



<b>Title</b>	Controlling speckle using lenses and free space
<b>Authors(s)</b>	Kelly, Damien P., Ward, Jennifer E., Gopinathan, Unnikrishnan, Sheridan, John T.
<b>Publication date</b>	2007-12-01
<b>Publication information</b>	Kelly, Damien P., Jennifer E. Ward, Unnikrishnan Gopinathan, and John T. Sheridan. "Controlling Speckle Using Lenses and Free Space." Optical Society of America, December 1, 2007. <a href="https://doi.org/10.1364/OL.32.003394">https://doi.org/10.1364/OL.32.003394</a> .
<b>Publisher</b>	Optical Society of America
<b>Item record/more information</b>	<a href="http://hdl.handle.net/10197/3345">http://hdl.handle.net/10197/3345</a>
<b>Publisher's statement</b>	This paper was published in Optics Letters and is made available as an electronic reprint with the permission of OSA. The paper can be found at the following URL on the OSA website: <a href="http://www.opticsinfobase.org/abstract.cfm?URI=ol-32-23-3394">http://www.opticsinfobase.org/abstract.cfm?URI=ol-32-23-3394</a> . Systematic or multiple reproduction or distribution to multiple locations via electronic or other means is prohibited and is subject to penalties under law.
<b>Publisher's version (DOI)</b>	10.1364/OL.32.003394

Downloaded 2026-05-01 23:50:15

The UCD community has made this article openly available. Please share how this access benefits you. Your story matters! (@ucd\_oa)



© Some rights reserved. For more information

# Controlling speckle using lenses and free space

Damien P. Kelly,<sup>1</sup> Jennifer E. Ward,<sup>2</sup> Unnikrishnan Gopinathan,<sup>2</sup> and John T. Sheridan<sup>2,\*</sup>

<sup>1</sup>Institut für Photonik & Zentrum für Mikro- und Nanostrukturen, Technische Universität Wien, A-1040 Wien, Austria

<sup>2</sup>School of Electrical, Electronic & Mechanical Engineering, College of Engineering, Mathematical and Physical Sciences, University College Dublin, Belfield, Dublin 4, Ireland

\*Corresponding author: john.sheridan@ucd.ie

Received June 13, 2007; revised August 18, 2007; accepted September 12, 2007;  
posted October 19, 2007 (Doc. ID 84141); published November 20, 2007

The correlation properties of speckle fields are studied for general paraxial systems. The previous studies on lateral and longitudinal speckle size for the case of free-space propagation (Fresnel transform) are generalized to the case of the linear canonical transform. These results have implications for the control of speckle size, through appropriate design of optical systems, with particular relevance for speckle interferometry.

© 2007 Optical Society of America

OCIS codes: 070.2580, 030.6140, 110.6150.

Coherent light reflected from an optically rough surface produces a grainy interference pattern known as a speckle field. A speckle field carries information about the surface it originates from, so much so that changes to the surface profile through applied stress [1,2], motion [3–7], or object surface slopes [8] can be detected by monitoring the resultant variation in the field. While an imaging system is often used for speckle metrology, alternative optical systems [described by using the linear canonical transform (LCT)], have been shown to be useful for estimating simultaneous in-plane translation and surface tilting [3–7], as well as allowing the user greater control over sensitivity and dynamic range of the measuring system. Speckle decorrelation [4,5] effects can also be minimized.

An important characteristic of speckle fields is the so called lateral and longitudinal speckle size. Studies have shown that this speckle size is dependent on the propagation distance,  $z$  (from the illuminated rough surface), the wavelength of light,  $\lambda$ , and the size of the illuminating spot [9,10] or scattering aperture [11,12]. These studies use a Fresnel transform (FST) to propagate the light field from the optically rough surface or between two phase-space planes [13]. By using the LCT instead of the FST we derive the corresponding speckle size expressions for general paraxial systems. We proceed by following the work of Leuschake and Kirchner [9]. However, for brevity we only present a 1D analysis.

A slit,  $p(x_0)$ , at an optically rough surface is illuminated by coherent radiation. The reflected light passes through an optical system consisting of an arbitrary number of lenses and distances of free space to the output plane [4,5]. Such a system (we assume the lenses are effectively infinite in extent) can be described using a LCT [4,5,14], and the field in the output plane is

$$U_i(x) = \frac{1}{\sqrt{j\lambda B_i}} \int_{-\infty}^{\infty} u(x_0)p(x_0) \times \exp\left[\frac{j\pi}{\lambda B_i}(D_i x^2 - 2x_0 x + A_i x_0^2)\right] dx_0, \quad (1)$$

where the parameters  $A_i$ ,  $B_i$ ,  $D_i$  are related to the particular  $i$ th optical system [4,5],  $A_i D_i - B_i C_i = 1$ , indicating a lossless transform, and  $u(x_0)$  is the field at the rough surface. As in [9], we wish to find the autocorrelation function of the intensity of the light distribution,  $I_i(x) = |U_i(x)|^2$ , which is given by the ensemble average

$$R(x, \tilde{x}) = \langle I_1(x) \rangle \langle I_2(\tilde{x}) \rangle + |J_A(x, \tilde{x})|^2, \quad (2)$$

where  $J_A(x, \tilde{x})$  is the mutual intensity of the fields after two different LCT transforms (LCTs 1 and 2) and is defined as

$$J_A(x, \tilde{x}) = \langle U_{\text{LCT1}}(x) U_{\text{LCT2}}^*(\tilde{x}) \rangle, \quad (3)$$

where the LCT1 and LCT2 subscripts refer to the particular LCT transform and \* denotes complex conjugate. We now make three assumptions:

(i) The speckle field is fully developed and  $\langle u(x_0) u^*(\tilde{x}_0) \rangle = C \delta(x_0 - \tilde{x}_0)$ , where  $C$  is a constant,

$$(ii) p(x_0) = \begin{cases} 1, & |x_0| \leq L/2 \\ 0, & \text{otherwise} \end{cases}$$

(iii)  $\langle I(x) \rangle$  and  $\langle I(\tilde{x}) \rangle$  are slowly varying and approximately equal to each other. Thus, a study of  $J_A(x, \tilde{x})$  is sufficient to describe the decorrelation of the speckle field [9].

With these assumptions and after some manipulation Eq. (3) can be rewritten as

$$|J_A(x, \tilde{x})| = \left| C \int_{-1}^1 \exp[j\alpha x_N] \exp(j\pi x_N^2) dx_N \right|, \quad (4a)$$

with dimensionless variables

$$\tau = \frac{\pi L^2 A_1}{4\lambda B_1} \left(1 - \frac{B_1}{A_1} \psi\right), \quad \alpha = \frac{\pi L(B_1 \gamma - \Delta_B x)}{\lambda B_1(B_1 + \Delta_B)}, \quad (4b)$$

where  $\psi = (A_1 + \Delta_A)/(B_1 + \Delta_B)$ ,  $\Delta_A = A_2 - A_1$ ,  $\Delta_B = B_2 - B_1$ ,  $\gamma = \tilde{x} - x$ ,  $x_N = x_0/(L/2)$ .

To represent Eq. (4a) in a normalized form, we introduce the correlation coefficient of intensity,  $\mu_I$ . From Eqs. (9) and (11) of [9] we find that

$$\mu_I(\alpha, \tau) = \left| \frac{|J_A(x, \tilde{x})|}{\sqrt{|J_A(x, x)||J_A(\tilde{x}, \tilde{x})|}} \right|^2. \quad (5)$$

Equations (4) and (5) are statistical relationships that describe the mutual correlation of LCT fields in different domains. This has been discussed in a different context elsewhere for fractional Fourier transform (FRT) systems [15].

An analytical solution for Eq. (5) exists, and we plot the results in a contour plot in Fig. 1. The function  $\mu_I(\alpha, \tau)$  varies from  $0 < \mu_I(\alpha, \tau) < 1$  in 11 steps. Clearly, as  $\alpha$  and  $\tau$  vary from zero,  $\mu_I$  decreases and the speckle field decorrelates. We now define when the speckle field is considered decorrelated by using two criterion; lateral and what we term phase-space speckle size. The phase-space speckle size can be considered a generalized longitudinal speckle size [9,10]; however, as we will see below, it can also define the maximum distance that a lens in some lossless optical system can be moved before the speckle field at the output plane decorrelates. Since, in general, (longitudinal) decorrelation can occur even though the input and output planes are stationary, we use the term phase-space speckle throughout the rest of this Letter.

**Lateral Speckle Size** ( $\Delta_A=0, \Delta_B=0$ ). Lateral speckle size refers to the maximum distance that the output plane can be translated along the  $x$  axis before the speckle fields are decorrelated. Importantly, the LCT system is left physically unchanged between observations. Therefore,  $\Delta_B=\Delta_A=0$ . From Eq. (4b), we see that in this case  $\tau=0$  and  $\alpha=\pi L\gamma/\lambda B_1$ . The first minimum of  $\mu_I(\alpha, 0)=|\sin(\alpha)/\alpha|^2$  occurs when  $\alpha_{\min}=\pi$ . Solving  $\mu_I(\alpha_{\min}, 0)$  for  $\gamma$ , we define the lateral speckle size,  $\gamma_{LS}$ , as

$$\gamma_{LS} = \lambda B_1 / L. \quad (6)$$

From Eq. (6) we see that the lateral speckle size can be controlled by changing the variable  $B_1$ , which, in turn can be achieved by using combinations of lenses and free space.

**Phase-Space (General Longitudinal) Speckle Size** ( $\gamma=0$ ). We now compare the speckle field at the output plane of two different LCT systems (i.e., in different domains). We assume that the output plane is not translated between observations; thus  $\gamma=0$ . Nevertheless, in general,  $\alpha \neq 0$ , and so  $\mu_I$  is still a function of both  $\alpha$  and  $\tau$ . To define when the field is correlated we first convert to polar coordinates  $\{\rho, \theta\}$ , such that  $\alpha = \rho \cos(\theta)$ ,  $\tau = \rho \sin(\theta)$ . For a particular value of  $\theta$ , the first local minimum of the function  $\mu_I(\rho \cos[\theta], \rho \sin[\theta])$  can be found numerically. Repeating this procedure for  $0 < \theta < 2\pi$  traces out what we propose to term the boundary of correlation (BOC), depicted in Fig. 1.  $\mu_I$  is not constant over the BOC but varies from a value of 0 at  $\{\alpha, \tau\} = \{3.14, 0\}$  to 0.24 at  $\{\alpha, \tau\} = \{2.01, 3.43\}$ . Any values of  $\alpha$  and  $\tau$  that lie outside this BOC are thus defined as being uncorrelated. We now examine the three separate systems: (i) the FST, (ii) a single paraxial lens system, and (iii) an optical FRT system.

**Fresnel Transform.** For the special case of free space propagation, the LCT parameters reduce to  $A_1=A_2=1, B_1=z, B_2=\tilde{z}$ . If we further assume that  $\tilde{z}z \approx z^2$  and  $x=0$ , then we find that  $\gamma_{LS}=\lambda z/L$  and  $\varepsilon_{SS}=7.31\lambda(z/L)^2$ , where  $\varepsilon_{SS}$  is the longitudinal speckle size. These results are consistent with those derived by Leuschke and Kirchner [9]. For an optical system with  $z=20$  cm,  $L=10$  cm, and  $\lambda=633$  nm,  $\gamma_{SS}=1.26 \mu\text{m}$ , and  $\varepsilon_{SS}=18 \mu\text{m}$ . Now we set  $x=8$  mm, and from Eq. (4b) we can see that  $\alpha$  is no longer zero; however, we do note that both  $\alpha$  and  $\tau$  vary as a function of the difference  $\tilde{z}-z$ . This can be conveniently expressed in the form of a parametric plot (referred to as the FST) which we overlay on Fig. 1 (solid black line). By examining the point of intersection of the FST with the BOC, a value for both  $\alpha$  and  $\tau$  can be found, which we refer to as the decorrelation point (DP). From Eq. (4b) an equivalent value for  $\varepsilon_{SS}$  can be calculated. In this case  $\varepsilon_{SS}=18.5 \mu\text{m} \sim 103\%$  of the on-axis value, indicating that the speckle field decorrelation is not strongly affected by  $x$ , once  $x < \pm 8$  mm.

**Single-Lens System.** We now consider a single-lens (SL) system with a fixed distance  $S$  between the input and output planes [5]. We first show that the lateral speckle size is dependent on the lens position. Then, with  $\gamma=0$ , we determine the phase-space speckle size; i.e., we examine how the speckle field in the output plane decorrelates as a function of lens position. The  $A$  and  $B$  parameters for this system are given by [5]

$$A_1 = 1 - \frac{z_l}{f}, \quad B_1 = z_l + (z_l - S) \left( 1 - \frac{z_l}{f} \right). \quad (7)$$

From Eq. (7) we can see that  $B_1$  is a function of the lens position,  $z_l$ . Using Eqs. (6) and (7), it is clear that  $\gamma_{LS}$  can be varied. In a real optical system ( $L=10$  cm,  $f=10$  cm, and  $\lambda=633$  nm) this means that  $1.15 < \gamma_{LS} \leq 79 \mu\text{m}$  when  $0.01 < z_l \leq 0.15$ .

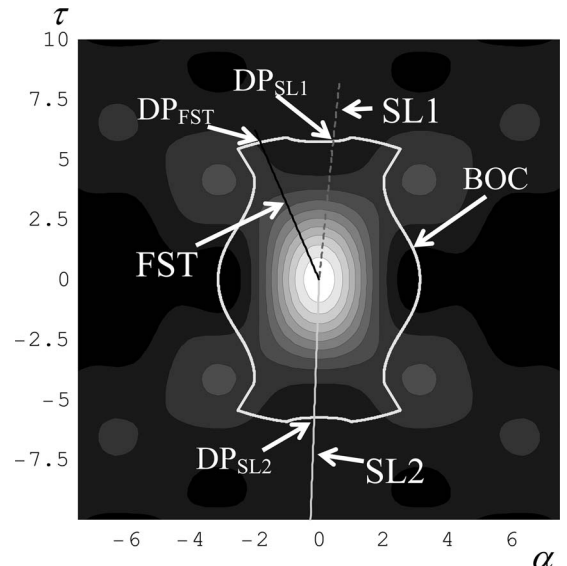


Fig. 1. Contour plot of  $\mu(\alpha, \tau)$  as a function of  $\tau$  and  $\alpha$ . SL, single lens; DP, decorrelation point.

We now wish to examine how the system decorrelates as a function of  $\Delta_A$  and  $\Delta_B$ . We introduce a new variable  $\Delta z_l$  defining the distance that the lens has been moved between observations. From Eqs. (4b) and (7),  $\Delta_A$  and  $\Delta_B$  can be expressed as a function of  $\Delta z_l$ . Setting  $L=10$  cm,  $\lambda=633$  nm,  $S=20$  cm, and  $f=10$  cm, we examine two cases: SL1 where  $z_l=17$  cm and  $x=0.1$  mm, and SL2 where  $z_l=5$  cm and  $x=-0.1$  mm. In Figs. 2 we plot the variation of  $\alpha$  and  $\tau$ , respectively, as a function of  $\Delta z_l$  for systems SL1 (dashed) and SL2 (solid). Both  $\alpha_{SL1}$  (dashed) and  $\alpha_{SL2}$  (solid) increase at similar rates; however,  $\tau_{SL2}$  (solid) increases more quickly than  $\tau_{SL1}$  (dashed), and so we expect that the SL2 system will decorrelate more quickly. The results in Fig. 2 have again been expressed as a parametric curve and overlaid on Fig. 2 (solid and dashed gray lines). The DPs for SL1 and SL2 are given by  $\{0.433, 5.81\}$  and  $\{-0.167, -5.69\}$ , respectively. By plotting two straight lines  $\tau=5.81$  and  $\tau=-5.69$  in Fig. 2, corresponding to the  $\tau$  coordinate of the DP, we can determine that SL1 and SL2 are decorrelated at  $\Delta z_l=14$   $\mu\text{m}$  and  $4.75$   $\mu\text{m}$ , respectively (it is completely equivalent to use the  $\alpha$  coordinate instead). We note that the amount the lens can be moved before total decorrelation depends on the initial position of the lens.

**Fractional Fourier Transform System.** Finally, we briefly examine phase-space speckle size for an FRT system, referring the reader to [6,16,17] for suitable definitions of  $A$  and  $B$ . Setting  $L=10$  cm,  $\lambda=488$  nm,  $f=10$  cm, and  $x=2$  mm, we construct an FRT system of fractional angle  $\phi=2.2$  rad (order 1.4) as in [6]. The variables  $\alpha$  and  $\tau$  again vary as a function of  $\Delta\phi$ , and it can be shown that total decorrelation occurs when  $\Delta\phi=27$   $\mu\text{rad}$ . A central assumption in the interpretation of the experimental results presented in [6] was that the speckle fields at the output of an FRT system of fractional angle  $2.2$  and  $\pi$  rad would be uncorrelated with respect to each other. We note that this assumption is supported by the analysis presented here.

In relation to the FRT it is interesting to note that, since there is no mathematical difference between an

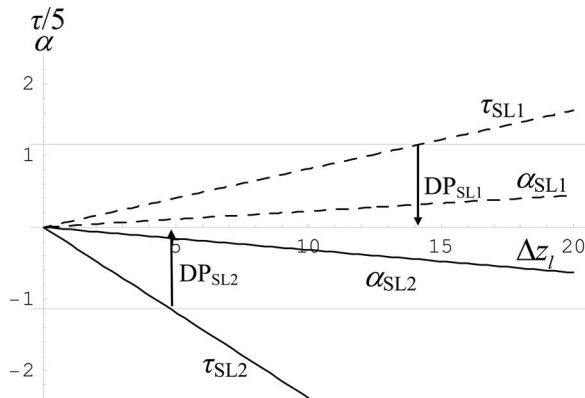


Fig. 2. (a) Variation of  $\alpha$  and  $\tau$  as a function of  $f\Delta z_l$  ( $\mu\text{m}$ ) for the SL.

FRT system of angle  $\beta$  and  $2\pi+\beta$ , we could expect speckle fields produced by two physically different (but lossless) optical systems to be perfectly correlated and repeat periodically every  $2\pi$ . This assumes, however, that no light is lost from the system, which may not be achievable in practice.

Obtaining high fringe contrast and low speckle noise in speckle interferometry requires balancing the lateral speckle size and camera pixel area with the expected error introduced by undesired in-plane displacement of the sample under investigation [18]. This is typically controlled by adjusting the size of the limiting aperture in the system [18]. If the limiting aperture is small, the system will be more sensitive to decorrelation introduced by an unintentional tilting of the sample under investigation [2,4,5]. The results derived in this Letter are applicable to apertures in optical systems if it can be assumed that the coherence area of the field in the aperture plane is very small compared with the size of the aperture area [4,5,13]. Using Eq. (6) we can control the lateral size of the speckle in the observation plane by using lenses and free space as well as the limiting aperture size. With this extra flexibility it may be possible to design a speckle interferometry system with both high fringe contrast and low speckle noise that allows the user control over the effect of errors introduced by unintentional tilting of the sample under investigation.

## References

1. P. K. Rastogi, *Digital Speckle Pattern Interferometry and Related Techniques* (Wiley, 2001).
2. T. Fricke-Begemann, *Appl. Opt.* **42**, 6783 (2003).
3. H. Tiziani, *Opt. Commun.* **5**, 271 (1972).
4. D. P. Kelly, J. E. Ward, B. M. Hennelly, U. Gopinathan, F. T. O'Neill, and J. T. Sheridan, *J. Opt. Soc. Am. A* **23**, 2861 (2006).
5. D. P. Kelly, J. E. Ward, U. Gopinathan, B. M. Hennelly, F. T. O'Neill, and J. T. Sheridan, *Opt. Lett.* **31**, 3444 (2006).
6. R. F. Patten, B. M. Hennelly, D. P. Kelly, F. T. O'Neill, Y. Liu, and J. T. Sheridan, *Opt. Lett.* **31**, 32 (2006).
7. J. M. Diazdelacruz, *Appl. Opt.* **44**, 2250 (2005).
8. P. Smíd, P. Horváth, and M. Hrabovský, *Appl. Opt.* **45**, 6932 (2006).
9. L. Leushacke and M. Kirchner, *J. Opt. Soc. Am. A* **7**, 827 (1990).
10. Q. Li and F. Chiang, *Appl. Opt.* **31**, 6287 (1992).
11. J. C. Dainty, in *Progress in Optics XIV*, E. Wolf, ed. (North-Holland, 1976).
12. J. W. Goodman, in *Statistical Optics* (Wiley, 1985), pp. 286–360.
13. H. T. Yura, S. G. Hanson, R. S. Hansen, and B. Rose, *J. Opt. Soc. Am. A* **16**, 1402 (1999).
14. S. A. Collins, *J. Opt. Soc. Am.* **60**, 1168 (1970).
15. M. F. Erden, H. M. Ozaktas, and D. Mendlovic, *J. Opt. Soc. Am. A* **13**, 1068 (1996).
16. T. Alieva, M. J. Bastiaans, and M. L. Calvo, *EURASIP J. Appl. Signal Process.* **10**, 1498 (2005).
17. S. Abe and J. T. Sheridan, *Opt. Lett.* **19**, 1801 (1994).
18. T. Yoshimura, M. Zhou, K. Yamahai, and Z. Liyan, *Appl. Opt.* **34**, 87 (1995).