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Authors(s)	Beiranvand, Arash, Cuffe, Paul
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A Topological Sorting Approach to Identify Coherent Cut-sets Within Power Grids

Arash Beiranvand, Paul Cuffe, *Member, IEEE*

***Abstract*—This paper proposes a new technique to identify sets of branches that form heavily loaded and potentially vulnerable flowgates within power grids. To this end, a directed acyclic graph is used to model the instantaneous state of power grids. One of the advantages of directed acyclic graphs is they allow the identification of where power flows are *coherent* e.g where power flows in a uniform direction along a set of branches that partition the network into two islands. This paper uses *topological sorts* to identify many sets of branches having this property. Definitions are provided for two new concepts, termed coherent cut-sets and coherent crack-sets, which are particular sets of branches extracted from a specific topological sort. Notably, there are numerous possible topological sorts for a directed acyclic graph and calculating distinctive topological sorts is challenging. In this paper a novel optimization algorithm is proposed to find multiple, diverse topological sorts each of which implies many cut-sets. The effectiveness of the proposed methods for enhancing grid observability and situational awareness is demonstrated using two standard test networks.**

***Index Terms*—power grids, graph theory, power flow, topological sorts, cut-sets**

I. Introduction

Real time risk observability of power flows in electrical networks is an important issue for control room operators and consequently has been widely investigated in the literature [1]–[3]. While steady state power flow is one of the most important tools to assess power grid operations [4], [5], analyzing the behaviours of power flows during emergencies like cascading failures can be challenging. Therefore, presenting an efficient method to provide real-time situational awareness of flowgate loadings in a rapidly changing system is necessary [6]. In this paper, a new concept, termed *coherent cut-sets*, is defined to offer insights into this problem. Using this new approach, sets of branches that are critical in terms of their instantaneous collective loading are readily identified. A coherent cut-set is defined as a set of branches that split the network into exactly two islands, with the restriction that all the traversing branches have the same flow directionality. Such cut-sets identify vulnerable bottlenecks within power grids and represent *seams* or *fault-lines* across which islanding seems likely.

Detecting these bottlenecks where collective power flows are loading substantial is important for network operators because these flowgates are vulnerable when facing branch outages and therefore, many methods have tried to make these areas observable for the operators in real-world networks [7]–[9]. The novel concept of coherent cut-sets enables the operators to effectively monitor these vulnerable flowgates within power grids in the face of rapidly changing condition e.g fluctuating power injections from variable renewable generators.

Graph theory can be a useful approach to assess power systems operation [8], [10], [11]. Power networks could be considered as complex networks and therefore, graph theory has been applied to analyze them in different aspects such as topological vulnerabilities [12]–[15]. To analyze topological vulnerabilities within power grids, power flow directions can be used to model the networks as a directed acyclic graph [12]. A directed acyclic graph refers to a directed graph which has no cycles e.g paths connecting any node back to itself [16]–[18]. The DC power assumptions imply a directed and acyclic flow graph for active power, which is exclusively transmitted from nodes of higher to lower voltage angle.

Using directed acyclic graphs for power systems analysis brings some important benefits, as these are well understood mathematical structures [19]. One such benefit is the ability to calculate topological sorts for the networks' nodes. A topological sort means assigning an integer rank to each node in a directed acyclic graph so that, for every directed edge from node u to node v, u must rank before v in the ordering [20]. As one can define various assumptions for finding the topological sorts of a directed acyclic graph (DAG), many valid topological sorts could be obtained for a DAG [21], [22]. Indeed, recalling the DC power assumptions, simply ordering the nodal voltage angles from highest to lowest gives a valid topological sort for the DAG representing a snap shot of a power system.

As the the size of a DAG increases, more and more valid topological sorts can be found [22]. Under such circumstances, finding all topological sorts becomes computationally infeasible. Therefore, presenting different algorithms and methods to find topological sorts for DAGs has been investigated [23], [24].

Concerning power grids, many studies have sought to find the most vulnerable branches of power grids in terms of topological centrality measures [25]–[27]. However, purely graph-based methods may lead to misleading results in terms of finding the most dangerous sets of branches for power grids and this is often due to the complex behaviors of power flows during emergencies like cascading failures [13]. In [12], [13] the authors investigate the abilities of the methods based on graph theory to assess cascading failure risk in electrical networks. Broadly speaking, the results given in [12], [13] show that purely graph-based methods embody real deficiencies for understanding emergency power flow behaviour in electrical networks.

For these reasons, any proposed graph-based methods must

A. Beiranvand (arash.beirandvand@ucdconnect.ie) and P. Cuffe are with the School of Electrical and Electronic Engineering, University College Dublin.
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take account of the physical realities of electrical power flow. A good example is [14], in which the authors seek to modify graph-based metrics to make them more applicable for power grids. For this purpose, some important electrical features of power grids like real power-flow allocation over branches and line flow limits are taken into account to update some usual structural metrics like node degree centrality and global efficiency. Global efficiency is one of the topological metrics for networks which seeks to gauge the holistic ability of a grid to transmit data (here active power) [28].

Concerning graph partitioning, many researchers have tried to suggest an effective method to partition graphs in the literature, either in steady state condition or dynamic state of power grids [29]–[33]. One of the most well-known methodologies to partition graphs is graph spectral analysis [29], [30]. These spectral methods, which are based on the eigenvalues of the graph Laplacian, can use the Fiedler vector [34] to partition the nodes into approximately equally sized islands, with the aim of minimizing the number of branches involved in the cut-set. Spectral partitioning techniques, and many other available methodologies in this area, do not typically aim to find *many* cut-sets for a graph, and instead focus on finding one specific cut-set in the graph, to meet some particular criteria. The suggested method in the present paper addresses this lacuna by proposing a new topological sorting approach that finds many cut-sets for a DAG, which furthermore have the useful property of coherence.

Detecting many vulnerable cut-sets in power grid can help the operators to have an overview about the contingencies in the grid and our method enables them to have such vital information.

The rest of this paper is organized as follows. In section II, the methods for finding topological sorts, coherent cut-sets and crack-sets are explained. In sections III and IV the sample grids and the simulation results and a discussion are presented. Section V concludes.

II. Methodology

In this section, a novel method to find many diverse topological sorts of a DAG is presented. These topological sorts can then be used to find many coherent cut-sets within the DAG. Therefore, at the first step, power grids should be modelled as a DAG.

A. Modelling power networks as directed acyclic graphs

To model a power grid as a digraph, the prevailing active power flows are used to set branch direction. The digraph's non-symmetric adjacency matrix A is built up by inserting a 1 for each pair of connected nodes i and j as follows:

$$\begin{cases} A_{ij} = 1 & if P_{ij} > 0 \\ A_{ji} = 1 & if P_{ij} < 0 \end{cases} \tag{1}$$

Where P_{ij} is the signed active power flow along the branch connecting i and j. These power flows could be obtained by simulation, state estimation or direct metering. Notably, these power flows depend on both the network's static topology and by the prevailing generator dispatch and load dispositions, and so represent a specific snapshot of the system's state.

The DC-PF power flow assumptions, recounted below, identify bus voltage angles as being the key determinant of active power flows through a grid:

$$P_{i,j} = \frac{\delta_i - \delta_j}{X_{ij}} \tag{2}$$

Where, δ_i and δ_j are the voltage angles of nodes i and j and X_{ij} is the reactance of the line which connects nodes i and j. These DC-PF power flow assumptions imply that active power exclusively flows from nodes of higher to lower voltage angles, and therefore they preclude the possibility of active power circulating in the network. This means that the system digraph built using equation 1 can be assumed to be acyclic i.e a DAG. It can be noted that simply ranking the nodes from highest to lowest voltage angle will provide one rather natural topological sort for the DAG. The following section describes a method to find other, less apparent topological sorts for the DAG.

B. Method to find distinctive topological sorts of directed acyclic graphs

1) Definition of topological sorts and coherent cut-sets: Consider a DAG $G = (V, E)$. A topological sort is the assignment of an unique integer rank to each node of G, with the requirement that for every pair of distinct vertices v_i and v_j in the ranking, if $v_i \longrightarrow v_j$ is an edge in G, then $R_i < R_j$ [20]. R_i and R_j are the ranks of node i and j respectively. A directed path cannot exist from a node to any node that precedes it in the sort.

The diagram in fig. 1 shows a small DAG, for which one could find many valid topological sorts. For example, one valid topological sort would be $\{1, 3, 5, 2, 4, 6, 7, 8\}$, as shown in table I. In this table, a red dashed line is shown which partitions the DAG into two islands, containing the nodes $\{1, 3, 5\}$ and $\{2, 4, 6, 7, 8\}$. The corresponding red dashed line is overlaid in fig. 1: this line identifies the three branches that must be removed to enforce this partition, the *cut-set*. Note that each of these branches is oriented from the upper to the lower island, and this property is guaranteed by the definition of a topological sort. Furthermore, every topological sort implies a network partitioning at each of its levels (i.e the dashed line in table I can be freely moved up and down) The many cut-sets that can be found in this way are likely to be heavily loaded, as their guaranteed coherence means that each branch additively contributes to the net cross-flow.

2) Optimization methodology to find topological sorts: This section describes a new algorithm which uses an integer optimization formulation to find many diverse topological sorts for a DAG. It should be noted that we emphasise the *diversity* of topological sorts because, after finding many possible sorts of a DAG using different assumptions, a lot of them could be quite similar. As similar topological sorts will imply similar cut-sets, this may obscure important bottlenecks in the grid. Therefore, an important aim for an algorithm for finding topological sorts should be obtaining heterogeneous topological sorts.

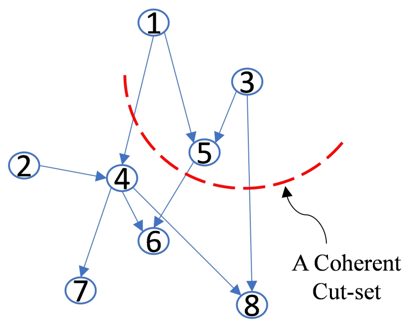

Fig. 1. A directed acyclic graph with a particular cut-set identified

TABLE I
EXAMPLE TOPOLOGICAL SORTING FOR THE SMALL DAG

Node label	Topological ranking
1	1^{st}
3	2^{nd}
5	3^{rd}
2	4^{th}
4	5^{th}
6	6^{th}
7	7^{th}
8	8^{th}

The decision variable is R_i which denotes the ranking of node i in the obtained topological sort. The key constraint for finding a topological sort is that if a node i sends power to j, then this directionality means that i must be ranked before j. Therefore, for any DAG with V nodes and E edges, the constraints of the problem are defined as follow:

$$R_i > R_j \quad \forall A_{i,j} = 1 \tag{3}$$

Where, R_i and R_j are the integer ranks of node i and node j in the obtained topological sort, and the number of these constraints is E. In the earlier example topological sort provided in table I, we would have $R_5 = 3$, as the node labelled 5 takes the 3^{rd} position in this sorting.

As a topological sort requires a unique integer ranking for each node, it is necessary to impose an `alldifferent` constraint [35] on the elements of R:

$$\texttt{alldifferent}(R_1 \ldots R_V) \tag{4}$$

The aim of the suggested method is to find numerous distinctive topological sorts. To this end, for each node i, considering the constraints mentioned in equation (1), we calculate two objective functions. The first one tries to maximize the rank of node i:

$$max\,(R_i) \tag{5}$$

The second objective function tries to minimize its rank:

$$min\,(R_i) \tag{6}$$

The solutions of each objective functions will give a topological sort e.g. a unique ranking for every node, which will be recorded by the algorithm.

The following pseudocode illustrates the procedure for iteratively applying the suggested algorithm to find many diverse topological sorts. By sequentially solving equations (5) and (6) for each node i, $2V$ topological sorts are found for each system, with each topological sort implying V cut-sets (recall table I). Therefore, the number of coherent cut-sets that this algorithm will identify is bounded by $2V^2$, however the actual number may be somewhat lower due to duplication and the fact that cut-sets creating more than two islands will be discarded for simplicity.

Input: An operational snapshot of the grid
Output: Many topological sorts of the grid's nodes

Build directed acyclic graph :
1: Extract all branches from power system
2: Detect sending node and receiving node for each branch, build matrix A
Build constraint set :
3: **for** $i, j \leq V$ **do**
4:   **if** $A_{i,j} = 1$ **then**
5:     add constraint $R_i > R_j$.
6:   **end if**
7: **end for**
Repeatedly solve the optimization :
8: **for** $i \leq V$ **do**
9:   **maximize** the rank of node i.
  s.t. all constraints obtained by line 5
10:   save sort R..
11:   **minimize** the rank of node i.
  s.t. all constraints obtained by line 5.
12:   save sort R.
13: **end for**

Topological sorting algorithms provide various time complexities depending on their methodologies. The most efficient time complexity for calculating one topological sort in a DAG $G = (V, E)$ is reported in [36] as follow:

$$O(|V| + |E|) \tag{7}$$

However, the proposed algorithm is distinct as, for each node i, it is searching for two specific topological sorts which minimize and maximize the rank of node i. Therefore, we suggest that the time complexity of the novel algorithm will likely exceed equation (7).

Regarding the feasibility of the suggested algorithm, there is at least one valid sort for the nodes which could be obtained from the voltage angles of the buses, as mentioned in section II-A. This important feature guarantees that there is at least one solution for the problem and the optimization is feasible.

C. Coherent cut-sets

Recall the definition of a coherent cut-set as a set of branches that split the network into two islands, with the restriction that all traversing branches have the same flow directionality. Returning to fig. 1, the red dash line shows a coherent cut-set

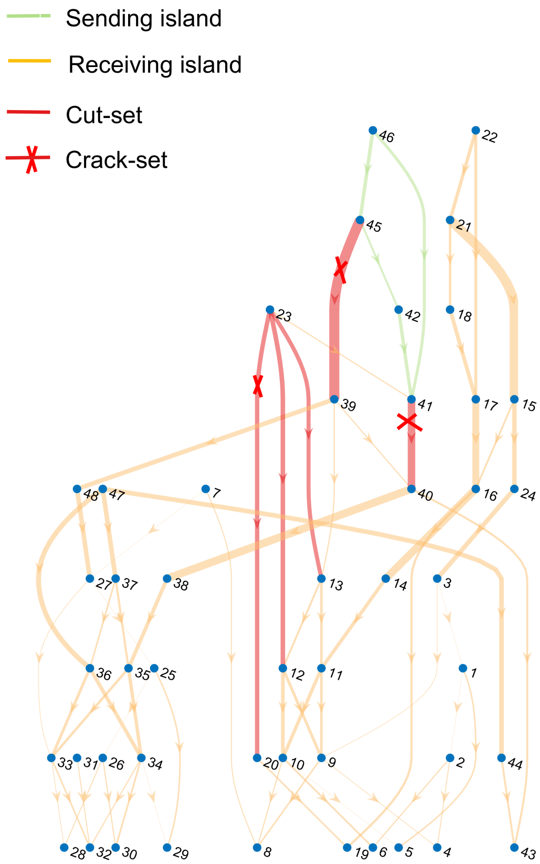

Fig. 2. The directed and acyclic graph of `case_ieee_rts_2_area`

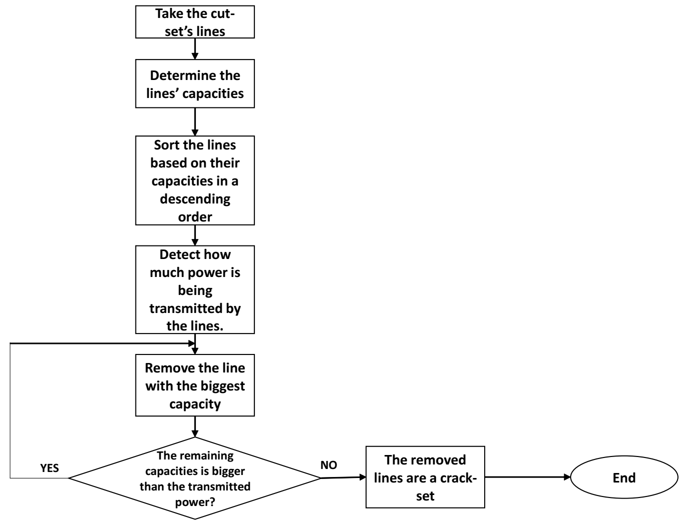

Fig. 3. The algorithm for extracting crack-sets from the cut-sets

for the DAG. As the edges between these two sub-grids together form a bridge connecting a sending and a receiving island, their flows are deemed to be *coherent*. After calculating each topological sort, it is trivial to find the coherent cut-sets it implies at each of its ordinal levels. To maintain clarity in the present work, only the levels in each topological sort which split the grid into *exactly two* islands will be used to define coherent cut-sets.

One can consider the two areas of a power grid separated by a coherent cut-set as a *sending island*, with a net power surplus, and a *receiving island*, with a net power deficit. This means that if the cut-set branches were removed, both islands would predictably suffer under- or over-frequency problems. Examples of these two zones can be seen in fig. 2, coloured green and orange, respectively. The total power carried by the branches in a cut-set determines how much power is being sent from the sending island to the receiving island.

To extend this analysis, we define a new normalized factor for each cut-set which is call *normalized power*:

$$\bar{P}(S_i) = \frac{\sum_{j\in S_i} P_j}{\sum_{j\in S_i} C_j} \tag{8}$$

Where, $\bar{P}(S_i)$ is the normalized power of cut-set S_i, P_j is the power transmitted by a line j within this cut-set, and C_j is this line's thermal capacity. This equation enables us to compare between the obtained coherent cut-sets in balanced way: it gives an aggregate percentage loading for the cut-set. Cut-sets with a high $\bar{P}(S_i)$ could be considered as heavily loaded flowgates which likely contribute to network congestion.

D. Coherent crack-sets

One useful observation stemming from the concept of coherent cut-set is the fact that, if some of the constituent branches of the cut-set are removed, the total transmitted power must continue to flow over the remaining branches.

Using this insight, another new concept is defined; *coherent crack-sets*. A coherent crack-set refers to the minimum set of branches that need to be removed from a cut-set so that the remaining branches' capacities are less than the amount of power which was being transmitted between the sending island and the receiving island. These crack-sets are very important in terms of operational vulnerability, because removing them guarantees that the other branches of the related cut-set will become overloaded (at least some of them) and subsequent events will occur which will result in splitting the grid into two separate parts, so that one side has surplus power and the other side is suffering from the shortage of active power. So, determining these crack-sets could help the operators to monitor the critical bottlenecks and contingencies in the system. To find the coherent crack-sets, the simple greedy algorithm presented in fig. 3 is applied.

By finding the coherent crack-sets of a power grid, we find the vulnerable sets of branches in the power grids $(N-X)$ securities. Removing these sets of branches will bring the grid to a dangerous operation point in which the grid may expect to face cascading failures.

E. Cascading failures simulation to validate the crack-sets analysis

To validate whether the cut-sets and crack-sets are usefully predictive of contingency power flow behaviour, cascading failures simulations are conducted. The applied technique to simulate cascading failures uses a practical and simple algorithm based on DC-PF to simulate cascading failures so that, at each steps of the cascading failures, overloaded branches are removed and the generation and demand balance is kept in the resulting islands by either shedding loads or adjusting the generation level. As the applied method takes account of line short-term overload limits, and can adjust generation/demand balance, its assumptions are somewhat different to those underpinning the crack-set algorithm in fig 3, and its more realistic results can be used to validate the crack-set potential damage estimate.

To this end, we make a comparison between two metrics for each sample grid. The first factor is titled *possible damage* which refers to the amount of power carried by the related cut-set j of a crack-set ($\sum_{j \in S_i} P_j$). It is called *possible* because it is predicted that removing the crack-set could result in a load-shedding equals to the power carried by the cut-set in the grid. The second factor is titled *simulated damage* which refers to the amount of load shedding actually arising after simulation of the removal of the branches in the crack-set.

III. Results

A. Test Platform

To evaluate the suggested method of this study, two sample grids are considered. These sample grids are `case_ieee_rts_2_area`, and `nesta_case118_ieee` from the repository at [37]. It should be noted that `case_ieee_rts_2_area` is built by joining two `nesta_case24_ieee_rts` from the repository at [37]. Also, the cascading failures simulation results suggested in section II-E are obtained by MATCASC [38]. MATCASC is an extension to MATPOWER which allows the simulation of cascading failures as explained in section II-E. Moreover, to obtain the distinctive topological sorts for the sample grids, CPLEX under GAMS [39] is applied which enables us to implement the mixed integer linear programme.

In this section, the obtained results after applying the suggested method for the two sample grids are presented thoroughly. Scripts and raw results are available at [40].

1) Simulation Speed and Scalability: The results in Table II show the performance of the suggested methodology and have been obtained using a personal desktop computer with Intel(R) Core(TM) i7-7700. The optimization formulation was not implemented with time performance in mind, however techniques such as parallelization [41] may offer gains here. Likewise, as the solver is called sequentially, the preceding topological sort could be used as a "warm start" for the next optimization problem.

B. Coherent cut-sets analysis

1) The results for `case_ieee_rts_2_area`*:* In fig. 2 the acyclic directed graph of `case_ieee_rts_2_area` is shown under one specific operation point. The branches determined by red colour are one coherent cut-set for the grid. As it is clear from this figure, these red branches are able to split the power network to two separate zones and simultaneously, their flow directions are uniform. These

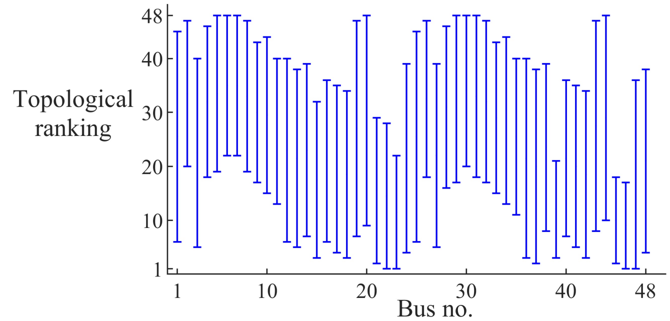

Fig. 4. The variation of ranking for each node in `case_ieee_rts_2_area`

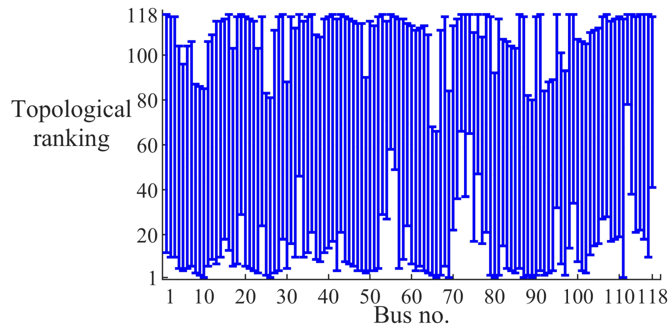

Fig. 5. The variation of ranking for each node in `nesta_case118_ieee`

branches are transmitting 1427 MW of active power from the sending to the receiving island.

To obtain the distinctive topological sorts of `case_ieee_rts_2_area` the optimization algorithm in equations (5) and (6) is sequentially applied.

The results in fig. 4 show the *range* of integer rankings R_i achieved for each node in this grid over many different solutions. As is obvious from this figure, the algorithm for finding the distinctive sorts is working so that, it puts each node at well separated maximum and minimum possible ranks to get a wide variety of distinctive topological sorts.

The cut-set analysis for `case_ieee_rts_2_area` can be found in fig. 6. This figure compares the cut-sets with each other, in terms of their sizes (which means the number of branches involved in each cut-set), and the outward power (which means the amount of power transmitted by the cut-sets from the sending island to the receiving island). This case study does not involve any cut-set with cardinality 9 or greater. It is notable, and perhaps relevant to a control room operator, that some sets of just three or four branches coherently carry over 1000 MW of active power.

The results in fig. 7 compare the normalized powers of all cut-sets with their related outward powers. The advantage of this figure is that the operators can observe how much each cut-set is transmitting power and how much this amount of power is close to the total capacities of the cut-set. This feature is potentially useful because a high normalised power loading may identify certain flowgates that are contributing to network congestion and market inefficiency. Note that the physics of power flow mean that a cut-set would typically not be able to

TABLE II
SCALABILITY AND THE PERFORMANCE OF THE PROPOSED METHODOLOGY

Sample grids	Number of calculated topological sorts	Number of unique topological sorts	Number of detected coherent cut-sets	Number of unique coherent cut-sets	The average time to find one cut-set (sec)
case_ieee_rts_2_area	96	93	4512	2229	0.40
nesta_case118_ieee	236	233	27612	23888	0.52

operate at near 100% of its notional capacity.

2) The results for `nesta_case118_ieee`*:* The plot in fig. 5 shows the achieved range of ranking for each node in this network. As is clear from this figure, the suggested method to find distinctive topological sorts is working and putting each node at minimum and maximum possible ranks that are quite separate.

Results in fig. 8 shows the cut-set analysis for this grid. Each dot is a specific cut-set. The novel method found 169 distinct coherent cut-sets for this grid under its particular operating point. Some of these cut-sets involved up to sixteen branches. In comparison with fig. 6, these cut-sets were typically less heavily loaded than those found for `case_ieee_rts_2_area`.

However, fig. 9 shows the normalized power analysis for `nesta_case118_ieee`, which is similar to fig. 7. This analysis indicates that cut-set thermal capacities are lower in `nesta_case118_ieee` than `case_ieee_rts_2_area`. Also, one can use figures 6 and 8 to detect highly congested cut-sets in the two grids.

C. Crack-sets analysis

Returning to the example of fig. 2: one can consider the red branches marked with X as a crack-set for this cut-set. This is due to this fact that after removing the crack-set's branches, the remaining capacities would be less than the 1025 MW transmitted, implying a power cascade will occur to the other branches in the cut-set.

1) The results for `case_ieee_rts_2_area`*:* The first sample grid is `case_ieee_rts_2_area`. The results in fig. 10 show the obtained results. This figure indicates that removing just two branches could interdict over 1000 MW of power flow: whereas removing three branches can interdict 1700 MW.

To more fully explore these worrisome vulnerabilities, the specific branches involved in the most dangerous crack-sets (the red frontier in fig. 10) are listed in Table III. From this table one can confirm that by simultaneously removing two branches such as (59, 64) or (59, 65), a substantial load-shedding (1047 MW) will occur. The grid is likewise quite vulnerable to a $(N-3)$ contingencies. Such important data could become critical for maintaining situational awareness as power flows rapidly fluctuate in the presence of variable renewable sources.

2) The results for `nesta_case118_ieee`*:* The results in fig. 11 is the crack-sets analysis for this network. Overall, this network appears more resilient against this type of $(N-X)$ contingency, in comparison with the results in fig. 10.

However, from the results presented in Table. IV, the first crack-set, which involves only one line, could be considered as a serious vulnerability for the grid because removing just line 38 could cause 673 MW of power flow to be interdicted.

TABLE III
CRACK-SET'S ANALYSIS FOR case_ieee_rts_2_area

Contingency type	Branches in most damaging crack-set(s)	Possible damage (MW)	Possible damage (P.U)
(N-1)	45	69	0.02
(N-2)	CS1: 59, 64 CS2: 59, 65	1047	0.30
(N-3)	53, 55, 56	1734	0.41
(N-4)	CS1: 52, 53, 54, 55 CS2: 48, 50, 54, 55 CS3: 48, 49, 50, 51	1744	0.41
(N-5)	17, 55, 56, 57, 60	1519	0.36

TABLE IV
CRACK-SETS ANALYSIS FOR nesta_case118_ieee

Contingency type	Branches in most damaging crack-set(s)	Possible damage (MW)	Possible damage (P.U)
(N-1)	38	673	0.20
(N-2)	59, 64	494	0.16
(N-3)	31, 38, 174	673	0.20
(N-4)	93, 94, 95, 100	1011	0.32
(N-5)	CS1: 90, 92, 93, 95, 100 CS2: 91, 92, 93, 95, 100	1069	0.34

One may deduce, using fig. 10 and fig. 11, that `case_ieee_rts_2_area` is more vulnerable in comparison with `nesta_case118_ieee`. To make a fair judgement about this problem we need the data available in the third columns of tables III and IV. As is clear from tables III and IV, `nesta_case118_ieee` could be more vulnerable than `case_ieee_rts_2_area` under attacks to one line and less vulnerable against attacks to two, three or four branches. For removals of five branches both grids have almost similar resilience. Therefore, the importance of the data available in tables III and IV is notable in terms of comparing two or more grids.

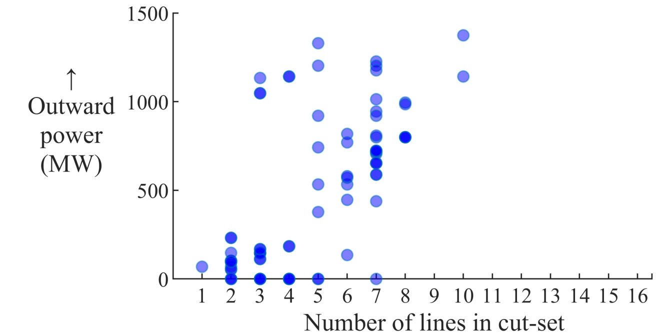

Fig. 6. The total coherent flows for the cut-sets found in `case_ieee_rts_2_area`

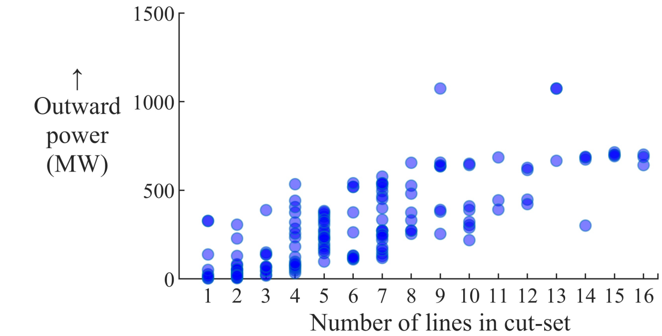

Fig. 8. The total coherent flows for the cut-sets found in `nesta_case118_ieee`

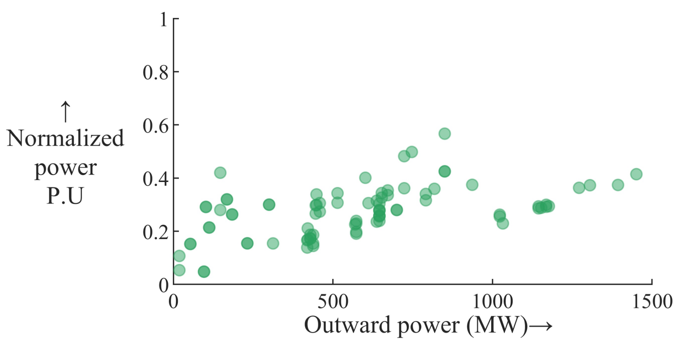

Fig. 7. The relationship between total and normalised power flows across the cutsets found in `case_ieee_rts_2_area`

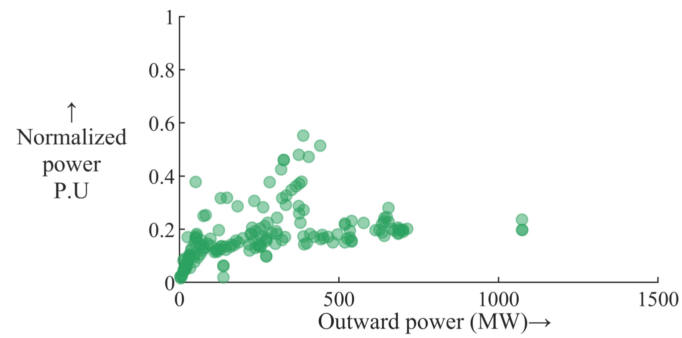

Fig. 9. The relationship between total and normalised power flows across the cutsets found in `nesta_case118_ieee`

D. MATCASC *results to validate to the crack-sets analysis*

1) Predictive value of crack-sets analysis: As mentioned in section.II-E, a cascading failure analysis is done to validate the crack-set analysis. To this end, the crack-sets are removed to calculate *simulated damage*, the concept presented in section II-E, for the grids. MATCASC [38] is applied to simulate the cascading failures and the damages. Then, *simulated damages* are compared with *possible damage*, another new concept defined in section II-E.

The results in figures 14 and 15 show these comparisons for `case_ieee_rts_2_area` and `nesta_case118_ieee` respectively. As is clear from both figures, removal of the majority of the crack-sets in both grids is sufficient invoke load shedding greater than, or equal to, the damages which were predicted. Although, in some cases the proposed method suggests crack-sets which are not dangerous as much as it is expected. This means that the intuitive crack-sets concept can offer meaningful insights into the complex non-linear behaviour of line overload cascades.

2) Comparison with Bompard technique: How much is the suggested crack-set analysis capable to detect the bottlenecks of the grids? To answer this important question the output of the crack-set analysis should be compared with the other available methods to find the critical lines in power grids [15], [42]. One of the well-known methodologies in this area is [15] which is based on a novel line centrality measure. We have implemented the method proposed in [15] for both sample grids of our paper and compared the obtained results with the results of our method presented in tables III and IV. To this end, the branches are ranked in terms of their criticality by the method of [15] and then $(N - X)$ contingency analysis is done by removing the top X critical branches and calculating the amount of resulting load-shedding. The amount of load-shedding is calculated by MATCASC. The $(N-X)$ contingencies implied by the Bompard method are then compared with the $(N - X)$ outages listed in Tables III and IV.

Figures 12, 13 show these comparisons for both sample grids. As is apparent, for all contingencies in `nesta_case118_ieee` and for most of the contingencies in `case_ieee_rts_2_area` the crack-set method has found more dangerous contingencies. This shows that, as an application of our method, the operator can have proper overview regarding the grids' vulnerabilities.

IV. Discussion

A. Some novel features of the proposed method

There are some other relevant points about this new method which should be mentioned. First of all, this new methodology is able to consider both topological and electrical features of power grids to assess the operation risk. This is due to this fact that the suggested method is based on prevailing power flows features which enable us to consider the topologies of the grids and the operation conditions simultaneously. Another important point is that the defined cut-sets are able to predict dynamic issues: after removing them, frequency deviations will arise in the sending and receiving islands.

Another point to note is the difficulty of intuitively identifying cut-sets. They may be geographically dispersed and

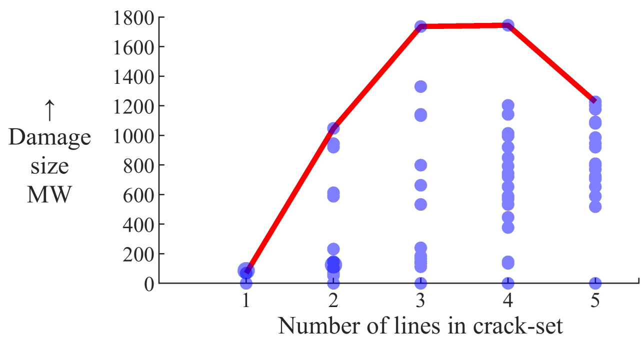

Fig. 10. The crack-sets analysis for `case_ieee_rts_2_area`

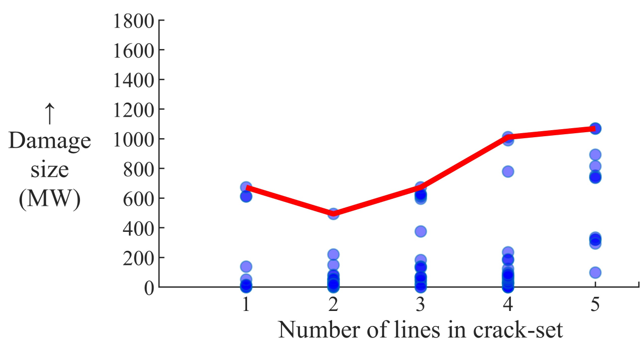

Fig. 11. The crack-sets analysis for `nesta_case118_ieee`

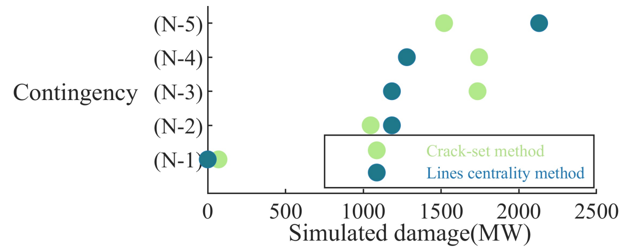

Fig. 12. Contingency comparison between the line centrality method and crack-set method for `case_ieee_rts_2_area`

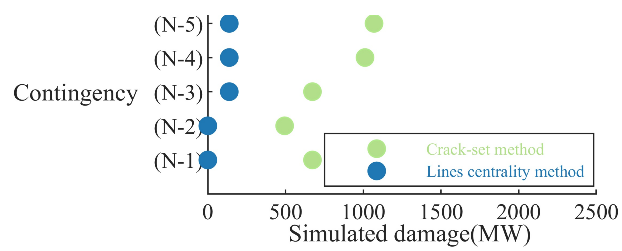

Fig. 13. Contingency comparison between the line centrality method and crack-set method for `nesta_case118_ieee`

far away from each other, but our suggested method is able to detect them effectively.The diagram [43] in fig. 16 shows a sample cut-set for `nesta_case118_ieee`. In this figure the red arrows are the cut-set's branches which also show the prevailing directions of active power flow and the two separated parts of the grid by the cut-set are shown by green and orange.

As it is clear, the cut-set's branches are physically far away from each other and not mutually connected. However, they are an important set of branches for the grid in this particular operational state and our proposed method has successfully found them. This diagram illustrates how identifying important cut-sets is not always intuitive and is challenging for the operators using pseudo-geographic single line diagrams: note that the layered graph drawing in fig 2 is an atypical diagram style which reflects the network's prevailing state, and thereby makes cut-sets seem substantially more apparent.

Finally, it should also be noted that in many cases, an outage of just one or two nodes could cause a number of branches to be removed [44], [45]. For instance, for `case_ieee_rts_2_area`, by removing just node 47 the branches 55, 56, 67 will be removed as well. These branches are a crack-set for the grid and removing them could interdict 1134 MW of cross-flow.

B. Comparison with the available graph portioning methods

As mentioned, graph partitioning has been widely investigated in the literature [29], [30], [33]. The extant methods in the literature generally provide just one specific cut-set in the graph, depending on the specific objective of each method for finding the cut-set. For example, one of the most common goals is to find a cut-set which splits the grid into two reasonably equal parts, in terms of the number of nodes in each side, with the restriction of minimizing the branches involved in the cut-set [29]. The result obtained from our method is clearly showing that in some cases the cut-sets which split the grid into a large and a small island represent real vulnerabilities for the grid. For example, for `case_ieee_rts_2_area`, our method has found a cut-set that includes branches (59, 64, 65) which splits the grid into two parts so that, one part includes only two nodes 45 and 46. Removing this cut-set will lead to a notable damage equal to 1047 MW.

Some very relevant research which focuses specifically on power grid partitioning is [30], which focuses on finding a specific cut-set which includes a minimum number of branches and causes maximum imbalance between two sub grids in terms of the generation level of each side. In this respect it is quite comparable to the present method, however it does not enumerate a wide number of different cut-sets, and so the present technique can be seen as an attempt to provide a more holistic view of potential congestion and vulnerabilities in a power system.

V. Conclusion

This paper outlined a new method to detect congested bottlenecks within power grids. A novel topological sorting algorithm was described, which uses a simple optimization formulation to find many diverse topological sorts for a directed acyclic graph. Each ordinal ranking in each of these topological sorts defines a cut-set that splits the networks into two islands, with the useful property that power flow directionality is coherent amongst all of the branches forming that cut-set.

Using this insight, two new terms were defined for analysing the instantaneous power flow disposition of a power system: coherent cut-sets and coherent crack-sets. The detection methodology was applied to two sample test systems and succeeded in

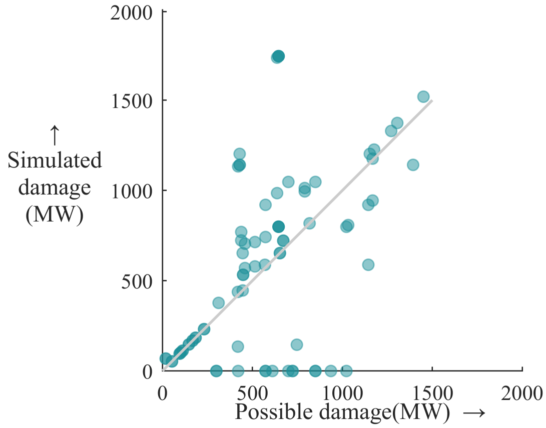

Fig. 14. MATCASC results comparison for `case_ieee_rts_2_area`

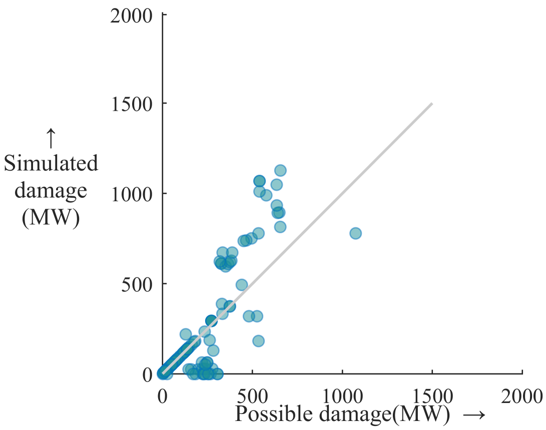

Fig. 15. MATCASC results comparison for `nesta_case118_ieee`

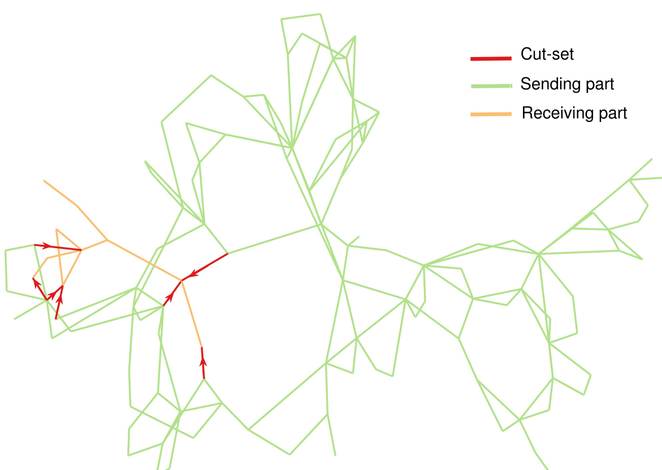

Fig. 16. A sample cut-set for `nesta_case118_ieee`

identifying heavily congested flowgates. Validating simulations indicated that the crack-set concept was useful for predicting the eventual damage stemming from a line-outage cascade. As future work in this area, improving the algorithm for finding the topological sorts and coherent cut-sets may help to decrease run-times and find more dangerous crack-sets within the grids in an operational context.

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