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Equilibrium and kinetic analysis of Phosphorus adsorption from aqueous  
solution using waste alum sludge

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## Abstract

1 Excess phosphorus (P) in wastewaters promotes eutrophication in receiving waterways. A  
2  
3 cost-effective method such as use of novel low-cost adsorbents for its adsorptive removal  
4  
5 would significantly reduce such impacts. Using batch experiments, the intrinsic dynamics of P  
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7 adsorption by waste alum sludge (an inevitable by-product of drinking water treatment plants)  
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9 was examined. Different models of adsorption were used to describe equilibrium and kinetic  
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11 data, calculate rate constants and determine the adsorption capacity. Results indicate that the  
12  
13 intraparticle rate constant increased from  $0.0075 \text{ mg g}^{-1} \text{ min}^{-1}$  at  $5 \text{ mg L}^{-1}$  to  $0.1795 \text{ mg g}^{-1}$   
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15  $\text{min}^{-1}$  at  $60 \text{ mg L}^{-1}$  indicating that more phosphate is adsorbed per gram.min at higher P  
16  
17 concentration. Further analyses indicate involvement of film and particle diffusion  
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19 mechanisms as rate controlling steps at lower and higher concentrations respectively. Mass  
20  
21 transfer coefficient obtained ranged from  $1.7 \times 10^{-6}$  to  $1.8 \times 10^{-8}$  indicating a rapid  
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23 transportation of phosphate molecules onto the alum sludge. These results further  
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25 demonstrates that alum sludge – hitherto thought of as undesirable waste, can be used as  
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27 novel adsorbent for P removal from wastewater through various applications, thus offsetting a  
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29 portion of the disposal costs while at the same time improving water quality in sensitive  
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31 watersheds.  
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**Keywords** Adsorption capacity, adsorption kinetics, adsorption models, alum sludge, phosphorus

## 1. Introduction

Phosphorus (P) is an essential, often limiting, nutrient for growth of organisms in most ecosystems and it is also a very important material for many industries. However, its extensive industrial use inevitably results in large amounts of P-bearing wastes which are usually discharged into municipal and industrial effluent streams. The need for efficient removal of P from wastewaters before discharge into watercourses is well understood. It is well known that the release of P from wastewaters into watercourses can cause severe pollution problems, such as eutrophication which frequently results in algal blooms, fish kills and loss of water resources. The cost of eutrophication to the UK water industry is estimated at > £15M per annum [1], while it is reported that about 30% of the Irish river channel length is polluted mainly from eutrophication [2]. Thus, the continued search for cost-effective methods for P removal remains a high priority in order to control and remedy eutrophication problems and maintain a sustainable green environment for the future.

In recent times, P removal using waste materials or by-products as novel adsorbents has been gaining increased attention. Reviews on such materials which include slag materials, burnt oil shale and ashes from the thermal incineration industry have been published [3, 4]. Another by-product which is a promising low-cost adsorbent for P removal from wastewaters is waste alum sludge. Alum sludge is a by-product of water treatment plants that use aluminium salts as primary coagulant and it is the most widely generated water treatment residual/sludge worldwide [5]. Alum sludge usually contains colloidal alum hydroxides which are often an amorphous species. These generally have larger surface area and greater reactivity toward anion adsorption than the corresponding crystalline mineral phases. Due to

1 the relative abundance of aluminium in the alum sludge and the high reactivity, they possess a  
2 high capacity for phosphorus removal through adsorption [5].  
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4  
5 Compared to other by-products that have been considered as adsorbent for P removal,  
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7 alum sludge possesses a distinct advantage because it is an easily and freely available by-  
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9 product in towns, cities and metropolis regions worldwide that utilize surface waters as a  
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11 drinking water source, and therefore, its availability is guaranteed. A number of researches  
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13 have been conducted to demonstrate the potential use of alum sludge as adsorbent for P  
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15 removal while its use as a P removing substrate in constructed and engineered wetland  
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17 systems is also being actively promoted and demonstrated [5, 6, 7, 8, 9, 10]. However, most  
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19 of these studies have been limited to the characterization and the determination of the P  
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21 adsorption capacity of the alum sludge. In this study, the intrinsic dynamics of P adsorption  
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23 from aqueous solution by the alum sludge is investigated. Such a study is important in  
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25 understanding the dynamics of P adsorption by the alum sludge and facilitating its efficient  
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27 use for P removal in practise.  
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## 36 **2 Materials and methods**

### 37 *2.1 P-adsorption experiments*

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41 Dewatered alum sludge cakes were collected from the largest water treatment plant in  
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43 Ireland, located in Co. Kildare, Ireland. The plant uses aluminium sulphate for reservoir water  
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45 flocculation. The alum sludge cakes were air-dried and ground to pass a 2-mm mesh sieve  
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47 prior to being used. The physico-chemical characteristics, the equilibrium time and optimal  
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49 dosage for P adsorption by the alum sludge have been determined and reported [6]. **Regarding**  
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51 **the metal composition of the dewatered alum sludge, the results show that the relative**  
52  
53 **proportion of aluminium, iron, arsenic, lead, titanium and zinc were 42.67, 3.34, 0.034, 0.005,**  
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55 **0.099 and 0.03 mg g<sup>-1</sup> respectively (metals analysed were based on their presence in the alum**  
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sludge from). It should however be noted that except for aluminium, the levels of most other metals in the alum sludge especially lead and zinc, are below their typical levels in uncontaminated soils [6]. Furthermore, a 42-week leaching analysis from a pilot field-scale study utilizing the alum sludge shows that although mainly aluminium leaching was observed in the first three weeks of operation, effluents concentrations of both total and soluble Al were all below the general regulatory guideline limit of 0.2 mg L<sup>-1</sup> for drinking water quality standard and effluent discharge [unpublished results]. It was therefore concluded that concerns about the leaching of aluminium or other metals should not restrict its use although periodical monitoring is recommended. Regarding the morphological structure of the alum sludge, X-ray diffraction analysis reveals no sharp diffraction characteristic, while SEM observation of the alum sludge shown in Figure 1 did not reveal any classical well-crystalline appearance on its surface suggesting that the alum sludge is amorphous.

A contact time of 48 hours and an optimal dosage of 10 g L<sup>-1</sup> were determined. To obtain equilibrium data for determining the P adsorption capacity and affecting factors, one gram of the alum sludge was equilibrated with 100 ml of P solution with concentrations ranging from 0 to 360 mg P L<sup>-1</sup> for 48 hours at three different pH levels (4, 7, 9). After the set time (48 hours), the mixture was withdrawn and filtered using a 0.45 µm Millipore membrane filter and analysed for residual P using a HACH DR/2400 spectrophotometer according to its standard operating procedures. The amount of P adsorbed by the alum sludge from the solution at equilibrium,  $q_e$  in mg g<sup>-1</sup>, was computed using Eq.1.

$$q_e = \frac{(C_o - C_e)V}{m} \quad \text{Eq. 1}$$

where  $C_o$  and  $C_e$  (both in mg L<sup>-1</sup>) are the initial and final P concentrations at equilibrium respectively. Data obtained were fitted to linearised forms of Langmiur, Freundlich, Dubinin-Radushkevich (D-R), Temkin, Frumkin and Harkins–Jura adsorption isotherm models in order to determine the adsorption parameters.

[INSERT FIG. 1 HERE]

The description of these adsorption isotherm models is widely available in scientific texts and literature and will not be repeated here. However, the adsorption isotherm equations, their corresponding linear forms and the parameters obtained from linearization are summarized and presented in Table 1, section 3.1.

## 2.2 Kinetics of P- adsorption by the alum sludge

To examine the kinetics of the P adsorption process, the batch technique was also used. A number of 250 ml bottles were filled with 100 ml of P solutions at four different P concentrations (5, 15, 30, and 60 mg L<sup>-1</sup>), and 1 g of the alum sludge was added to each bottle. The mixture was then agitated at 200 rpm on the rotary shaker. At predecided times, the mixtures were withdrawn, filtered and analysed for P uptake using Eq. 1. The kinetic data were then analysed as discussed below.

### 2.2.1 Adsorption rate constant

The kinetics of P adsorption process by the alum sludge was examined by the pseudo-first-order equation (Eq. 2): [11]

$$\frac{dq_t}{dt} = K_1(q_e - q_t) \quad \text{Eq. 2}$$

where  $q_t$  is the amount of adsorbate adsorbed at time  $t$  (mg g<sup>-1</sup>),  $K_1$  is the pseudo-first-order rate constant (min<sup>-1</sup>) and  $t$  the contact time (mins). Solving the equation, it can be written as

$$\log(q_e - q_t) = \log q_e - \frac{K_1}{2.303}t \quad \text{Eq. 3}$$

The kinetic data were also fitted to the pseudo-second-order equation which has the form:  
[12]

$$\frac{dq_t}{dt} = K_2(q_e - q_t)^2 \quad \text{Eq. 4}$$

where  $k_2$  is the pseudo-second-order rate constant ( $\text{g mg}^{-1} \text{min}^{-1}$ ). Solving the equation, it can be written as

$$\frac{t}{q_t} = \frac{1}{k_2 q_e^2} + \left( \frac{1}{q_e} \right) t \quad \text{Eq. 5}$$

The intraparticle diffusion model was also used [13, 14]

$$q = k_d t^{1/2} \quad \text{Eq. 6}$$

where  $k_d$  ( $\text{mg g}^{-1} \text{min}^{-1/2}$ ) is the rate constant for intraparticle diffusion. The initial rates of intraparticle diffusion can be obtained by linearization of Eq.6. The values were further supported by calculating the intraparticle diffusion coefficient  $\bar{D}$  ( $\text{cm}^2 \text{s}^{-1}$ ) using Eq. 7 [15]

$$\bar{D} = 0.03r^2/t_{1/2} \quad \text{Eq. 7}$$

where  $t_{1/2}$  (min) is the time for half of the adsorption and  $r$  is the average particle radius of the adsorbent particles.

### 2.2.2 Mass transfer

To further examine the kinetics of P adsorption and calculate the mass transfer coefficient  $\beta_L$ , experimental kinetic data were fitted to a mass transfer model [16]

$$\ln \left[ \frac{C_t}{C_o} - \frac{1}{1+mk} \right] = \ln \left[ \frac{mk}{1+mk} \right] - \left[ \frac{1+mk}{mk} \right] \beta_L S_s t \quad \text{Eq. 8}$$

where  $C_t$  and  $C_o$  (both in  $\text{mg L}^{-1}$ ) are the respective concentrations of P in solution at time  $t$  and zero and  $k$  (in  $\text{L g}^{-1}$ ) is a constant obtainable by multiplying the Langmuir constants  $Q_m$  and  $b$ .  $m$  ( $\text{g L}^{-1}$ ) and  $S_s$  ( $\text{cm}^{-1}$ ) are, respectively, the mass and outer surface area of the adsorbent particles per unit volume of particle free solution and  $\beta_L$  ( $\text{cm}^{-1}$ ) is the mass transfer coefficient. The values of  $m$  and  $S_s$  are calculated using the following equations:

$$m = \frac{W}{V} \quad \text{Eq. 9}$$

$$S_s = \frac{6m}{(1 - \varepsilon_p)d_p\rho_p} \quad \text{Eq. 10}$$

where  $W$  is the weight of the adsorbent (g),  $V$  is the volume of particle free adsorbate solution (L),  $d_p$  is the particle diameter (cm),  $\rho_p$  is the density of the adsorbent (g/cm), and  $\varepsilon_p$  is the porosity of the adsorbent particle.  $\beta_L$  was determined by linearizing Eq. 8.

### 2.2.3 Rate expression

For the proper understanding of the experimental data, it is necessary to identify the rate controlling step governing the overall adsorption process. Boyd et al. [17] and Reichenberg [18] gave the expression of Eq. 11

$$F = 1 - \frac{6}{\pi^2} \sum_1^{\infty} \left( \frac{1}{n^2} \right) \exp(-n^2 B_t) \quad \text{Eq. 11}$$

Where  $F$  is the fractional attainment of equilibrium at time 't' and is obtained from Eq. 12 and  $n$  is Freundlich constant of the on going adsorption process.

$$F = \frac{Q_t}{Q_{\infty}} \quad \text{Eq. 12}$$

where  $Q_t$  and  $Q_{\infty}$  are the amount of uptake at time  $t$  and equilibrium, respectively. For every value of  $F$ , corresponding pre-calculated value of  $B_t$  was derived from the Reichenberg's table [17]. Thereafter, plots of time versus  $B_t$  were obtained at different initial  $P$  concentrations. The linearity of  $B_t$  versus  $t$  is employed to distinguish between the film and particle controlled mechanism [19]. To evaluate the data, the slopes of the straight line plots obtained at the different concentrations were taken as  $B$ , the time constant, and this was subsequently used for the evaluation of the values of  $D_i$  using Eq. 13

$$D_i = \frac{Br_o^2}{\pi^2} \quad \text{Eq. 13}$$

where  $D_i$  is the effective diffusion coefficient of adsorbate in adsorbent phases,  $r_o$  is the radius of the adsorbent particle assumed to be spherical and  $n$  is an integer.

### 3 Results and discussion

#### 3.1 Determination of adsorption isotherm constant and maximum adsorption capacity

The fitting of the experimental data to the linear forms of the six adsorption isotherm is shown in Fig. 2a, b, c, d, e and f for Langmuir, Freundlich, D-R, Temkin, Frumkin and Harkins–Jura models respectively. The parameters obtained from the linearization alongside the model equations and their linear forms are shown in Table 1. The results of the fitting of the experimental data to the Langmuir and Freundlich models have been discussed elsewhere [6] and would only be mentioned here briefly.  $R^2$  were found to be highest for the Langmuir plots ranging from 0.97 to 0.98 indicating its best fit. The Freundlich, D-R, Temkin and Frumkin plots also gave high  $R^2$  values ranging from 0.81 to 0.97 with the exception of D-R plot at pH 9 which gave a  $R^2$  value of 0.65. However, the H-J model was not analysed further since it appears to be poorly fitted ( $R^2$ :0.61 to 0.69).

**[INSERT FIG. 2 HERE]**

**[INSERT TABLE 1 HERE]**

The maximum adsorption capacity evaluated from the Langmuir isotherm plot was determined to be 31.9  $\text{mg P g}^{-1}$  at pH 4 and the value of the adsorption capacity decreased with an increase in pH from 4 to 9. Previous investigations indicate that the adsorption capacity varied from 3.3  $\text{mg P g}^{-1}$  to 23.9  $\text{mg P g}^{-1}$  depending on the alum sludge and test conditions such as initial concentration, pH and age of the alum sludge [20, 21]. In

1 comparison to the maximum adsorption capacity of other materials cited in literature as  
2 determined from the linearised plot of the Langmuir model [9, 10, 22], the alum sludge used  
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4 in this study can be seen to have a significant and comparable P adsorption capacity. For  
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6 instance, Sakadevan and Bavor [22] used the Langmuir model to determine the maximum P  
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8 adsorption capacity of soils, slags and zeolites and obtained a range of values from 0.93 to  
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10 44.3 mg P g<sup>-1</sup>. Similarly, in two separate studies using water treatment plant residual for P  
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12 adsorption, Kim et al. [9] and Ippolito et al. [10] calculated removal capacities of 25 mg P g<sup>-1</sup>  
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14 and 12.5 mg P g<sup>-1</sup> respectively using the Langmuir model. Thus, with P adsorption capacity  
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16 values ranging from 10.2 mg P g<sup>-1</sup> to 31.9 mg P g<sup>-1</sup>, the alum sludge used in this study can be  
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18 seen to have a reasonable and high P adsorption capacity as compared to the capacity of other  
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20 adsorbents reported in literature.  
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26 The D-R plot of log q<sub>e</sub> vs ε<sup>2</sup> at the different pH is shown in Fig. 2c and the constants  
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28 parameters determined are also listed in Table 1. The mean adsorption energy, E, which gives  
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30 information about chemical and physical adsorption was found to be in the range 17.7 to 21.3  
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32 KJ mol<sup>-1</sup>. This is bigger than the energy range of adsorption reaction (8-16 KJ mol<sup>-1</sup>) [23], and  
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34 thus the adsorption of P onto the dewatered alum sludge cakes may also involve chemical  
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36 adsorption. The Temkin plot (Fig. 2d) and the parameters obtained listed in Table 1 indicate  
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38 that the constant B<sub>1</sub> which represents the heat of adsorption decreased with an increase in pH.  
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40 This may also suggest an increase in adsorption activity at the low pH. The positive values of  
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42 constant 'a' in the Fumkin model indicate that there is attractive interaction between the  
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44 adsorbed species [23], i.e. the phosphate molecules and the alum sludge.  
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### 54 **3.2 Adsorption rate constant studies**

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56 The experimental data were fitted to both the first and second order adsorption models  
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58 with correlation coefficients of R<sup>2</sup> = 0.92-0.97 and R<sup>2</sup> = 0.99 obtained respectively (Fig. 3a,  
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1 b). Although high correlation coefficients were obtained from the fitting of the data to the first  
2 and second-order models, several authors have shown that the adsorption of P fits better with  
3 the second-order-adsorption model than the first order [24, 25, 26]. From the slopes derived  
4 from the straight line plot of the first and second order adsorption model, respective values of  
5 the rate constants of adsorption,  $k_1$  and  $k_2$  respectively, at different initial P concentration  
6 were determined.  
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14 Values of the rate constants of adsorption at P concentrations of 5, 15, 30 and 60 mg L<sup>-1</sup>  
15 were determined to be  $5.0 \times 10^{-2}$ ,  $2.0 \times 10^{-2}$ ,  $1.4 \times 10^{-2}$  and  $1.1 \times 10^{-2}$  min<sup>-1</sup>, and  $8.3 \times 10^{-1}$ ,  $1.3 \times 10^{-1}$ ,  
16  $0.5 \times 10^{-1}$  and  $0.2 \times 10^{-1}$  g mg<sup>-1</sup> min<sup>-1</sup> for the first and second order models respectively.  
17  
18 Similarly, the  $q_e$  values were determined to be 0.11, 0.44, 1.06 and 2.30, and 0.51, 1.45, 2.54  
19 and 4.47 mg g<sup>-1</sup> for first and second order models respectively. In both cases, the rate  
20 constants were found to decrease with increasing initial P concentration while  $q_e$  values  
21 increased with increase in initial P concentration. This indicates that the contact time  
22 necessary to achieve equilibrium might be affected by the initial P concentration, with more  
23 time being necessary to reach equilibrium at the higher values of the initial P concentration.  
24 However, the P uptake increases with increase in initial P concentration. This same trend was  
25 reported for P adsorption by Namasivayam and Sangeetha [24]. In this study, at an initial P  
26 concentration of 30 mg P L<sup>-1</sup>, values of 0.014 min<sup>-1</sup> and 0.05 g mg<sup>-1</sup>min<sup>-1</sup> were obtained for  $k_1$   
27 and  $k_2$  respectively. At the same initial P concentration of 30 mg P L<sup>-1</sup>, Ozacar [27] and  
28 Namasivayam and Sangeetha [24] reported values of 0.0544 min<sup>-1</sup> and 0.016 min<sup>-1</sup> for  $k_1$   
29 respectively, while Namasivayam and Sangeetha [24] reported  $k_2$  value of 0.048 g mg<sup>-1</sup> min<sup>-1</sup>.  
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[INSERT FIG 3 HERE]

1 Thus it can be seen that the values obtained in this study compares favourably well with  
2 those obtained in other studies. To probe further into the rate of internal mass transfer, data  
3 was fitted to the intraparticle diffusion model (Fig. 3c). An excellent fit of the experimental  
4 data to the intraparticle diffusion model should show a strong linear relationship with the  
5 straight line passing through the origin. However, this is apparently not the case even though  
6 there is a linear relationship as suggested by the plots at the respective concentrations (Fig.  
7 3c). This indicates that, whereas particle diffusion mechanism is involved, it may not be the  
8 rate-limiting mechanism. As can be deduced from Fig. 3c, increasing the phosphate  
9 concentration in the solution promoted the diffusion in the alum sludge particles and this  
10 resulted in an increase in the intraparticle diffusion rate. The rate constants as calculated from  
11 the regression equation increased from  $0.0075 \text{ mg g}^{-1} \text{ min}^{-1}$  at  $5 \text{ mg L}^{-1}$  to  $0.1795 \text{ mg g}^{-1} \text{ min}^{-1}$   
12 at  $60 \text{ mg L}^{-1}$  indicating that more phosphate is adsorbed per gram.min at the higher P  
13 concentration. At this stage, it can be suggested that the adsorption of phosphate probably  
14 takes place via surface exchange reaction until the surface functional sites are fully occupied.  
15 Thereafter, the phosphate molecules diffuse into the pores of the alum sludge for further  
16 reactions. This has also been suggested as possible kinetic mechanism for the adsorption of P  
17 by water treatment residuals from recent studies [10]. They postulated that P may be initially  
18 adsorbed as an outer sphere complex or found in the diffuse ion swarm near individual  
19 particles.

20 To lend credence to this, results from this study further highlight the better linearity  
21 obtained at higher P concentration ( $R^2 = 0.62, 0.85, 0.96$  and  $0.90$  for P concentration values  
22 of  $5, 15, 30$  and  $60 \text{ mg L}^{-1}$ ). This suggests that intraparticle diffusion may be a predominant  
23 mechanism for P adsorption at high P concentration. It can be reasoned that the higher  
24 concentration provides a higher gradient which drives the ions into the pores. According to  
25 Walker et al. [28], the initial stage of the intraparticle diffusion represents external mass

1 transfer, followed by intraparticle diffusion into the macro, meso and micro-pore structure of  
2 the adsorbent. In this study, the plots obtained do suggest to some extent, the presence of  
3 intraparticle diffusion (pore diffusion), but it also indicates that it is not the only rate  
4 controlling step.  
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9 The calculated values of  $\bar{D}$  obtained in this study at different initial P concentrations of 5,  
10 15, 30 and 60  $\text{mg L}^{-1}$  were  $1.98 \times 10^{-10}$ ,  $6.67 \times 10^{-11}$ ,  $2.67 \times 10^{-11}$ , and  $2.22 \times 10^{-11} \text{ cm}^2 \text{ s}^{-1}$ .  
11  
12 According to Michelsen et al. [29], a  $\bar{D}$  value of the order of  $10^{-11}$  is indicative of  
13 intraparticle diffusion as the rate limiting step. Based on this, it can be suggested that  
14 intraparticle diffusion predominates at the high initial P values. Namasivayam and Sangeetha  
15 [24] in their study on P adsorption onto  $\text{ZnCl}_2$  activated coir pith carbon reported  $\bar{D}$  values in  
16 the order of  $10^{-12}$  to  $10^{-13} \text{ cm}^2 \text{ s}^{-1}$  and concluded that the P adsorption follows a pore diffusion  
17 process. Analysis of experimental data obtained in this study however suggests that such  
18 intraparticle (pore) diffusion predominates at high P concentrations.  
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### 32 **3.3 Mass transfer and rate determining step**

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34 The experimental data were also well fitted to the mass transfer model (Fig. 4) with fairly  
35 and comparative low values of  $\beta_L$  obtained. The  $\beta_L$  values ranged from  $1.7 \times 10^{-6}$  to  $1.8 \times$   
36  $10^{-8}$  and the values were found to decrease with increasing initial P concentration. The fairly  
37 low values of  $\beta_L$  obtained indicate a rapid transportation of the adsorbate from bulk to solid  
38 phase and it reflects the good efficiency of the adsorbent towards the adsorbate [30, 31]. The  
39 fitting of the experimental data to the model of Boyd and Reichenberg [17, 18] is shown in  
40 Fig. 5. The linear plots clearly indicate that at low concentration ( $5 \leq P \leq 15 \text{ mg P L}^{-1}$ ), the  
41 linearity of the plot deviates, whereas at higher concentrations, ( $30 \leq P \leq 60 \text{ mg P L}^{-1}$ ), the  
42 linearity is maintained.  
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[INSERT FIGURES 4 AND 5 HERE]

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5 This phenomenon suggests the involvement of film diffusion and particle diffusion  
6  
7 mechanisms as the rate controlling steps at lower and higher concentrations respectively. To  
8  
9 further evaluate the data, the constant B was analysed. Values of B obtained compare  
10  
11 favourably with those reported in literature [11] and also indicate that with increase in  
12  
13 concentration, the rate of intraparticle diffusion also increases.  
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### 19 **Conclusion**

20  
21 This study further demonstrates that alum sludge, a widely available by-product of water  
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23 treatment plants can be used a cost-effective adsorbent for P in aqueous solutions. Analysis of  
24  
25 adsorption data obtained in this study suggests that P adsorption takes place via surface  
26  
27 exchange reaction until the surface functional sites are fully occupied. Thereafter, the P  
28  
29 molecules diffuse into the pores of the alum sludge for further reactions. Higher rate constants  
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31 were obtained at higher P concentrations indicating that more P is adsorbed per gram.min at  
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33 higher concentration and the values of the mass transfer coefficient obtained indicate a rapid  
34  
35 transportation of P onto the alum sludge. Wider, practical application of the results can  
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37 include the use of the alum sludge to enhance phosphorus removal in treatment systems such  
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39 as constructed and engineered wetlands.  
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Table 1 Adsorption isotherm model equations, corresponding linear forms and parameters obtained from linearization

Model	Equation	Linear form	Parameters obtained from linearization			
			pH 4	pH 7	pH 9	
Langmuir	$q_e = \frac{Q_0 b C_e}{1 + b C_e}$	$\frac{C_e}{q_e} = \frac{a_L}{K_L} C_e + \frac{1}{K_L}$	$Q_0$ (mg-P/g)	31.9	23.0	10.2
			$b$ (L/mg)	0.027	0.025	0.024
			$R^2$	0.98	0.97	0.97
Freundlich	$q_e = K_F C_e^{1/n}$	$\log q_e = \log K_F + \frac{1}{n} \log C_e$	$k_f$ (L/g)	0.67	0.53	0.41
			$n$	1.13	1.24	1.24
			$R^2$	0.91	0.93	0.88
D-R	$q_e = q_m e^{-k\varepsilon^2}$	$\ln q_e = \ln q_m - k\varepsilon^2$	$q_m$ (mg-P/g)	5.23	3.32	2.69
			$K$ (mol <sup>2</sup> /kJ <sup>2</sup> )	0.0011	0.0013	0.0016
			$E$ (kJ/mol)	21.3	19.6	17.7
			$R^2$	0.84	0.81	0.65
Temkin	$q_e = \frac{RT}{b} \ln(k_T C_e)$	$q_e = B_1 \ln k_T + B_1 \ln C_e$	$B_1$	10.35	7.82	5.62
			$k_T$ (L/mg)	0.93	0.82	0.82
			$R^2$	0.93	0.94	0.97
Frumkin	$\frac{\theta}{1-\theta} e^{-2a\theta} = K C_e$	$\log \left[ \left( \frac{\theta}{1-\theta} \right) \frac{1}{C_e} \right] = \log K + 2a\theta$	$a$	1.27	0.62	1.04
			$\ln k$	-1.32	-1.03	-1.03
			$R^2$	0.96	0.97	0.97
H-J	$\frac{1}{q_e^2} = \left( \frac{B}{A} \right) - \left( \frac{1}{A} \right) \log C_e$	$\frac{1}{q_e^2} = \left( \frac{B}{A} \right) - \left( \frac{1}{A} \right) \log C_e$	$A$	1.78	12.8	13.7
			$B$	1.52	1.82	2.1
			$R^2$	0.62	0.61	0.69

## Figure captions

**Fig. 1** SEM image of the dewatered alum sludge (using LEO 1530 VP, Leo Ltd, Germany)

**Fig. 2** Adsorption plots for phosphate adsorption onto alum sludge at three different pH values using linearised forms of (a) Langmuir (b) Freundlich (c) D-R (d) Temkin (e) Frumkin and (f) Harkins – Jura model

**Fig. 3** Sorption kinetics of P onto alum sludge at four different initial P concentrations using (a) first (b) second order and (c) intraparticle diffusion models (initial solution pH = 7)

**Fig. 4** Plot of  $\text{Log } A^*$  versus time for the mass transfer of phosphorus onto alum sludge at different initial concentrations.  $A^* = \left[ \frac{C_t}{C_o} - \frac{1}{1+mk} \right]$  (initial solution pH = 7)

**Fig. 5** Plot of  $B_t$  versus time for the adsorption of phosphorus onto alum sludge at different initial concentrations (initial solution pH = 7)

Figure 1



Figure 2

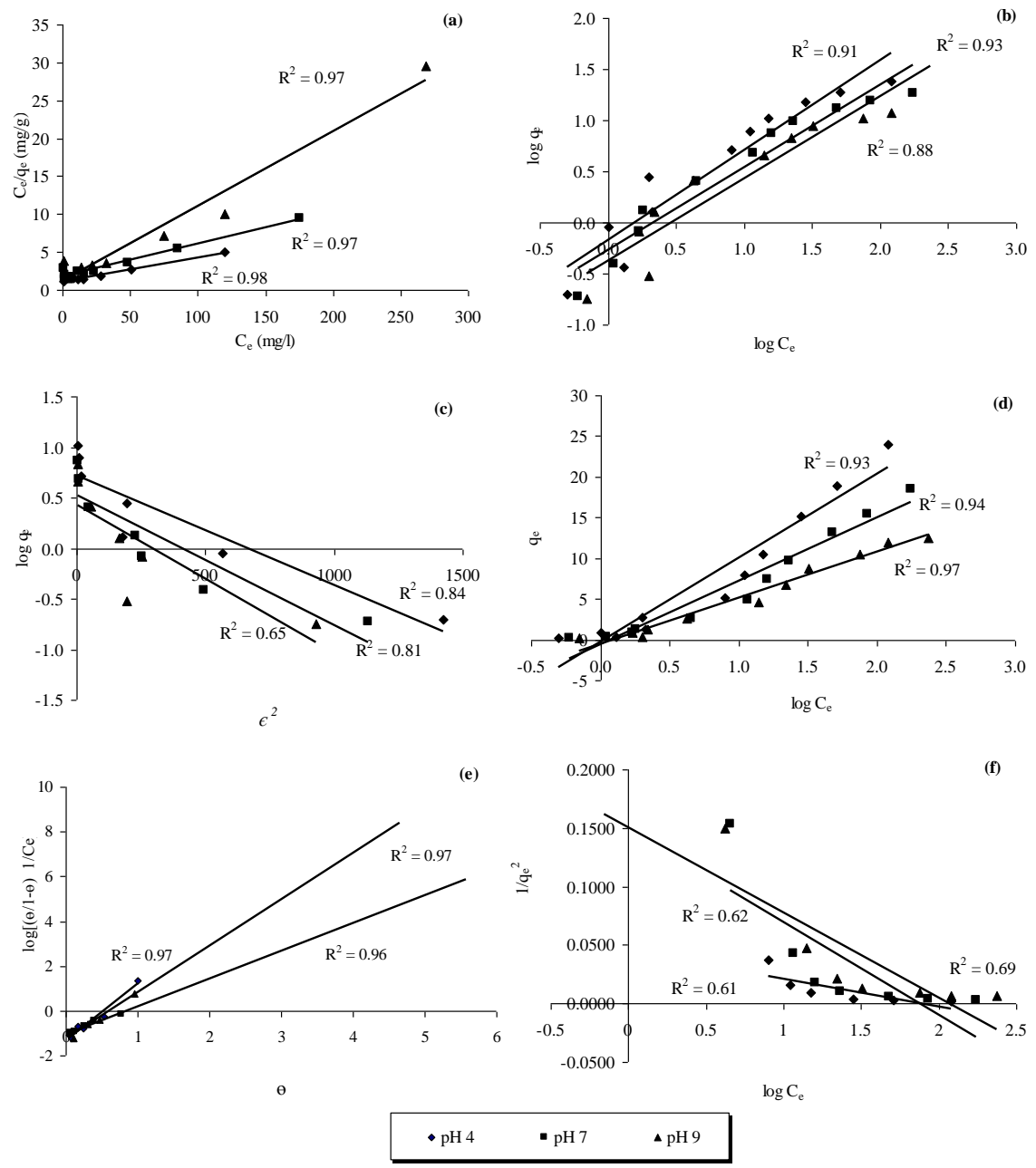


Fig. 2

Figure 3

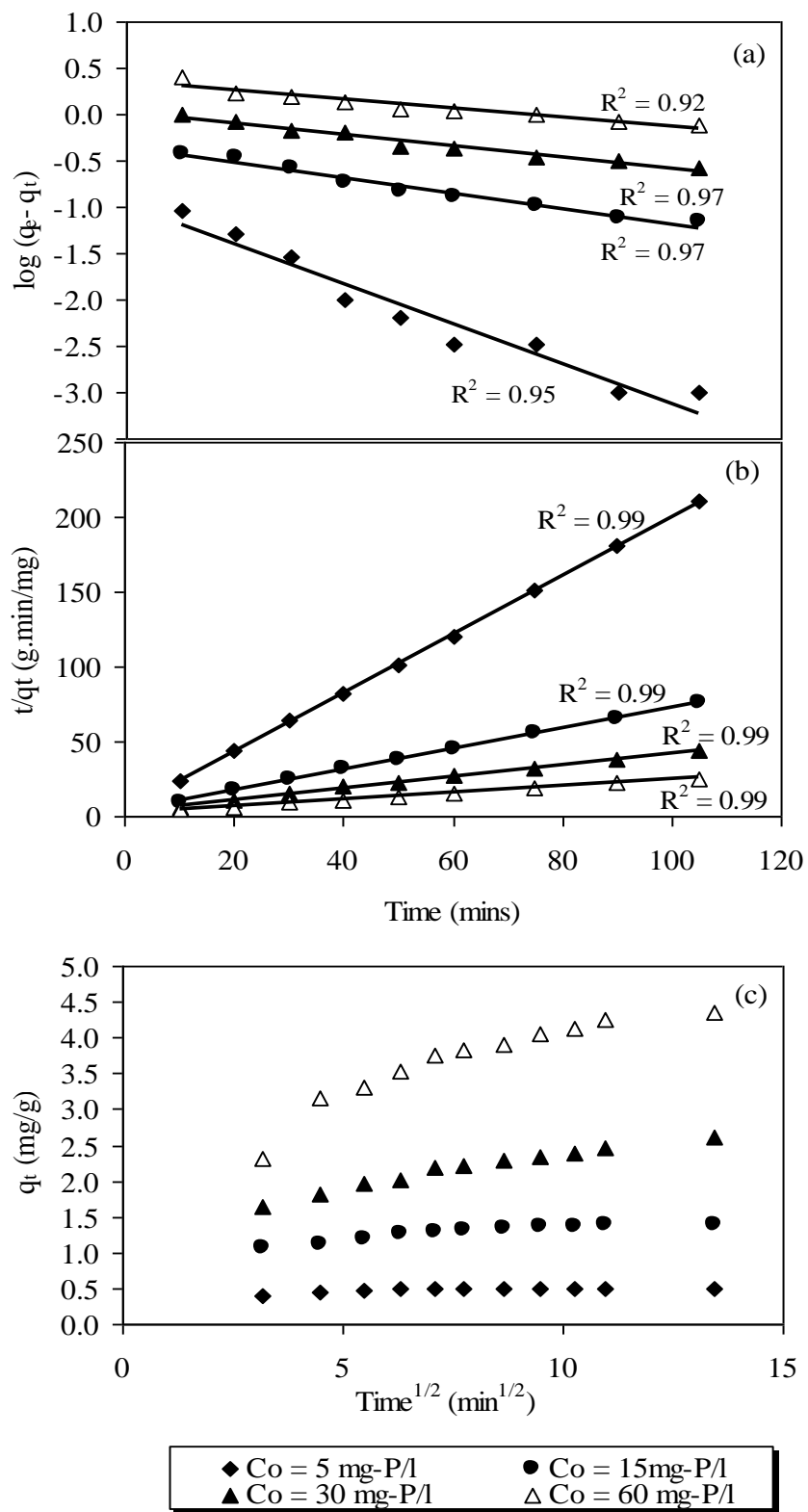


Fig. 3

Figure 4

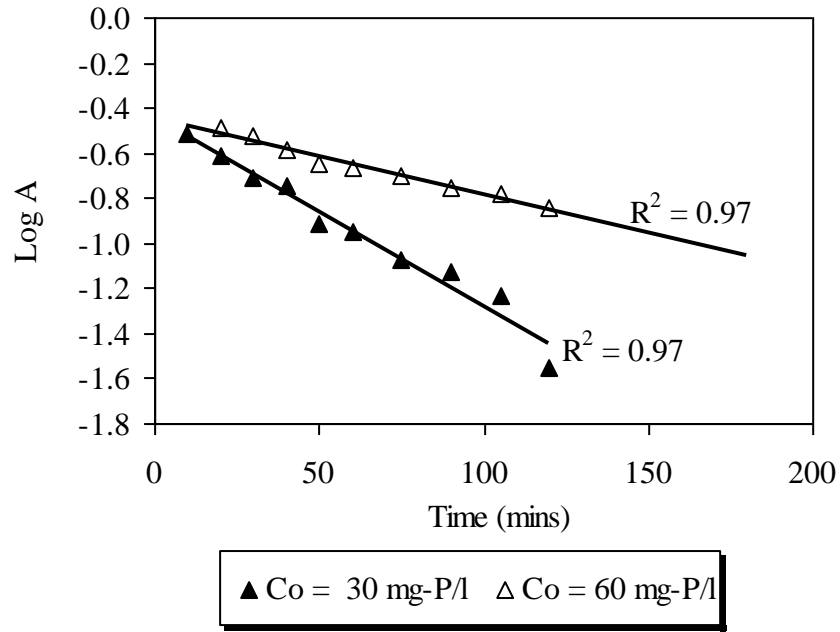


Fig. 4

Figure 5

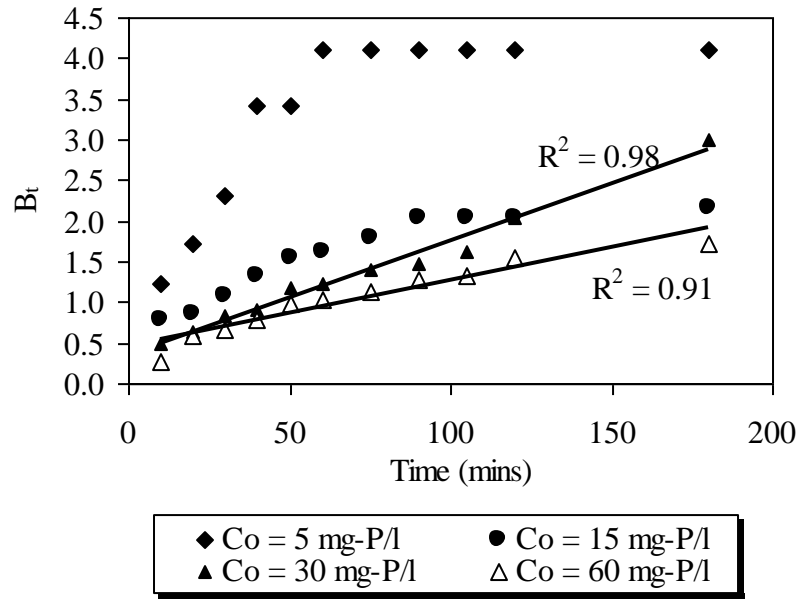


Fig. 5