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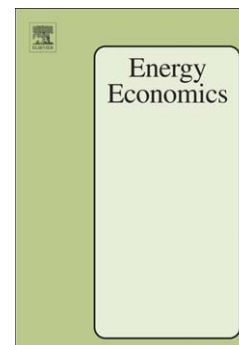
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When to invest in carbon capture and storage technology: a mathematical model

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Abstract

We present two models of the optimal investment decision in carbon capture and storage technology (CCS)-one where the carbon price is deterministic (based on the newly introduced carbon floor price in Great Britain) and one where the carbon price is stochastic (based on the ETS permit price in the rest of Europe). A novel feature of this work is that in both models investment costs are time dependent which adds an extra dimension to the decision problem. Our deterministic model allows for quite general dependence on carbon price and consideration of time to build and simple calculus techniques determine the optimal time to invest. We then analyse the effect of carbon price volatility on the optimal investment decision by solving a Bellman equation with an infinite planning horizon. We find that increasing the carbon price volatility increases the critical investment threshold and that adoption of this technology is not optimal at current prices, in agreement with other works. However reducing carbon price volatility by switching from carbon permits to taxes or by introducing a carbon floor as in Great Britain would accelerate the adoption of carbon abatement technologies such as CCS.

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Highlights:

- Analytic solution for the critical ETS permit price for optimal investment in CCS.
- Solution for the optimal time for investment in CCS in GB subject to Carbon Floor.
- Time varying Investment cost included.
- Not optimal to invest at current ETS prices.
- ETS permit price volatility increases the optimal investment threshold.
- Discussion of the merits of a tax based system over the current quota based ETS.

Key words:

- Carbon Capture and Storage
- Emission Trading Scheme
- Carbon Floor
- Real Options

1 Introduction

The European Union introduced its Emission Trading Scheme (ETS), a system in which CO₂ emission permits are traded, in 2005 as a key ingredient in its plan to adhere to the Kyoto Protocol on emission reduction. The idea was that by creating a market for emission permits cleaner technologies would be rewarded at the expense of heavy emitters. This measure was intended to accelerate investment in electricity generation from renewable sources and therefore move Europe towards becoming a low carbon emissions region. For more information on the ETS see [Abadie and Chamorro (2008)] for example.

However, renewable sources of generation tend to be intermittent so there is still a role for traditional fossil based generation to maintain system stability. The relative abundance of coal compared to other fossil fuels makes it an attractive option for electricity generation. However it is amongst the largest producers of CO₂ per unit of electricity generated so that if emitters are to be penalised through the need for ETS permits, coal loses some of its appeal. One attractive approach, in theory, is to capture the carbon released during combustion and store it permanently. There has been a huge research effort into this technique but at present there is still no commercially operating carbon capture and storage (CCS) unit anywhere in the world.

The goal of this paper is to analyse the investment decision in CCS, and determine analytically the optimal time to invest, in a region with volatile emission costs (such as a permit-based system like the ETS) and also the decision facing the investor in a region where the cost of emissions evolves deterministically (such as in a tax-based system). We will explicitly take into account decreasing investment costs as the technology matures.

The carbon floor mechanism introduced in Great Britain (GB) in April 2013 means that electricity producers in GB are effectively subject to a deterministically evolving tax rather than a stochastically evolving allowance price such as the ETS. The current level of the ETS is approx. €5/tCO₂. The lower bound on Carbon to be paid by generators in GB is currently £16/tCO₂ rising linearly to £30/tCO₂ in 2020 and rising again to £70/tCO₂ by 2030. Since the ETS permit price is significantly less than the carbon floor price and the fact that reforms of the ETS aimed at raising it's level are slow, the price of carbon emissions by fossil fuel based electricity generators in GB will be effectively deterministic. For more information on the carbon floor mechanism in GB see [Curtis et al. (2013)] for example.

As noted above, the carbon floor price has introduced an effectively deterministic carbon price into GB. Without a carbon price floor mechanism in place, power plants in the rest of Europe are subject to the stochastically evolving ETS permit price. Despite

the current low ETS price, a number of proposals have been put forward to raise the ETS price and penalise heavy polluters. One such mechanism, called “back-loading”, involves the withdrawal of a large proportion of the ETS permits in the hope that this will increase the price of the permits in the short term before they are reintroduced at a later date. However, the ETS permit price will still be volatile so to model the investment decision facing non-GB European Power plants more sophisticated techniques of stochastic calculus will need to be employed.

A number of authors have addressed the question of when it is optimal to invest in CCS given carbon price and electricity price uncertainty. In [Fuss et al. (2008)] both types of uncertainty are included in a numerical model with a finite planning horizon of 50 years. In their model the CCS unit may be switched on and off depending on which state is optimal. Their profit function is a linear function of electricity, heat and carbon price and other costs. They then solve numerically a Bellman equation to determine the optimal time to invest in CCS so that the sum of discounted expected future profits is maximised.

Another thorough numerical analysis of the problem is given in [Abadie and Chamorro (2008)]. Again the electricity price and carbon price follow correlated stochastic processes (in both papers the carbon prices follow geometric Brownian motion) and there is a finite planning horizon and the problem is solved using a two-dimensional binomial lattice to obtain the optimal investment rule.

In [Heydari et al. (2012)] an analytical model was presented in which the authors solved a partial differential equation to determine the optimal investment boundary under fuel price and carbon price uncertainty (electricity price was found not to affect the option value of the retrofit of a coal fired power plant since the outputs of the plant pre and post retrofit were taken to be the same). They also (numerically) value the option to invest in full CCS (approx. 85-95% of carbon emissions captured) and partial CCS (approx. 45-65% of carbon emissions captured) and find that if price volatilities are low enough the investment region is dichotomous so that for a given fuel price investment is optimal in Full CCS (Partial CCS) if the carbon price increases (decreases) sufficiently. It was assumed in this work that investment costs remain fixed.

The literature on CCS has identified the lowering of investment costs as crucial to the large-scale deployment of CCS technology. In [Herzog (2011)] it was noted that the first several CCS plants would likely be more expensive, typical of the introduction of a new technology. In [Riahi et al. (2004)] the situation was compared to the past experience of installing scrubbers to control sulfur dioxide emissions from power plants. A ‘learning curve’ for CCS was quantified in comparison with the sulfur dioxide case with investment costs greatly reduced as the technology matures. The importance of including time dependent investment costs in any model of CCS uptake is further il-

illustrated in the two-period model in [Hoel and Jensen (2012)] where it was concluded that cost reductions in CCS may be more desirable than cost reductions associated with renewable energy, from a welfare perspective.

The decision the investor faces today, based on estimates of total investment cost, will be different to the decision faced in the future if investment costs have fallen. Since quantifying the value of waiting for more favourable conditions before investing is one of the key strengths of the real options approach, we believe that incorporating time dependent costs is an important advancement on other approaches that ignore this issue and use fixed costs (see [Heydari et al. (2012)] for example).

We are not aware of any other research comparing the optimal investment decisions for CCS retrofitting using a carbon price process that models the deterministic carbon floor in GB and another than models the stochastic ETS price. This work is timely as reform of the ETS is needed if it is to meet its goal of driving Europe towards becoming a low carbon emissions region and the newly introduced carbon floor mechanism in GB promises to provide guaranteed incentives to generators to reduce emissions.

We expect to find it optimal to invest in CCS much sooner in GB than in the rest of Europe, which has obvious policy implications if policy makers still see a role for coal-based electricity generation in Europe. The reason for this expectation is two-fold. Firstly, the current ETS price is much lower than the current value of the carbon floor price. Secondly, we expect to find that increasing the volatility of the ETS price will increase the critical investment threshold. Modelling uncertainty is thus fundamental to this problem.

As in all the works mentioned above, we will model the ETS permit price as geometric Brownian motion. The volatility of the process takes into account the inherent uncertainty of a tradable allowance permit and also the uncertainty in expectations over future emissions policy.

In this work we first model the investment decision facing the investor in GB. We model this as a deterministic problem for the reasons outlined above in connection to the carbon floor. We obtain the optimal time to invest that maximises the net present value (NPV) of the option taking into account a time to build of one year and assuming that no revenue is received during this year. A numerical example for a hypothetical baseload coal plant illustrates this result in Section 2.2. We then model the decision facing an investor in the rest of Europe subject to a stochastically evolving ETS permit price in Section 2.3 and find the critical investment threshold of the ETS price above which it is optimal to invest (assuming that the CCS unit may be built instantaneously). A numerical example for a hypothetical baseload plant in Europe (excluding GB) follows in section 2.4. We conclude this work with a summary and discussion of our results in Section 3.

In both of our models our investment cost function varies with time and in this respect provides a valuable addition to the literature on this topic. Our models are analytic and compliment numerical approaches, as analytic formulae allow greater clarity about the contribution of various factors to the investment decision.

2 When to invest in CCS - a free boundary problem

We are interested in determining analytically the optimal time for a new coal plant to retrofit a carbon capture and storage unit with and without carbon price uncertainty. To do this we maximize the net present value (NPV) of the investment option.

2.1 The CCS investment decision in GB: Deterministic Case

Let P_o denote the profit function for the coal plant without the CCS unit upgrade and P_n denote the profit function for the upgraded plant, both depending on the carbon price C . If the time of investment in CCS is taken to be T (an unknown) then we can write the NPV of the asset as

$$W(C) = \int_0^T P_o(C(t))e^{-rt} dt + \int_{T+1}^{40} P_n(C(t))e^{-rt} dt - I(T)e^{-rT} \quad (2.1)$$

where $I(T)$ is the investment cost function, and r is the discount rate. We have assumed that it takes one year to build the CCS unit and that during this time there is no profit flow (hence the lower bound in the second integral is $T + 1$ rather than T). Also we are assuming that the lifetime of the plant is 40 years.

We assume that the investment cost is irreversible since, as noted in [Abadie and Chamorro (2008)], the CCS unit has a limited range of uses and cannot be installed at another power plant so that, as noted [Pindyck (2007)] and [Abadie and Chamorro (2008)], there is an opportunity cost associated with the investment. We model the investment cost as a once-off payment, for a discussion of the case of a multi-stage investment see [Dixit and Pindyck (1994)].

We believe it is interesting to consider the case where investment costs decrease over time. This gives an explicit value to waiting. Our intuition tells us that at current prices the baseload plant considered in the example below will be more profitable without a CCS unit (since it doesn't have to store the carbon or operate and maintain the CCS unit) but that over time as the carbon floor price increases it will become less profitable and CCS more attractive, especially if the investment costs are decreasing over time as

the technology matures. We determine below the optimal time to invest which balances these competing factors not examined in other works with constant investment costs.

Assuming that C evolves deterministically, (2.1) may be differentiated with respect to T which yields

$$\frac{dW}{dT}(T) = (P_o(C(T)) - P_n(C(T+1))e^{-r} + rI(T) - I'(T)) e^{-rT} \quad (2.2)$$

so that W is extremised¹ when

$$P_n(C(T+1))e^{-r} - P_o(C(T)) = rI(T) - I'(T). \quad (2.3)$$

We now choose a functional form for P_o and P_n . As in other works (see [Heydari et al. (2012)] and [Fuss et al. (2008)] for example) we choose P to be a linear function of C and choose it to have the same functional form before and after retrofitting. That is, we choose:

$$P_o = \alpha_o - q_o C(t)$$

and

$$P_n = \alpha_n - q_n C(t).$$

Choosing the linear form above has the advantage that the parameters have a clear interpretation as gross revenue before and after retro-fitting (α_o and α_n respectively) and the cost to the plant of carbon emissions ($q_o C(t)$ and $q_n C(t)$). We have chosen to omit the fuel price and concentrate on the cost of carbon emissions and decreasing investment costs. ([Heydari et al. (2012)]) choose a GBM drift rate of 0.04 for the fuel price which is not captured in our results which assume a constant fuel price. Volatility of the coal price is typically low ([Heydari et al. (2012)]) choose it to be 0.05 compared to a volatility of 0.47 for the carbon price) so we believe our model, with the annual constant fuel consumption costs absorbed into α_o and α_n is still a good approximation with the added benefit of realistic investment costs that decrease over time.

We choose the profit function to have the same form after retrofitting to isolate the effect of the retrofit and for ease of interpretation of parameter choices (we choose $q_n < q_o$ since less carbon is emitted by the upgraded plant and $\alpha_n < \alpha_o$ since there are extra costs such as carbon storage and extra operation and maintenance costs for the upgraded plant that will reduce its gross revenue).

¹We will determine whether this extremum is a maximum or a minimum by first determining the extremising T and then checking that other values of T give a lower value to (2.1). We do this because the function $C(t)$ that we use to model the carbon floor is not differentiable at $t = 7$ i.e. in 2020 when the slope of the carbon floor function increases.

2.2 A numerical example of the investment decision in GB

We will assume we are dealing with a baseload coal plant throughout the lifetime of the plant.²

Suppose we have a Super Critical Pulverised Coal (SCPC) power plant with 500MW capacity, an 80% capacity factor and an average CO₂ emission rate of 800g/kWh (these characteristics are taken from [Abadie and Chamorro (2008)]). Assuming that 5% of the electricity output is consumed by ancillary units this gives a total annual output of 3,328,800MWh. Combining this with the CO₂ emission rate gives 2,663,040 ton/year of CO₂ emitted per year.

It is clear that the emissions cost to the plant is then 2,663,040 ton/year \times Average Carbon price €/ton = $q_o C$. Since 90% of the CO₂ is captured once the plant has been upgraded we have $q_n = q_o/10$. Following [Abadie and Chamorro (2008)] once more we take the cost of storage and transportation of the CO₂ to be €7.35/ton giving an annual cost of $2.663 \times 10^6 \text{ ton/year} \times 0.90 \times 7.35 \text{ Euro/ton} = €17.62 \text{ M/year}$.

Operation and maintenance cost of the CCS unit are taken to be 1.348 €/MWh giving a total annual cost of 4.49M €/year. So the total extra cost of running the CCS unit is approximately 22M €/year. This will provide a lower bound on $\Delta\alpha := \alpha_o - \alpha_n$ since it ignores the revenue depletion from the reduction in output of the CCS unit (in [Abadie and Chamorro (2008)] it is assumed that there's a 20% loss of the plant's output due to the presence of the CCS unit). Finally we take our investment time dependent investment cost function $I(T) = €214.5 \times 10^6 \exp(-0.0202T)$ so that $I(T)$ decreases by 2% per year, as in [Abadie and Chamorro (2008)].

We will choose the discount rate to be $r = 0.06$.

A summary of all the parameters used in this example is given in Table 1.

Table 1: Parameters used to determine the optimal time to invest in CCS in GB.

Parameter	Value
Discount rate r	0.06
Carbon Floor Price (GB) in 2013	£16/tCO ₂ =€18.82/tCO ₂
Carbon Floor Price (GB) in 2020	£30/tCO ₂ =€35.29/tCO ₂
Carbon Floor Price (GB) in 2030	£70/tCO ₂ =€82.35/tCO ₂
Investment Cost Function $I(T)$	€214.5 \times 10 ⁶ exp(-0.0202T)
Difference in Plant gross revenues $\Delta\alpha$	€(22 + 50) \times 10 ⁶ /y
Difference in emission coupling Constants Δq	2.397 \times 10 ⁶ tCO ₂ /y

²For a high carbon price scenario this is unrealistic since coal plants without CCS are heavily penalised since they emit more carbon than gas plants and may not be dispatched as baseload.

We believe it is more meaningful to choose a value for $\Delta\alpha := \alpha_o - \alpha_n$ rather than α_o and α_n separately since we know that the gross revenue of the retrofitted plant will be less, in our model, than that of the plant without the retrofit due to the costs incurred to transport and store the carbon and also the increased operation and maintenance costs and the revenue depletion due to the reduced output of the plant (the CCS unit uses electricity), $\text{€}50 \times 10^6/\text{y}$ in this example. We need only choose a value for the difference $\Delta\alpha$ if we make the approximation $\exp(-r) \approx .94 \approx 1$ i.e.

$$0 = P_n(C(T+1))e^{-r} - P_o(C(T)) - rI(T) + I'(T) \quad (2.4)$$

$$\approx P_n(C(T+1)) - P_o(C(T)) - rI(T) + I'(T). \quad (2.5)$$

We choose the functional form for $C(t)$ to match the carbon floor prices (in euros)

$$C(t) = \begin{cases} (18.82 + 2.353t) \text{ €}/\text{tCO}_2, & \text{if } 0 \leq t \leq 7 \\ (35.29 + 4.70(t-7)) \text{ €}/\text{tCO}_2, & \text{if } t > 7. \end{cases} \quad (2.6)$$

Solving (2.5) numerically using the parameters in Table 1 gives an approximate value of 7.31 years for the optimal time to invest. That is, if one invests after 7.31 years (i.e. during 2020) the NPV of the investment option will be maximised.

2.3 The investment decision in the rest of Europe:

Power plants in GB are still subject to the ETS, however the high level of the carbon floor price means that it is highly unlikely that the ETS permit price will rise above the carbon floor price, thus there is a deterministic carbon floor price in GB at present.

Without the (high level of) the carbon floor price, the carbon permit price in the rest of Europe (the ETS permit price) has a stochastic component. It is affected by the supply and demand for permits and also by uncertainties in European emissions policy. There is widespread consensus that the ETS in its current form is not working efficiently to penalise heavy polluters and a number of proposals have been discussed with the aim of increasing the ETS permit price. This suggests modelling the ETS permit price as a stochastic process with positive drift and significant volatility.

Following [Abadie and Chamorro (2008)] and [Heydari et al. (2012)], we model the ETS permit price $C(t)$ as geometric Brownian motion (GBM):

$$dC = \mu C dt + \sigma C dz$$

where μ is the constant drift rate, σ is the constant volatility and z describes a Wiener/Brownian process.

In [Pindyck (1999)] it was noted that although energy prices in the long-run tend to be mean-reverting, the rate of mean reversion is low so that GBM may be a good approximation. Since the current low level of 5€/tCO₂ is unsustainable as Europe looks to meet its emission targets, we have chosen not to calibrate the drift and volatility parameters of our GBM ETS permit price from historical data (since this would yield a negative drift). Instead we choose in the numerical example that follows a high positive drift rate ($\mu = 0.05$) and vary the volatility parameter.

In the case of a stochastically varying carbon price we may no longer differentiate (2.1) to obtain the optimal T , since T is now a random variable. Instead we obtain it indirectly by computing the critical threshold C^* above which it is optimal to invest immediately. Our approach will be to solve a Bellman equation, sometimes called the Hamilton-Jacobi-Bellman equation, derived from (2.7) below by application of Ito's lemma, to obtain the critical threshold for investment, the "free-boundary" C^* . This procedure has been carried out, for example, in [McDonald and Siegel (1986)], [Dixit and Pindyck (1994)] and [Pindyck (2002)].

The introduction of volatility into the optimal investment problem considerably complicates the solution process. For this reason we make two further simplifying assumptions. Firstly, we assume that the CCS unit may be built instantaneously, once the decision to invest has been made, with no loss of revenue. The plant's profit functions are thus integrated over the continuous lifetime of the plant (unlike in the deterministic model above where the interval $(T, T+1)$ was omitted as it represented the time during which the CCS unit was being built). For a stochastic model with time to build included see [Majd and Pindyck (1987)].

Secondly, we now assume that the plant has an infinite lifetime and consequently that the option to invest doesn't expire (this assumption was also made in [Heydari et al. (2012)]). It is not unusual for a coal plant to have a lifetime of 60 years by which time the discounting of future profit flows in (2.1) dramatically reduces their contribution to the NPV integral. As $t \rightarrow \infty$ this contribution tends to zero and the NPV integral, (2.7) below, will converge to a finite value (provided P_o and P_n take the same functional linear forms as above and also that with C following GBM that we also have $\mu < r$ so that the integral converges).

This assumption simplifies the resulting Bellman equation by removing the time derivative that would be present if the integral for W had a finite time horizon or there was explicit calendar time dependence. Whilst it is often possible to solve the Partial differential equation that results from the Bellman equation with two GBM processes in this framework (see [Heydari et al. (2012)]), it is more challenging to solve problems

with a finite time horizon where numerical approaches tend to be used (even standard numerical approaches such as shooting, Runge-Kutta and finite difference schemes are problematic for Bellman equations for the reasons outlined in [Dangl and Wirl (2004)]).

In our time-independent Bellman equation we are modelling the decision to invest or wait which is the same in every period except that the state variable $C(t)$ and $I(T)$ have changed.

So, in the case where C follows GBM we have

$$W(C) = \mathcal{E}_0 \left(\int_0^T P_o(C(t))e^{-rt} dt + \int_T^\infty P_n(C(t))e^{-rt} dt - I(T)e^{-rT} \right) \quad (2.7)$$

where \mathcal{E}_0 denotes the expected value based on information available at time $t = 0$.

The presence of a T dependent investment cost $I(T)$ prevents us from simply applying a Bellman equation derived from the integral. However, this term may be taken inside the integral. Then we have

$$W(C) = \mathcal{E}_0 \left(\int_0^T P_o(C(t))e^{-rt} dt + \int_T^\infty [P_n(C(t)) + I'(t) - rI(t)]e^{-rt} dt \right) \quad (2.8)$$

since

$$-I(T)e^{-rT} = \int_T^\infty \frac{d}{dt}(I(t)e^{-rt}) dt = \int_T^\infty (I' - rI)e^{-rt} dt.$$

To avoid the explicit introduction of calendar time into the integral we define

$$\frac{d}{dt}I := I'(t) := -\xi I \quad (2.9)$$

where $\xi > 0$ measures the rate at which investment costs decrease, so now the problem depends on I and C only and not explicitly on t (i.e. $T = T(I, C)$). Now we can apply a standard Bellman equation to the regions before and after investment to determine the ‘free-boundary’ C^* , the trigger price above which it is optimal to invest. The general Bellman equation reads

$$rW(C) = \hat{P}(C) + \frac{1}{dt}\mathcal{E}_0[dW(C)], \quad (2.10)$$

where \hat{P} denotes the profit flow in the interval dt in the pre and post investment regions in (2.7) above, $\mathcal{E}_0[dW(C)]$ denotes the expected capital gain and r is the discount rate.

Once more we assume that the profit function of the coal plant without CCS, P_o , may be written as

$$P_o = \alpha_o - q_o C(t)$$

and that the profit function of the coal plant with CCS retrofitted is given by

$$P_n = \alpha_n - q_n C(t),$$

where $\alpha_{o,n}$ and $q_{o,n}$ are constants throughout the lifetime of the plant.

Substituting this choice of profit function into the general Bellman equation (2.10) in the pre-investment region and expanding $\mathcal{E}_0[dW(C)]$ according to Ito's Lemma gives

$$rW^o = P_o + \frac{\sigma^2 C^2}{2} W_{cc} + \mu C W_c \quad (2.11)$$

which, choosing $P_o = \alpha_o - q_o C$, has the solution

$$W^o = A_1 C^m + A_2 C^{m'} + \frac{\alpha_o}{r} - \frac{q_o C}{r - \mu},$$

where m and m' are the roots of $\hat{m}(\hat{m} - 1)\frac{\sigma^2}{2} + \mu\hat{m} - r = 0$. If $r > \mu > 0$ then we know that one root is positive, m say, whilst the other, m' , is negative. The first two terms in this solution represent the value of the option to wait before investing whilst the last two terms are particular solutions of the integral defining W , (2.7), (of course the integral must be a solution of the Bellman equation derived from it). Since we require $W^o(C = 0)$ to be finite we can set $A_2 = 0$. So the value of the option to invest is given by $A_1 C^m$ and is unknown at this stage since the constant A_1 has not been determined. In the region where it is optimal to invest ($C > C^*$) we have

$$rW^n = P_n - (\xi + r)I(t) + \frac{\sigma^2 C^2}{2} W_{cc} + \mu C W_c. \quad (2.12)$$

The solution of this equation is

$$W^n = B_1 C^m + B_2 C^{m'} + \frac{\alpha_n - \xi I(t)}{r} - I(t) - \frac{q_n C}{r - \mu}.$$

This time it is clear that there is no value to the option to delay, since we are in the investment optimal region. Thus we take $B_1 = B_2 = 0$ since we are not modelling an option value of disinvestment (the investment is irreversible). See [Dixit (1989)] for an example where the option to disinvest is included.

The final two boundary conditions are the value matching condition on the free boundary

$$W^o(C^*) = W^n(C^*)$$

and the smooth pasting condition

$$W_c^o(C^*) = W_c^n(C^*).$$

If W^o and W^n did not match continuously and tangentially at C^* , then it would be optimal to invest at a different point to C^* (i.e. it would not be optimal to invest above C^* and wait below C^*). This is proven by contradiction in [Dixit and Pindyck (1994)]. (Note that our investment decision is akin to deciding the optimal time to exercise an American option. In this case the value matching and smooth pasting boundary conditions ensure that there are no arbitrage opportunities). Together these two boundary conditions determine the constant A_1 and the free-boundary C^* .

Applying the value matching and smooth pasting boundary conditions we find that the free boundary is given by:

$$C^* = \frac{m}{m-1} \left(\frac{\alpha_o - \alpha_n - I'(t)}{r} + I(t) \right) \frac{r - \mu}{q_o - q_n} \quad (2.13)$$

after substituting $I'(t)$ for $-\xi I(t)$. This expression for C^* is the main result of this work. For values of $C > C^*$ it is optimal to invest immediately whilst for values of $C < C^*$ it is optimal to wait. The implicit time dependence of C^* is a major advance on previous works which assume unrealistic fixed investment costs over time. Of course the methodology at arriving at this formula is not restricted to modelling CCS investment but rather is well suited to modelling situations where it is desirable to ‘switch’ optimally under exogenous uncertainty-the functional form of the profit functions being critical in describing a particular switching scenario.

Note that we can already see a value to waiting to invest since $(-I'(t)) > 0$ behaves like extra revenue for the plant that has not yet been upgraded (α_o/r).

For an investment cost decreasing with time, $I'(t) < 0$, C^* tends to a positive constant proportional to the difference in gross revenues $\Delta\alpha := \alpha_o - \alpha_n$ as $I(t)$ tends to zero. This makes intuitive sense since without investment costs the decision to invest will be made on the basis of the relative attractiveness of the retrofit as described by the parameters α_o , α_n , q_o and q_n , r , μ and σ .

C^* tends to the same non-zero constant described above in the case where the investment costs are increasing with time according to $I'(t) = rI(t)$. In this case the discounted investment cost appearing in the integral (2.7) is T-independent leaving only the constant $I(t=0)$ which does not affect the optimisation.

However, if the rate of growth of the investment cost is greater than r then the NPV integral and also C^* tend to $-\infty$, so we must exclude this case from our analysis on the grounds that W is not bounded.

We now apply techniques of comparative statics to determine how C^* changes as its parameters vary.

We first study how this investment threshold changes as the carbon permit price volatility is varied, this standard argument may be found in more detail in [Dixit and Pindyck (1994)], for example. First note that the only term in (2.13) that depends on the volatility σ is m , where recall that m is the positive root of the fundamental quadratic

$$Q(\hat{m}) = \hat{m}(\hat{m} - 1)\frac{\sigma^2}{2} + \mu\hat{m} - r = 0 \quad (2.14)$$

The coefficient of \hat{m} in $Q(\hat{m})$ is positive so $Q(\hat{m})$ describes an upward pointing parabola tending to ∞ as $\hat{m} \rightarrow \pm\infty$. Now $Q(1) < 0$ since we are assuming $\mu < r$, and $Q(0) < 0$. Therefore the graph of Q crosses the horizontal axis at one point to the right of 1 and at one point to the left of zero. Thus at the positive root $\hat{m} = m > 1$. We are interested in how m changes as the volatility is varied since m is the only parameter in (2.13) that depends on σ . For this we follow [Dixit and Pindyck (1994)] and take the total derivative of (2.14) with respect to σ to find

$$\frac{\partial Q}{\partial \hat{m}} \frac{\partial \hat{m}}{\partial \sigma} + \frac{\partial Q}{\partial \sigma} = 0 \quad (2.15)$$

with all derivatives evaluated at the positive root m . Since $Q(\hat{m})$ is an upward-pointing parabola, at m we have $\frac{\partial Q}{\partial \hat{m}} > 0$. Furthermore

$$\frac{\partial Q}{\partial \sigma} = \sigma \hat{m}(\hat{m} - 1) > 0$$

at $m > 1$.

So we conclude that $\frac{\partial m}{\partial \sigma} < 0$ so that as σ increases, m decreases and in particular $\frac{m}{m-1}$ increases. So an increase in the volatility of the ETS permit price will push up the critical threshold for optimal investment C^* (see Figure 1 and the numerical example that follows).

Note that expanding the explicit formula for the positive root m in a power series in σ and taking the limit $\sigma \rightarrow 0$ we find that $m = \frac{r}{\mu}$. In Appendix A we verify that the deterministic limit of (2.13) gives the value of T that maximises the deterministic limit of (2.7).

Now we define the difference in gross revenues of the plants $\Delta\alpha = \alpha_o - \alpha_n$ and the difference in the carbon coupling constants of the plants $\Delta q = q_o - q_n$. Using (2.13) it follows that

$$\frac{\partial C^*}{\partial \Delta\alpha} > 0 \quad (2.16)$$

and

$$\frac{\partial C^*}{\partial \Delta q} < 0. \quad (2.17)$$

Now (2.16) tells us that as we increase the difference between the gross revenues of the plants the optimal investment boundary increases and it's optimal to invest later, since in this case the plant without the upgrade has an increased relative advantage over the upgraded plant in terms of gross revenue. Likewise, (2.17) tells us that if the plant without the upgrade emits more CO₂ then the plant with the upgrade has a relative advantage and so the investment threshold decreases and it's optimal to invest earlier.

In Appendix B we plot C^* varying $\Delta\alpha$ (Figure 2) and varying Δq (Figure 3) for the range of parameters used in the the numerical example in the next section.

2.4 A numerical example of the investment decision in the rest of Europe

To illustrate the utility of our expression for C^* we will give a numerical example. We assume again we are dealing with a baseload coal plant throughout the lifetime of the plant.

We will choose the discount rate $r = 0.06$ and the carbon permit drift $\mu = 0.05$. In fact, as noted in [Abadie and Chamorro (2008)], some authors recommend using a much higher discount rate-as high as 14.8% in [Rubin et al. (2007)] to reflect the higher risk involved in CCS investments. For fixed $0 < \mu < r$, the higher the discount rate, the faster the convergence of the NPV integral and so our approach of using an infinite planning horizon becomes more similar to a finite horizon problem.

To consider the effects of carbon price volatility on the investment timing decision we plot the free boundary C^* with Δq as above and with $\Delta\alpha = \text{€}(22 + 50) \times 10^6$ i.e. assuming 22M€/y cost to transport and store the carbon and revenue depletion of €50 M due to the reduced output of the upgraded plant (the comparative statics result (2.16) tells us that increasing $\Delta\alpha$ pushes up the investment boundary C^*). We plot C^* for $\sigma = 0$, the deterministic case, and for $\sigma = 0.3$, the stochastic case. In the deterministic case the intersection of the deterministic carbon price curve and the free boundary C^* gives the maximum of the NPV (not shown).

A summary of all the parameters used in this example is given in Table 2.

In the stochastic case the time of optimal investment depends on the sample path of our GBM. However we can still define the optimal 'switching time' as

$$T_s := \inf\{t > 0 : C(t) \geq C^*(t)\}$$

where inf stands for infimum or greatest lower bound, see [Mosino (2012)] for example. For presentation purposes we take the the deterministic path followed by C as our

Table 2: Parameters used in Figure 1 to plot the optimal investment boundary C^* (2.13)

Parameter	Value
Discount rate r	0.06
Drift rate μ	0.05
Volatility (Deterministic scenario) σ	0
Volatility (Stochastic scenario) σ	0.3
Initial Carbon Price $C(t = 0)$	5€/tCO ₂
Investment Cost Function $I(T)$	€214.5 × 10 ⁶ exp(−0.0202T)
Difference in Plant gross revenues $\Delta\alpha$	€(22 + 50) × 10 ⁶ /y
Difference in emission coupling Constants Δq	2.397 × 10 ⁶ tCO ₂ /y

reference path, even in the stochastic case. Figure 1 clearly demonstrates the effect of volatility on the optimal decision choice. When $\sigma = 0$ we recover the deterministic solution and the intersection of C and C^* gives the maximum of the NPV. Adding volatility to the ETS price drives the free-boundary C^* upwards hence delaying the optimal decision to invest further, in agreement with the comparative statics of the previous section.

For the range of parameters chosen, investment in carbon capture technology is not optimal in the normal lifetime of an SCPC power plant which we take to be 40 years and assuming that the investor will not invest after the 35 year period as they will want to recoup their investment (for the reference path chosen T_s is approximately 38 years for the deterministic scenario and approximately 50 years for the stochastic scenario).

The significant difference between predicted investment timing in these two scenarios illustrates the sensitivity of investment to the expected volatility of carbon prices. This volatility assumption, in turn, depends upon the form of climate policy that is in place and the way the policy is operated. In particular, if climate policy relies on tradable carbon permits, as in the ETS, expected price volatility will likely be higher than if carbon taxes are used. Both mechanisms can give rise to some carbon price volatility, but taxes tend to change more slowly and predictably than the prices of permits. Permit systems are intended to guarantee a quantity of carbon abatement but must allow price variation to achieve this.

Figure 1 illustrates the point that putting structures in place that reduce the volatility of the ETS price (such as a carbon tax or binding carbon price floor) will lead to earlier investment in abatement technologies than maintaining volatile carbon prices.

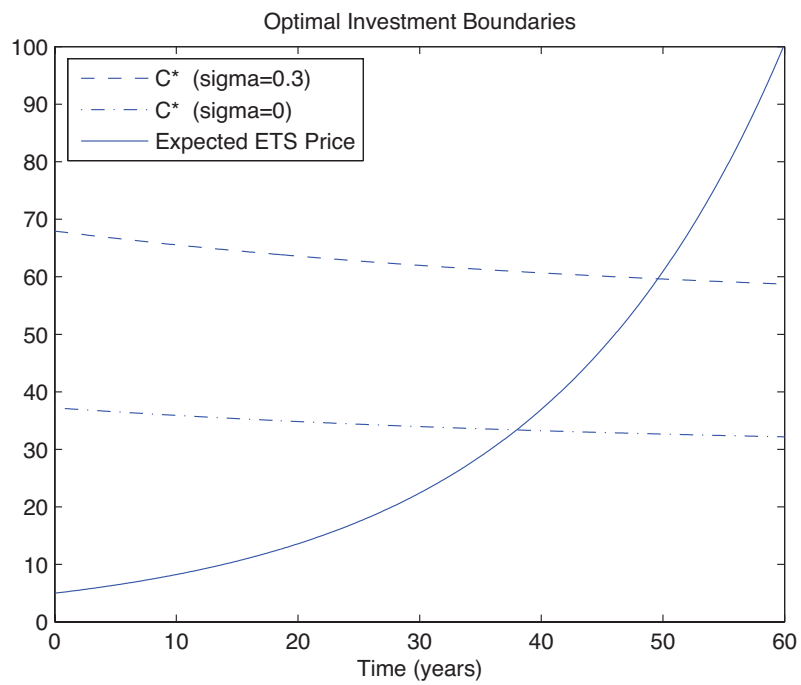


Figure 1: The intersection of the expected ETS price with the critical boundary curve C^* with and without ETS price volatility.

3 Discussion

In this work we have determined the optimal time to invest in CCS technology in Great Britain where the carbon floor price effectively removes emission cost volatility to fossil-based electricity generators. We also determined the optimal time to invest in CCS in the rest of Europe where the cost of emissions to electricity generators is volatile. We improved upon existing literature in our analytical models by including time dependent costs so that the investment decision has an added component, namely the value of waiting for investment costs to decrease sufficiently.

In our deterministic model for the investment decision in GB-simple calculus techniques were sufficient to determine the investment time that maximised the net present value of the option which we found to be during 2020 in our model with the carbon price fully determined by the carbon floor price.

In contrast the optimal investment time for our stochastic model of the investment decision in the rest of Europe was highly sensitive to the level of volatility in the ETS permit price (assumed to follow geometric Brownian motion). With volatility level of $\sigma = 0.3$ the optimal investment time was post 2060 (leaving little or no time for the investor to recoup the investment cost) and approximately 2050 when $\sigma = 0$, both times taken with respect to the expected ETS permit price path as explained above.

This work has policy implications in the area of maximally incentivising investment in carbon abatement technologies such as CCS. It contributes to the ongoing debate on reform of the ETS by providing evidence on the merits of a tax-based system to accelerate optimal investment in carbon abatement technologies. We have shown that different investment timing decisions are optimal depending on the level of volatility in the ETS price. Carbon taxes and tax-type climate policy mechanisms such as the newly introduced carbon floor in Great Britain can substantially reduce the uncertainty in the carbon price and push the critical value of C for investment, C^* , lower. They are likely to be more effective than permit-based policies for the policy priority of encouraging investment in abatement technologies. Of course, their wider efficiency properties depend upon the tax/carbon price floor being set at an appropriate level and on the policy being harmonised across as many jurisdictions as possible.

As noted in [Abadie and Chamorro (2008)] different methodologies and parameters used in the existing literature will lead to different estimates of the optimal investment threshold. The main contribution of this paper is an analytic model of the CCS investment decision with the important advancement of the inclusion of time dependent investment cost in regions where the carbon price is based on a tax and in regions where it is based on a tradable permit. We reproduce the qualitative features of previous research, namely that volatility increases the investment threshold and that at current ETS permit prices

it is not optimal to invest in CCS.

Appendix A

In this appendix we verify that in the limit as $\sigma \rightarrow 0$, that the integral (2.7) is maximised by the $\sigma \rightarrow 0$ limit of (2.13).

In this limit W becomes:

$$W(C) = \int_0^T P_o(C(t))e^{-rt} dt + \int_T^\infty P_n(C(t))e^{-rt} dt - I(T)e^{-rT}. \quad (3.18)$$

This may be differentiated with respect to T which yields

$$\frac{dW}{dT}(T) = (P_o(C(T)) - P_n(C(T)) + rI(T) - I'(T)) e^{-rT} \quad (3.19)$$

with an extremal value of T found by setting it equal to zero.

To determine whether this extremal value is a maximum or a minimum we differentiate once again with respect to T

$$\frac{d^2W}{dT^2}(T) = -r(P_o(C(T)) - P_n(C(T)) + rI(T) - I'(T)) e^{-rT} \quad (3.20)$$

$$+ \left(\left(\frac{dP_o}{dC} - \frac{dP_n}{dC} \right) \frac{dC}{dT}(T) + (-I''(T) + rI'(T)) \right) e^{-rT}. \quad (3.21)$$

The first term in brackets on the right hand side above vanishes at an extremal value (it is equal to $-r \frac{dW}{dT}$). Since we are assuming the profits of the plant that has not been upgraded, P_o , fall off faster with increasing carbon price C than the upgraded plant's profits, P_n , we have that

$$\left(\frac{dP_o}{dC} - \frac{dP_n}{dC} \right) = -\Delta q < 0.$$

In the deterministic limit of an ETS price following geometric Brownian motion we have exponential growth

$$dC = \mu C dt,$$

where μ is the constant drift rate taken to be positive to model an increasing ETS price, i.e.

$$\frac{dC}{dT} = \mu C > 0.$$

Furthermore, we assume that as the technology matures the investment costs will decrease so that for a convex decreasing investment cost function $I(T)$

$$(-I''(T) + rI'(T)) < 0.$$

Therefore, for quite general choices of $P_{o,n}$ and $I(T)$ we find

$$\frac{d^2W}{dT^2} < 0$$

at an extremal value and therefore the NPV is maximised at the value of T obtained from setting (3.19) equal to zero.

Substituting our linear functional forms for P_o and P_n into

$$P_n(C(T)) - P_o(C(T)) = rI(T) - I'(T)$$

and rearranging terms we are left with the deterministic free boundary equation for C^* (noting that $m \rightarrow r/\mu$ as $\sigma \rightarrow 0$).

Appendix B

In Figure 2 we plot the optimal investment boundary in the stochastic case ($\sigma = 0.3$) for three values of the revenue depletion due to the reduced output of the CCS retrofitted plant, namely, for $\Delta\alpha = (22 + 25) \times 10^6 \text{€}/y$, $\Delta\alpha = (22 + 50) \times 10^6 \text{€}/y$ (the reference value used in figure 1) and for $\Delta\alpha = (22 + 75) \times 10^6 \text{€}/y$. For a higher gross revenue of the retrofitted plant compared to reference value the optimal investment threshold decreases whilst for lower values of the gross revenue of the retrofitted plant the optimal investment threshold increases, in agreement with our comparative statics (2.16).

In Figure 3 we can clearly see that the more CO_2 is emitted (i.e. the less that is captured) the higher the investment threshold, again in agreement with (2.17). Note that we have ignored the fact that the investment costs for a retrofit capturing less than 90% of the carbon (the reference value $q_n = q_o/10$) would be much less and therefore would push the investment threshold downwards.

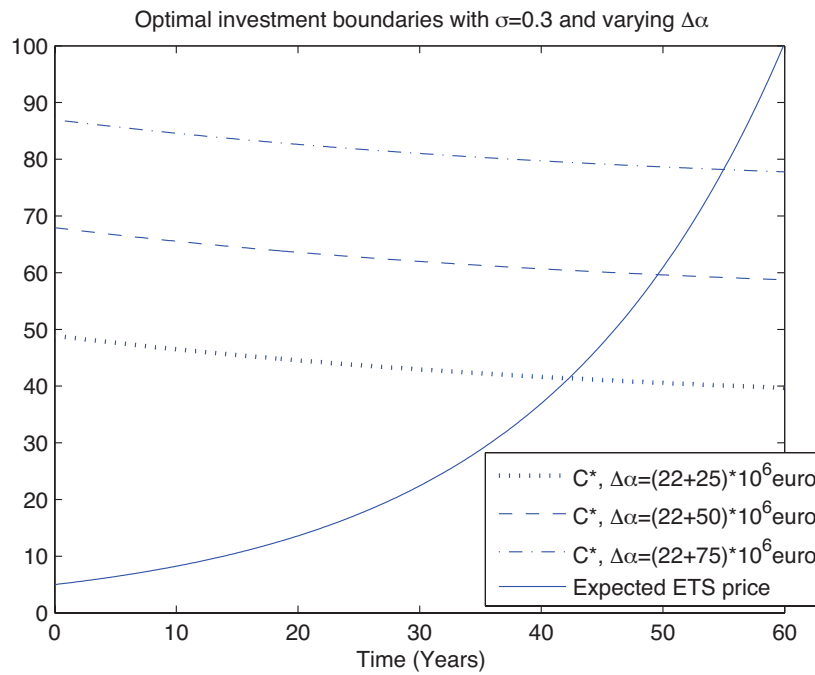


Figure 2: The intersection of the expected ETS price with the critical boundary curve C^* for three values of $\Delta\alpha$.

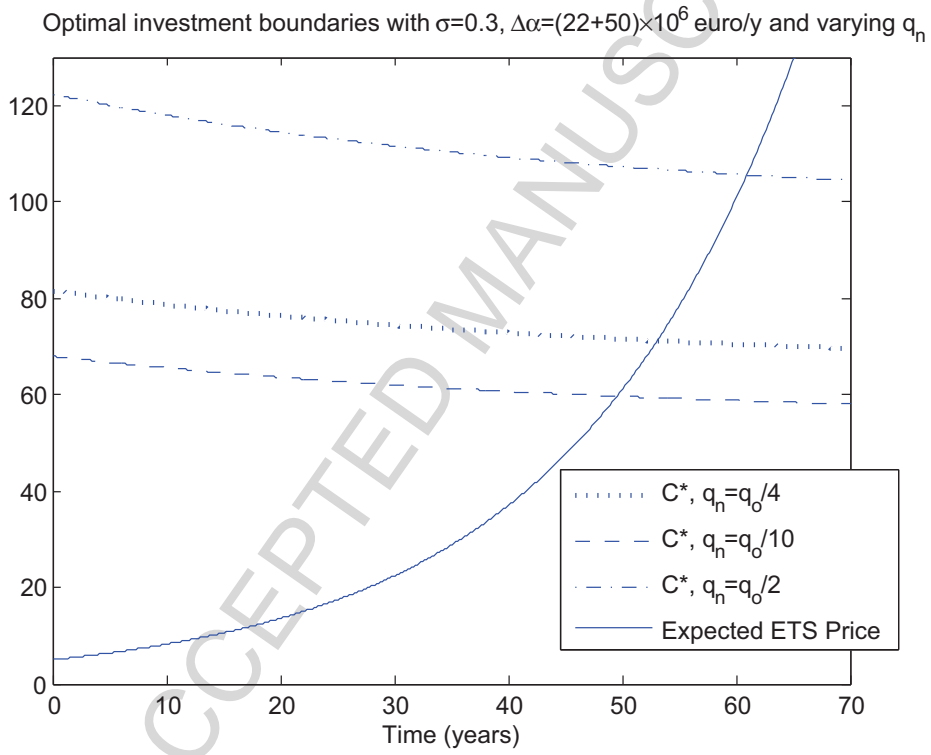


Figure 3: The intersection of the expected ETS price with the critical boundary curve C^* for three values of Δq .

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