction decreases until it

becomes zero after the critical voltage values of bus  $i$ , i.e.,  $\underline{V}_{i,c}$ ,  $\overline{V}_{i,c}$ . The upper and lower critical values of voltage in bus  $i$ , are defined as follows:

$$\begin{aligned}\overline{V}_{i,c} &= (1 + \bar{\epsilon}_i^V) \times \overline{V}_{i,s} \\ \underline{V}_{i,c} &= (1 - \underline{\epsilon}_i^V) \times \underline{V}_{i,s}\end{aligned}\quad (5)$$

Where,  $\bar{\epsilon}_i^V$  and  $\underline{\epsilon}_i^V$  are the upper and lower limits of permissible voltage constraint dissatisfaction in bus  $i$ , respectively.

The membership function of the voltage constraint satisfaction can be mathematically represented as:

$$\mu_{i,t,d}^V = \begin{cases} \frac{V_{i,t,dl} - \underline{V}_{i,c}}{\underline{V}_{i,s} - \underline{V}_{i,c}} & \underline{V}_{i,c} \leq V_{i,t,dl} \leq \underline{V}_{i,s} \\ 1 & \underline{V}_{i,s} \leq V_{i,t,dl} \leq \overline{V}_{i,s} \\ \frac{V_{i,t,dl} - \overline{V}_{i,c}}{\overline{V}_{i,s} - \overline{V}_{i,c}} & \overline{V}_{i,s} \leq V_{i,t,dl} \leq \overline{V}_{i,c} \\ 0 & \text{else} \end{cases}\quad (6)$$

The minimum value of voltage constraint satisfaction, i.e.  $\mu_{i,t,d}^V$ , over all buses of the network, can provide information about the overall voltage condition in year  $t$  and demand level  $dl$ , as follows:

$$\mu_{t,d}^V = \min_{i=1:N_b} (\mu_{i,t,d}^V) \quad (7)$$

The values obtained from (7) show the condition of voltage constraint satisfaction for overall network, in demand level  $dl$  and year  $t$ . Since there is more than one demand level in a real system, the planner will have different satisfaction levels of voltage constraint for a given network. To obtain an index which shows the condition of the network in year  $t$ , it is proposed in this work to calculate the weighted average of satisfaction of voltage over the demand levels, as follows:

$$\mu_t^V = \frac{\sum_{dl=1}^{N_{dl}} \tau_{dl} \times \mu_{t,d}^V}{\sum_{dl=1}^{N_{dl}} \tau_{dl}} \quad (8)$$

In (8), if the network does not fully satisfy the voltage constraints in demand level  $dl$  but the duration of this dissatisfaction is short, the voltage constraint satisfaction is not very degraded in the whole year  $t$ .

*Thermal limit of feeders and Substation.* To maintain the security of the feeders and the substation, the flow of current/energy passing through them should be kept below the

feeders/substation capacity limit. The safe operating limit of feeder  $\ell$  until year  $t$ , i.e.  $\bar{I}_{\ell,s}^t$ , is calculated as follows:

$$\bar{I}_{\ell,s}^t = \bar{I}_{\ell}^{t=0} + Cap_{\ell} \times \sum_{i=1}^t \gamma_i^{\ell} \quad (9)$$

Where,  $Cap_{\ell} \times \sum_{i=1}^t \gamma_i^{\ell}$  represents the added capacity of feeder  $\ell$  due to the investments made until year  $t$ .

The critical operating limit of feeder  $\ell$  until year  $t$ , i.e.  $\bar{I}_{\ell,c}^t$ , is calculated as follows:

$$\bar{I}_{\ell,c}^t = (1 + \bar{\epsilon}_{\ell}^I) \times \bar{I}_{\ell,s}^t \quad (10)$$

Where,  $\bar{\epsilon}_{\ell}^I$  is the maximum accepted toleration for dissatisfaction of thermal limit constraint of feeder  $\ell$ .

A strictly monotonically decreasing and continuous function is considered for modeling the satisfaction of this limit of feeder  $\ell$ , as depicted in Fig.2 and formulated as follows:

$$\mu_{\ell,t,d}^I = \begin{cases} 1 & I_{\ell,t,d} \leq \bar{I}_{\ell,s}^t \\ \frac{I_{\ell,t,d} - \bar{I}_{\ell,c}^t}{\bar{I}_{\ell,s}^t - \bar{I}_{\ell,c}^t} & \bar{I}_{\ell}^t \leq I_{\ell,t,d} \leq \bar{I}_{\ell,c}^t \\ 0 & I_{\ell,t,d} \geq \bar{I}_{\ell,c}^t \end{cases} \quad (11)$$

The minimum value of thermal capacity constraint satisfaction, i.e.  $\mu_{\ell,t,d}^I$ , over all feeders of the network, can provide information about the overall feeder condition in year  $t$  and demand level  $dl$ , as follows:

$$\mu_{t,d}^I = \min_{\ell=1:N_{\ell}} (\mu_{\ell,t,d}^I) \quad (12)$$

An index is needed to reflect the overall performance of the system regarding the thermal limits of feeders, in year  $t$ . The average weighted value of  $\mu_{t,d}^I$  over all demand levels can provide such information as follows:

$$\mu_t^I = \frac{\sum_{dl=1}^{N_{dl}} \mu_{t,d}^I \times \tau_{dl}}{\sum_{dl=1}^{N_{dl}} \tau_{dl}} \quad (13)$$

The capacity of substation until year  $t$ , i.e.  $\bar{S}_{tr}^t$ , is equal to the initial capacity plus the added capacity until year  $t$ , as follows:

$$\bar{S}_{tr,s}^t = \bar{S}_{tr,s}^{t=0} + Cap_{tr} \times \sum_{i=1}^t \psi_i^{tr} \quad (14)$$

Where,  $Cap_{tr} \times \sum_{i=1}^t \psi_i^{tr}$  represents the added capacity of substation resulting from adding new transformers until year t. The upper critical operating limit of substation, i.e.  $\bar{S}_{tr,c}^t$ , is calculated as follows:

$$\bar{S}_{tr,c}^t = (1 + \bar{\epsilon}_{tr}) \times \bar{S}_{tr,s}^t \quad (15)$$

Where,  $\bar{\epsilon}_{tr}$  is the maximum permissible dissatisfaction of thermal limit constraint of substation.

The satisfaction of this constraint is calculated as follows:

$$\mu_{t,dl}^{S^{grid}} = \begin{cases} 1 & S_{t,dl}^{grid} \leq \bar{S}_{tr,s} \\ \frac{S_{t,dl}^{grid} - \bar{S}_{tr,c}}{\bar{S}_{tr,s} - \bar{S}_{tr,c}} & \bar{S}_{tr,s} \leq S_{t,dl}^{grid} \leq \bar{S}_{tr,c} \\ 0 & S_{t,dl}^{grid} \geq \bar{S}_{tr,c} \end{cases} \quad (16)$$

$$\mu_t^{S^{grid}} = \frac{\sum_{dl=1}^{N_{dl}} \tau_{dl} \times \mu_{t,dl}^{S^{grid}}}{\sum_{dl=1}^{N_{dl}} \tau_{dl}}$$

The maximum permissible dissatisfaction of each soft constraint is determined by planner, based on his experience and operating condition of the system under study.

### 2.3. Objective Functions

The proposed model minimizes two objective functions, namely, total costs and technical dissatisfaction, as follows:

$$\begin{aligned} & \min \{OF_1, OF_2\} \\ & \text{subject to: (1)} \rightarrow (16) \end{aligned}$$

The objective functions are formulated next.

#### 2.3.1. Total Costs

The first objective function, i.e.,  $OF_1$ , to be minimized is the total costs which includes the cost of electricity purchased from the grid, the installation and the operating costs of the DG units and finally the reinforcement costs of the distribution network. The cost of purchasing electricity from the grid, i.e. GC, can be determined as:

$$GC = \sum_{t=1}^T \sum_{dl=1}^{N_{dl}} PLF_{dl} \times \rho \times P_{t,dl}^{grid} \times \tau_{dl} \times \frac{1}{(1+d)^t} \quad (17)$$

Installation costs of the DG units, i.e. DGIC, can be calculated as:

$$DGIC = \sum_{t=1}^T \sum_{i=1}^{N_b} \sum_{dg} \xi_{i,t}^{dg} \times IC_{dg} \times \frac{1}{(1+d)^t} \quad (18)$$

The operating costs of the DG units, i.e. DGOC, can be calculated as:

$$DGOC = \sum_{t=1}^T \sum_{i=1}^{N_b} \sum_{dg} \sum_{dl=1}^{N_{dl}} \tau_{dl} \times OC_{dg} \times P_{i,t,dl}^{dg} \times \frac{1}{(1+d)^t} \quad (19)$$

The reinforcement cost of the distribution network is the sum of all costs paid for installation and operation of new feeders and transformers. The total feeder reinforcement cost, i.e. LC, and substation reinforcement cost, i.e. SC, are calculated as follows:

$$LC = \sum_{i=1}^T \sum_{\ell=1}^{N_{\ell}} C_{\ell} \times d_{\ell} \times \gamma_{i,t}^{\ell} \times \frac{1}{(1+d)^t} \quad (20)$$

$$SC = \sum_{i=1}^T C_{tr} \times \psi_{i,t}^{tr} \times \frac{1}{(1+d)^t}$$

Thus,  $OF_1$  is defined as:

$$OF_1 = GC + DGIC + DGOC + LC + SC \quad (21)$$

### 2.3.2. Technical constraints satisfaction

The second objective function to be minimized is the dissatisfaction of technical constraints. The average technical dissatisfaction, denoted by  $ATD_t$ , is defined as the maximum average dissatisfaction of all technical constraints as follows:

$$ATD_t = 1 - \min \left\{ \mu_t^V, \mu_t^I, \mu_t^{Sgrid} \right\} \quad (22)$$

To demonstrate the severity of technical constraint violation, another index is proposed here called Maximum Technical Dissatisfaction in year t, i.e.  $MTD_t$ , as follows:

$$MTD_t = 1 - \min \left\{ \mu_{t,dl}^V, \mu_{t,dl}^I, \mu_{t,dl}^{Sgrid} \right\} \quad (23)$$

The weighted average of severity and average technical dissatisfaction in year t is calculated and the objective function to be minimized is proposed here as the maximum value of this quantity over the planning horizon as:

$$OF_2 = \max_t (w_1 \times ATD_t + w_2 \times MTD_t) \quad (24)$$

Where,  $w_1$  and  $w_2$  are the weight factors reflecting the importance of the average technical dissatisfaction, i.e.  $ATD_t$ , and the severity of technical dissatisfaction over the planning horizon, i.e.  $MTD_t$ , respectively. These factors are specified by the planner.

### 3. The proposed solution Method

The problem formulated in Section 2, is a mixed integer non-linear multi-objective problem. These kinds of problems can be solved by defining a set of weights representing the priorities of objectives and transforming it into a single-objective problem. The shortcomings of this method [18] has been the motivation for using Pareto optimality concept. The heuristic search methods are able to easily deal with more than one objective function in a single run by finding a set of non-dominated solutions instead of a single solution. The principles of multi-objective optimization are as follows: Suppose  $F(X)$  is the vector of objective functions, and  $H(X)$  and  $G(X)$  represent equality and inequality constraints, respectively. A multi-objective optimization problem is formulated as follows:

$$\min \quad F(X) = [f_1(X), \dots, f_{N_O}(X)] \quad (25)$$

*Subject to:*

$$\{G(X) = \bar{0}, H(X) \leq \bar{0}\}$$

$$X = [x_1, \dots, x_m]$$

Suppose  $X_1$  and  $X_2$  belong to the solution space.  $X_1$  dominates  $X_2$  if:

$$\forall k \in \{1 \dots N_O\} f_k(X_1) \leq f_k(X_2) \quad (26)$$

$$\exists k' \in \{1 \dots N_O\} f_{k'}(X_1) < f_{k'}(X_2)$$

Each solution is checked to find if it is dominated by any other solution or not. If a solution is found which is not dominated by any other solution, it belongs to the first Pareto front, i.e. FN=1. The solutions of the first Pareto front are removed and remaining solutions are checked for the conditions of (26) to find the solutions of second Pareto front, i.e. FN=2. The process is repeated for the remaining fronts. In this context, the Non-dominated Sorting Genetic Algorithm (NSGA-II) [12] and Immune Algorithm [25] have been applied to multi-objective optimization in power systems planning applications. In the present

work, a hybrid Immune-Genetic Algorithm (IGA) is proposed to find the Pareto optimal front of the solution space. The proposed algorithm strengthens the Immune algorithm by incorporating the crossover operator of Genetic Algorithm (GA), for better exploration of solution space. The solution algorithm proposed here consists of two stages. In the first stage, a hybrid Immune-GA method is proposed and the solutions which form the Pareto optimal front are found and in the second stage, the best solution is selected considering the planner's preferences. Both stages are described as follows:

### *3.1. Stage I: Finding the Pareto optimal front using hybrid Immune-GA method*

The Immune Algorithm (IA), first introduced in [26], is inspired by the immune system of human body. When external particles (antigens) enter into the human body, the immune cells (antibodies) have to detect and remove them. The antibodies are randomly generated by immune system and the ones with better match to the antigens are selected and reproduced (colonized) [27]. This idea is used to deal with optimization problems by considering the objective functions and the constraints as antigens while the solutions construct the antibodies [28]. The Immune Algorithm is an iterative process which creates an initial solution and tries to improve its performance through three operators namely, affinity factor, hyper mutation and clonal selection [29]. The affinity factor is a measure of fitness for each solution which shows how antibodies (solutions) have detected (optimized) the antibodies (objective functions and constraints). The hyper mutation operator is the same as mutation operator in Genetic Algorithm (GA) [27], but in IA, the probability of mutation is proportional to the inverse value of affinity factor of the solution. This means that if the affinity factor of a solution is low, it will be more mutated to explore the solution space and vice versa. The clonal selection is an operator to give a chance of reproduction to each solution. This chance is proportional to the affinity factor of each solution. The concept of fitness in multi-objective optimization is different with single objective optimization because more than one objective should be optimized. The Pareto optimality [18], is used to provide a pseudo fitness value for solution  $n$ , i.e.  $X_n$ , to be used as its affinity factor, i.e.  $AF_n$ . The  $AF_n$  should be defined in a way that effectively reflect two important aspects of multi-objective optimization namely, the ability of  $X_n$  in minimizing the objective functions and also maintaining the diversity among the solutions

and the ability of solution in minimizing the objective functions. This is done by sorting the solutions into different Pareto optimal fronts [18]. The process of fitness assignment is as follows: all of the solutions are sorted to find out the Pareto front they belong. This will determine the front number of each solution, i.e.  $FN$ . To evaluate the diversity of the solutions found in each Pareto front, global diversity factor, i.e.  $GD$ , is introduced and calculated. This factor shows the average distance of solutions in a given Pareto front. Since there are more than one objective function, a local diversity factor for solution  $n$  regarding objective function  $k$ , i.e.  $LD_n^k$ , is defined here as :

$$LD_n^k = \sum_{X_m \in FN_n} \frac{|f_k(X_n) - f_k(X_m)|}{MD_k} \quad (27)$$

Where, in (27), the summation is done over all solution existing in the same Pareto front as  $X_n$ . The  $MD_k$  is the maximum distance between the solutions of the mentioned Pareto front, regarding just the  $k^{th}$  objective function. Then  $LD_n^k$ , is normalized by dividing it by the maximum value of  $LD_n^k$  over all solutions in the mentioned Pareto front as :

$$LD_n^k = \frac{LD_n^k}{\max(LD_n^k)} \quad (28)$$

The global diversity factor for each solution is then calculated as the average of its local diversities as follows:

$$GD_n = \sum_{k=1}^{N_O} \frac{LD_n^k}{N_O} \quad (29)$$

Having  $FN_n$  and  $GD_n$  in hand, the affinity factor of solution  $n$ , is defined as follows:

$$AF_n = w_3 \times (FN_n)^{-1} + w_4 \times GD_n \quad (30)$$

The first term in (30) guides the population toward the lower Pareto optimal fronts and the second term insures the diversity among the solutions. In order to calculate the global diversity of the  $n^{th}$  solution, i.e.  $GD_n$ , a local diversity factor, i.e.  $LD_n^k$ , is defined for each objective function [18]. In initial iterations, a small number of solutions belong to the first Pareto front, so getting closer to Pareto optimal front is more important than maintaining the diversity among them. It is necessary to enable the algorithm in distinguishing between the solutions in different Pareto fronts,  $w_3$  and  $w_4$  in (30) are adaptively selected which guarantees that the solution belonging to a lower Pareto front

has a bigger affinity factor than a solution belonging to an upper front level ( $w_3$  is bigger than  $w_4$  in the initial iterations) and when most of the solutions are in the Pareto optimal front,  $w_4$  is chosen bigger than  $w_3$  to maintain the diversity among the solutions. To do so, each antibody, is a vector containing the investment decision for DG units and network. the steps of the first stage of the solution algorithm are as follows:

Step 1. Generate N initial random solutions.

Step 2. Set iteration=1.

Step 3. Calculate  $OF_1, OF_2$  for each solution.

Step 4. Sort the solutions based on the Pareto front they belong to and the global diversity of each solution using (29).

Step 5. Calculate the affinity factor using (30) for each antibody.

Step 6. Save the best N antibodies in the memory.

Step 7. If the stopping criterion is met, go to step (13), else, continue.

Step 8. Set the cloning counter, i.e.  $m=1$ .

Step 9. Select two antibodies of memory, i.e.  $X_p, X_q$  based on their affinity factors, using roulette wheel method.

Step 10. Determine the cloning number, i.e.  $K_m$ , and the mutation probability, i.e.  $\varsigma_m$ , as follows:

$$K_m = \text{round}(\beta \times N \times \frac{AF_p + AF_q}{2\max(AF_n)}) \quad (31)$$

$$\varsigma_m = \varsigma^{\max} \times \frac{AF_p + AF_q}{2\max(AF_n)}$$

Where, *round* is a function which returns the nearest integer value,  $\beta$  is a control parameter for regulating the number of reproduction in cloning process,  $\varsigma^{\max}$  is the maximum mutation probability.

Step 11. Clone the selected two antibodies  $K_m$  times and generate  $2K_m$  new antibodies and save them.

Step 12. Check if  $m < N$ , then increase cloning counter by one and go to step 9, else construct the new population of antibodies using the union of old and new antibodies, increase iteration by one and go to step 3.

Step 13. End.

The flowchart of the two stages of the proposed method is depicted in Fig.3.

### 3.2. Stage II (Selecting ‘the best’ solution)

The ultimate goal of the planner is to choose the “most preferred” solution among the Pareto optimal front. A fuzzy satisfying method [30] is used in this paper to find the ‘the best’ solution. The principles of this method are as follows: for each solution in the Pareto optimal front,  $X_n$ , a membership function is defined as  $\mu^{f_k(X_n)}$ . This value, which varies between 0 to 1, shows the level of which  $X_n$  belongs to the set that minimizes the objective function  $f_k$ . A linear membership function [31] is used in the present work for all objective functions, as follows:

$$\mu^{f_k(X_n)} = \begin{cases} 0 & f_k(X_n) > f_k^{max} \\ \frac{f_k^{max} - f_k(X_n)}{f_k^{max} - f_k^{min}} & f_k^{min} \leq f_k(X_n) \leq f_k^{max} \\ 1 & f_k(X_n) < f_k^{min} \end{cases} \quad (32)$$

A conservative decision maker tries to maximize minimum satisfaction among all objectives or minimize the maximum dissatisfaction [30]. The final solution can then be found as:

$$\max_{n=1:N_p} \left( \min_{k=1:N_o} \mu^{f_k(X_n)} \right) \quad (33)$$

## 4. Simulation Results

The proposed methodology is applied to an actual distribution network which is shown in Fig.4. This system has 573 sections and 180 load points. The average load and power factor at each load point are 55.5 kW and 0.9285, respectively. This network is fed through a 20kV substation with,  $\bar{S}_{tr,s}^{t=0} = 20$  MVA. The options for reinforcing the network are as follows: transformers with a capacity of  $Cap_{tr}=10$  MVA and a cost of  $C_{tr}=0.2$  Million \$ for each; replacing the feeders at a cost of  $C_\ell=0.15$  Million \$/km [32]. In this paper, the DG technology is assumed to be Gas turbine [17] but this is not limiting the ability of the model for considering other DG technologies. Four demand levels, i.e., minimum, medium, base and high are considered here with  $DLF_{dl}$  are 0.75, 0.87, 1 and 1.25 respectively; the  $PLF_{dl}$  values associated to these demand levels are 0.65, 0.82, 1

and 1.65, respectively; The duration of each demand level, i.e.  $\tau_{dl}$ , is 2920, 2920, 2847 and 73 hours, respectively; the stopping criterion is reaching to a maximum number of iterations. Other simulation assumptions and characteristics of the DG units [33, 34] are presented in Table. 1. The formulated problem was implemented in MATLAB [35] and solved using the proposed two-stage algorithm and 40 non-inferior solutions are found. The maximum and minimum values of all objectives (of proposed dynamic model) are shown in Table 2. The variation of individual cost terms of (21) are shown in Fig. 5. The grid cost, i.e. GC, increases with the decrease of DG investment. This means that if DNO invests in DG units he can expect reduction of costs he should pay to main grid for purchasing energy. In this section, the effect of soft constraint handling of the problem is investigated and then the following comparisons are made: first, the obtained results of the proposed model and those obtained from other planning models are compared. Secondly, the proposed solution algorithm is compared with other heuristic techniques, as follows:

#### 4.1. Effect of soft constraint handling of the problem

The purpose of this section is to investigate the effect of considering the voltage limit and feeder/substation constraint as soft constraints instead of hard constraints. This analysis will help the planner to understand how much money should be spent to improve the technical condition of the network. The proposed model enables the planner to handle the degree of softness of the constraints. If the mentioned constraints should be fully satisfied ( $\bar{\epsilon}_i^V, \underline{\epsilon}_i^V, \underline{\epsilon}_\ell^I, \bar{\epsilon}_\ell^I, \bar{\epsilon}_{tr} = 0$ ), then  $OF_2$  will be equal to zero (no technical dissatisfaction). On the other hand, the planner may be interested to know the effect of relaxation of these constraints (to some degree) on the other objective function, i.e.  $OF_1$ . To compare both cases, the planner can search the Pareto optimal front for the solution with maximum and minimum relaxation of the soft constraints. The solution which has  $OF_2 = 0$ , represents the hard constraint modeling and has  $OF_1 = 4.0474 \times 10^7$ \$. The maximum value of  $OF_2$  is 0.9757 which means the maximum relaxation of the soft constraints. The total cost associated to this solution is  $OF_1 = 3.5323 \times 10^7$ \$. This means, the maximum relaxation of soft constraints allows the planner reduce the total costs up to  $5.1506 \times 10^6$ \$. This is the maximum economic benefit the DNO can get by soft handling voltage and feeder

limit constraints. However, the DNO is allowed to decide about the degree of relaxation of these constraints.

#### 4.2. Comparing the proposed dynamic model with other planning models

The purpose of this section is comparing the ability of proposed dynamic model and other planning models of the literature. The proposed model is compared with five other planning models which are listed in Table. 6. The models named A, B and C are static because they do not consider the timing of investment and assume that all investment decisions are done at the beginning of the planning horizon. The pareto optimal front found by these models and the non-inferior solutions of the proposed model are depicted in Fig.6. As it is obvious in Fig.6, model B and C can not reach to  $OF_2$  lower than 0.5 in the given iterations. Since models B and C use just one of the planning options, so it was predictable that they can not compete with model A which use both DG units and network reinforcement simultaneously. As can be seen in Fig.6, for every solution proposed in Pareto front of A, B and C models, the planner can find a solution in the Pareto optimal front of the proposed model with lower objective functions. This means that the solutions found by static models are dominated (see 26) by at least one solution of proposed model. The same comparison can be done between the proposed model and the model D and E which consider timing of investment and are dynamic ones. The model D just uses network reinforcement and model E considers just DG units. As it can be concluded from Fig.7, all solutions provided by model D and E are dominated by the solutions of proposed model. In order to make the analysis more sensible, the results obtained by the model C and the proposed model are quantitatively compared as follows: suppose that the planner is looking for a solution which has a technical dissatisfaction less than a certain level. Let's assume that this limit is 0.25, determined by the planner based on the requirements of the system under study. In Fig.7, all of the solutions located to the left of  $OF_2 = 0.25$ , are accepted for the planner. Both of the models can provide such a solution, but the question is "which one should be chosen?". The most logical answer to this question is that if the only important criteria is satisfying the condition  $OF_2 < 0.25$ , selecting the minimum cost among the qualified solutions would be the best choice. The values of objective functions associated to solutions found by each model (proposed model

and static model C) are as shown in Table 3. The technical satisfaction in the solution proposed by static model is better than the dynamic model. The technical dissatisfaction, it is reduced by  $0.2416 - 0.2235 = 0.0181$  and the total cost is increased by  $3.799 \times 10^6$ \$. This means if the only criterion is  $OF_2 < 0.25$ , the solution obtained by proposed model is cheaper than the solution of static model and using the proposed model reduces the cost up to  $3.799 \times 10^6$ \$. In the last case, the selection of the best solution was biased toward just one of the objective functions, i.e.  $OF_2$ , but in some cases, it is needed to consider both objectives. If the planner is going to make a tradeoff between the satisfaction of both objectives, the method introduced in section 3.2 should be used. The (33) is applied on the solutions found by both models and the best solutions and the satisfaction degrees of each objective function are given for each model, in Table 4. The proposed planning schemes of both models are given in Table 5. The best solution of model C, proposes to invest in DG units and also in network reinforcement. In this solution, 8 DG units will be installed in the network. All of the investments are taken place in first year as specified in Table 5. On the other hand, the best solution of the proposed model, uses both of planning options, DG and network. It proposes to install 7 DGs in the system but the investment is done during the planning horizon. Analyzing the satisfaction levels of both objectives given in Table 4, shows that the solution obtained by dynamic model, has a better performance in minimizing both objectives.

#### 4.3. Comparing the IGA with IA and GA methods

The Pareto optimal front of the solutions is found using IGA method. To investigate the value of this algorithm, it is needed to be compared to other heuristic methods. Since the IGA is a hybrid of immune algorithm and GA, the comparison is done between IGA, GA and IA. To compare the performance of any two search methods, it is needed to compare their performance in finding the best solutions. In single objective problems, it is done by comparing the best solution found by each algorithm. In multi-objective problems, the comparison should be made between the Pareto optimal fronts found by each algorithm. The ability of solutions in dominating (see (26)) the others, is a measure of their performance. The formulated problem is solved using both GA and Immune algorithm. The number of population and maximum iteration for all of the three algorithms

are considered the same as each other and the Pareto optimal front of all of them are depicted in Fig.8. The number of non-inferior solutions found by GA, is 37 and for IA is 35. The variation range of objective functions, in both GA and IA are is the same as IGA, but as it is clear in Fig.8 the solutions found by IGA dominate the solutions of GA and IA.

## 5. Conclusions

This paper presents a dynamic multi-objective formulation of Distribution network expansion planning problem integrated with DG options and an Immune-GA based method to solve the formulated problem. The proposed two-stage algorithm finds the non-dominated solutions by simultaneous minimization of total costs and technical dissatisfaction in the first stage and uses a fuzzy satisfying method to select the best solution from the candidate set in the second stage. The novel planning model is applied to an actual distribution network and its flexibility and effectiveness is demonstrated through different studies and comparative analyses. The results show the Pareto optimal front found by formulated model and solved by IGA, is more efficient than other studied alternative models and solution methods. It should be noted that the proposed model can be directly used in power market model in which the DNO is authorized for DG integration in addition to the network reinforcement. However, in power market models where the DG investment is done by independent investors instead of DNO, the provided information would also be useful as an economical and technical signal for regulators. It can be used for regulating the incentives to encourage the private section to invest in what DG technology and where, to be more beneficial. The research is in under way to consider the uncertainties associated to DG units and electricity price in the future work.

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- Figure 1: Membership function of technical satisfaction of voltage constraints
- Figure 2: Membership function of thermal capacity for feeders and substation
- Figure 3: The flowchart of the two stages of the proposed model
- Figure 4: Single-line diagram of an actual 574-node distribution network
- Figure 5: Variations of different cost terms in pareto optimal front
- Figure 6: Comparison of Pareto optimal front found by proposed model to other static models
- Figure 7: Comparison of Pareto optimal front found by proposed model to other dynamic models
- Figure 8: Comparison of Pareto optimal front found by IGA, GA and IA

Table 1: Data used in the study

Parameter	Unit	Value
$T$	year	10
$N_p$		100
$w_1 = w_2$		0.5
$\overline{S}_{tr}^{t=0}$	MVA	40
$\rho$	\$/MWh.	70
$OC_{dg}$	\$/MWh	50
$IC_{dg}$	\$/MVA	500000
$\alpha$	%	3.5
$\beta$	%	10
d	%	12
$\overline{V}_{i,s}$	Pu	1.05
$\underline{V}_{i,s}$	Pu	0.95
$\overline{\epsilon}_i^V = \underline{\epsilon}_i^V$	%	2
$\overline{\epsilon}_\ell^I = \underline{\epsilon}_\ell^I$	%	3
$\overline{\epsilon}_{tr}$	%	5
$\zeta^{max}$	%	5
Maximum iteration		1000

Table 2: Variation ranges of objective functions in Pareto optimal front

	$OF_1(10^7\$)$	$OF_2$
$f_k^{min}$	3.5323	0
$f_k^{max}$	4.0474	0.9757

Table 3: Comparison between the solutions with  $OF_2 < 0.25$  in model C and proposed dynamic model

Model	$OF_1(10^7\$)$	$OF_2$
Proposed model	3.8933	0.2139
Model C	4.2732	0.2098

Table 4: Comparison between the best solutions in other models and proposed model

Model	$OF_1(10^7\$)$	$OF_2$	$\mu^{f_2}$	$\mu^{f_1}$
Proposed model	3.7806	0.50417	0.4833	0.5180
Model C	4.1086	0.50417	0.4833	0

Table 5: Investment plans in different cases

Planning models	DG			Network Reinforcement			
	$\xi_{t,i}^{dg}$	i	t	SC( $\times 10^6\$$ )	LC( $\times 10^6\$$ )	t	
Proposed model	1	37	1	0	0.0944	1	
	1	50	1	0	0.3812	2	
	1	180	1	0	0.7364	3	
	1	288	1	0	1.1471	4	
	1	324	1	0	0.2387	5	
	1	37	3	0.2	1.2994	6	
	1	73	5	0	0.2422	7	
	1	467	7	0	0.8504	8	
					0	0.2326	9
					0	1.3917	10
Model C	1	324	1	0.2	6.8102	1	
	1	180	1				
	1	324	1				
	1	252	1				
	1	360	1				
	1	467	1				
	1	288	1				
	1	50	1				

Table 6: Different models of DG and distribution planning

Model	Timing	Planning options	
		DG	Network reinforcement
A	N	N	Y
B	N	Y	N
C	N	Y	Y
D	Y	N	Y
E	Y	Y	N
Proposed model	Y	Y	Y

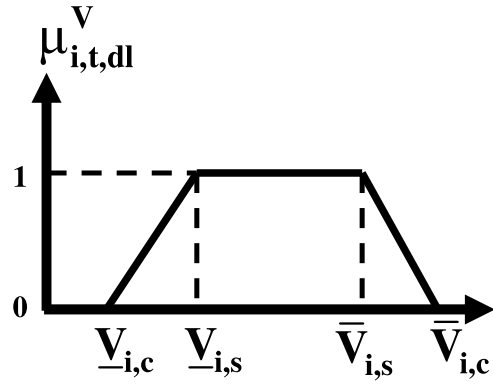


Figure 1: Membership function of technical satisfaction of voltage constraints

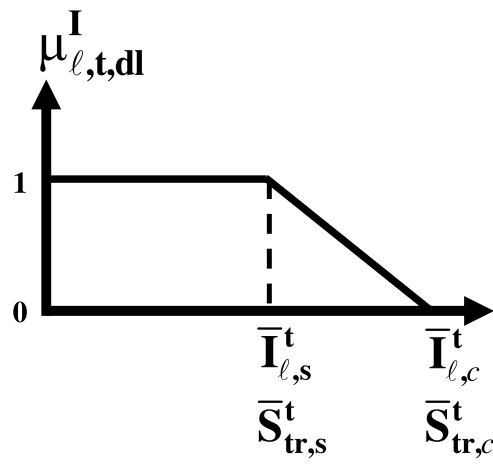


Figure 2: Membership function of thermal capacity for feeders and substation

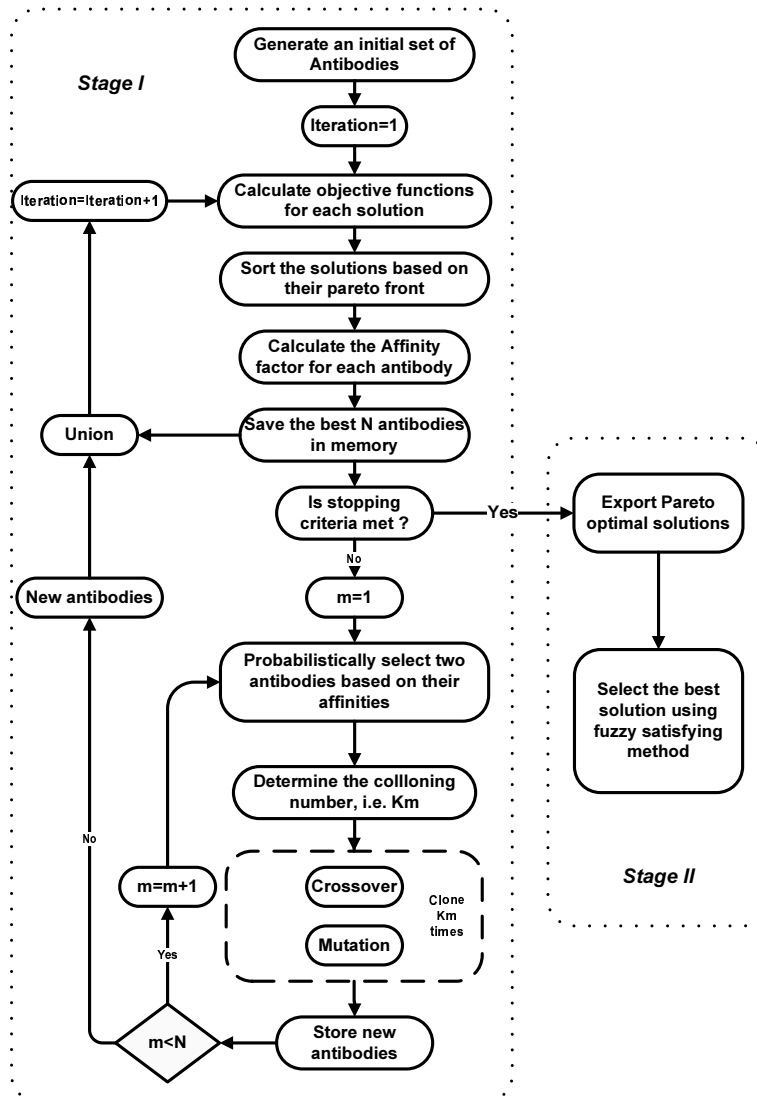


Figure 3: The flowchart of the two stages of the proposed model