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Authors(s)	Sugathapala, Inosha, Hanif, Muhammad Fainan, Lorenzo, Beatriz, Glisic, Savo, Juntti, Markku, Tran, Le-Nam
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Topology Adaptive Sum Rate Maximization in the Downlink of Dynamic Wireless Networks

Inosha Sugathapala, *Student Member, IEEE*, Muhammad Fainan Hanif, Beatriz Lorenzo, *Member, IEEE*, Savo Glisic, *Senior Member, IEEE*, Markku Juntti, *Senior Member, IEEE* and Le-Nam Tran, *Member, IEEE*

Abstract—Dynamic network architectures (DNAs) have been developed under the assumption that some terminals can be converted into temporary access points (APs) anytime when connected to the Internet. In this paper, we consider the problem of assigning a group of users to a set of potential APs with the aim to maximize the downlink system throughput of DNA networks, subject to total transmit power and users' quality of service (QoS) constraints. In our first method, we relax the integer optimization variables to be continuous. The resulting non-convex continuous optimization problem is solved using successive convex approximation framework to arrive at a sequence of second-order cone programs (SOCPs). In the next method, the selection process is viewed as finding a sparsity constrained solution to our problem of sum rate maximization. It is demonstrated in numerical results that while the first approach has better data rates for dense networks, the sparsity oriented method has a superior speed of convergence. Moreover, for the scenarios considered, in addition to comprehensively outperforming some well-known approaches, our algorithms yield data rates close to those obtained by branch and bound method.

Index Terms—DNA networks, user association, SOCP, throughput maximization, beamforming, convex optimization, branch and bound algorithm, exhaustive search.

I. INTRODUCTION

Successful development of wireless communication networks greatly depends on efficient utilization of the available resources. In dynamic network architectures (DNAs) some users share their connectivity and act as access points (APs) to serve other users in their vicinity without additional network infrastructure cost [2], [3]. The introduction of high-processing capacity wireless devices such as smart phones offers such

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I. Sugathapala, S. Glisic, and M. Juntti are with the Centre for Wireless Communications, Faculty of Information Technology and Electrical Engineering, University of Oulu, Finland, Emails: {inosha.sugathapala, savo.glisic, markku.juntti}@oulu.fi.

B. Lorenzo was with the Centre for wireless Communications, Faculty of Information Technology and Electrical Engineering, University of Oulu, Finland. She is now with the University of Vigo, Spain, Email: borenzo@gti.uvigo.es.

M. F. Hanif was with the Centre for wireless Communication, Faculty of Information Technology and Electrical Engineering, University of Oulu, Finland. He has also been with the School of Computing and Communications, Lancaster University, United Kingdom. He is currently associated with the University of Lahore, Lahore, Pakistan, Email: mfh21@uclive.ac.nz.

L.-N. Tran is with the School of Electrical and Electronic Engineering, University College Dublin, Ireland. Email: nam.tran@ucd.ie.

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an opportunity. This is entirely possible by plug and play reference architectures that are based on some well-known protocols such as universal plug and play (UPnP) and devices profile for web services (DPWS) [4]. These protocols can control and exchange information within the network through a central server. The large numbers of users and their dynamic availability make the DNA highly adaptive to traffic variations in the network. Thus, the DNA can support the variations of the network topology (who transmits to whom) to meet the traffic demands, the feature referred to as topology adaptiveness [2]. Besides, it is suitable for low cost ubiquitous Internet connectivity. Conventional small/pico cells have their own infrastructure with fixed dedicated access points. Their deployment normally ignores the dynamic traffic fluctuations that render a significant part of this infrastructure unutilized in space and time increasing the energy and infrastructure cost.

Therefore, optimizing the number of active APs in DNA is highly important to maintain the system performance and the quality of service (QoS). Wireless beamforming techniques have gradually matured to a level that they can be integrated into many wireless systems. When equipped with multiple antennas, the APs provide more degrees of freedom, which can be exploited to improve the spectral efficiency of the system through spatial reuse of the channels. To this end, we need to optimize the beamforming weights in accordance with a given design criterion. A joint design of user-AP connection and beamformers results in the best solution.

A. Related works

Practical system design requires careful consideration of numerous performance metrics. Tan *et al.* [5] highlight three interconnected important metrics in multiple input multiple output (MIMO) networks. Therein, throughput is one of the key performance measures and will be studied in this paper as well. Prior works in [6]–[12] present different methods to maximize throughput in various network models. Shi *et al.* [6] propose an iterative algorithm to maximize the total sum rate of MIMO networks by means of weighted mean square error. Kaleva *et al.* in [7] and Choi *et al.* in [8] propose algorithms based on successive convex approximation (SCA) method and zero gradient method, respectively. Further, Wang *et al.* [9] apply zero forcing technique for throughput maximization in MISO networks while Li *et al.* [10] propose a dual link algorithm for MIMO networks. In particular, Zheng *et al.* in [11] and Lai *et al.* in [12] establish a duality through a virtual dual single-input multiple-output (SIMO)

network which is then used to develop algorithms to maximize the sum rates in cognitive networks. These algorithms are numerically shown to achieve throughput close to that attained by a globally optimal solution. We remark, however, that all these algorithms are dedicated to a fixed user-AP association.

Funabiki *et al.* [13] suggest that maximizing the number of active APs in the network may not always result in higher QoS or network throughput unless proper user-AP selection is performed. Thus, for our DNA scenario where the numbers of users and available APs can change in time and space, the need of an user-AP selection mechanism in order to ensure the users' QoS requirements is more relevant. The authors in [14]–[16] claim that received signal strength indicator (RSSI) is unable to provide higher performance in IEEE 802.11 networks due to several reasons, even though it is the most common criterion to select the best AP. Perkins and Velaga [14] propose a method to address the drawbacks of this scheme. It is based on frame error rate and airtime cost for various packet size categories and the new potential available bandwidth after client association is completed, which relies on the predicted traffic load. Further, Athanasiou *et al.* [16] present channel-quality-based association mechanism to attain higher system throughput with smaller transmission delay. A preliminary investigation of the basics of access point problem from different perspectives (game theoretic, etc.) relying on our initial work [1] is given in [17, Ch. 12]. In [18], a method is given to maximize throughput through a joint user-AP association algorithm, which achieves higher data rates than the existing works. For a similar system model, a simple method based on path loss was proposed in [19]. In [20], a throughput model based on the network condition sensed by the AP is proposed. In addition to the system-wide data rates considered in the present paper, other metrics for user-AP selection have also been studied in the literature, such as the potential bandwidth a user can obtain [21], or the packet error rate with respect to the number of associated users [22].

For cellular wireless networks, the problem of joint optimization of power and AP selection has been investigated in several pioneering works. For example, the Hanly [23] as well as Yates and Huang [24] consider the problem of joint user-base station association and power optimization for uplink transmission. Hanley [23] assumes static users that select the best AP to optimize their powers. A hybrid non-cooperative game model for CDMA systems to optimize the power and connection such that the required signal to interference plus noise ratio (SINR) is guaranteed is studied in [24]. In [25], the problem of joint AP selection and power allocation for a multi-carrier wireless network with multiple APs and mobile users is investigated. Further, Nguyen *et al.* [26] propose non-convex quadratically constrained quadratic programming based algorithm to optimize power and the user association for coordinated multi-point transmission or reception system. Shen and Yu [27] present pricing based user association algorithm for SISO by considering dual domain optimization. Li *et al.* [28] propose a queue aware algorithm for cloud radio access networks. Particularly, Hong *et al.* [25] also highlight some problems regarding the use of non-cooperative game theoretical approaches to solve resource allocation problems

in wireless networks. Different from these earlier studies, we propose algorithms to address the joint AP selection and beamformer design problem using convex approximations.

B. Contributions

We consider the downlink transmission in DNA networks where multiple potential APs (e.g., laptops, tablets), each equipped with multiple antennas send data to a set of single-antenna users. Furthermore, we also assume that a user can receive data from only one AP. The problem of interest is to maximize the system throughput by jointly designing the beamformers and choosing properly an AP for each user. We assume that the AP is connected to the backbone such that the data for the single-antenna user is available via a backhaul link. In the sequel, this problem is referred to as joint throughput maximization and user association problem (JTMUA). In a broader context, our contributions include, but are not limited to, the following.

- We first formulate the JTMUA problem by introducing binary selection variables. Even for a fixed user-AP connection setup, the resulting problem becomes a throughput maximization which is known to be NP-hard [29]. To approximately solve this problem, a standard continuous relaxation method is used where the selection variables are relaxed to be continuous. Even with this relaxation, the resulting problem is still non-convex. Thus, we resort to the concave-convex procedure (CCP) to tackle this non-convex relaxed problem. To make this possible, we express the involving non-convex constraints as a difference of two convex functions. In fact, the CCP can be seen as a special case of the SCA framework [30] (also known as majorization-minimization (MM)), where a locally tight approximation of the non-convex optimization problem is solved at each iteration [31]. Our contribution in this regard is to develop a second-order cone program (SOCP) in each iteration of the proposed iterative procedure. In our numerical experiments, the relaxed selection variables are found to converge to nearly binary values.
- By viewing the selection problem as searching for sparse matrices corresponding to individual users, we study the JTMUA problem from another perspective. Instead of introducing binary selection variables, sparsity constraints are imposed in the beamformer design problem. This method is motivated by the increasing interest in compressed sensing applications in wireless communications. For example, the idea was considered in [32]–[34] as an antenna subset selection problem. In our sparsity based approach, the connection between users and APs is selected based on the non-zero beamforming vectors generated by our algorithm.
- The SCA framework used in both approaches is critically dependent on the initialization of iterative algorithms developed using this strategy. We have proposed an efficient initialization technique that formalizes this procedure and is independent of heuristic approaches.
- Our mathematical framework is flexible enough to incorporate the admission control (AC) problem. A sparsity

constrained version of the AC problem is formulated and it is shown that the proposed SCA approach can be utilized to solve the AC problem as well.

- In addition, complexity estimates of our algorithms are presented, including conditions for convergence. Last but not least, the proposed procedures are numerically tested from several different aspects in the numerical results. For instance, the impact of the channel estimation errors is quantified, comparison with exhaustive search or branch and bound algorithms is performed, etc.

The rest of the paper is organized as follows. The network model and the problem formulation are illustrated in Section II. Section III discusses the proposed approaches and the complexity and convergence properties of the algorithms. The numerical results are shown in Section IV. Finally, Section V concludes the paper.

Notations: Boldface lower and upper case letters are used to denote vectors and matrices, respectively. $\text{Re}(\mathbf{x})$ and $\text{Im}(\mathbf{x})$ represent the real and imaginary parts of a complex vector \mathbf{x} , respectively. $\mathbb{R}^{m \times n}$ and $\mathbb{C}^{m \times n}$ represent the space of real and complex matrices of dimension given in superscript, respectively; \mathbf{X}^T and \mathbf{X}^H are the transpose and Hermitian transpose of \mathbf{X} , respectively; $\mathbf{1}_{m \times n}$ denotes the $m \times n$ matrix of all 1s. The absolute value of a scalar y is defined by $|y|$, and $\|\mathbf{y}\|_2$ represents the Euclidean norm of a vector \mathbf{y} . For two vectors \mathbf{x} and \mathbf{y} of the same size, their inner product is denoted by $\langle \mathbf{x}, \mathbf{y} \rangle$, i.e., $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{y}$; $[\mathbf{x}]_i$ denotes the i th element of a vector \mathbf{x} and $(\mathbf{X})_i$ represents the i th row of a matrix \mathbf{X} . For sets \mathcal{A} and \mathcal{B} , the relative complement of \mathcal{A} in \mathcal{B} is denoted $\mathcal{B} \setminus \mathcal{A}$.

II. NETWORK MODEL AND PROBLEM FORMULATION

A. Network Model

At a given time instant we consider a network consisting of K potential access points, each equipped with T transmit antennas, and N single-antenna users. It is assumed that each user is served by one AP and data is not shared among APs, as in coordinated beamforming mode in LTE networks. The APs are equipped with linear precoding capability and the transmission is using beamforming. The APs communicate with the users in a single hop mode, as illustrated in Fig. 1 for a simple network model. To minimize inter-user interference and maximize its own data rates, a user has to select the most appropriate AP among all possible connections.

B. Problem Formulation

Let $\mathcal{K} = \{1, 2, \dots, K\}$ and $\mathcal{N} = \{1, 2, \dots, N\}$ denote the sets of APs and users, respectively. The channel between AP $i \in \mathcal{K}$ and user $j \in \mathcal{N}$ is denoted by a (row) complex vector $\mathbf{h}_{ij} \in \mathbb{C}^{1 \times T}$. The transmitted data symbol for user j is linearly weighted by a column vector $\mathbf{w}_{ij} \in \mathbb{C}^{T \times 1}$ before being transmitted from AP i . For ease of description we define $\mathbf{w} \triangleq [\mathbf{w}_{11}^T, \dots, \mathbf{w}_{NK}^T]^T \in \mathbb{C}^{(NK) \times 1}$ which is the vector including all the beamformers. To simplify the considered problem we assume that the received signal at an arbitrary user undergoes frequency-flat fading and log normal shadowing. It

is worth mentioning that although we focus on the downlink scenario, the proposed methods also apply to the uplink case. At a given time instance, each transceiver acts either as a user or as an AP, and, thus, there is no self-interference induced in the considered network. In our model, if selected, all K APs are ready for transmission. The complexity of the system under consideration increases with K and N . Therefore, in order to make the optimization process feasible in the network with high traffic dynamics, network clustering may be introduced to reduce the size of the network under consideration [35]–[37]. Cluster is different from a small cell. Specifically, a cluster consists of multiple potential access points that may enter or leave the systems in a dynamic way. Small clusters will have relatively moderate values of parameters K and N , as shown in Fig. 2. A central server controls the communication within each cluster. The proposed algorithms in this paper are centralized and thus require full channel state information (CSI) of the network. Basically for a full duplex, CSI is estimated at the receiver and then fed back to an AP by a dedicated control channel. In time division half duplex system, the AP can estimate the CSI from uplink pilots using the so-called channel reciprocity. All the APs then can transmit CSI to the server where central processing is carried out using little overheads defines in UPnP or DPWS protocols. Further, users in adjacent clusters will transmit on a different frequencies, so there will be no interference between their transmissions. Therefore, we will focus on a single-cluster scenario and the overall system performance can be estimated by aggregating the performance of each cluster.

Let s_{ij} be a binary selection variable that represents the connection status between AP i and user j . That is

$$s_{ij} = \begin{cases} 1 & \text{if user } j \text{ is served by AP } i, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Since each user must be served only by one AP, we have the following constraint

$$\sum_{i \in \mathcal{K}} s_{ij} = 1, \quad \forall j \in \mathcal{N}. \quad (2)$$

For notational simplicity, we define $\mathbf{s}_j = [s_{1j}, s_{2j}, \dots, s_{Kj}]^T \in \{0, 1\}^K$ to be the vector consisting of K binary selection variables associated with user j , and $\mathbf{s} = [s_1^T, s_2^T, \dots, s_N^T]^T$ to be the vector comprising all KN binary selection variables introduced in (1). With the above notations, the data rate of the user j can be written as

$$R_j = \log(1 + \gamma_j(\mathbf{w}, \mathbf{s})) \quad (3)$$

where

$$\gamma_j(\mathbf{w}, \mathbf{s}) = \frac{\sum_{i \in \mathcal{K}} s_{ij} |\mathbf{h}_{ij} \mathbf{w}_{ij}|^2}{\sigma_j^2 + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{N} \setminus \{j\}} s_{kt} |\mathbf{h}_{kj} \mathbf{w}_{kt}|^2} \quad (4)$$

and σ_j^2 is the variance of the additive white Gaussian background noise (AWGN). In order to better understand the expression in (4), we show that it indeed reduces to the actual expression of data rate of user j for a given set of binary variables s_{ij} satisfying (1) and (2). Suppose AP m is selected for user j , i.e., $s_{mj} = 1$ and $s_{ij} = 0$ for $i \neq m$. Then the summation in the numerator of (4) is reduced

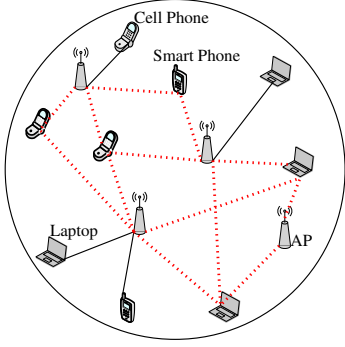


Fig. 1. An example scenario of our system model. The red dotted lines indicate potential connections between a user and APs and black solid lines refer to the selected connection between a user and an AP.

to $|\mathbf{h}_{m_j} \mathbf{w}_{m_j}|^2$, which is the desired signal part. Next, let $J_k = \{t | s_{kt} = 1 \ \& \ t \in \mathcal{N}\}$, i.e., J_k refers to the set of users served by AP k . Then (3) reduces to

$$R_j = \log \left(1 + \frac{|\mathbf{h}_{m_j} \mathbf{w}_{m_j}|^2}{\sigma_j^2 + \sum_{l \in J_m \setminus j} |\mathbf{h}_{m_j} \mathbf{w}_{m_l}|^2 + \sum_{k \in \mathcal{K} \setminus \{m\}} \sum_{l \in J_k \setminus j} |\mathbf{h}_{k_j} \mathbf{w}_{k_l}|^2} \right). \quad (5)$$

We can see that the denominator encompasses all interference caused by other users to user j . In the context of cellular networks, the first and second summations in the denominator are called intra- and inter-cell interference, respectively. Note that (5) is in fact the actual rate of user j for a given set of selection variables s_{ij} .

The power budget of each AP is subject to the following constraint

$$\sum_{j \in \mathcal{N}} s_{ij} \|\mathbf{w}_{ij}\|_2^2 \leq p_i^{\max}, \quad \forall i \in \mathcal{K} \quad (6)$$

where p_i^{\max} is the maximum power of AP i . Moreover, we have assumed that the transmitted symbols are temporally white, and hence, the power constraint is reflected via the beamforming vectors. We can now state the JTMUA problem as

$$(\mathcal{P}) \triangleq \begin{cases} \max_{\mathbf{s}, \mathbf{w}} & \sum_{j \in \mathcal{N}} \log(1 + \gamma_j(\mathbf{w}, \mathbf{s})) & (7a) \\ \text{s.t.} & \gamma_j^{\min} \leq \gamma_j(\mathbf{w}, \mathbf{s}), \quad \forall j \in \mathcal{N} & (7b) \\ & \sum_{j \in \mathcal{N}} s_{ij} \|\mathbf{w}_{ij}\|_2^2 \leq p_i^{\max}, \quad \forall i \in \mathcal{K} & (7c) \\ & \mathbf{1}_{1 \times K} \mathbf{s}_j = 1, \quad \forall j \in \mathcal{N} & (7d) \\ & \mathbf{s} \in \{0, 1\}^{KN}. & (7e) \end{cases}$$

In (7b), we impose the constraint SINR that is larger than or equal to a threshold γ_j^{\min} in order to maintain some degree of fairness among users. The problem (\mathcal{P}) is an instance of mixed-integer nonlinear program. Generally, this problem is difficult to solve even for a small network. More explicitly, problem (\mathcal{P}) is computationally intractable, even for fixed selection variables, the problem is known to be NP-hard [38]. However, under some special conditions such as quasi-invertibility, the above paper proved that the throughput maximization problem can be convexified and thus, polynomial time optimal algorithm is possible [12]. Before proceeding further we provide a simple result that is used to establish the

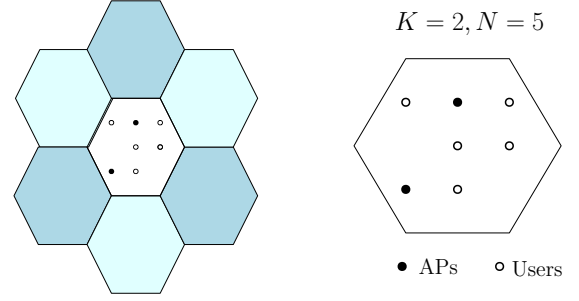


Fig. 2. Illustration of a highly dense network (left) possibly partitioned into different clusters of smaller sizes and distinct frequencies (right) for which the proposed algorithms can handle real-time traffic. The figure demonstrates a frequency reuse factor of three for network partitioning but a higher frequency reuse factor can be applied if the interference situation is severe.

convergence of the iterative algorithms proposed in the next section.

Proposition 1: The optimal objective of (\mathcal{P}) is bounded from above by $\sum_{j \in \mathcal{N}} \log(1 + \bar{\gamma}_j)$, where $\bar{\gamma}_j = \sum_{i \in \mathcal{K}} \|\mathbf{h}_{ij}\|_2^2 p_i^{\max} / \sigma_j^2$.

Proof: The proof is simple. By ignoring the interference term in (4) we have $\gamma_j(\mathbf{w}, \mathbf{s}) \leq \sum_{i \in \mathcal{K}} s_{ij} |\mathbf{h}_{ij} \mathbf{w}_{ij}|^2 / \sigma_j^2$. Next, we can write $s_{ij} |\mathbf{h}_{ij} \mathbf{w}_{ij}|^2 = |\mathbf{h}_{ij} (\sqrt{s_{ij}} \mathbf{w}_{ij})|^2$ and apply the Cauchy-Schwarz inequality to obtain $\sum_{i \in \mathcal{K}} s_{ij} |\mathbf{h}_{ij} \mathbf{w}_{ij}|^2 \leq \sum_{i \in \mathcal{K}} \|\mathbf{h}_{ij}^T\|_2^2 s_{ij} \|\mathbf{w}_{ij}\|_2^2$. The proof immediately follows by noting that $s_{ij} \|\mathbf{w}_{ij}\|_2^2 \leq p_i^{\max}$ for all $i \in \mathcal{K}$ due to (7c). ■

III. PROPOSED SOLUTIONS TO (\mathcal{P})

A. Globally Optimal Solution by Exhaustive Search

Despite its computational challenge, (\mathcal{P}) can be solved to global optimality thanks to its specific structure. Specifically, all possibilities of selection variables s_{ij} 's (which are K^N in number) that satisfy (7d) and (7e) are considered. For each case, the resulting problem becomes the sum rate maximization for which global optimization methods presented in [39], [40] can be used to find an optimal solution. Then the users-APs assignment combination, which produces the best sum rate, is retained as the global solution. Thus, \mathcal{P} is solved globally. We remark that these global optimization methods are based on the branch and bound technique, and have non-polynomial time complexity. Obviously, this exhaustive search method requires prohibitively high computational complexity and it is only used for benchmarking purpose in this paper. For more practically appealing applications such as those that occur in dynamic network architectures, we propose two low-complexity solutions in the following.

B. Continuous Relaxation

To tackle the discrete part of the JTMUA problem we first consider a continuous relaxation of the AP selection variables. Relaxing the discrete constraints to lie in continuous sets is a standard simplification in combinatorial optimization theory. For our case, it can be considered as a first step towards

approximating the NP-hard optimization problem. To this end, we consider the following continuous relaxation of (\mathcal{P})

$$(\tilde{\mathcal{P}}) \triangleq \begin{cases} \max_{\mathbf{s}, \mathbf{w}, \mathbf{v}} & \sum_{j=1}^N \log v_j & (8a) \\ \text{s.t.} & v_j \leq 1 + \gamma_j(\mathbf{w}, \mathbf{s}), \forall j \in \mathcal{N} & (8b) \\ & \gamma_j^{\min} \leq \gamma_j(\mathbf{w}, \mathbf{s}), \forall j \in \mathcal{N} & (8c) \\ & \sum_{j \in \mathcal{N}} s_{ij} \|\mathbf{w}_{ij}\|_2^2 \leq p_i^{\max}, \forall i \in \mathcal{K} & (8d) \\ & \mathbf{1}_{1 \times K} \mathbf{s}_j = 1, \forall j \in \mathcal{N} & (8e) \\ & 0 \leq \mathbf{s} \leq 1 & (8f) \end{cases}$$

where $\mathbf{v} \in \mathbb{R}^{N \times 1}$ is a newly introduced optimization variable. To arrive at $(\tilde{\mathcal{P}})$ two steps have been carried out. First, $\mathbf{v} \in \mathbb{R}^{N \times 1}$ is introduced in (8a) and (8b), which does not affect the optimality. The reason is that, (8b) holds with equality for optimal solutions; otherwise we can always increase v_j without violating the constraint and obtain a strictly higher objective. A similar manipulation was also done in [41], [42]. Second, simply relaxing \mathbf{s} to be continuous as in (8f) results in $(\tilde{\mathcal{P}})$.

It is obvious that the difficulty in solving $(\tilde{\mathcal{P}})$ is due to the nonconvex constraints in (8b), (8c) and (8d). In light of the SCA method, convex approximations of these constraints are required. Towards this end, recall that every twice differentiable function can be written as a difference of (two) convex functions on any compact convex set [43, Proposition 3.2]. Proceeding further, note that (8b) and (8c) can be replaced by the system of the following three constraints without loss of optimality

$$z_j \geq \sigma_j^2 + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{N} \setminus \{j\}} s_{kt} |\mathbf{h}_{kj} \mathbf{w}_{kt}|^2, \forall j \in \mathcal{N} \quad (9a)$$

$$(v_i - 1)z_j \leq \sum_{i \in \mathcal{K}} s_{ij} |\mathbf{h}_{ij} \mathbf{w}_{ij}|^2, \forall j \in \mathcal{N} \quad (9b)$$

$$\gamma_j^{\min} z_j \leq \sum_{i \in \mathcal{K}} s_{ij} |\mathbf{h}_{ij} \mathbf{w}_{ij}|^2, \forall j \in \mathcal{N} \quad (9c)$$

where $\mathbf{z} \in \mathbb{R}^{N \times 1}$ is a newly introduced variable. The equivalence can be easily proven from the definition of $\gamma_j(\mathbf{w}, \mathbf{s})$ in (4). Specifically, if the above constraints are met, then so are (8b) and (8c). Conversely, if (8b) and (8c) hold then we can simply set $z_j = \sigma_j^2 + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{N} \setminus \{j\}} s_{kt} |\mathbf{h}_{kj} \mathbf{w}_{kt}|^2$, and the constraints in (9) are immediately satisfied. This justifies the equivalence of the constraints in (8b) and (8c), and the system in (9).

Let us deal with (9a) first. To do this, an auxiliary vector $\boldsymbol{\beta}_{jt} \in \mathbb{R}^{K \times 1}$ is introduced for each pair of users $j \neq t$, and equivalently (9a) expands to the following system

$$z_j \geq \sigma_j^2 + \sum_{t \in \mathcal{N} \setminus \{j\}} \mathbf{s}_t^T \boldsymbol{\beta}_{jt}, \forall j \in \mathcal{N} \quad (10a)$$

$$|\boldsymbol{\beta}_{jt}|_k \geq |\mathbf{h}_{kj} \mathbf{w}_{kt}|^2, \quad \forall t \in \mathcal{N} \setminus \{j\}, \forall k \in \mathcal{K}. \quad (10b)$$

The above maneuver is similar to the move executed to transform (8b) and (8c) to (9). In the same way, (9b) and (9c) can be equivalently rewritten as the set of two inequalities given below

$$(v_j - 1)z_j \leq \mathbf{s}_j^T \mathbf{t}_j, \quad \forall j \in \mathcal{N} \quad (11a)$$

$$|\mathbf{t}_j|_i \leq |\mathbf{h}_{ij} \mathbf{w}_{ij}|^2, \quad \forall j \in \mathcal{N}, \forall i \in \mathcal{K} \quad (11b)$$

$$\gamma_j^{\min} z_j \leq \mathbf{s}_j^T \mathbf{t}_j, \quad \forall j \in \mathcal{N} \quad (11c)$$

where $\mathbf{t}_j \in \mathbb{R}^{K \times 1}$, $j \in \mathcal{N}$, is an auxiliary variable. Therefore, (8b) and (8c) are equivalently represented with (10) and (11). Similarly, we can equivalently rewrite (8d) as

$$\|\mathbf{w}_{ij}\|_2^2 \leq u_{ij}, \quad \forall j \in \mathcal{N}, \forall i \in \mathcal{K} \quad (12a)$$

$$\tilde{\mathbf{s}}_i^T \mathbf{u}_i \leq p_i^{\max}, \quad \forall i \in \mathcal{K} \quad (12b)$$

where $\mathbf{u}_i \in \mathbb{R}^{N \times 1}$, $i \in \mathcal{K}$ and $\mathbf{u} \triangleq [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_K] \in \mathbb{R}^{KN \times 1}$ are new optimization variables and $\tilde{\mathbf{s}}_i \triangleq [s_{i1}, s_{i2}, \dots, s_{iN}]^T$. Note that $\tilde{\mathbf{s}}_i$ in (12b) is not a newly introduced variable, it denotes the vector of selection variables associated with AP i .

Before proceeding further, we remark that (10), (11) and (12) are the equivalent reformulation of non-convex constraints (8b)-(8d) in $(\tilde{\mathcal{P}})$. Note that (10b) and (12a) are convex and SOC representable. Clearly, the non-convexity in the remaining constraints from (10) to (12) is due to the inner product of the two involved variables. To apply the CCP, we use the well-known equality $4\langle \mathbf{x}, \mathbf{y} \rangle = \|\mathbf{x} + \mathbf{y}\|_2^2 - \|\mathbf{x} - \mathbf{y}\|_2^2$ to write the inner product as difference of two quadratic functions. In this regard we can *equivalently* rearrange all the non-convex constraints (10a), (11) and (12b) as

$$\sum_{t \in \mathcal{N} \setminus \{j\}} \|\mathbf{s}_t + \boldsymbol{\beta}_{jt}\|_2^2 - \sum_{t \in \mathcal{N} \setminus \{j\}} \|\mathbf{s}_t - \boldsymbol{\beta}_{jt}\|_2^2 \leq 4(z_j - \sigma_j^2), \quad (13a)$$

$$(z_j + v_j - 1)^2 - (z_j - v_j + 1)^2 - \|\mathbf{s}_j + \mathbf{t}_j\|_2^2 + \|\mathbf{s}_j - \mathbf{t}_j\|_2^2 \leq 0, \quad (13b)$$

$$|\mathbf{t}_j|_i - \text{Re}(\mathbf{h}_{ij} \mathbf{w}_{ij})^2 - \text{Im}(\mathbf{h}_{ij} \mathbf{w}_{ij})^2 \leq 0, \quad (13c)$$

$$4\gamma_j^{\min} z_j - \|\mathbf{s}_j + \mathbf{t}_j\|_2^2 + \|\mathbf{s}_j - \mathbf{t}_j\|_2^2 \leq 0, \quad (13d)$$

$$\|\tilde{\mathbf{s}}_i + \mathbf{u}_i\|_2^2 - \|\tilde{\mathbf{s}}_i - \mathbf{u}_i\|_2^2 \leq 4p_i^{\max}, \quad (13e)$$

respectively. Now after these manipulations, the problem in (8) can be equivalently written as (14), where the optimization variables $\mathbf{s}, \mathbf{w}, \mathbf{v}, \boldsymbol{\beta}, \mathbf{t}$ and \mathbf{u} have already been defined in the mathematical developments presented above. It should be stressed at this point that (14) is still equivalent to (8), and hence, still non-convex. A systematic scheme of convex approximation of this problem follows below.

Obviously, the constraints in (13) are non-convex due to the concave term $-\|\cdot\|_2^2$. In the light of the CCP, this concave part is linearized to obtain a convex approximation [31]. For the description purpose, let us denote by $x^{(n)}$ the value of an optimization variable x after n iterations of the proposed iterative algorithm described below in **Algorithm 1**. In iteration $n+1$, by the first order Taylor series expansion, all the constraints in (13) are approximated respectively as (15).

The non-convex constraints (8b)-(8d) have been approximated by (10b), (12a) and (15). In summary, the convex problem in the $(n+1)$ th iteration of the first proposed algorithm is given by

$$(\tilde{\mathcal{P}}_{n+1}) \triangleq \begin{cases} \max_{\mathbf{s}, \mathbf{w}, \mathbf{u}, \mathbf{t}, \mathbf{z}, \mathbf{v}, \boldsymbol{\beta}} & \left(\prod_{j=1}^N v_j \right)^{\frac{1}{N}} & (16a) \\ \text{s.t.} & (10b), (12a), (15), & (16b) \\ & \mathbf{1}_{1 \times K} \mathbf{s}_j = 1, \forall j \in \mathcal{N}, & (16c) \\ & 0 \leq \mathbf{s} \leq 1. & (16d) \end{cases}$$

Note that to arrive at $(\tilde{\mathcal{P}}_{n+1})$, we have used the fact that maximizing $\sum_{j \in \mathcal{N}} \log v_j$ is equivalent to maximizing

$$\begin{aligned}
(\tilde{\mathcal{P}}) \triangleq \left\{ \begin{array}{l} \max_{\mathbf{s}, \mathbf{w}, \mathbf{v}, \beta, \mathbf{t}, \mathbf{u}} \quad \sum_{j=1}^N \log v_j \\ \text{s.t.} \quad \sum_{t \in \mathcal{N} \setminus \{j\}} \|\mathbf{s}_t + \beta_{jt}\|_2^2 - \sum_{t \in \mathcal{N} \setminus \{j\}} \|\mathbf{s}_t - \beta_{jt}\|_2^2 \leq 4(z_j - \sigma_j^2), \quad \forall j \in \mathcal{N} \\ [\beta_{jt}]_k \geq |\mathbf{h}_{kj} \mathbf{w}_{kt}|^2, \quad \forall t \in \mathcal{N} \setminus \{j\}, \forall k \in \mathcal{K} \\ (z_j + v_j - 1)^2 - (z_j - v_j + 1)^2 - \|\mathbf{s}_j + \mathbf{t}_j\|_2^2 + \|\mathbf{s}_j - \mathbf{t}_j\|_2^2 \leq 0, \quad \forall j \in \mathcal{N} \\ [\mathbf{t}_j]_i - \text{Re}(\mathbf{h}_{ij} \mathbf{w}_{ij})^2 - \text{Im}(\mathbf{h}_{ij} \mathbf{w}_{ij})^2 \leq 0, \quad \forall j \in \mathcal{N}, \forall i \in \mathcal{K} \\ 4\gamma_j^{\min} z_j - \|\mathbf{s}_j + \mathbf{t}_j\|_2^2 + \|\mathbf{s}_j - \mathbf{t}_j\|_2^2 \leq 0, \quad \forall j \in \mathcal{N} \\ \|\mathbf{w}_{ij}\|_2^2 \leq u_{ij}, \quad \forall j \in \mathcal{N}, \forall i \in \mathcal{K} \\ \|\tilde{\mathbf{s}}_i + \mathbf{u}_i\|_2^2 - \|\tilde{\mathbf{s}}_i - \mathbf{u}_i\|_2^2 \leq 4p_i^{\max}, \quad \forall i \in \mathcal{K} \\ \mathbf{1}_{1 \times K} \mathbf{s}_j = \mathbf{1}, \quad \forall j \in \mathcal{N} \\ 0 \leq \mathbf{s} \leq \mathbf{1}. \end{array} \right. \quad \begin{array}{l} (14a) \\ (14b) \\ (14c) \\ (14d) \\ (14e) \\ (14f) \\ (14g) \\ (14h) \\ (14i) \\ (14j) \end{array}
\end{aligned}$$

$$\sum_{t \in \mathcal{N} \setminus \{j\}} \|\mathbf{s}_t + \beta_{jt}\|_2^2 \leq \sum_{t \in \mathcal{N} \setminus \{j\}} \|\mathbf{s}_t^{(n)} - \beta_{jt}^{(n)}\|_2^2 + 2\langle \mathbf{s}_t^{(n)} - \beta_{jt}^{(n)}, \mathbf{s}_t - \mathbf{s}_t^{(n)} - \beta_{jt} + \beta_{jt}^{(n)} \rangle + 4(z_j - \sigma_j^2), \quad (15a)$$

$$(v_j - 1 + z_j)^2 + \|\mathbf{s}_j - \mathbf{t}_j\|_2^2 \leq (v_j^{(n)} - 1 - z_j^{(n)})^2 + 2\langle v_j^{(n)} - 1 - z_j^{(n)}, v_j - v_j^{(n)} - z_j + z_j^{(n)} \rangle + \|\mathbf{s}_j^{(n)} + \mathbf{t}_j^{(n)}\|_2^2 + 2\langle \mathbf{s}_j^{(n)} + \mathbf{t}_j^{(n)}, \mathbf{s}_j - \mathbf{s}_j^{(n)} + \mathbf{t}_j - \mathbf{t}_j^{(n)} \rangle, \quad (15b)$$

$$[\mathbf{t}_j]_i \leq |\mathbf{h}_{ij} \mathbf{w}_{ij}^{(n)}|^2 + 2\text{Re}(\mathbf{w}_{ij}^{(n)H} \mathbf{h}_{ij}^H \mathbf{h}_{ij} (\mathbf{w}_{ij} - \mathbf{w}_{ij}^{(n)})), \quad (15c)$$

$$4\gamma_j^{\min} z_j + \|\mathbf{s}_j - \mathbf{t}_j\|_2^2 \leq \|\mathbf{s}_j^{(n)} + \mathbf{t}_j^{(n)}\|_2^2 + 2\langle \mathbf{s}_j^{(n)} + \mathbf{t}_j^{(n)}, \mathbf{s}_j - \mathbf{s}_j^{(n)} + \mathbf{t}_j - \mathbf{t}_j^{(n)} \rangle, \quad (15d)$$

$$\|\tilde{\mathbf{s}}_i + \mathbf{u}_i\|_2^2 \leq \|\tilde{\mathbf{s}}_i^{(n)} - \mathbf{u}_i^{(n)}\|_2^2 + 2\langle \tilde{\mathbf{s}}_i^{(n)} - \mathbf{u}_i^{(n)}, \tilde{\mathbf{s}}_i - \tilde{\mathbf{s}}_i^{(n)} - \mathbf{u}_i + \mathbf{u}_i^{(n)} \rangle + 4p_i^{\max}. \quad (15e)$$

$(\prod_{j=1}^N v_j)^{\frac{1}{N}}$, which is concave in \mathbf{v} , and the concavity of the objective function follows due to the monotonicity of the $\log(\cdot)$ function. Moreover, the geometric mean of \mathbf{v} in (16a) admits an SOC presentation [44, Sect. 2.3.5] and thus $(\tilde{\mathcal{P}}_{n+1})$ can be cast as an SOCP. To conclude this section, the proposed algorithm for solving $(\tilde{\mathcal{P}})$ is outlined in **Algorithm 1**, which is also referred to as the CR-based algorithm in the sequel of the paper. After solving the **Algorithm 1**, a post-processing procedure is normally required since the relaxed variables can be non-binary. Interestingly, we find numerically that the CR-based algorithm produces nearly binary values at convergence. More precisely, there exists a group of relaxed variables with values very close to zero which makes the post-processing trivial. More numerical details are provided in Sect. IV. Note that to find the actual beamforming vectors we need to re-run **Algorithm 1** for the fixed selection vector \mathbf{s} obtained from the rounding process.

Remarks: We note that the CCP has a long history and has been successfully applied in a variety of scenarios (see [31], [45] and references therein). By linearizing the concave part, it is shown that the optimal solution at the current iteration is also feasible to the problem encountered in the next iteration. Thus, the objective is always improved until convergence. A more detailed convergence analysis is presented in Sect. III-E.

C. Group Sparsity (GS)-based Design

In the first method we introduced binary variables to represent the connection status of APs and users. In the second method we will solve the JTMUA problem by viewing it

as sparsity constrained optimization problem. This method is inspired from the theory of sparse representation of signals, which has received growing interest recently. To proceed, let us for the moment fix the selection variables vector $\mathbf{s} = \mathbf{1}_{K \times J}$, and thus the rate expression in (3) reduces to

$$R_j = \log(1 + \tilde{\gamma}_j(\mathbf{w})), \quad (18)$$

where

$$\tilde{\gamma}_j(\mathbf{w}) = \frac{\sum_{i \in \mathcal{K}} |\mathbf{h}_{ij} \mathbf{w}_{ij}|^2}{\sigma_j^2 + \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{N} \setminus \{j\}} |\mathbf{h}_{kj} \mathbf{w}_{kt}|^2}. \quad (19)$$

To make R_j a valid expression for the data rate of user j as in (5), in the numerator of the fraction in (19), all beamformers \mathbf{w}_{ij} 's related to user j must be made zero except the one associated with the selected AP. To clarify our point, let us define a matrix Ψ_j for user j as

$$\Psi_j = \begin{bmatrix} [\mathbf{w}_{1j}]_1 \cdots [\mathbf{w}_{1j}]_T \\ \vdots \\ [\mathbf{w}_{mj}]_1 \cdots [\mathbf{w}_{mj}]_T \\ \vdots \\ [\mathbf{w}_{ij}]_1 \cdots [\mathbf{w}_{ij}]_T \\ \vdots \\ [\mathbf{w}_{Kj}]_1 \cdots [\mathbf{w}_{Kj}]_T \end{bmatrix} \in \mathbb{C}^{K \times T} \quad (20)$$

where the i th row of Ψ_j corresponds to the set of beamformers associated with the i th AP.

Now, in order to select an appropriate AP to communicate with user j , all rows of Ψ_j should be nulled out except, say, the m th one (shown in gray color). In mathematical programming, it is equivalent to imposing some degree of sparsity on Ψ_j . More precisely, we need to impose group sparsity on Ψ_j , so

Algorithm 1 Continuous relaxation (CR) approach for solving (\mathcal{P})

Initialization:

- 1: Set $n = 0$ and generate $\mathbf{s}^{(0)} \in \{0, 1\}$ such that the constraints in (2) are satisfied. To increase the chance of obtaining a feasible solution, a reasonable way is to assign users to the APs with higher channel gains.
- 2: With the given $\mathbf{s}^{(0)}$, consider the following feasibility problem

$$\text{find } \mathbf{w} \quad (17a)$$

$$\text{s.t. } \gamma_j^{\min} \leq \gamma_j(\mathbf{w}, \mathbf{s}^{(0)}), \quad \forall j \in \mathcal{N} \quad (17b)$$

$$\sum_{j \in \mathcal{N}} s_{ij}^{(0)} \|\mathbf{w}_{ij}\|_2^2 \leq p_i^{\max}, \quad \forall i \in \mathcal{K}. \quad (17c)$$

Note that for a given $\mathbf{s}^{(0)}$, the constraints in the above feasibility problem are SOC representable.

- 3: If (17) is feasible, then use the obtained \mathbf{w} , along with $\mathbf{s}^{(0)}$, to calculate $\beta^{(0)}$, $\mathbf{t}^{(0)}$, $\mathbf{v}^{(0)}$, $\mathbf{z}^{(0)}$ and $\mathbf{u}^{(0)}$ by setting the inequalities in the constraints in which they appear to be equalities. If (17) is infeasible, regenerate $\mathbf{s}^{(0)}$ and repeat step 2 until a feasible point is achieved. More on the initialization aspect appears in Section III-D.

Main loop:

- 4: **repeat**
- 5: Solve $(\tilde{\mathcal{P}}_{n+1})$ to find an optimal solution and denote the optimal solution as $(\mathbf{w}^*, \beta^*, \mathbf{t}^*, \mathbf{v}^*, \mathbf{z}^*, \mathbf{u}^*, \mathbf{s}^*)$.
- 6: Update $(\mathbf{w}^{(n+1)}, \beta^{(n+1)}, \mathbf{t}^{(n+1)}, \mathbf{v}^{(n+1)}, \mathbf{z}^{(n+1)}, \mathbf{u}^{(n+1)}, \mathbf{s}^{(n+1)}) = (\mathbf{w}^*, \beta^*, \mathbf{t}^*, \mathbf{v}^*, \mathbf{z}^*, \mathbf{u}^*, \mathbf{s}^*)$
- 7: $n \rightarrow n + 1$
- 8: **until convergence**

that all rows of Ψ_j except one are encouraged to be zero as discussed in [46], [47]. This can be done by introducing a mixed l_1/l_q norm for matrix Ψ_j for $q > 1$. Specifically, the mixed l_1/l_q norm acts as the l_q norm for the rows of Ψ_j and the l_1 norm for the columns of Ψ_j . We note that if $q = 1$ then the mixed l_1/l_q norm is actually the sum of the absolute values of all entries of Ψ_j and, thus, no group sparsity is achieved. The values, $q = 2$ and $q = \infty$ are the most commonly used for group sparsity inducing norm. However, as discussed in [47], $q = \infty$ could lead to undesired solutions, such as components in a row having equal magnitude. Therefore, we consider $q = 2$ in this paper and, the mixed l_1/l_2 norm metric for Ψ_j is defined as

$$\|\Psi_j\|_{1,2} = \sum_i \|(\Psi_j)_i\|_2 = \sum_i (\sum_t \|\mathbf{w}_{ij}\|_2^2)^{1/2}. \quad (21)$$

In the second proposed approach, instead of (\mathcal{P}) , we consider the continuous optimization problem given by

$$(\hat{\mathcal{P}}) \triangleq \begin{cases} \max_{\mathbf{w}} & \sum_{j \in \mathcal{N}} \log(1 + \tilde{\gamma}_j(\mathbf{w})) - \rho \|\Psi_j\|_{1,2} & (22a) \\ \text{s.t.} & \gamma_j^{\min} \leq \tilde{\gamma}_j(\mathbf{w}), \quad \forall j \in \mathcal{N} & (22b) \\ & \sum_{j \in \mathcal{N}} \|\mathbf{w}_{ij}\|_2^2 \leq p_i^{\max}, \quad \forall i \in \mathcal{K} & (22c) \end{cases}$$

where ρ is a positive constant controlling the degree of sparsity of the solution. Problem $(\hat{\mathcal{P}})$ is still non-convex, but we can follow similar steps as those used in developing CR-based

algorithm to solve it. In what follows, the notations defined in the CR-based algorithm will be reused, unless otherwise mentioned. First, rewrite (22) as

$$\max_{\mathbf{w}, \mathbf{v}} \sum_{j \in \mathcal{N}} \log v_j - \rho \|\Psi_j\|_{1,2} \quad (23a)$$

$$\text{s.t. } v_j \leq 1 + \tilde{\gamma}_j(\mathbf{w}), \quad \forall j \in \mathcal{N} \quad (23b)$$

$$(22b), (22c) \quad (23c)$$

where $\mathbf{v} \in \mathbb{R}^{N \times 1}$. Note that (22c) is a convex constraint and (22b) and (23b) are similar to (8c) and (8b). Thus, (22b) and (23b) can be rewritten as in (9), but without including \mathbf{s} , i.e.,

$$z_j \geq \sigma_j^2 + \sum_{t \in \mathcal{N} \setminus \{j\}} |\mathbf{h}_j \mathbf{w}_t|^2, \quad \forall j \in \mathcal{N} \quad (24a)$$

$$(v_j - 1)z_j \leq |\mathbf{h}_j \mathbf{w}_j|^2, \quad \forall j \in \mathcal{N} \quad (24b)$$

$$\gamma_j^{\min} z_j \leq |\mathbf{h}_j \mathbf{w}_j|^2, \quad \forall j \in \mathcal{N} \quad (24c)$$

where we have defined $\mathbf{h}_j \triangleq [\mathbf{h}_{j,1}^T, \mathbf{h}_{j,2}^T, \dots, \mathbf{h}_{j,K}^T] \in \mathbb{C}^{1 \times (KT)}$ as the aggregated channel including the channels from all APs to user j , and $\mathbf{w}_j = [\mathbf{w}_{j,1}^T, \mathbf{w}_{j,2}^T, \dots, \mathbf{w}_{j,K}^T]^T \in \mathbb{C}^{KT \times 1}$ to be the vector stacking all the beamformers associated with user j . Observe that (24a) is convex and according to the SCA principle followed in (9), we can replace (24b) and (24c) as

$$(v_j - 1 + z_j)^2 \leq 2(v_j - v_j^{(n)} - z_j + z_j^{(n)})(v_j^{(n)} - 1 - z_j^{(n)}) + |\mathbf{h}_j \mathbf{w}_j^{(n)}|^2 + (v_j^{(n)} - 1 - z_j^{(n)})^2 + 2\text{Re}(\mathbf{w}_{ij}^{(n)H} \mathbf{h}_j^H \mathbf{h}_j (\mathbf{w}_j - \mathbf{w}_j^{(n)})) \quad (25)$$

and

$$\gamma_j^{\min} z_j \leq |\mathbf{h}_j \mathbf{w}_j^{(n)}|^2 + 2\text{Re}(\mathbf{w}_{ij}^{(n)H} \mathbf{h}_j^H \mathbf{h}_j (\mathbf{w}_j - \mathbf{w}_j^{(n)})), \quad (26)$$

respectively. Using the same arguments as in [32], we can show that there exists ρ' such that $\max_{\mathbf{w}, \mathbf{v}, \mathbf{z}} \{\sum_{j \in \mathcal{N}} \log v_j - \rho \|\Psi_j\|_{1,2} | (22c), (24a), (25), (26) \}$ is equivalent to $\max_{\mathbf{w}, \mathbf{v}, \mathbf{z}} \{(\prod_{j \in \mathcal{N}} v_j)^{1/N} - \rho' \|\Psi_j\|_{1,2} | (22c), (24a), (25), (26) \}$. In summary, the convex problem in iteration $n + 1$ of the second proposed iterative method is given by

$$(\hat{\mathcal{P}}_{n+1}) \triangleq \begin{cases} \max_{\mathbf{w}, \mathbf{v}, \mathbf{z}} & (\prod_{j \in \mathcal{N}} v_j)^{1/N} - \sum_{j=1}^N \rho' \|\Psi_j\|_{1,2} & (27a) \\ \text{s.t.} & (22c), (24a), (25), (26). & (27b) \end{cases}$$

The formulation in (27) is an SOCP. The group sparsity-based method is outlined in **Algorithm 2**, which is also referred to as the GS-based algorithm in the rest of the paper. In order to find an appropriate AP for a particular user j , a post-processing procedure, outlined in **Algorithm 2** above, is also required after the iterative process has converged. The problem of optimizing the number of active APs is not considered in this paper. Such a selection process will involve iteratively optimizing the set of active APs and their beamformers, which will be elaborated in future works. Before proceeding further we remark that if a user is allowed to associate with several APs, then a sparsity-based approach is more preferable. The reason is that a few rows of matrix Ψ can be nonzero and post-processing is trivial.

Algorithm 2 Group sparsity based approach for the JTMUA problem

Initialization:

- 1: Set $n = 0$ and find a feasible solution $(\mathbf{w}^{(0)}, \mathbf{v}^{(0)}, \mathbf{z}^{(0)})$ to $(\tilde{\mathcal{P}})$ using the steps described in the initialization stage of **Algorithm 1**. Alternatively, the method presented in Section III-D can be used for this purpose.

Main loop:

- 2: **repeat**
- 3: Solve $(\hat{\mathcal{P}}_{n+1})$ and denote an optimal solution as $(\mathbf{w}^*, \mathbf{v}^*, \mathbf{z}^*)$.
- 4: Update $(\mathbf{w}^{(n+1)}, \mathbf{v}^{(n+1)}, \mathbf{z}^{(n+1)}) = (\mathbf{w}^*, \mathbf{v}^*, \mathbf{z}^*)$ and $n \rightarrow n + 1$.
- 5: **until convergence**

Post-processing:

- 6: For each user $j \in \mathcal{N}$ calculate the l_2 norm of each row in Ψ_j^∞ , where Ψ_j^∞ denotes the value of Ψ_j at the convergence of the iterative process. Then the selected AP for user j is the one associated with the row having the largest l_2 norm. For numerical experiments considered in Section IV, it is seen that the l_2 norm of the selected row is higher than 0.01, while that of the others is nearly zero.

D. Feasible Initialization of Algorithms 1 and 2

In general, one of the challenges in applying SCA framework to solve a non-convex problem is to find a feasible point to start the iterative procedure. In some problems such as the one in [42], this can be done easily by scaling/rescaling operation. Unfortunately, such simple manipulations do not apply to the considered problem, and thus finding an initial point to start **Algorithms 1** and **2** is nontrivial. For this reason, a heuristic way to find a feasible point for the CR-based algorithm is presented here. Specifically, we first set the elements of $\mathbf{s}^{(0)}$ to be 0 or 1 based on a given user-AP association which is determined by path loss, i.e. users are assigned to the AP that has the smallest path loss. For this set of $\mathbf{s}^{(0)}$, the problem of finding a set of beamformers that satisfy (7b) and (7c) becomes a second order cone feasibility problem which can be solved easily by conic solvers. If a set of beamformers feasible to (7b) and (7c) are found, then we can start the CR-based algorithm, otherwise we need to regenerate $\mathbf{s}^{(0)}$ and repeat the process until a feasible point is found. Similar arguments also apply to the GS-based algorithm. In this subsection a more efficient method to find an initial point for the two proposed algorithms is described. Towards this end, let us write a generic convex formulation of the problem in iteration $k + 1$ of the proposed methods (i.e., (16) for **Algorithm 1** and (22) for **Algorithm 2**) in a compact form as

$$\max_{\mathbf{x}} f(\mathbf{x}) \quad (28a)$$

$$\text{s.t.} \quad g_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \dots, m \quad (28b)$$

$$\tilde{g}_p(\mathbf{x}; \mathbf{x}^{(k)}) \leq 0, \quad p = 1, 2, \dots, n \quad (28c)$$

where $f(\mathbf{x})$ is concave, $g_i(\mathbf{x})$ is convex and, $\tilde{g}_p(\mathbf{x}; \mathbf{x}^{(k)})$ is a convex approximation of a non-convex constraint around point $\mathbf{x}^{(k)}$. Note that $\mathbf{x}^{(k)}$ denotes the value of the optimization

variable in the SCA's iteration k . In general, it is difficult to generate $\mathbf{x}^{(0)}$ such that problem (28) is feasible. To overcome this issue, we use the method introduced in [48] and consider a regularized version of (28) given by

$$\max_{\mathbf{x}, \alpha \geq 0} f(\mathbf{x}) - \gamma \sum_{j=1}^{m+n} \alpha_j \quad (29a)$$

$$\text{s.t.} \quad g_i(\mathbf{x}) \leq \alpha_i, \quad i = 1, 2, \dots, m \quad (29b)$$

$$\tilde{g}_p(\mathbf{x}; \mathbf{x}^{(k)}) \leq \alpha_{m+p}, \quad p = 1, 2, \dots, n \quad (29c)$$

where $\gamma > 0$ is a constant and $\alpha \geq 0$ is a vector of $m+n$ newly introduced variables. The idea is to introduce an optimization variable for each constraint in problem (28) so that (29) is always feasible for any $\mathbf{x}^{(0)}$. This is true since we can find sufficiently large $\alpha \geq 0$ for a given $\mathbf{x}^{(0)}$. That is to say, an iterative algorithm based on SCA method applied to (29) can start from any $\mathbf{x}^{(0)}$. By the maximization of the objective in (29a), α will eventually decrease to zero after some iterations. Thus, (29) and (28) are then equivalent and the obtained value of \mathbf{x} is also feasible to (28), which can be used to initialize **Algorithms 1** and **2**.

We note that the parameter γ controls the rate at which the auxiliary variables α s converge to zero. If γ is chosen too large, the maximization enforces α to decrease quickly as feasibility is emphasized. In contrast, if γ is small, the maximization focuses on optimizing the original objective and, thus, α decreases slowly. Balancing between the optimality and the infeasibility plays an important role.

E. Complexity and Convergence Analysis

Complexity Analysis: As mentioned earlier, our original problem (7) is a NP-hard problem even with fixed APs. That is, the JTMUA problem has exponential complexity in the worst-case. In this section the complexity of the proposed SCA based algorithms is discussed. For complexity comparison purpose, the linear constraints in both algorithms are ignored, since, compared to the SOC constraints, they have minor contribution to the overall computational cost. It is difficult, if not impossible, to provide an analytical bound on the number of iterations for **Algorithms 1** and **2** to converge. Therefore, we present the complexity for solving $(\tilde{\mathcal{P}}_n)$ and $(\hat{\mathcal{P}}_n)$ instead. To this end, we recall a result presented in [44, Sect. 4.6.2], which states that the worst-case arithmetic complexity of solving an SOCP problem is $\mathcal{O}((m+1)^{1/2}n(n^2+m+\sum_{i=1}^m k_i^2))$, where n is the number of variables, m is the number of SOC constraints, and k_i is the dimension of the i th SOC constraint.¹ For $(\tilde{\mathcal{P}}_n)$ and $(\hat{\mathcal{P}}_n)$, it can be seen that $n \gg k_i, \forall i$, and thus we can simplify the complexity estimate to $\mathcal{O}((m+1)^{1/2}n(n^2+m))$. Applying this result to $(\tilde{\mathcal{P}}_n)$ and $(\hat{\mathcal{P}}_n)$, the complexity of the SOCP in **Algorithms 1** and **2** in each iteration is given by $\mathcal{O}((KN+K+N+1)^{1/2}(KN+KNT+N)((KN+KNT+N)^2+KN+N+K))$ and $\mathcal{O}((K+N+1)^{1/2}(KNT+N)((KNT+N)^2+K+N))$, respectively. Obviously, the complexity of the GS-based algorithm is lower than that of CR-based algorithm, since the GS-based algorithm has less variables and constraints compared to CR-based algorithm.

¹Here we skip the term $\mathcal{O}(\log(1/\epsilon))$ which depends on solution accuracy $\epsilon > 0$.

It is again emphasized that the overall complexity of each algorithm depends on the number of iterations SCA takes to converge, which is hard to predict analytically. In Sect. IV, it is numerically observed that the number of iterations that the CR-based algorithm takes to converge is larger than that of the GS-based algorithm.

Convergence Analysis: The convergence analysis of the two proposed algorithms is similar. Thus, a unified representation will be followed for this purpose in the sequel. Let us generally express the non-convex design problem seen in **Algorithm 1** and **2** as

$$\max_{\mathbf{x} \geq 0} f(\mathbf{x}) \quad (30a)$$

$$\text{s.t. } g_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \dots, m \quad (30b)$$

$$h_p(\mathbf{x}) \leq 0, \quad p = 1, 2, \dots, n. \quad (30c)$$

where $g_i(\mathbf{x})$ is convex and $h_p(\mathbf{x})$ is non-convex. Further the non-convex function $h_p(\mathbf{x})$ is expressed as a difference of two convex functions: $h_p(\mathbf{x}) = h_p^1(\mathbf{x}) - h_p^2(\mathbf{x})$. The convex approximation $\tilde{g}_p(\mathbf{x}; \mathbf{x}^{(k)})$ is found as

$$\tilde{g}_p(\mathbf{x}; \mathbf{x}^{(k)}) = h_p^1(\mathbf{x}) - h_p^2(\mathbf{x}^{(k)}) - \nabla h_p^2(\mathbf{x}^{(k)})(\mathbf{x} - \mathbf{x}^{(k)}) \quad (31)$$

where $\nabla(\cdot)$ denotes the gradient operator. Note that due to the linearization, we have $h_p(\mathbf{x}) \leq \tilde{g}_p(\mathbf{x}; \mathbf{x}^{(k)})$ for all \mathbf{x} .

Let \mathcal{X}_k , $\{\mathbf{x}^{(k)}\}$, and $\{f(\mathbf{x}^{(k)})\}$ represent the convex feasible set, the optimal solution, and the optimal objective value at iteration k of **Algorithm 1** or **2**, respectively. Further, let \mathcal{X} be the non-convex feasible set for the corresponding original problem. Then the following properties hold [49]

- $\mathcal{X}_k \subseteq \mathcal{X}$. This is easily seen because $h_p(\mathbf{x}) \leq \tilde{g}_p(\mathbf{x}; \mathbf{x}^{(k-1)}) \leq 0$, which implies that if \mathbf{x} is feasible to $(\tilde{\mathcal{P}}_k)$, it is also feasible to the original problem, i.e., $\mathbf{x} \in \mathcal{X}$.
- $\mathbf{x}^{(k)} \in \mathcal{X}_k \cap \mathcal{X}_{k+1}$. That $\mathbf{x}^{(k)} \in \mathcal{X}_k$ is obvious. At iteration $k+1$, the constraint becomes $\tilde{g}_p(\mathbf{x}; \mathbf{x}^{(k)}) \leq 0$. It is clear from (31) that when $\mathbf{x} = \mathbf{x}^{(k)}$, then $\tilde{g}_p(\mathbf{x}^{(k)}; \mathbf{x}^{(k)}) = h_p(\mathbf{x}^{(k)}) \leq 0$. This simply implies that $\mathbf{x}^{(k)}$ is feasible to $(\tilde{\mathcal{P}}_{k+1})$, i.e., $\mathbf{x}^{(k)} \in \mathcal{X}_{k+1}$.
- $f(\mathbf{x}^{(k+1)}) \geq f(\mathbf{x}^{(k)})$. This is in fact a direct consequence of the two above properties. Since $\mathbf{x}^{(k)}$ is a feasible solution of $(\tilde{\mathcal{P}}_{k+1})$, its objective function value $f(\mathbf{x}^{(k)})$ is no larger than the optimal value $f(\mathbf{x}^{(k+1)})$.

In summary, **Algorithm 1** or **2** generate a non-decreasing sequence objective $f(\mathbf{x}^{(k)})$. Further, the feasible set of the original problem is bounded and thus $f(\mathbf{x}^{(k)})$ is guaranteed to converge.

We remark that strict increase in $\{f(\mathbf{x}^{(k)})\}$ cannot be achieved in **Algorithm 1** or **2**. The reason is that $\{f(\mathbf{x})\}$ is not strictly convex with respect to \mathbf{x} . Consequently, the convergence of iterates $\{\mathbf{x}^{(k)}\}$ is not generally guaranteed. To achieve a stronger convergence result, consider a modified formulation of $(\tilde{\mathcal{P}}_n)$ or $(\tilde{\mathcal{P}}_n)$, in which the objective $f(\mathbf{x})$ is replaced by $f(\mathbf{x}) - \frac{1}{2c} \|\mathbf{x} - \mathbf{x}^{(k-1)}\|_2^2$, where c is a positive scalar parameter. The idea of adding a quadratic term is based on the proximal minimization algorithm [50, Sect 3.4.3]. With this modified formulation, a stronger result as stated in the following lemma, can be established.

Lemma 1: The sequence $\{\mathbf{x}^{(k)}\}$ generated by the modified version of **Algorithm 1** and **2** converges to a stationary solution of $(\tilde{\mathcal{P}})$ or $(\tilde{\mathcal{P}})$, respectively.

Proof: Let us show the convergence to the stationarity of $(\tilde{\mathcal{P}})$. Since $\mathbf{x}^{(k)}$ is the optimal solution to $\tilde{\mathcal{P}}_k$ and due to the proximal operation, it holds that

$$f(\mathbf{x}^{(k)}) - \frac{1}{2c} \|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_2^2 \geq f(\mathbf{x}) - \frac{1}{2c} \|\mathbf{x} - \mathbf{x}^{(k-1)}\|_2^2, \quad \forall \mathbf{x} \in \mathcal{X}_k. \quad (32)$$

We have shown that $\mathbf{x}^{(n-1)}$ is feasible to \mathcal{X}_n , and by setting $\mathbf{x} = \mathbf{x}^{(k-1)}$ in the above inequality we obtain

$$f(\mathbf{x}^{(k)}) - f(\mathbf{x}^{(k-1)}) \geq \frac{1}{2c} \|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_2^2. \quad (33)$$

The inequality in (33) implies that the objective sequence $\{f(\mathbf{x}^{(k)})\}$ strictly increases. Since $\{f(\mathbf{x}^{(k)})\}$ is bounded from above as proved earlier, (33) also implies that

$$\lim_{k \rightarrow \infty} \frac{1}{2c} \|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_2^2 \rightarrow 0. \quad (34)$$

Now the rest of the proof follows the same arguments as those in [49, Proposition 3.2], and thus is omitted here for brevity. ■

We note that the parameter c should not be too small, because otherwise the algorithm converges slowly. From (34), it is easily seen that Lemma 1 still holds if the value of c is adaptively changed with the iterative process, but is not allowed to become vanishingly small. In practice, we only need to consider adding the quadratic term if a strict increase in the objective value in the current iteration is not achieved.

F. Extension to the Admission Control Problem

In DNA networks, dynamic APs can come and leave unpredictably. With the reduction of degrees of freedom due to these leaving APs, serving all users in the system with their QoS demands is not always possible. In such cases some users have to be dropped which gives rise to the admission control problem. Nevertheless, it should be ensured that the sum rate with remaining users is not negatively affected. From a feasibility viewpoint, we can simply drop the user with the highest QoS requirement, and then the one with the second highest QoS requirement, and keep on doing so until the resulting problem (\mathcal{P}) is feasible. However, this trivial way is a channel-unaware scheme and thus, the sum rate of the remaining users may be very small. In this subsection a procedure to modify the above proposed algorithms to obtain a better sub-optimal solution is presented. First we introduce a binary optimization variable $a_j \in \{0, 1\}$ corresponding to user j to denote if the user should be dropped or not. For the AC problem, we change connectivity constraint for user j in (7d) to the following

$$\mathbf{1}_{1 \times K} \mathbf{s}_j = a_j. \quad (35)$$

That is, if $a_j = 1$, user j should be served by at least one AP, and if $a_j = 0$, user j is simply dropped. Now consider the following problem

$$(\mathcal{P}^{\text{AC}}) \triangleq \begin{cases} \max_{\mathbf{s}, \mathbf{w}, \mathbf{a}} \left(\prod_{j=1}^N (1 + \gamma_j(\mathbf{w}, \mathbf{s})) \right)^{1/N} - \zeta \sum_{j=1}^N a_j & (36a) \\ \text{s.t. } a_j \gamma_j^{\min} \leq \gamma_j(\mathbf{w}, \mathbf{s}), \forall j \in \mathcal{N} & (36b) \\ 0 \leq \mathbf{a} \leq 1 & (36c) \\ (7c), (7e), (35) & (36d) \end{cases}$$

where $\mathbf{a} \in \mathbb{R}^{N \times 1}$ are called the soft QoS requirements of the users, and ζ is a positive constant to strike a balance between sum rate maximization and problem feasibility. In (36c), \mathbf{a} has been relaxed to take values on the interval $[0, 1]$ so that continuous optimization methods used in the previously proposed algorithms can be applied. Note that the term $\sum_{j \in \mathcal{N}} a_j$ in the objective (36a) is in fact the l_1 norm of \mathbf{a} , and by minimizing $\sum_j a_j$, sparsity in \mathbf{a} is promoted. We also remark that $(\mathcal{P}^{\text{AC}})$ is always feasible which can be easily seen by simply setting $\mathbf{a} = 0$. To solve $(\mathcal{P}^{\text{AC}})$ we can use the approximations presented in the previous subsections. In relation to CR-based algorithm, the only change that needs to be made is for (15d). Specifically, for $(\mathcal{P}^{\text{AC}})$, (15d) becomes

$$\begin{aligned} & \gamma_j^{\min} (a_j + z_j)^2 + \|(\mathbf{s}_j - \mathbf{t}_j)\|_2^2 \leq \|\mathbf{s}_j^{(n)} + \mathbf{t}_j^{(n)}\|_2^2 + \\ & 2(\mathbf{s}_j^{(n)} + \mathbf{t}_j^{(n)}, \mathbf{s}_j - \mathbf{s}_j^{(n)} + \mathbf{t}_j - \mathbf{t}_j^{(n)}) + \gamma_j^{\min} (a_j^{(n)} - z_j^{(n)})^2 + \\ & 2\gamma_j^{\min} (a_j^{(n)} - z_j^{(n)})(a_j - a_j^{(n)} - z_j + z_j^{(n)}). \end{aligned} \quad (37)$$

As a result, $(\mathcal{P}^{\text{AC}})$ is optimized by solving the following convex problem in iteration $n + 1$

$$(\tilde{\mathcal{P}}_{n+1}^{\text{AC}}) \triangleq \begin{cases} \max_{\mathbf{s}, \mathbf{w}, \mathbf{u}, \mathbf{t}, \mathbf{z}, \mathbf{v}, \beta, \mathbf{a}} \left(\prod_{j=1}^N v_j \right)^{\frac{1}{N}} - \zeta \sum_{j=1}^N a_j & (38a) \\ \text{s.t. } (10b), (15a), (15b), (37), (12a), \\ (15e), (16d), (35), (36c). & (38b) \end{cases}$$

After the iterative process converges, the terms $\{a_j \gamma_j^{\min}\}$ in the soft QoS constraints are examined. The user with the smallest value is dropped first, and **Algorithms 1** and **2** are executed to solve (\mathcal{P}) . If solved, then we can stop and report the beamformers and the achieved sum rate. If not, we drop the user with the largest soft QoS requirement among the remaining ones and repeat these steps until (\mathcal{P}) becomes feasible.

GS-based algorithm can also be modified in quite a similar manner. Recall that Ψ_j defined in (20) is the matrix including all beamformers associated with user j . To remove some users we force Ψ_j to be zero for some j 's. Let us define a matrix $\Phi = [\text{vec}(\Psi_1), \text{vec}(\Psi_2), \dots, \text{vec}(\Psi_N)]^T$, where $\text{vec}(\cdot)$ denotes the vectorization operation. That is, the m th row of Φ contains all the beamformers associated with user m . Now the admission control problem can be viewed as forcing some rows of Φ to zero. For this purpose, we can again apply the mixed l_1/l_2 norm to Φ as

$$\|\Phi\|_{1,2} = \sum_m \|(\Phi)_m\|_2 = \sum_{m \in \mathcal{N}} \left(\sum_{k \in \mathcal{K}} \|\mathbf{w}_{km}\|^2 \right)^{1/2}. \quad (39)$$

Due to the introduction of \mathbf{a} , the approximation in (26) needs to be changed accordingly. In fact, (26) becomes

$$\begin{aligned} \gamma_j^{\min} (z_j + a_j)^2 & \leq |\mathbf{h}_j \mathbf{w}_j^{(n)}|^2 + 2\text{Re}(\mathbf{w}_j^{(n)H} \mathbf{h}_j^H \mathbf{h}_j (\mathbf{w}_j - \mathbf{w}_j^{(n)})) + \\ \gamma_j^{\min} (z_j^{(n)} - a_j^{(n)})^2 & + 2\gamma_j^{\min} (z_j^{(n)} - a_j^{(n)})(z_j - z_j^{(n)} - a_j + a_j^{(n)}). \end{aligned} \quad (40)$$

In summary, the convex subproblem in iteration $n + 1$ for the sparsity-inducing norm based iterative algorithm is given by

$$(\hat{\mathcal{P}}_{n+1}^{\text{AC}}) \triangleq \begin{cases} \max_{\mathbf{w}, \mathbf{v}, \mathbf{z}, \mathbf{a}} \left(\prod_{j=1}^N v_j \right)^{\frac{1}{N}} - \sum_{j=1}^N \rho' \|\Psi_j\|_{1,2} - \rho'' \|\Phi\|_{1,2} & (41a) \\ \text{s.t. } (22c), (24a), (25), (36c), (40) & (41b) \end{cases}$$

where $\rho'' > 0$ is a small positive parameter that controls the degree of sparsity of Φ .

IV. NUMERICAL RESULTS

In this section, we numerically evaluate the performance of the proposed algorithms. In our simulation model, we consider one cluster with different numbers of users and APs². We set SINR threshold of all users to $\gamma^{\min} = 0$ dB and the variance of AWGN, $\sigma^2 = 1$. In our simulations, the channel vector from AP i to user j is generated as $\mathbf{h}_{ij} = \sqrt{\eta\gamma} \tilde{\mathbf{h}}_{ij}$, where η represents log-normal shadowing with a standard deviation of 8 dB, γ denotes the path loss, and $\tilde{\mathbf{h}}_{ij}$ is distributed as $\mathcal{CN}(\mathbf{0}, \mathbf{I})$. The path loss is modeled as $\gamma = 10^{-\kappa/10}$, where κ is given in dB by $30.5 + 35 \log(d)$ and d is the distance in meters [51]. The maximum transmission power p^{\max} is fixed to 42 dBm for all APs. For GS-based algorithm, the parameter ρ' in (41a) is set to 2, unless otherwise stated. The proposed algorithms are implemented in MATLAB environment using the conic solver SeDuMi [52] through the parser CVX [53]. The stopping criterion for **Algorithms 1** and **2** is when the increase in the objective values between two successive iterations is less than $\epsilon = 10^{-4}$.

In the first set of numerical experiments, we study the convergence rate of the two proposed algorithms. Specifically, Fig. 3 illustrates the convergence of the CR-based algorithm for a system with $K = N = T = 2$ and two different channel realizations, which are randomly generated without considering the path loss and shadowing, i.e., $\eta = \gamma = 1$. For each channel realization, the CR-based algorithm is started with three different initial points. The first two are generated according to steps 1-3 in **Algorithm 1** (shown by the blue and black curves). For the two channel realizations considered, **Algorithm 1** is first initialized by setting $s_{11}^{(0)} = s_{22}^{(0)} = 1$ and $s_{12}^{(0)} = s_{21}^{(0)} = 1$, respectively. The second initial point is generated by setting $s_{21}^{(0)} = s_{22}^{(0)} = 1$ for both channel realizations (marked by the blue curves in Fig. 3). The red color curves for both channel instances represent the initialization method given in Sec. III-D. It can be observed that the convergence rate depends on the initial points but the same sum rate is achieved on convergence. The CR-based algorithm seems to be stabilized after first 5–15 steps (cf. the black dashed curve)

²For large-scale networks we can use the concept of partitioning as shown in Fig. 2 and the performance of the whole network is expected to be the aggregated performance of each cluster.

but at this stage, the relaxed variables $s_{ij} \approx 0.5$. If the CR-based algorithm runs further, it finally produces nearly a binary solution. Interestingly, it is numerically seen that **Algorithm 1** yields \mathbf{s} close to binary in all cases, meaning that the rounding-off procedure (due to the continuous relaxation in (8f)) is trivial. It is difficult, if not impossible, to provide an analytical explanation to this numerical observation, but our intuition is as follows. Compared to the standard formulation of an assignment problem, we have additional inequality constraints, for instance, (8c). Hence, our problem can be regarded as a generalized version of the assignment problem that also involves inequality constraints. When being relaxed to be continuous, $s_{m,j}$'s act as an AP-sharing factor among users. Now in order to maximize the system throughput, user-AP association should be in such a way that signal part is maximized and inter-user interference is minimized. This physical mechanism perhaps encourages association variables to take on binary values. In addition to this argument, we note that relaxing the discrete selection variables to lie in a continuous interval has an upper bounding effect when a maximization problem is solved. At the same time the SCA solves a non-convex problem by solving a series of convex problems, the feasibility sets of which are *subsets* of the original problem's feasibility set. Hence, by solving the given problem using SCA with continuous selection variables, we end up being quite close to the optimal, and hence quite close to obtaining binary a solution. To analyze this point further, we consider the same formulation (i.e., (7)) that used to develop Algorithm 1, but optimize beamformers and selection variables in an alternating manner. Particular, for given selection variables $\{s_{ij}\}$, we use the SCA principle to find the optimal beamformers $\{\mathbf{w}_{ij}\}$ for the resulting user-AP associations. Then, for the obtained beamformers, we again apply the SCA to find the best user-AP association. The convex approximate constraints in each step are exactly the same as Sect. III-A, except either $\{s_{ij}\}$ or $\{\mathbf{w}_{ij}\}$ is treated as constant, accordingly. Our observation is that the value of s_{ij} is proportional to the norm of the corresponding beamformer. Specifically, if $\|\mathbf{w}_{ij}\|_2$ is large, then so is s_{ij} . However, $\{s_{ij}\}$ do not converge to binary values at convergence. Hence, it is the formulation of the problem coupled with the SCA method which promotes the binary nature of selection variables.

In Fig. 4, the convergence results of **Algorithm 2** are provided for $K = 4$, $T = 3$ with different values of N and ρ' . The objective value returned at each iteration of **Algorithm 2** with different starting points and $\eta = \gamma = 1$ is shown. The legends 'IM1' and 'IM2' (also differentiated by solid and dashed curves) in Fig. 4 represent initialization methods proposed in Sec III-D and step 1 of the GS-based algorithm, respectively. Regardless of the initialization, the GS-based algorithm converges to the same objective value with similar convergence speed. We note that red and blue curves in Fig. 4 represent results for the same channel realization but different penalty values ρ' . Fig. 4 shows that the convergence rate of **Algorithm 2** is nearly independent of the choice of penalty values ρ' . After the GS-based algorithm converges, the beamformers obtained are used to calculate the actual sum rate (and of course ρ' is ignored in the objective). Further, convergence

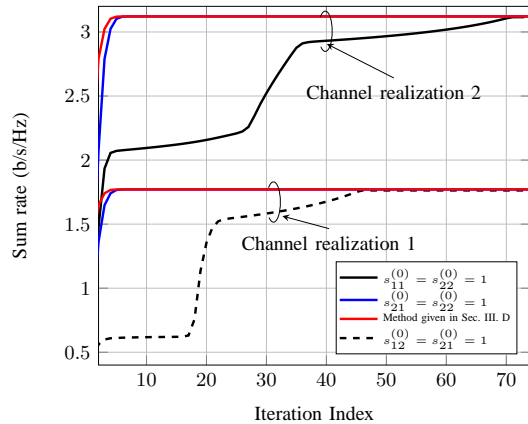


Fig. 3. Convergence of **Algorithm 1** with different initialization points for two random channel realizations. The blue and black curves refer to the case when the initial points are generated using steps 1-3 of **Algorithm 1**. The red curves represent the flexible method of Sec. III-D.

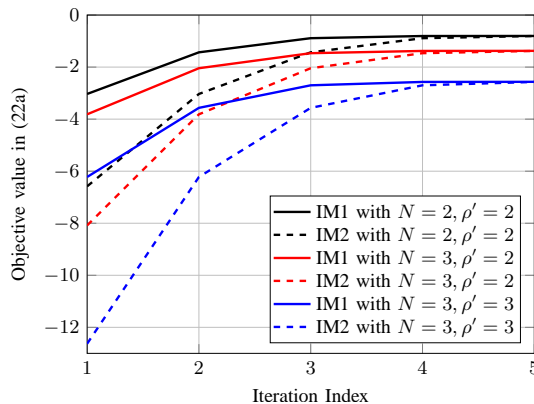


Fig. 4. Convergence behavior of **Algorithm 2** with different N and penalty values. Continuous solid and dashed curves correspond to the initialization methods proposed in section III-D and step 1 of **Algorithm 2**, respectively.

TABLE I
RUN TIME (SEC.) COMPARISON FOR DIFFERENT CONFIGURATIONS.

	$N = 4$			$K = 3$	
	$K = 5$	$K = 4$	$K = 3$	$N = 3$	$N = 2$
Algorithm 1	380	310	220	180	72.5
Algorithm 2	48.8	35.6	25.6	16.3	7.8

rate of the GS-based algorithm is relatively independent of the number of users, N in the system. Fig. 4 also shows that, compared to the CR-based algorithm, the GS-based algorithm converges much faster. This may be attributed to the absence of binary variables s_{ij} in **Algorithm 2** that may take more iterations to stabilize.

Table I shows the average run time required to execute the algorithms with $T = 3$. We created the simulation code on MATLAB using CVX as a parser to solve the SOCP at each iteration of **Algorithms 1** and **2**. The codes are run on a workstation installed with Intel® Core i5-4300U @ 1.9 GHz Processor and 8GB RAM. It is seen that the complexity of **Algorithms 1** and **2** increases with the problem size and, as expected, the GS-based algorithm is much faster than the CR-based algorithm.

We consider three different network settings to compare our proposed algorithms with an optimal solution obtained by a brute force algorithm. For all cases, the number of users N in the system is set to be $N = 2$, and the number of APs is taken as $K = 2, 4$ and $K = 10$, respectively. We note that in the first setup, the numbers of APs and users are equal. Thus, the degree of AP selection is quite limited. Such a scenario actually resembles a dense network. On the other hand, there are more APs than users in the second and third setup. Thus, a user will have more degrees of freedom to select an appropriate AP. We refer to a network similar to above setting as a sparse one. Due to the excessive computation time of the optimal solution, the average sum rates in Fig. 5 are calculated over only 100 channel realizations. We can see that both algorithms deviate only 3%–7% from the optimal performance for all three scenarios we consider, meaning performances are very good even for relatively large networks but with much less complexity compared to the brute force algorithm. In particular, the CR-based algorithm tends to outperform the GS-based algorithm for a dense network where the degree of selection is limited, i.e., where the number of APs is small. On the other hand, when the number of APs is comparable to or larger than that of users, there will be a link of the most favorable channel conditions among all APs. Thus, the network tends to contain a sparse solution for such cases. As a result, the GS-based algorithm becomes more efficient as it aims at finding a high-performance sparse solution. This observation will be further elaborated in the following experiments.

The average sum rate of **Algorithms 1** and **2** as a function of the number of users N for different choices of K and T is illustrated in Fig. 6. Here, blue solid and red dashed curves represent **Algorithms 1** and **2**, respectively. The sum rate is averaged over 5000 channel realizations and small-scale fading is considered for the sake of simplicity. First, for small N , (i.e., fewer users than APs), we observe that the sum rate increases with N . The reason is that a user has sufficient degrees of freedom to select the best APs, without causing much interference to the others. As a result, multiuser diversity can be exploited to improve the achievable sum rate. However, when $N > K$, i.e., (more users than APs), the system has a limited degree of freedom in terms of AP selection, which results in a severe interference situation. Therefore, serving more users will make worse the interference situation worse. This decreases the sum rate as can be seen in Fig. 6. We also observe that, a higher sum rate can be achieved by a larger number of transmission antennas and APs, which is easily understood. Again, in Fig. 6, the CR-based algorithm performs poorly compared to the GS-based algorithm when the number of users is small, and vice versa when the number of users is large. Specifically, the central server can select the CR-based algorithm over the GS-based algorithm if $K \leq N$.

Next, we study the sensitivity of **Algorithm 1** towards channel estimation errors. For this purpose, cumulative distribution functions (CDFs) of the sum rate in the presence of channel estimation errors are shown in Fig. 7. First, a set of channel realizations for a system with $K = 5, N = 5, T = 3$ is generated, which is referred to as the perfect CSI. The CR-

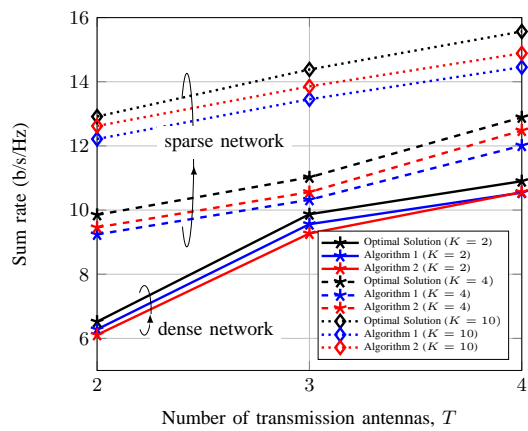


Fig. 5. Sum rate comparison of **Algorithms 1** and **2** with the optimal solution for different network settings.

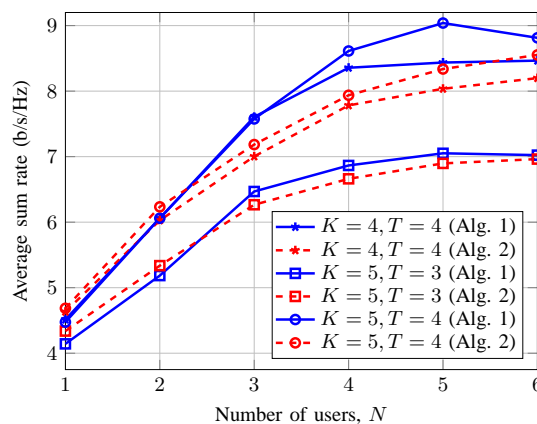


Fig. 6. Comparison of the achievable average sum rates of the two algorithms for different system parameters.

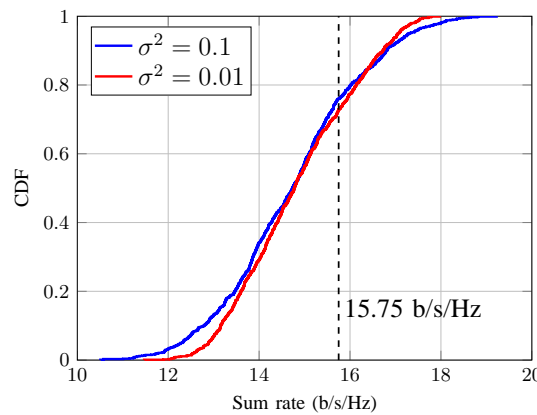


Fig. 7. CDF of the sum rate with Gaussian channel estimation errors having zero mean and different variances. System has been analyzed for 10000 channel estimation errors.

based algorithm is applied to this perfect channel estimate, which results in a sum rate of 15.75 bps/Hz (shown by the dotted line in Fig. 7). To compute an empirical CDF, we model the actual channel vectors as the sum of the perfect CSI and channel estimation errors which are assumed to follow Gaussian distribution with zero mean and variance of σ^2 . The

obtained beamformers are then used to calculate the sum rate for each erroneous CSI and each curve in Fig. 7 is obtained by considering 10000 sets of channel errors. It is seen that due to the presence of CSI errors, the rates tend to fluctuate around the perfect CSI rate of 15.75 bps/Hz. When σ^2 increases, CDF is more spread which means the sum rate deviates further from the expected rate.

For the next set of simulations, we consider a cluster with two static APs (named as S1, S2), three dynamic APs (named as D3-D5), and five users (named as U1-U5), in a hexagonal area of radius 75 m as shown in Fig. 8. Log-normal shadowing and path loss are considered with the parameters described at the beginning of the Sec. IV. The dynamic state of the network is analyzed by varying the number of available dynamic APs with time. At a given time instant t , it is assumed that there will be m active dynamic APs, where $m \in \{0, \dots, 3\}$.

Fig. 9 shows the convergence of the CR-based algorithm with dynamic behavior of the system taken into account. In each system configuration, we use 40 iterations for illustration. After 40 iterations, the system reconsiders the configuration to identify the changes in the network. If there is any change (different N or K), **Algorithm 1** is re-executed so that the system can be optimized for the new configuration. This explains the sharp fall seen in Fig. 9 after 40 and 80 iterations. As shown in the figure, we first fix the number of users and then activate the dynamic APs D3, D4 and D5 in order. In every configuration, depending on the status of the user association, the elements of the selection vector are $s_{ij} \in [0.01, 0.03]$ or $[0.97, 0.99]$ after convergence of the algorithm. That is to say, the relaxed variables are nearly binary after convergence. The system starts converging within a few iterations, indicating the efficiency of the joint optimization process.

In Fig. 10, we plot the sum rate versus the number of active dynamic APs for different algorithms. Note that, for a given number of APs m , there are $\binom{3}{m} = \frac{3!}{m!(3-m)!}$ different ways to choose m APs. For a particular scheme, two types of sum rate performances are reported in Fig. 10. The first one is the average sum rate of all possible cases, and the second one is the maximum sum rate among these. The two performances are differentiated by the suffix (avg) and (max) in the legends shown in Fig. 10. In particular we compare the proposed algorithms with the simple method described in [19], which is referred to as the PL-based method. The PL-based model consists of a system where the user-AP association is simply determined by the path loss, i.e., users are associated with their nearest AP. Note that for a given user-AP association, there are several methods to find the beamformers. For a fair comparison, we apply a branch and bound method to compute optimal beamformers, and employ the PL-based method for user-AP association. In other words, we benchmark **Algorithms 1** and **2** with the best achievable performance of the PL-based method. It can be seen that **Algorithms 1** and **2** yield a gain of 11%–26% and 17%–39%, respectively, in terms of the average sum rates over the PL-based method. Similarly, the corresponding gains offered by **Algorithms 1** and **2** for maximum sum-rates over the PL-based method are 5%–26% and 10%–39%, respectively. For **Algorithms 1** and **2**, there is a slight difference between the maximum and average sum

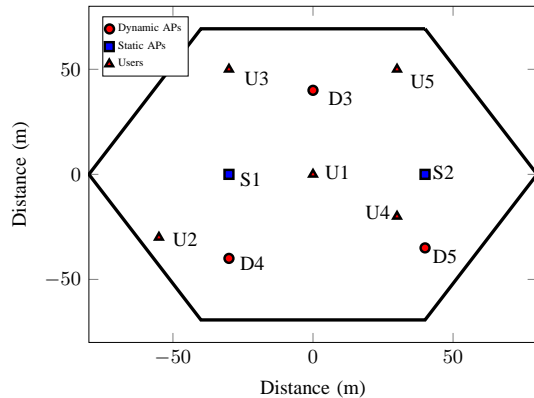


Fig. 8. A more realistic system model with static and dynamic APs used in Figs. 9,10, and 5. The axes display the distance between users and APs in meters.

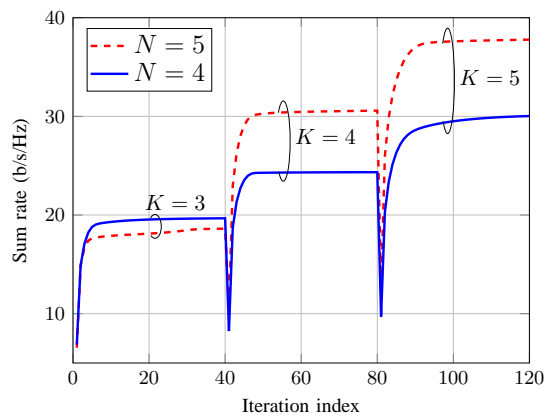


Fig. 9. The achieved sum rate of **Algorithm 1** with iterations when the system parameters K and N dynamically change.

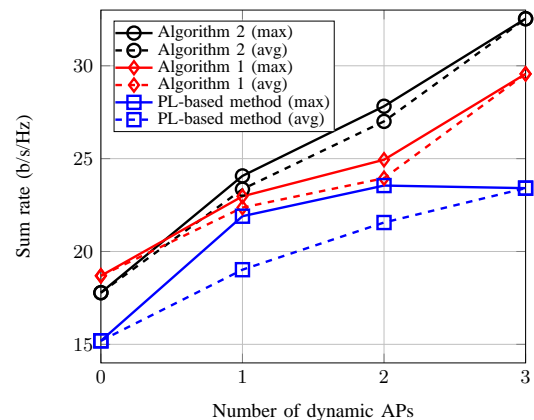


Fig. 10. Sum rate comparison of different algorithms with the number of dynamic APs. The positions of users, static and dynamic APs are shown in Fig. 8.

rate, while the difference is sharper for the PL-based method. This basically implies that, by jointly optimizing user-AP connection and beamformers, the effects of the locations of the APs are minimized. We also see that as more dynamic APs enter the system, the GS-based algorithm turns superior to the CR-based algorithm since the resulting network becomes

more sparse. This observation is in fact consistent with the ones reported in the previous experiments.

In the last experiment, we investigate the dynamic behavior of the network and the performance of the proposed admission control algorithms, as shown in Fig. 11. In particular, we consider the same network setup as the one described in Fig. 8, but SINR thresholds $\{\gamma_k^{\min}\}$ are modified. Specifically, $\gamma_1^{\min}, \gamma_2^{\min}, \gamma_4^{\min}$ are set to 5.22 dB, while γ_3^{\min} and γ_5^{\min} are set to 7 dB and 3 dB, respectively. Performance is analyzed for two cases, first with (S1, S2, D4) and second with (S1, S2, D3). For the two scenarios, the problem (\mathcal{P}) is feasible when all APs are active, and the resulting sum rate is labeled by ‘All APs active’ in Fig. 11. To study the proposed AC algorithms, D4 is switched off in the first setup and S1 in the second one. Since the remaining two APs cannot provide sufficient diversity gains to serve all five users, the resulting problem (\mathcal{P}) becomes infeasible, and admission control mechanism is thus required. Numerically it is observed that the two proposed AC algorithms using formulations in (38) and (41) drop U5 in the first scenario and U3 in the second one to make the corresponding system feasible again. We then execute **Algorithms 1** and **2** on the remaining four users to find the beamformers and the resulting sum rates, which are indicated by ‘Proposed AC Alg.’ in Fig. 11. We compare it to the simple method of dropping the users with the highest QoS demand, referred to as ‘QoS-based Alg.’ in Fig. 11. We remark that the proposed AC algorithms are not guaranteed to drop the least number of users, since the optimal solution for this problem is not known. It is also seen that the system performance is not only dependent on the number of active APs but also on the location and connection of the active APs.

V. CONCLUSION

Two iterative algorithms have been proposed to optimize the user-AP assignment and the beamformers in the downlink of DNA networks. In the first method, the binary optimization variables are simply relaxed to be continuous and an iterative procedure based on the SCA framework to arrive at a sequence of SOCPs is found. In the second method, the selection process is formulated as finding a solution to a sparsity constrained optimization problem. Our extensive numerical results suggest that the first approach is more effective in terms of the achievable data rate for dense networks, although its convergence rate is relatively sensitive to the initial point. On the other hand, the second approach is shown to achieve faster convergence, and a higher sum rate performance for sparse networks, wherein the degree of selection is high. In particular, the CR-based algorithm and the GS-based algorithm perform comparatively well against the optimal solution that has a prohibitively high complexity. Moreover, the two proposed algorithms have been demonstrated to outperform the existing method which is simply based on the path loss between APs and users for a quite realistic scenario. The effects of mobility on the efficiency of these algorithms and finding an optimal solution for the AC problem will be studied in future work. Also the impact of backhaul capacity constraints in supporting the data sharing to the APs will be addressed.

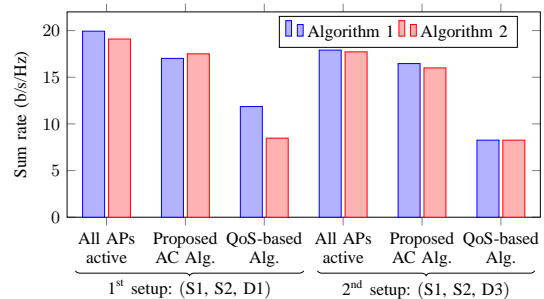


Fig. 11. Performance evaluation of the proposed admission control algorithms for different network setups.

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Inosha Sugathapala (S'13) received her B. Sc. degree in Electrical and Electronic Engineering from the University of Peradeniya, Sri Lanka in 2007 and the M. Eng. degree from the Asian Institute of Technology, Thailand in 2011. She is currently working on her Dr. Tech. degree in Communications Engineering in University of Oulu, Finland. Inosha is also a member of the research staff of the Centre for Wireless Communications (CWC), Oulu, Finland. Her main research interests are in resource optimization, network sharing and multi-antenna networks.



Muhammad Fainan Hanif received the Ph.D. degree in electrical and electronic engineering from the University of Canterbury, Christchurch, New Zealand, in 2011. In 2012 and 2013, he was as a postdoctoral researcher at the Centre for Wireless Communications, the University of Oulu, Finland. Then in 2015 he worked as a Research Associate within the Computing and Communications Department of Lancaster University, United Kingdom. At present, he is an Associate Professor at the Department of Electrical Engineering, the University of Lahore, Lahore, Pakistan. His research interests include convex optimization applications in signal processing and modern communication systems.



Beatriz Lorenzo received her M.Sc. degree in telecommunication engineering from the University of Vigo, Spain, in 2008, and her Ph.D. degree from the University of Oulu, Finland, in 2012. Since 2014, she has worked as a senior researcher at Atlantic Research Center for Information and Communication Technologies, University of Vigo. She was awarded a Fulbright Visiting Scholar Fellowship at the University of Florida (2016-2017). Her research interests include wireless networks, network architectures and protocol design, mobile computing, optimization, and network economics.



Savo Glisic is a Professor of Telecommunications at University of Oulu, Finland, head of the networking research group, and Director of Institute for Networking Sciences. He was Visiting Scientist at Cranfield Institute of Technology, Cranfield, U.K. (1976-1977) and University of California, San Diego (1986-1987). He has been active in the field of wireless communications for 30 years and has published a number of papers and books. The latest book "Advanced wireless Networks: 5G/6G joint design of technology and business models" John Wiley &

Sons, 2015, covers the enabling technologies for the definition of incoming 5G systems.

He is running an extensive doctoral program in the field of wireless networking (www.telecomlab.oulu.fi/kurssit/networks/). His research interest is in the area of network optimization theory, network topology control and graph theory, cognitive networks and game theory, radio resource management, QoS and queuing theory, networks information theory, protocol design, advanced routing and network coding, relaying, cellular, WLAN, ad hoc, sensor, active and bio inspired networks with emphasis on genetic algorithms and stochastic geometry. The latest interest is in the area of spectra sharing, robust heterogeneous network design, Artificial Intelligence (AI), Inter System Networking (ISN), block chains and complex networks theory. He is doing research within the WiFiUS collaborative program between the NSF in US and Finnish Academy as well as on a number of research projects sponsored by EU FP7 program. He is active within 5G ppp association preparing the projects for Horizon 2020 calls. Dr. Glisic has served as the Technical Program Chairman of the third IEEE ISSSTA'94, the eighth IEEE PIMRC'97, and IEEE ICC'01. He was Director of IEEE ComSoc MD programs.



Markku Juntti (S'93-M'98-SM'04) received his M.Sc. (EE) and Dr.Sc. (EE) degrees from University of Oulu, Oulu, Finland in 1993 and 1997, respectively.

Dr. Juntti was with University of Oulu in 1992–98. In academic year 1994–95, he was a Visiting Scholar at Rice University, Houston, Texas. In 1999–2000, he was a Senior Specialist with Nokia Networks. Dr. Juntti has been a professor of communications engineering since 2000 at University of Oulu, Centre for Wireless Communications (CWC), where he leads the Communications Signal Processing (CSP) Research Group. He also serves as Head of CWC – Radio Technologies (RT) Research Unit. His research interests include signal processing for wireless networks as well as communication and information theory. He is an author or co-author in some 450 papers published in international journals and conference records as well as in books *WCDMA for UMTS*, *Signal Processing Handbook* and *5G Wireless Technologies*. Dr. Juntti is also an Adjunct Professor at Department of Electrical and Computer Engineering, Rice University, Houston, Texas, USA.

Dr. Juntti is an Editor of IEEE TRANSACTIONS ON COMMUNICATIONS and was an Associate Editor for IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY in 2002–2008. He was Secretary of IEEE Communication Society Finland Chapter in 1996–97 and the Chairman for years 2000–01. He has been Secretary of the Technical Program Committee (TPC) of the 2001 IEEE International Conference on Communications (ICC 2001), and the Co-Chair of the Technical Program Committee of 2004 Nordic Radio Symposium and 2006 IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC 2006), and the General Chair of 2011 IEEE Communication Theory Workshop (CTW 2011). He has served as Co-Chair of the Signal Processing for Communications Symposium of Globecom 2014 Signal Processing for Communications Symposium and IEEE GlobalSIP 2016 Symposium on Transceivers and Signal Processing for 5G Wireless and mm-Wave Systems.



Le-Nam Tran (M'10-SM'17) received the B.S. degree in electrical engineering from Ho Chi Minh City University of Technology, Ho Chi Minh City, Vietnam, in 2003 and the M.S. and Ph.D. degrees in radio engineering from Kyung Hee University, Seoul, Korea, in 2006 and 2009, respectively. He is currently a Lecturer/Assistant Professor at the School of Electrical and Electronic Engineering, University College Dublin, Ireland. Prior to this, he was a Lecturer at the Department of Electronic Engineering, Maynooth University, Co. Kildare, Ireland.

From 2010 to 2014, he had held postdoc positions at the Signal Processing Laboratory, ACCESS Linnaeus Centre, KTH Royal Institute of Technology, Stockholm, Sweden (2010-2011), and at Centre for Wireless Communications and the Department of Communications Engineering, University of Oulu, Finland (2011-2014). His research interests are mainly on applications of optimization techniques on wireless communications design. Some recent topics include energy-efficient communications, cloud radio access networks, massive MIMO, and full-duplex transmission. He has authored or co-authored in some 80 papers published in international journals and conference proceedings.

Dr. Tran is an Associate Editor of EURASIP Journal on Wireless Communications and Networking. He was Symposium Co-Chair of Cognitive Computing and Networking Symposium of International Conference on Computing, Networking and Communication (ICNC 2016).