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A Numerical Investigation of Spherical Void Growth in an Elastic-Plastic Continuum

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Abstract

Significant toughening of structural epoxy adhesives has been achieved with the addition of nano and micro-scale particles. However, the toughening mechanisms introduced by the addition of these particles is not very well understood. The ultimate aim of this research is to develop an understanding of the toughening mechanisms present and investigate the parameters which affect the degree of toughening, i.e. particle size, particle volume fraction and particle distribution to guide future adhesives development. The current work examines the growth of a single void in an elastic-plastic material as a function of constraint and compares the results with the predictions of the classic Rice & Tracey model.

Keywords: void growth, modelling, finite volume

1. Introduction

Significant toughening of structural adhesives is attainable with addition of nano and/or micro particles[1, 2, 3]. Tapered Double Cantilever Beam (TDCB) experiments, conducted at University College Dublin (UCD), have observed a significant dependence of the fracture toughness of these adhesives on bond gap thickness[5]. In conjunction with this change in fracture toughness, scanning electron-microscopy (SEM) of the fracture surface has also revealed corresponding changes in void evolution as the bond gap is varied. Classical analysis suggests the change in toughness may be attributed to a physical constraint of the size to which the plastic zone around a crack tip may develop[6]. However, simulation of these TDCB tests using finite volume stress analysis has found that little plasticity develops in the bulk adhesive

layer and is instead concentrated in the fracture process zone. The change in fracture toughness and void evolution present can be attributed to the change in triaxiality at different bond gap thicknesses and the results agree quite well with the void growth model of Rice & Tracey[4]. The variance of void growth with triaxiality is investigated here.

2. Rice & Tracey model

Rice & Tracey[4] considered a spherical void in a rigid, perfectly plastic solid and derived an exponential dependence of void growth on stress triaxiality or constraint defined as,

$$D = 0.283 \exp(1.5H), \quad H = \frac{\sigma_{hyd}}{\sigma_{eq}}, \quad D = \frac{\dot{R}}{\dot{\epsilon}_{eq}R_0} \quad (1)$$

$$\sigma_{hyd} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3}, \quad \sigma_{eq} = \sqrt{\frac{2}{3}s : s}, \quad s = \sigma - I\sigma_{hyd} \quad (2)$$

where D is the dilatational amplification factor, H is the constraint, ϵ_{eq} is the equivalent macroscopic strain, R is the average void radius and R_0 is initial void radius.

This exponential dependance of the void growth amplification factor with constraint is investigated subsequently with numerical simulations to determine its appropriateness and validity in this case.

3. Numerical Model

Simulations are conducted with the open source finite volume package OpenFOAM (Open Field Operation And Manipulation). OpenFOAM is a general 3D based, object-oriented C++ computational continuum mechanics library. The work considered here is concerned with modelling in 3D a void in an elastic-plastic material with a view to verifying the exponential dependence of void growth on the macroscopic stress triaxiality in the system in accordance with the Rice & Tracey model. A unit cell consisting of a single void is considered. Due to three planes of symmetry, only one eighth of the geometry is modelled. The ratio of the void radius to the outer dimension of the cell is 1 : 15 which was considered a suitable approximation to the infinite domain considered by Rice & Tracey. A hexahedral mesh is constructed in

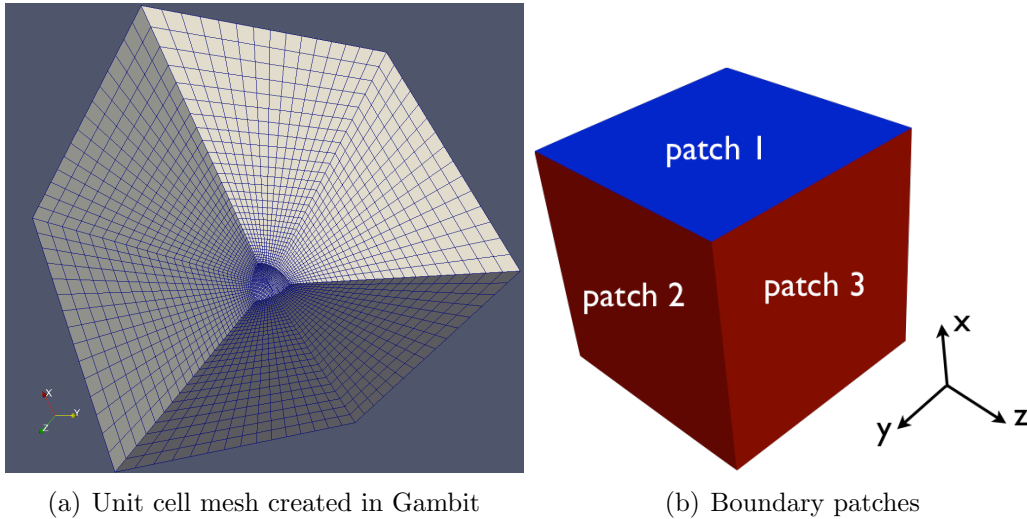


Figure 1: Unit cell

Gambit, exported as an Fluent mesh file and then imported into OpenFOAM using the OpenFOAM utility `fluentMeshToFoam`, see Figure 1(a).

The material was modelled using a homogeneous isotropic elastic-plastic constitutive model given by:

$$\sigma = \begin{cases} E\epsilon & \text{for } \epsilon < \epsilon_o \\ \sigma_o(\frac{\epsilon}{\epsilon_o})^n & \text{for } \epsilon \geq \epsilon_o, n = 0, 0.1, 0.2, 0.3 \end{cases} \quad (3)$$

where E is Young's Modulus, ϵ is strain, and σ_o and ϵ_o are the stress and strain at yield, respectively, with values of 8.4 GPA and 28 MPa being used for Young's Modulus and yield stress, respectively.

As can be seen in Figure 1(b), a normal displacement is applied to the top patch, patch 1 (shown in blue), with unit normal in the x direction and a normal stress is applied to patches 2 & 3, with unit normals in the y and z directions, respectively (red patches), with the void surface set as traction free. The value of the normal stress applied on patches 2 & 3 is calculated as a function of the traction on patch 1 resulting from the prescribed displacement in each time step giving a constant value of macroscopic constraint throughout the simulation.

By writing the constraint, H , in terms of the principal stresses, an ex-

pression for σ_2 can be derived in terms of σ_1 as follows:

$$\begin{aligned} H &= \frac{\sigma_{hyd}}{\sigma_{eq}} = \left(\frac{\sigma_1 + 2\sigma_2}{3}\right) / \sqrt{(\sigma_1 - \sigma_2)^2}, \text{ assuming } \sigma_3 = \sigma_2 \\ &= \frac{\sigma_1 + 2\sigma_2}{3(\sigma_1 - \sigma_2)}, \text{ assuming } \sigma_1 > \sigma_2 \end{aligned} \quad (4)$$

$$\text{Rearranging, } \sigma_2 = \sigma_3 = \left(\frac{3H - 1}{2 + 3H}\right) \sigma_1 = \lambda\sigma_1 \quad (5)$$

When calculating σ_2 (and thus σ_3), the value of σ_1 from the previous time step was used in order to reduce the numerical overhead with a negligible effect on the solution as the loading rate was sufficiently small.

4. Results

For direct comparison with the Rice & Tracey model, initial cases use a perfectly plastic material model (strain hardening exponent, $n = 0$) with values of macroscopic constraint achieved ranging from $\frac{1}{3}$ to 3. It was observed that the computed void growth rates did not agree well with the Rice & Tracey model, with significantly larger values of void growth present for all values of triaxiality compared to the analytical solution as seen in Figure 2(a).

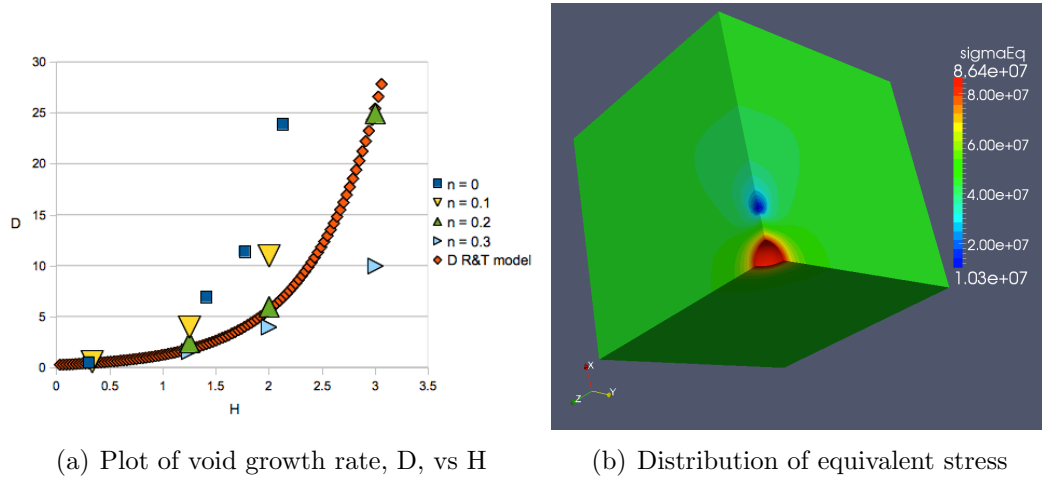


Figure 2: Simulation results

Dependence of void growth on stress triaxiality for different material models was also investigated. Strain hardening exponents, n , equal to 0.1, 0.2 and 0.3 were considered. As seen in Figure 2(a), increasing plastic modulus acts to suppress void growth, as void growth for a particular value of constraint, H , decreases with increasing strain hardening exponent, n . However, a significant dependence of void growth on constraint can be seen in all cases. A typical distribution of the equivalent stress can be seen in Figure 2(b).

5. Conclusions

Numerical analysis of void growth as a function of constraint, H , was performed using the finite volume based package OpenFOAM. Results clearly show significant dependence of void growth with constraint for all material models considered but for a perfectly plastic material model, the results do not adhere to the classical void growth model of Rice & Tracey. Increasing the degree of strain hardening present in the material model acted to suppress void growth with a strain hardening exponent, n , equal to 0.2 giving excellent agreement with the Rice & Tracey model.

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