



<b>Title</b>	Robust Frequency Divider for Power System Online Monitoring and Control
<b>Authors(s)</b>	Zhao, Junbo, Mili, Lamine, Milano, Federico
<b>Publication date</b>	2018-07
<b>Publication information</b>	Zhao, Junbo, Lamine Mili, and Federico Milano. "Robust Frequency Divider for Power System Online Monitoring and Control." IEE, July 2018. <a href="https://doi.org/10.1109/TPWRS.2017.2785348">https://doi.org/10.1109/TPWRS.2017.2785348</a> .
<b>Publisher</b>	IEE
<b>Item record/more information</b>	<a href="http://hdl.handle.net/10197/10009">http://hdl.handle.net/10197/10009</a>
<b>Publisher's statement</b>	© 2018 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.
<b>Publisher's version (DOI)</b>	10.1109/TPWRS.2017.2785348

Downloaded 2026-05-01 23:37:20

The UCD community has made this article openly available. Please share how this access benefits you. Your story matters! (@ucd\_oa)



© Some rights reserved. For more information

# Robust Frequency Divider for Power System Online Monitoring and Control

Junbo Zhao, *Student Member, IEEE*, Lamine Mili, *Fellow, IEEE*, Federico Milano, *Fellow, IEEE*

**Abstract**—Accurate local bus frequency is essential for power system frequency regulation provided by distributed energy sources, flexible loads and among others. This paper proposes a robust frequency divider (RFD) for online bus frequency estimation. **Our RFD is independent of the load models, and the knowledge of swing equation parameters, transmission line parameters and local PMU measurements at each generator terminal bus is sufficient.** In addition, it is able to handle several types of data quality issues, such as measurement noise, gross measurement errors, cyber attacks and measurement losses. Furthermore, the proposed RFD contains the decentralized estimation of local generator rotor speeds and the centralized bus frequency estimation, which resembles the structural of the decentralized/hierarchical control scheme. This enables RFD for very large-scale system applications. Specifically, we decouple each generator from the rest of the system by treating metered real power injection as inputs and the frequency measurements provided by PMU as outputs; then a robust unscented Kalman filter-based dynamic state estimator is proposed for local generator rotor speed estimation; finally, these rotor speeds are transmitted to control center for bus frequency estimation. Numerical results carried out on the IEEE 39-bus system demonstrate the effectiveness and robustness of the proposed method.

**Index Terms**—Frequency estimation, robust statistics, decentralized estimation, dynamic state estimation, unscented Kalman filter, power system dynamics and stability.

## I. INTRODUCTION

WITH the increasing penetration of renewable energy-based generations, the total inertia of the synchronous power system is reduced significantly. As a result, the traditional capacity-based requirements for primary reserve definitions may not be able to satisfy the frequency RoCoF and nadir limits [1], [2]. To tackle this potential frequency instability issue, it is expected by the transmission system operators (TSOs) that the wind farms [3], flexible loads [5], and energy storage devices [6] would provide frequency regulations. However, to enable effective frequency regulations, reliable and accurate knowledge of the local bus frequency is a prerequisite.

In the literature, the numerical derivative of the voltage phase angle provided by Phasor Measurement Unit (PMU) through a washout filter is used to define the local bus frequency [7], [8]. However, as shown by Radman *et al.* [9], it may produce physically implausible spikes in frequency

due to the numerical derivatives and consequently may exhibit instabilities if controls are taken based on them. An alternative way to obtain local bus frequency is through the phase-locked loop (PLL) technique [10]. But it may be unreliable in presence of large step-input speed changes, and has problems in presence of harmonics, unbalance, etc. Furthermore, only a few load buses/substations have PMUs or PLLs installed, which may prevent the system from taking full advantages of local frequency controls. Finally, measurements provided by PMUs and PLLs are always subject to noise or even gross errors, communication losses, etc [4]. For example, it is shown in [5] that the measurement noise affects the performance of the frequency regulator significantly, not to mention the gross errors, measurement losses, etc. To address these issues, a model dependent analytical expression of bus frequency is proposed in [11] assuming comprehensive and accurate models of the system. Milano and Ortega [12] improved that approach and proposed a transient stability simulation-based frequency divider. The main idea underlying this method is to solve a steady-state boundary value problem, where the boundary conditions are given by synchronous generator rotor speeds. However, both methods assume accurate power system dynamic models for time-domain simulations, which is difficult to achieve in practice. In addition, the time-domain simulations of large-scale power systems are computational demanding, which may prevent them for the online control applications.

This paper proposes a robust frequency divider (RFD) for online bus frequency estimation. Our RFD is not dependent on load models, and the knowledge of swing equation parameters, transmission system line parameters and local PMU measurements is sufficient. In addition, it is able to filter out measurement noise, suppress gross measurement errors, handle cyber attacks as well as measurement losses. To develop the proposed RFD, it is observed that to obtain an accurate estimation of bus frequency, accurate generator rotor speeds are required. In the meantime, complicated generator and load models should be avoided. To this end, we propose to divide RFD into two subproblems, namely the decentralized estimation of local generator rotor speed using local measurements and the centralized bus frequency estimation. To address the first problem, we decouple each generator from the rest of the power system by treating metered real power injection as model inputs and the frequency measurements provided by PMU, local meter devices or Frequency monitoring Network devices [13] as outputs. As a result, only the swing equation is required for rotor speed estimation and no detailed generator model is assumed. Since the local measurements can be subject to data quality issue when implementing an

Junbo Zhao and Lamine Mili are supported in part by the U.S. National Science Foundation under Grant ECCS-1711191. Federico Milano is supported in part by the European Commission under the RESERVE Consortium (grant No. 727481) and EC Marie Skłodowska-Curie CIG No. PCIG14-GA-2013-630811. He is also funded by the Science Foundation Ireland, under Investigator Programme, Grant No. SFI/15/IA/3074.

Junbo Zhao and Lamine Mili are with the Bradley Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University, Falls Church, VA 22043, USA (e-mail: zjunbo@vt.edu, lmili@vt.edu).

Federico Milano is with the School of Electrical and Electronic Engineering, University College Dublin, Ireland. (email: federico.milano@ucd.ie).

estimator, a robust unscented Kalman filter-based dynamic state estimator is proposed. Next, the local estimates are transmitted to the control center for bus frequency estimations, yielding two benefits: 1) the requirement of communication bandwidth is decreased notably as only estimated rotor speeds are communicated instead of voltage and current phasors; 2) the wide-area generator rotor angles are available for operator to achieve better situational awareness and carry out other applications, such as oscillatory modes monitoring, rotor angle stability analysis, etc. Last but not the least, thanks to the decentralized and centralized estimation scheme, the proposed method is suitable for designing controllers of very large-scale power systems.

The remainder of the paper is organized as follows. Section II presents the problem formulation. Section III describes the proposed RFD in detail and Section IV shows and analyzes the simulation results. Finally, Section V concludes the paper.

## II. PROBLEM FORMULATION

In this section, the analytical relationship between bus frequency and generator rotor speeds will be presented first; then the limitations of this approach will be discussed thoroughly, and finally the problem statement will be declared.

### A. Analytical Relationship between Bus Frequency and Generator Rotor Speeds

When a disturbance occurs, such as transmission line faults, load shedding or generator tripping, power mismatch appears between the mechanical torque and electrical power at the generator terminal buses. As a consequence, the generator rotor speeds will deviate from their nominal values. To resynchronize generator with the rest of the power system, an increase or a decrease in the rotor speed is actuated, which causes rotor angle oscillations as well. Due to such oscillations, the voltage phase angles of the buses that are adjacent to generators will encounter changes, which in turn causes a power mismatch. In this way, the electro-mechanical oscillations will be propagated throughout the entire power system with limited speed. Since we are interested in electro-mechanical oscillations and the propagation speed of such oscillations is much lower than that of the wave, the transient effects of wave propagation are therefore neglected. Based on the analysis above, it is clear that the spatial variations of the system frequency are characterized by synchronous generator rotor speeds. Those frequency variations at each bus of the system are of vital importance for designing local controllers to enhance the frequency regulation capability of a power system with high penetration renewable energy integrations.

Note that electro-mechanical oscillations can be characterized by the magnitude and phase angle modulations of voltages and currents as well since they are corresponding to the movement of rotors of electric machines around the synchronous speed [14], [15]. Thus, to estimate bus frequency of a transmission system, we first need to analyze the relationship of the voltage or current phasors between generators and system buses. This relationship can be expressed by the

balanced current injection formula shown as follows:

$$\begin{bmatrix} \mathbf{I}_G \\ \mathbf{I}_B \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{GG} & \mathbf{Y}_{GB} \\ \mathbf{Y}_{BG} & \mathbf{Y}_{BB} + \mathbf{Y}_{B0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_G \\ \mathbf{V}_B \end{bmatrix}, \quad (1)$$

where  $\mathbf{I}_G$  are generator current injections;  $\mathbf{V}_G$  are generator internal electromotive forces (emfs);  $\mathbf{I}_B$  and  $\mathbf{V}_B$  are current and voltage injections of the network buses, respectively;  $\mathbf{Y}_{BB}$  is the power network admittance matrix;  $\mathbf{Y}_{GG}$ ,  $\mathbf{Y}_{GB}$  and  $\mathbf{Y}_{BG}$  are admittance matrices calculated by including the internal impedances of the synchronous generators;  $\mathbf{Y}_{B0}$  is a diagonal matrix, which takes into account the internal impedances of synchronous generators at the generator buses. Since the load current injections are negligible compared with that of the synchronous generators [12], [18], (1) can be rewritten as

$$\begin{bmatrix} \mathbf{I}_G \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{GG} & \mathbf{Y}_{GB} \\ \mathbf{Y}_{BG} & \mathbf{Y}_{BB} + \mathbf{Y}_{B0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_G \\ \mathbf{V}_B \end{bmatrix}. \quad (2)$$

By taking simple algebraic operations on the second row of (2), we can derive the relationship between bus voltage vector  $\mathbf{V}_B$  and the emfs of generators as follows

$$\mathbf{V}_B = -(\mathbf{Y}_{BB} + \mathbf{Y}_{B0})^{-1} \mathbf{Y}_{BG} \mathbf{V}_G = \mathbf{D} \mathbf{V}_G. \quad (3)$$

Taking time derivatives on both sides of (3) in rotating reference frame, i.e., dq frame, we get

$$\frac{d\mathbf{V}_B}{dt} + j\omega_0 \mathbf{V}_B = \mathbf{D} \cdot \frac{d\mathbf{V}_G}{dt} + j\omega_0 \mathbf{D} \mathbf{V}_G, \quad (4)$$

where  $\omega_0$  is the nominal rotor speed. For more details of deriving (4), please see [12]. Define  $\Delta\omega_B = \omega_B - \omega_0$ ,  $\Delta\omega_G = \omega_G - \omega_0$ , the analytical relationship between bus frequency and generator rotor speeds can be derived from (4), which is expressed as follows [12]

$$\omega_B = \omega_0 + \mathbf{D} (\omega_G - \omega_0). \quad (5)$$

It can be observed from (5) that the bus frequency is correlated with each synchronous generator in the system, but the degree of participation of each generator on bus frequency is determined by the transmission parameters. To speed up the calculation of bus frequency from (5) without a relevant loss of accuracy, the conductances of transmission lines utilized to calculate  $\mathbf{D}$  can be neglected [12].

### B. Limitations and Problem Statement

When implementing (5) to estimate bus frequency, there are two possible ways: i) the dynamic simulation based-approach and ii) the measurement-based approach. In the former approach, the transient stability program is used to obtain the rotor speed of each generator, followed by the calculations of the bus frequencies through (5). However, to obtain good time-domain simulation results, accurate and detailed generator and load models are required, which may be difficult to achieve in practice. In addition, the time-domain simulations of large-scale power systems are computational demanding, which may not be suitable the online control applications. By contrast, the measurement-based approach does not have such issues, but the metered generator rotor speeds and frequencies by PMUs or Frequency monitoring Network devices are assumed to be of high quality. However, this assumption may not hold true for

practical power systems as the PMU measurements are usually subject to noise or even impulsive noise, gross errors, cyber attacks, communication losses, etc. Under those conditions, this approach will produce significantly biased results and subsequently the control actions based on them may make the system even worse. Furthermore, it should be noted that the frequency of each bus is correlated with most generators, thus if just one rotor speed measurement is corrupted, its error may propagate to many other bus frequency estimations. It is thus indispensable to make sure that every generator rotor speed is of good accuracy.

*Problem statement:* given a limited number of PMUs installed at the terminal bus of each generator, a robust frequency divider is developed to address the data quality issues of PMU measurements; in the meantime, it should be model independent so as to mitigate the strong assumptions on the generator and load models, and finally, it should be fast to calculate and suitable for large-scale system online control applications. Note that in this paper, the rotor speed and frequency of each generator at its terminal bus are assumed to be monitored by PMUs. This is a reasonable assumption due to several reasons: 1) online monitoring of generators plays a major role in power system operation and control, thus it is given a high priority for PMU placement according to the NERC PMU placement standard [16]; 2) it is required by NERC standard [17] to have PMUs installed at the point of interconnection for power plant model validation and verification.

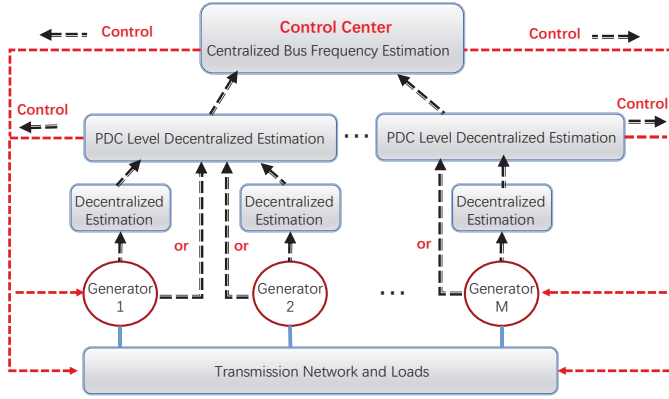


Fig. 1: Proposed decentralized-centralized bus frequency estimation framework.

### III. PROPOSED ROBUST FREQUENCY DIVIDER

By looking at (5), one may easily come up with the idea that this equation can be formulated as a regression problem, where the generator rotor speeds are measurements provided by PMUs while the frequency of each bus is the unknown state vector to be estimated. However, since the number of rows of the matrix  $D$  is larger than that of the columns, (5) cannot be treated as an estimation problem. Therefore, alternative approaches should be developed.

Note that to obtain an accurate estimation of bus frequency, accurate generator rotor speeds are required. Thus, if measurement quality issues can be addressed locally at the generator bus through a local robust estimator, we are able to obtain

accurate bus frequency estimates. To this end, we propose a decentralized-centralized bus frequency estimation framework shown in Fig. 1. Particularly, we first perform the robust unscented Kalman filter-based dynamic state estimator at the local generator substation or phasor data concentrator (PDC) level by using the proposed model decoupling approach; then these local estimates (generator rotor speeds and angles) are transmitted to control center for bus frequency estimation through (5). Note that if the decentralized dynamic state estimation at each generator substation is costly, the data of those generators associated with PMU measurements will be transmitted to the PDC or regional system operating center for decentralized estimation. After that, local controls can be initiated if required, otherwise, they will be further communicated to the control center for bus frequency estimation and coordinated control. In the following subsections, we will elaborate on each of the block.

#### A. Generator Model Decoupling Approach

Due to power unbalance and other control actions, the rotor of the generator will accelerate or decelerate. Those electro-mechanical dynamics can be captured by the swing equations shown as follows [18],

$$\frac{d\delta}{dt} = \omega - \omega_0, \quad (6)$$

$$\frac{2H}{\omega_0} \frac{d\omega}{dt} = T_M - P_e - D_p(\omega - \omega_0), \quad (7)$$

where  $\delta$  is the rotor angle;  $H$  is the generator inertia constant;  $T_M$  and  $P_e$  are the generator mechanical power and electrical power outputs, respectively; it is assumed that  $T_M$  remains the same value as that in steady-state condition during transient process, which is reasonable as the time constant of the governor is very large;  $D_p$  is the generator damping constant. Note that the generator first swing dynamics are affected by  $P_e$  significantly. In other words, the generator swing equation is coupled with its dq windings and the rest of the system through  $P_e$ . For example, in the two axis generator model,  $P_e = E'_d I_d + E'_q I_q + (X'_q - X'_d) I_d I_q$ , where  $E'_d$  and  $E'_q$  are the d-axis and q-axis transient voltages, respectively;  $X'_d$  and  $X'_q$  are generator d-axis and q-axis transient reactance, respectively;  $I_d$  and  $I_q$  are the d and q axis currents, respectively. For more detailed model, the expression is different.

Motivated by the aforementioned analysis, if  $P_e$  is measured and taken as the model input while the measured frequency by PMUs is treated as outputs, the swing equation can be decoupled from the rest of the system. Furthermore, no information of the d and q damper windings is required. By doing that, no complex generator model is assumed. The physical mean of this decoupling approach can be explained as follows: when there is a disturbance at one point of the power system, synchronous generators will response to it through actions on the rotors; these responses reveal themselves in their real power injections and frequency. In other words, the generator swing dynamics are coupled with the rest of the system at the point of connection, and its interactions with the rest of the system are through the real power injections and frequency. If real power injections and frequency are measured by a PMU,

its swing responses to the disturbance are captured completely and no other system information is required.

### B. Proposed Robust Decentralized Dynamic State Estimator

The model decoupling approach enables a generator swing equation to be decoupled from the rest of the system model, which in turn allows us to rely only on local measurements to estimate the rotor speed and angle of a generator. The discrete-time state representation of the  $i$ th synchronous generator is

$$\mathbf{x}_k^i = \mathbf{f}_i(\mathbf{x}_{k-1}^i, \mathbf{u}_k^i) + \mathbf{w}_k^i, \quad (8)$$

$$\mathbf{z}_k^i = \mathbf{h}_i(\mathbf{x}_k^i, \mathbf{u}_k^i) + \mathbf{v}_k^i, \quad (9)$$

where  $\mathbf{x}_k^i$  is the state vector, including the generator rotor speed  $\omega_i$  and rotor angle  $\delta_i$ ;  $\mathbf{z}_k^i$  is the measurement vector that contains rotor speed  $z_{k1}$  provided by PMUs and frequency  $z_{k2}$  by local metering devices or frequency monitoring network devices [13];  $\mathbf{f}_i(\cdot)$  represents the discrete-time form of (6) and (7) while  $\mathbf{z}_k^i = [z_{k1} \ z_{k2}]^T$  and  $z_{k1} = \omega_i + v_{k1}$ ,  $z_{k2} = (1 + \Delta\omega_i)f_0 + v_{k2}$ ;  $f_0$  is the nominal system frequency;  $\mathbf{h}_i(\cdot) = \mathbf{H}_k^i \mathbf{x}_k^i$  and  $\mathbf{H}_k^i$  is a constant matrix that can be derived directly from the measurement equations of  $z_{k1}$  and  $z_{k2}$ ;  $\mathbf{w}_k^i$  and  $\mathbf{v}_k^i = [v_{k1} \ v_{k2}]^T$  are the process and observation noise, respectively; they are assumed to be Gaussian with zero mean and covariance matrices  $\mathbf{Q}_k^i$  and  $\mathbf{R}_k^i$ , respectively;  $\mathbf{u}_k^i$  is the input that contains the real power injection of the  $i$ th generator.

Based on the derived discrete-time state space equations (8) and (9), the dynamic state estimator (DSE) can be used to estimate the generator rotor speed and angle using local measurements. In this paper, the UKF is chosen as the basic DSE as it achieves a more balanced performance between computational efficiency and ability to cope with strong system nonlinearities than the extended Kalman filter, or the particle filter [19]. However, UKF has been proved to be sensitive to gross errors, cyber attacks and loss of measurements, etc [20]. To handle these issues, a robust Generalized Maximum-likelihood-type UKF (GM-UKF) is proposed. It consists of four major steps, namely a batch-mode regression form step, a robust pre-whitening step, a robust regression state estimation step, and a robust error covariance matrix updating step. In the following subsections, we will discuss them in detail. Note that the index  $i$  is neglected for simplicity but without the loss of generality.

1) *Derive Batch-Mode Regression Model:* Given a state estimate at time step  $k-1$ ,  $\hat{\mathbf{x}}_{k-1|k-1} \in \mathbb{R}^{n \times 1}$ , having a covariance matrix given by  $\mathbf{P}_{k-1|k-1}^{xx}$ , its statistics are captured by  $2n$  weighted sigma points defined as [19]

$$\chi_{k-1|k-1}^i = \hat{\mathbf{x}}_{k-1|k-1} \pm \left( \sqrt{n\mathbf{P}_{k-1|k-1}^{xx}} \right)_i, \quad (10)$$

with weights  $w_i = \frac{1}{2n}$ ,  $i = 1, \dots, 2n$ , where  $n$  is the number state variables for each generator. Then, each sigma point is propagated through the nonlinear system process model (8), yielding a set of transformed samples expressed as

$$\mathbf{x}_{k|k-1}^i = \mathbf{f}\left(\chi_{k-1|k-1}^i\right). \quad (11)$$

Next, the predicted sample mean and sample covariance matrix of the state vector are calculated by

$$\begin{aligned} \hat{\mathbf{x}}_{k|k-1} &= \sum_{i=1}^{2n} w_i \chi_{k|k-1}^i, \\ \mathbf{P}_{k|k-1}^{xx} &= \sum_{i=1}^{2n} w_i (\chi_{k|k-1}^i - \hat{\mathbf{x}}_{k|k-1})(\chi_{k|k-1}^i - \hat{\mathbf{x}}_{k|k-1})^T + \mathbf{Q}_k. \end{aligned} \quad (12)$$

We define  $\hat{\mathbf{x}}_{k|k-1} = \mathbf{x}_k - \Delta_k$ , where  $\mathbf{x}_k$  is the true state vector;  $\Delta_k$  is the prediction error and  $\mathbb{E}[\Delta_k \Delta_k^T] = \mathbf{P}_{k|k-1}^{xx}$ . By processing the predictions and observations simultaneously, we have the following batch-mode regression form

$$\begin{bmatrix} \mathbf{z}_k \\ \hat{\mathbf{x}}_{k|k-1} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_k \\ \mathbf{I} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \mathbf{v}_k \\ -\Delta_k \end{bmatrix} \quad (14)$$

which can be rewritten in a compact form

$$\tilde{\mathbf{z}}_k = \widetilde{\mathbf{H}}_k \mathbf{x}_k + \tilde{\mathbf{e}}_k, \quad (15)$$

and the error covariance matrix is

$$\mathbf{W}_k = \mathbb{E}[\tilde{\mathbf{e}}_k \tilde{\mathbf{e}}_k^T] = \begin{bmatrix} \mathbf{R}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{k|k-1}^{xx} \end{bmatrix} = \mathbf{S}_k \mathbf{S}_k^T, \quad (16)$$

where  $\mathbf{I}$  is an identity matrix;  $\mathbf{S}_k$  is calculated by the Cholesky decomposition technique.

2) *Perform Robust Pre-whitening:* Before carrying out a robust regression, the state prediction errors of the batch-mode regression form need to be uncorrelated. This can be done by pre-multiplying  $\mathbf{S}_k^{-1}$  on both sides of (15), yielding

$$\mathbf{S}_k^{-1} \tilde{\mathbf{z}}_k = \mathbf{S}_k^{-1} \widetilde{\mathbf{H}}_k \mathbf{x}_k + \mathbf{S}_k^{-1} \tilde{\mathbf{e}}_k, \quad (17)$$

which can be further organized to the compact form

$$\mathbf{y}_k = \mathbf{A}_k \mathbf{x}_k + \boldsymbol{\xi}_k, \quad (18)$$

where  $\mathbb{E}[\boldsymbol{\xi}_k \boldsymbol{\xi}_k^T] = \mathbf{I}$ . However, if outliers occur, the use of  $\mathbf{S}_k^{-1}$  for prewhitening will cause negative smearing effect. To handle this issue, we first detect and downweight the outliers by means of weights calculated using the projection statistics (PS) [21] and a statical test applied to them. Those weights contribute to the robust prewhitening and their functionals will be shown later in the objective function. Specifically, we apply the PS to a 2-dimensional matrix  $\mathbf{Z}$  that contains serially correlated samples of the innovations and of the predicted state variables. Formally, we have

$$\mathbf{Z} = \begin{bmatrix} \mathbf{z}_{k-1} - \mathbf{H}_k \hat{\mathbf{x}}_{k-1|k-2} & \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \\ \hat{\mathbf{x}}_{k-1|k-2} & \hat{\mathbf{x}}_{k|k-1} \end{bmatrix}, \quad (19)$$

where  $\mathbf{z}_{k-1} - \mathbf{H}_k \hat{\mathbf{x}}_{k-1|k-2}$  and  $\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}$  are the innovation vectors while  $\hat{\mathbf{x}}_{k-1|k-2}$  and  $\hat{\mathbf{x}}_{k|k-1}$  are the predicted state vectors at time instants  $k-1$  and  $k$ , respectively. The PS values of the predictions and of the innovations are separately calculated because the values taken by the former and the latter are centered around different points. The implementation of PS can be found in [21].

Once the PS values are calculated, they are compared to a threshold to identify outliers. According to our previous work [20], [21],  $\mathbf{Z}$  can be shown to follow a bivariate Gaussian probability distribution and the calculated PS values using  $\mathbf{Z}$  follow a chi-square distribution with degree of freedom 2. As

a result, the outliers can be flagged if their PS values satisfy  $PS_i > \chi_{2,0.975}^2$  at a significance level 97.5% in the statistical test, and are further downweighted via

$$\varpi_i = \min(1, d^2/PS_i^2), \quad (20)$$

where the parameter  $d$  is set as 1.5 to yield good statistical efficiency at Gaussian distribution.

*Remark:* Except for the occurrence of outliers in rotor speed measurements, outliers may occur in the measured real power injections as well, and consequently, yielding incorrect predicted states, called innovation outliers. In such condition, the predicted state corresponding to incorrect real power injection will be flagged as outliers. Since the local measurement redundancy is not high, we will not downweight it directly, instead we propose to replace the current real power injection by its previous value and obtain the new state predictions. By doing that, we can achieve better statistical efficiency.

3) *Carry out Robust Regression:* To address the data quality issues, we develop a robust GM-estimator that minimizes the following objective function:

$$J(\mathbf{x}_k) = \sum_{i=1}^l \varpi_i^2 \rho(r_{S_i}), \quad (21)$$

where  $l = m + n$  and  $m$  is the number of measurements;  $\varpi_i$  is calculated by (20);  $r_{S_i} = r_i/s\varpi_i$  is the standardized residual;  $r_i = y_i - \mathbf{a}_i^T \hat{\mathbf{x}}$  is the residual, where  $\mathbf{a}_i^T$  is the  $i$ th row vector of the matrix  $\mathbf{A}_k$ ;  $s = 1.4826 \cdot b_m \cdot \text{median}_i |r_i|$  is the robust scale estimate;  $b_m$  is a correction factor;  $\rho(\cdot)$  is the convex Huber- $\rho$  function, that is

$$\rho(r_{S_i}) = \begin{cases} \frac{1}{2}r_{S_i}^2, & \text{for } |r_{S_i}| < \lambda \\ \lambda|r_{S_i}| - \lambda^2/2, & \text{elsewhere} \end{cases}, \quad (22)$$

where the parameter  $\lambda$  is typically chosen to be between 1.5 and 3 to achieve high statistical efficiency in the literature [22].

To minimize (21), the following necessary condition must be satisfied

$$\frac{\partial J(\mathbf{x}_k)}{\partial \mathbf{x}_k} = \sum_{i=1}^l -\frac{\varpi_i \mathbf{a}_i}{s} \psi(r_{S_i}) = \mathbf{0}, \quad (23)$$

where  $\psi(r_{S_i}) = \partial \rho(r_{S_i})/\partial r_{S_i}$  is the so-called  $\psi$ -function. By dividing and multiplying the standardized residual  $r_{S_i}$  to both sides of (23) and putting it in a matrix form, we get

$$\mathbf{A}_k^T \mathbf{\Lambda} (\mathbf{y}_k - \mathbf{A}_k \mathbf{x}_k) = \mathbf{0}, \quad (24)$$

where  $\mathbf{\Lambda} = \text{diag}(q(r_{S_i}))$  and  $q(r_{S_i}) = \psi(r_{S_i})/r_{S_i}$ . By using the IRLS algorithm [21], the state estimates at the  $j$  iteration can be calculated

$$\Delta \hat{\mathbf{x}}_{k|k}^{(j+1)} = \left( \mathbf{A}_k^T \mathbf{\Lambda}^{(j)} \mathbf{A}_k \right)^{-1} \mathbf{A}_k^T \mathbf{\Lambda}^{(j)} \mathbf{y}_k, \quad (25)$$

where  $\Delta \hat{\mathbf{x}}_{k|k}^{(j+1)} = \hat{\mathbf{x}}_{k|k}^{(j+1)} - \hat{\mathbf{x}}_{k|k}^{(j)}$ . The algorithm converges when  $\left\| \Delta \hat{\mathbf{x}}_{k|k}^{(j+1)} \right\|_{\infty} \leq 10^{-2}$ .

4) *Update Error Covariance Matrix:* After the convergence of the algorithm, the estimation error covariance matrix  $\mathbf{P}_{k|k}^{xx}$  of the GM-UKF needs to be updated so that the state prediction at the next time sample can be performed. Following the work from [20], [21], we derive the estimation error covariance matrix of our GM-UKF as

$$\mathbf{P}_{k|k}^{xx} = \frac{\mathbb{E}_F[\psi^2(r_{S_i})]}{\{\mathbb{E}_F[\psi'(r_{S_i})]\}^2} (\mathbf{A}_k^T \mathbf{A}_k)^{-1} (\mathbf{A}_k^T \mathbf{Q}_{\varpi} \mathbf{A}_k) (\mathbf{A}_k^T \mathbf{A}_k)^{-1} \quad (26)$$

where  $\mathbf{Q}_{\varpi} = \text{diag}(\varpi_i^2)$ .

*Remark:* the proposed robust DSE is supposed to be performed for each generator substation. It can be implemented at the control center as well if all the generator data and PMU measurements are transmitted from local substations to it. However, there exist several concerns by doing so, such as increased communication burden that is discussed in the next subsection and the delayed local control, etc. Indeed, if all the calculations are performed at the control center while some local controls are required at this period, the estimated bus frequency for those local controls can be delayed; by contrast, our robust DSE is first conducted locally using local PMU measurements and its estimated rotor speeds and angles can be used for local controls; in the meantime, they can be transmitted to control center for bus frequency estimation and coordinated control. In practice, if it is costly to implement the decentralized DSE for each generator substation, we can do it at the local phasor data concentrator (PDC) level or regional system level. This can still save a lot of communication burden and enable the timely local control actions compared with the fully centralized strategy.

### C. Bus Frequency Estimation

When the rotor speed and rotor angle of each generator are obtained, they need to be communicated to the control center for bus frequency estimation. Depending on the applications, there are two ways to communicate the estimation results.

- If the control center is only interested in monitoring and regulating the system frequency, the rotor speed of each generator is transmitted and the bus frequency is estimated using (5). Compared with the conventional strategy, that is, all the measured voltage magnitudes and angles, current magnitudes and angles, and frequency by PMUs are communicated, the communication burden of our proposed approach is only 20% of it;
- If the control center are interested in both frequency and rotor angle stability monitoring and control, rotor speed and angle estimates are communicated. In this case, it only requires 40% communication burden of the conventional strategy.

Note that the total computing time of the proposed approach consists of two parts: the decentralized DSE and the projection of the rotor speed to bus frequency through (5). Since each robust DSE is performed locally and its estimates are communicated to the control center through its communication link, the proposed DSE is independent of the size of the power system. The only concern of the proposed RFD for very large-scale power system online applications is that  $\mathbf{D}$  becomes rather dense, which requires a lot of computer

memory. However, this is not a problem if we use the sparse matrices  $B_{BB}$ ,  $B_{G0}$  and  $B_{BG}$  instead of  $D$  for the projection of rotor speeds to bus frequencies (see equation (20) in [12]). As a result, the computational burden of this step is negligible.

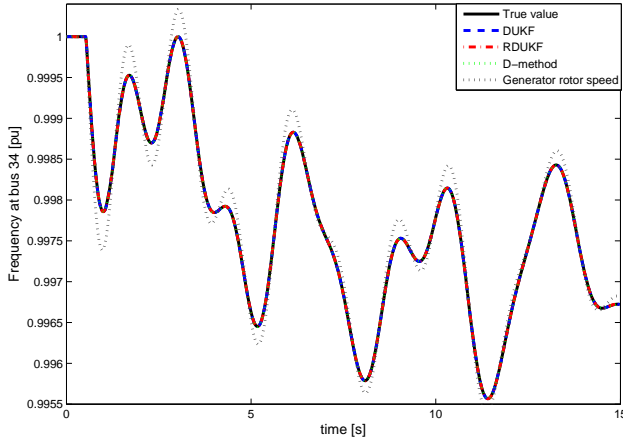


Fig. 2: Comparing the estimated frequency at bus 34 by DUKF, RDUKF and D-method with normal measurement noise in the IEEE 39-bus system.

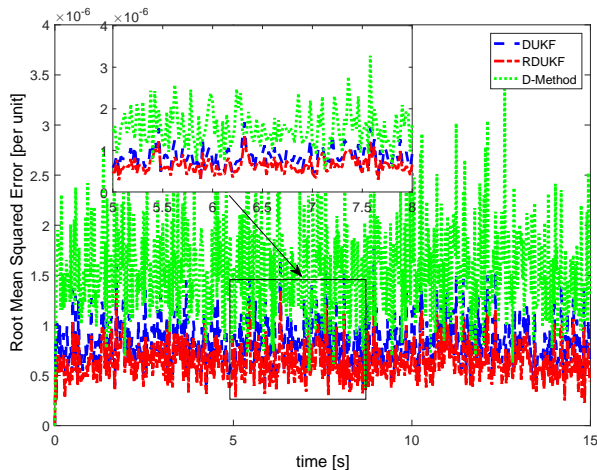


Fig. 3: RMSE of DUKF, RDUKF and D-method with normal measurement noise in the IEEE 39-bus system.

*Remark:* The reason that the complicated generator model is not required has been discussed in the model decoupling section. We discuss here how the proposed RFD is not dependent on load model. It is well-known that the power system dynamics are different if different load models are assumed. However, although system dynamic behaviors are different, they are reflected on the variations of the rotor speeds of synchronous generators. Since the proposed RFD is based on such variations, load models have been implicitly taken into account. This conclusion has also been verified by extensive simulation results in [12].

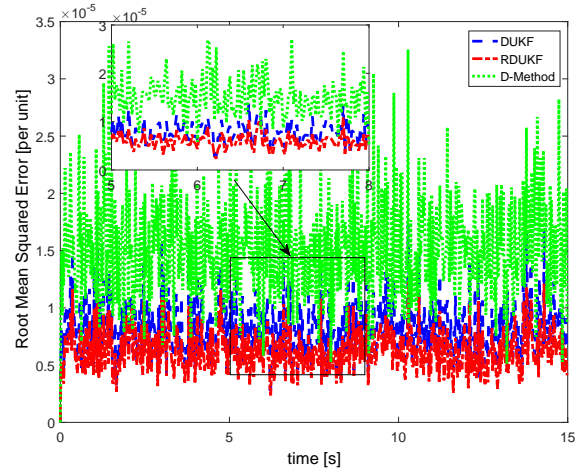


Fig. 4: RMSE of DUKF, RDUKF and D-method with large measurement noise in the IEEE 39-bus system, where the variance is changed from  $10^{-6}$  to  $10^{-4}$ .

#### IV. NUMERICAL RESULTS

In this section, extensive simulations on the IEEE 39-bus test system will be carried out to demonstrate the effectiveness and robustness of the proposed RFD. Specifically, at  $t=0.5s$ , the Generator 4 connected to bus 33 is tripped to simulate system disturbance. **The transient stability simulations are performed to generate measurements and true state variables using the Matlab-based software PST with some revisions [23]. The fourth order Ruge-Kutta approach is adopted with integration step  $t=1/120s$  to solve differential and algebraic equations.** The measured real power injection of each generator is taken as model input, while the measured generator rotor speed and frequency by PMU are treated as outputs/measurements. A random Gaussian variable with zero mean and variance equal to  $10^{-6}$  is assumed for system process noise. The generator model assumed for transient simulation is the detailed two-axis generator model, whose parameter values are taken from [24]. The root-mean-squared error (RMSE) of all bus frequencies is used as the performance index while the estimated frequency at bus 34 is taken for illustration. Note that, Generator 5 is connected to bus 34. The proposed non-robust UKF based method will be called DUKF, and the proposed robust UKF based method is called RDUKF while the original proposal [12] that works on  $D$  matrix directly will be called the D-method.

##### A. Estimation Results with Noisy Measurements

All the methods are tested with noisy measurements. Normal and large noise are considered; their noise standard deviations are assumed to be  $10^{-6}$  and  $10^{-4}$ , respectively. The results are shown in Figs. 2- 4, where in Fig. 2 the rotor speed of Generator 5 is shown as well. From Fig. 2, we observe that the rotor speed of Generator 5 is different from its terminal bus frequency. This difference is caused by two factors: (i) generator internal impedance and (ii) severity of the transient (e.g., how much rotor speeds differ from each other). On the other hand, by observing both Figs. 2 and 3, it is found that the D-method is one of the most sensitive method to measurement noise while our DUKF and RDUKF approaches are able to

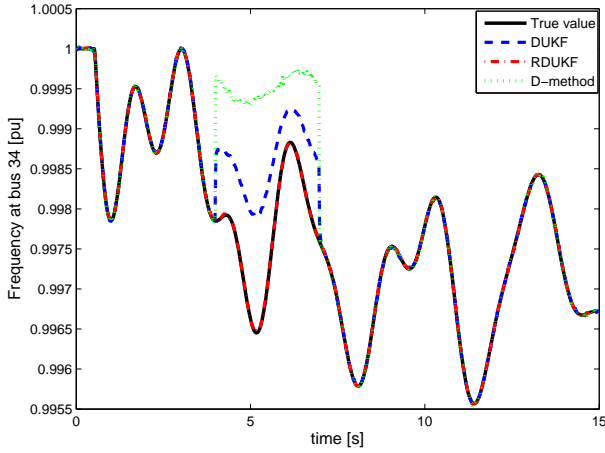


Fig. 5: Estimated frequency at bus 34 by DUKF, RDUKF and D-method with observation outliers in the IEEE 39-bus system, where the measured rotor speed of Generator 5 is contaminated with 20% error from  $t=4s$  to  $t=6s$ .

filter them out. This is because in D-method, the noise is directly propagated through the rotor speed measurements to the bus frequency. By contrast, our methods adopt the UKF to filter out the noise, yielding better performance. When we increase the variance of the measurement noise from  $10^{-6}$  to  $10^{-4}$ , the results of D-method become even worse while our methods can achieve comparable performance as those in the former case (see Fig. 4).

### B. Impact of Observation and Innovation Outliers

Due to cyber attacks, imperfect phasor synchronization, the saturation of metering current transformers or by metering **Couple Capacitor Voltage Transformers** (CCVTs), to name a few, gross errors can occur in the PMU measurements [20]. As for our decentralized DSE-based bus frequency estimation problem, there are two ways to induce outliers: i) the measured rotor speed by PMUs is contaminated with gross error, which is called observation outlier; ii) since the real power injection measured by PMUs can be contaminated with gross error, treating it as model input can yield incorrect rotor speed predictions, which is called innovation outlier. Note that, as D-method is working directly with rotor speed measurements, it is affected by observation outliers while being independent of the innovation outlier caused by incorrect real power injection measurements. To this end, two cases are considered:

*Case 1:* the measured rotor speed of Generator 5 is contaminated with 20% error from  $t=4s$  to  $t=6s$  to simulate observation outlier.

*Case 2:* the measured real power injection of Generator 5 is contaminated with 30% error from  $t=3s$  to  $t=6s$  to simulate innovation outlier.

The test results for Case 1 are shown in Figs. 5 and 6. From these two figures, we find that the estimation results of the DUKF and the D-method are significantly biased in the presence of observation outliers. DUKF is less sensitive to the observation outlier compared with the D-method. By contrast, our RDUKF is able to suppress the observation

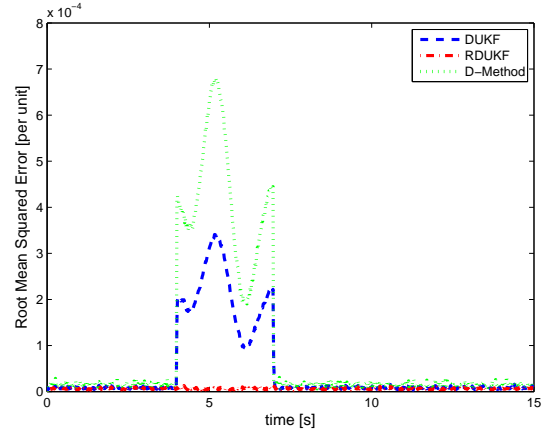


Fig. 6: RMSE of DUKF, RDUKF and D-method with observation outliers in the IEEE 39-bus system, where the measured rotor speed of Generator 5 is contaminated with 20% error from  $t=4s$  to  $t=6s$ .

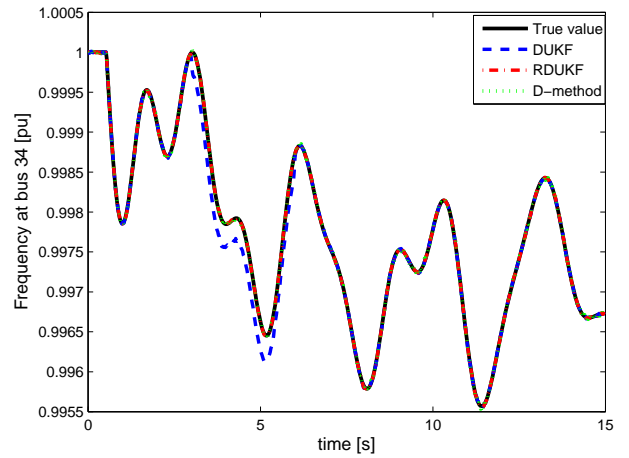


Fig. 7: Estimated frequency at bus 34 by DUKF, RDUKF and D-method with innovation outliers in the IEEE 39-bus system, where the measured real power injection of Generator 5 is contaminated with 30% error from  $t=3s$  to  $t=6s$ .

outliers thanks to the robustness provided by PS and the GM-estimator, yielding negligible bias of the estimation. It should be noted that due to the smearing effect of applying (5) for bus frequency estimation, the estimated frequencies at many buses are affected by the incorrect rotor speed of the generator 5. This however does not happen in our RDUKF.

The test results for Case 2 are shown in Figs. 7 and 8. As expected, the results of DUKF are biased in the presence of innovation outliers while the D-method is not affected. Due to the robustness of the proposed DSE, this innovation outlier has been suppressed. By comparison, RDUKF still outperforms D-method, yielding the best results.

### C. Loss of PMU Measurements

Due to the failures of communication links between PMU and phasor data concentrator or cyber attacks, the PMU placed at Bus 34, where Generator 5 is connected, is assumed to lose its measurements from  $t=3s$  to  $t=6s$ . Therefore, the

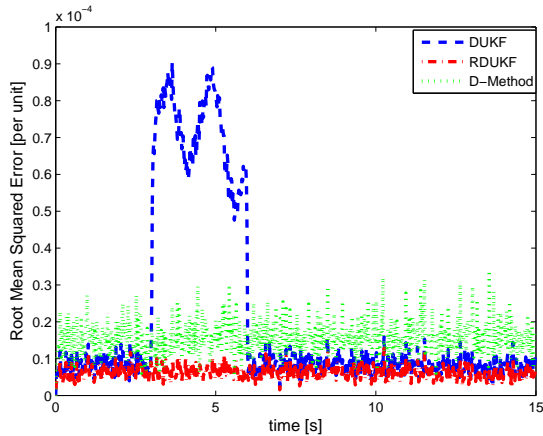


Fig. 8: RMSE of DUKF, RDUKF and D-method with innovation outliers in the IEEE 39-bus system, where the measured real power injection of Generator 5 is contaminated with 30% error from  $t=3s$  to  $t=6s$ .

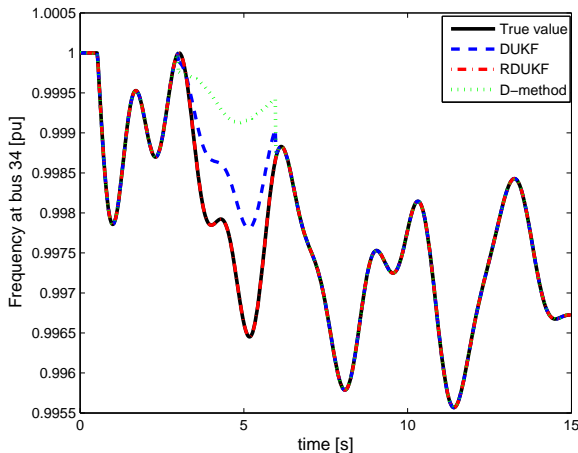


Fig. 9: Estimated frequency at bus 34 by DUKF, RDUKF and D-method with measurement losses from  $t=3s$  to  $t=6s$  in the IEEE 39-bus system.

measurement set becomes unavailable during this time interval and their values are set equal to zero for simulation purpose. Please note that in this extreme case, both predicted and measured rotor speeds will be flagged as outliers. To enable the estimation of bus frequency by the proposed method, we advocate to either recover the missing data using [25] or perform short-term forecasting of the PMUs using their spatial and temporal correlations [26]. The test results are presented in Figs. 9 and 10. It can be seen from these two figures that the estimated bus frequencies of both the DUKF and the D-method are biased significantly. But the DUKF approach is less sensitive to the measurement losses than the D-method. As a result, the proposed RDUKF can always track the bus frequency reliably and accurately. It should be noted that the two mitigation approaches [25], [26] can be used in the situation that the measurements are temporally lost (a few seconds). For longer period, they may not be valid. Otherwise, the operator will be warned and the decentralized DSE stops

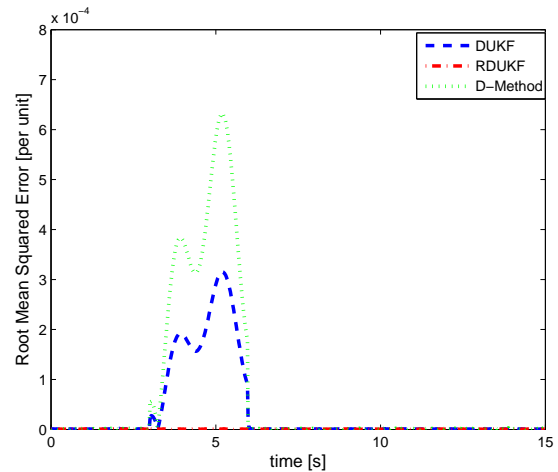


Fig. 10: RMSE of DUKF, RDUKF and D-method with measurement losses from  $t=3s$  to  $t=6s$  in the IEEE 39-bus system.

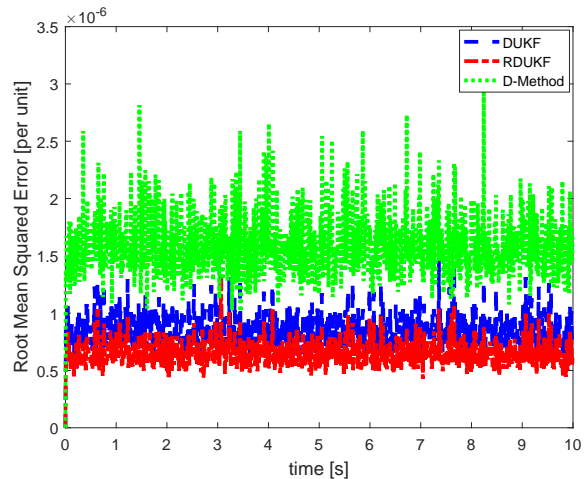


Fig. 11: RMSE of DUKF, RDUKF and D-method with normal measurement noise in the IEEE 145-bus system.

before further careful investigations are done.

#### D. Results on Large-Scale Systems

To demonstrate the applicability of the proposed robust frequency divider for large-scale system, the 50-machine IEEE 145-bus system is used. The dynamic data can be found through [27]. The generator located at bus 106 is tripped at  $t=0.5s$  to simulate the system disturbance. The following three scenarios are considered and tested:

- Scenario 1: only normal Gaussian noise is added to the simulated data like the test done in Section IV-A;
- Scenario 2: 10% of the measured generator rotor speeds is contaminated with 20% error from  $t=2s$  to  $t=5s$ ;
- Scenario 3: 50% of the measured generator rotor speeds are lost from  $t=3s$  to  $t=6s$ .

The test results of all three scenarios are displayed in Figs. 11-13, where the RMSE of scenario 1 and the estimated frequencies of bus 73 under scenarios 2 and 3 are represented accordingly. Note that bus 73 is close to the tripped generator.

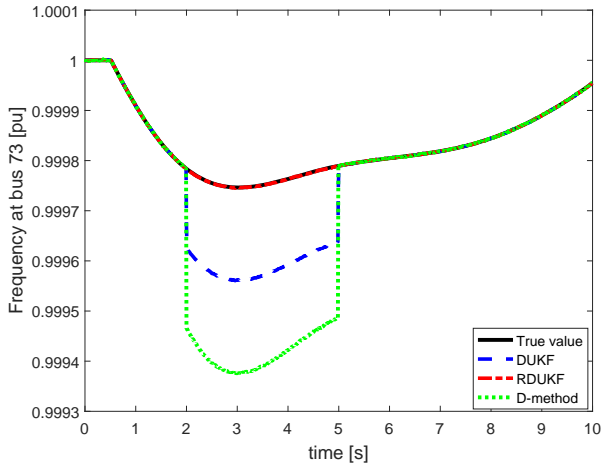


Fig. 12: Estimated frequency at bus 73 by DUKF, RDUKF and D-method with outliers in the IEEE 145-bus system, where 10% of the measured generator rotor speeds is contaminated with 20% error from  $t=2s$  to  $t=5s$ .

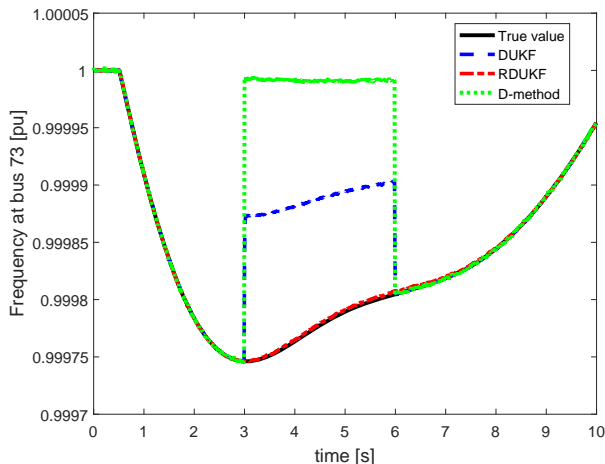


Fig. 13: Estimated frequency at bus 73 by DUKF, RDUKF and D-method with measurement losses in the IEEE 145-bus system, where 50% of the measured generator rotor speeds are lost from  $t=3s$  to  $t=6s$ .

Based on these figures, we conclude that our proposed robust frequency estimator still outperforms the other two alternatives in a larger-scale system. These results are consistent with those for IEEE 39-bus system. It is interesting to find that even without using missing data recovering approach [25], [26], our proposed method is able to rely on good PMU measurements and the predicted dynamic state variables for filtering, yielding good estimation results.

#### E. Computational Efficiency

To validate the capability of the proposed method for online estimation, that is, to be compatible with PMU sampling rate, its computational efficiency is analyzed. All cases and scenarios simulated in the previous sections are considered. All the tests are performed on a PC with Intel Core i5, 2.50 GHz, 8GB of RAM. The average computing time of each method for every PMU sample is displayed in the

TABLE I: Average Computing Times of The Three Methods At Each PMU Sample

Scenarios	D-method	DUKF	RDUKF
Section IV-A	0.02ms	0.15ms	0.37ms
Section IV-B Case 1	0.022ms	0.14ms	0.45ms
Section IV-B Case 2	0.021ms	0.16ms	0.46ms
Section IV-C	0.021ms	0.15ms	0.46ms
Scenario 1	0.08ms	0.66ms	1.6ms
Scenario 2	0.09ms	0.78ms	2.1ms
Scenario 3	0.085ms	0.79ms	2.14ms

Table I. We observe from this table that all methods have comparative computational efficiency and their computing times are much lower than the PMU sampling period, which is 16.7ms for 60 samples/s. On the other hand, although RDUKF is the most time consuming method compared with RDUKF and D-method, its computing time is negligible for practical applications considering the PMU sampling speed. Note that the decentralized DSE for each generator can be carried out independently and in a parallel manner, which are very fast to calculate and independent of the scale of the power system. For very large-scale power systems,  $D$  becomes rather dense and a numerical stable and computational efficient approach proposed in [12] is used to project the rotor speeds to bus frequencies. The computational burden of this step has been shown to be negligible [12]. In conclusion, the proposed method is suitable for large-scale power system online applications.

#### V. CONCLUSION

A robust frequency divider (RFD) is proposed to estimate the frequency of each bus in a power system. Our RFD consists of two steps, namely the estimation of generator rotor speeds through a robust decentralized UKF using local measurements, and the projection of all generator rotor speeds to bus frequency. The proposed RFD is model independent, and the knowledge of local PMU measurements at each generator terminal bus and transmission system line parameters is sufficient. Furthermore, the proposed RFD is able to filter out measurement noise, suppress gross measurement errors, handle cyber attacks as well as measurement losses. Extensive results carried out on the IEEE 39-bus system demonstrate the effectiveness and the robustness of the proposed method. A possible issue of the proposed RFD is that the decentralized and centralized scheme may produce some delays for the estimated bus frequencies used for controllers. However, thanks to the advancement of control techniques, the time delays can be effectively mitigated [28], [29]. In the future work, we will design this type of robust frequency regulator based on our RFD.

#### REFERENCES

- [1] National Grid Frequency Response Working Group, Frequency Response Technical Sub-Group Report, Nov. 2011, Tech. Rep. [Online].
- [2] J. Van de Vyver, J. D. M. De Koning, B. Meersman, L. Vandeveldel, T. L. Vandoorn "Droop control as an alternative inertial response strategy for the synthetic inertia on wind turbines," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1129-1138, 2016.
- [3] H. Ye, P. Pei, Z. Qi, "Analytical modeling of inertial and droop responses from a wind farm for short-term frequency regulation in power systems," *IEEE Trans. Power Syst.*, vol. 31, no. 5, pp. 3414-3423, 2016.

- [4] J. B. Zhao, G. Zhang, M. L. Scala, Z. Wang, "Enhanced robustness of state estimator to bad data processing through multi-innovation analysis," *IEEE Trans. Industrial Informatics*, vol. 13, no. 4, pp. 1610-1619, 2017.
- [5] C. Zhao, U. Topcu, S. H. Low, "Optimal load control via frequency measurement and neighborhood area communication," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3576-3587, 2013.
- [6] Y. Wen, W. Li, G. Huang, X. Liu, "Frequency dynamics constrained unit commitment with battery energy storage," *IEEE Trans. Power Syst.*, Vol. 31, no. 6, pp. 5115-5125, 2016.
- [7] IEEE Task Force on Load Representation for Dynamic Performance, "Load representation for dynamic performance analysis of power systems," *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 472-482, 1993.
- [8] L. Wang, D. Z. Fang, T. S. Chung, "New techniques for enhancing accuracy of EMTP/TSP hybrid simulation algorithm," in *Proc. IEEE Int. Conf. Electr. Utility Deregulation, Restructuring Power Technol.*, pp. 734-739, 2004.
- [9] G. Radman, M. A. Tabrizi, "Simulation of wide area frequency measurement from phasor measurement units (PMUs) or frequency disturbance recorders (FDRs)," Oct. 2010 [Online].
- [10] M. Lai, M. Nakano, G. Hsieh, "Application of fuzzy logic in the phase-locked loop speed control of induction motor drive," *IEEE Trans. Industrial Electronics*, vol. 43, no. 6, pp. 630-639, 1996.
- [11] J. Nutaro, V. Protopopescu, "Calculating frequency at loads in simulations of electro-mechanical transients," *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 233-240, 2012.
- [12] F. Milano, A. Ortega, "Frequency divider," *IEEE Trans. Power Syst.*, Vol. 32, no. 2, pp. 1493-1501, 2017.
- [13] Y. Zhang, P. Markham, *et al.*, "Wide-area frequency monitoring network (FNET) architecture and applications," *IEEE Trans. Smart Grid*, vol. 1, no. 2, pp. 159-167, 2010.
- [14] A. G. Phadke, B. Kasztenny, "Synchronized phasor and frequency measurement under transient conditions," *IEEE Trans. Power Deliv.*, Vol. 24, no. 1, pp. 89-95, 2009.
- [15] E. Ahad, M. Kezunovic, "Impact of electromechanical wave oscillations propagation on protection schemes," *Electric Power Systems Research*, Vol. 138, pp. 85-91, 2016.
- [16] NERC Reliability Guideline, "PMU Placement and Installation," 2016.
- [17] NERC Reliability Guideline, "Power Plant Dynamic Model Verification using PMUs," 2016.
- [18] P. Sauer, M. A. Pai. *Power system dynamics and stability*. Urbana, 1997.
- [19] S. Julier, J. K. Uhlmann, "Unscented filtering and nonlinear estimation," *Proceedings of the IEEE*, Vol. 92, no. 3, pp. 401-422, 2004.
- [20] J. B. Zhao, M. Netto, L. Mili, "A robust iterated extended Kalman filter for power system dynamic state estimation", *IEEE Trans. Power Syst.*, vol. 32, no. 4, pp. 3205-3216, 2017.
- [21] M. Gandhi, L. Mili, "Robust Kalman filter based on a generalized maximum-likelihood-type estimator," *IEEE Trans. Signal Processing*, vol. 58, no. 5, pp. 2509-2520, 2010.
- [22] P. J. Huber, *Robust Statistics*. New York: Wiley, 1981.
- [23] J. H. Chow, K. W. Cheung, "A toolbox for power system dynamics and control engineering education and research," *IEEE Trans. Power Syst.*, vol. 7, no. 4, pp. 1559-1564, 1992.
- [24] IEEE PES TF on Benchmark System for Stability Controls, "Benchmark systems for small-signal stability analysis and control," Aug. 2015.
- [25] P. Gao, M. Wang, *et al.*, "Missing data recovery by exploiting low-dimensionality in power systems synchrophasor measurements," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1006-1013, 2016.
- [26] Y. Chakhchoukh, V. Vittal, G. T. Heydt, "PMU based state estimation by integrating correlation," *IEEE Trans. Power Syst.*, vol. 29, no. 2, pp. 617-626, 2014.
- [27] <https://www2.ee.washington.edu/research/pstca/>
- [28] S. Prasad, S. Purwar, N. Kishor, "H-infinity based non-linear sliding mode controller for frequency regulation in interconnected power systems with constant and time-varying delays," *IET Generation, Transmission & Distribution*, vol. 10, no. 11, pp. 2771-2784, 2016.
- [29] T. Ramachandran, M. H. Nazari, S. Grijalva, M. Egerstedt, "Overcoming communication delays in distributed frequency regulation," *IEEE Trans. Power Syst.*, Vol. 31, no. 4, pp. 2965-2973, 2016.