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Smart Grid Topology Designs*

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ABSTRACT

This paper addresses supports for evolving design demands of electricity low voltage networks in urban areas. Innovations in how electricity is generated and supplied are required to support transformation of energy systems in response to climate change. We describe a MIP model to support grid upgrade decisions in the context of an energy community in an existing urban setting. We evaluate the MIP model on an adaption of an IEEE radial network benchmark instance augmented with geographic data. We present interesting computational results which suggest additional arcs to be added. Our results highlight potential research opportunities for the network optimisation community to facilitate the desired energy systems transformation challenge.

1 INTRODUCTION

The methods of electrical energy production and distribution are changing in response to climate change concerns and as technological innovations create new opportunities. Consumers can now generate electricity through rooftop photovoltaic (PV) panels, and small rooftop wind turbines [2]. End-users equipped with renewable energy generation are turning pro-active in the distribution system and becoming a so called “prosumer”. In future electricity distribution models, any member of the network could potentially generate electricity. We consider the context of an energy community, a geographically close grouping in an urban setting, who wish to collaborate together to share electricity in their local area.

Many challenges and opportunities exist to achieving a transformation of the energy system. In this paper we focus on the problem of deciding how to upgrade an existing local low voltage network to facilitate the operation of the energy community. We contribute a MIP formulation to determine which additional edges could be added to upgrade a distribution system tree topology to form a meshed topology.

We evaluate our model on a 37 node IEEE radial test feeder system [7] under a number of scenarios. We augment the test system with geographic information to create realistic renewable energy test instances. Our results show that the problem becomes more challenging as more prosumers participate in the energy community.

2 ENERGY SYSTEM TRANSFORMATION

The EU Commission’s “Clean Energy for All Europeans” package aims to drive a transformation of the energy system to ensure clean, secure and efficient energy in response to the needs for climate change mitigation actions [4]. Demand for electricity by an end user is currently managed in many jurisdictions through their relationship with an electricity (energy) supplier. Suppliers meet their own total needs by buying from a centrally managed pool. Electricity generators sell the output of their plants to the pool, the electricity can be generated by renewable sources such as wind, or from fossil or nuclear fuels. The electricity is transported from the generators’ sites over the transmission system and finally over the distribution system to the end-users buildings. Approximately 8 - 15% of the power generated is lost through heat loss during transport and distribution This motivates the desire to locate generation nearer to demand sites. The move to more sustainable practices further motivates the focus on increasing the use of Renewable Energy Sources (RES), and decreasing the reliance on fossil and nuclear fuels.

The future decentralised distribution network will be required to facilitate new market practices where certain end-users become electricity generators. Therefore if formerly all the end-users were consumers, now some of them are becoming prosumers. One concept being explored is that of an energy collective to allow participants a more proactive role in power system operation. An energy collective can be viewed as a community-based electricity market structure where prosumers are allowed to share energy at community level [9]. Prosumers may generate more electricity than their needs at some times and may wish to make their excess electricity available to either to their supplier, or in this case, to the local energy community network. At other times they may be self satisfied, or may not produce enough and need to buy electricity from their supplier, or preferably to buy the excess renewable electricity of their neighbours in the energy community network.

The connection topology of traditional centrally controlled electricity grids are generally tree distribution networks. Figure 1 shows the IEEE 37 node radial test feeder topology. The symbol adjacent to node 799 is a type of transformer which acts as on/off switch. The symbol between nodes 709 and 775 is a transformer to control voltage levels. We have added a compass rose to show how we interpret the direction orientation of the test network.

As community energy collectives evolve, upgrade of the local low voltage distribution network may be warranted. The evolution of electricity grid tree topologies mirrors that of telecommunications networks, when connectivity constraints were added to meet reliability concerns. In turn ring bounds can be considered to limit flow (and loss) in network cycles [3, 6]. Many of the (telecommunications) network design models and solution techniques are transferable to address the needs of smart grid topology design. Similar ideas in adapting network topology design models are used in wind farm cabling problems in [5].

An additional challenge to understanding the requirements of future local electricity networks is the move toward the electrification of heat and transport in climate change mitigation actions. These demands will push the demand for electricity upwards

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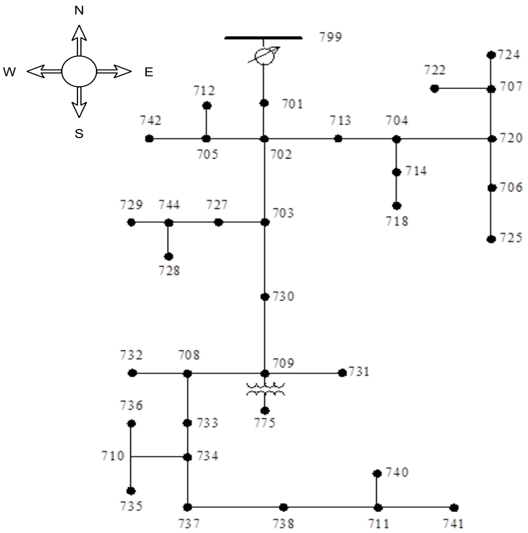


Figure 1: IEEE 37 node radial test network.

and may require significant network reinforcement. In contrast, retrofitting of building with modern (thermo) efficient materials and the use of more efficient white goods counter balance somewhat to decrease electricity (and total energy) demand. Estimates of the uptake of RES further complicate estimates of future network needs. An understanding of potential electricity flow in energy collectives will provide increased understanding for transmission and distribution system operators.

3 SMART GRID LOW VOLTAGE UPGRADE MIP MODEL

Consider an existent electricity low voltage distribution network in an urban area modelled as $G = (N, A)$ for a set of n locations $N = \{1, \ldots, n\}$ such that the topology is a tree rooted at a substation n_0. Electricity flows according to the laws of physics and can be controlled by controller devices. Historically electricity flowed from the substation in response to consumer demand so that graphs were considered to be directed. We make some simplifying assumptions to handle nonlinear alternating current flow. Smart wire technology in development may make these assumptions realistic in the near future [8].

Consider that the set $N \backslash n_0$ is partitioned into sets C of the end-users who remain consumers and set P of the new prosumers. Electricity can flow from the prosumer back into the distribution network without the need for additional arcs, the flow can be controlled and monitored by switching devices. Hence we can assume the existent network is modelled by $\bar{G} = (N, E)$

We consider a time horizon T. Each end-user $i \in N \setminus n_0$ consumes a certain amount EC_i^t of energy at time t. We treat the substation root node as a prosumer in the sense that they can both provide and accept electricity. Each prosumer $i \in P$ generates a certain amount EG_i^t of energy at time t. At each time t, the energy demand Q_i^t for each consumer $i \in C$ is $Q_i^t = -EC_i^t$. At each time t, the energy demand Q_i^t for each prosumer $i \in P$ is $Q_i^t = EG_i^t - EC_i^t$.If this value is zero the prosumer is self satisfied. If $Q_i^t > 0$ the prosumer has an excess of electricity and sells electricity to the community network. If $Q_i^t < 0$ the prosumer has insufficient electricity and buys electricity, preferably from the community network but otherwise from their supplier.

We assume that the community network needs can be satisfied. Therefore at each time, the substation node n_0 either provides or accepts energy:

$\sum_{i \in N \backslash \{n_0\}} Q_i^t < 0$ or $\sum_{i \in N \backslash \{n_0\}} Q_i^t > 0$, respectively.

Set $Q_{n_0}^t = \sum_{i \in N \backslash \{n_0\}} Q_i^t$. Set $\bar{Q} = \max_{t \in T} \sum_{i \in N} |Q_i^t|$ to be the maximum amount of electricity transported in any connection.

We take possible energy losses into account. The overall losses between the substation and consumers can be modelled by a percentage loss factor $L \in [8, 15]\%$.

To obtain the MILP model consider the following decision variables:

Topological binary integer variables x_{ij} indicate whether the arc (i, j) is selected to be included in the new decentralised network. Flow variables y_{ij}^t indicate the amount of electricity transported from location i to location j at time t. Let the constants a_{ij} take value 1 if arc $(i, j) \in E$, meaning that it is already installed and belongs to the distribution network, or take value 0 if the arc $(i, j) \notin E$, it is not installed.

$$\min \quad \sum_{(i,j) \in A} c_{ij} x_{ij} + \sum_{t \in T} \sum_{(i,j) \in A} y_{ij}^t \tag{1}$$

subject to

$$\sum_{i \in N} (a_{ij} + x_{ij}) \geq 1 \qquad j \in C \tag{2}$$

$$\sum_{i \in N} (a_{ij} + x_{ij}) \geq 1, \qquad j \in P \tag{3}$$

$$\sum_{i \in N} (a_{ji} + x_{ji}) \geq 1, \qquad j \in P \tag{4}$$

$$\sum_{i \in N} y_{ij}^t + (1 + L) Q_j^t = \sum_{i \in N} y_{ji}^t, \qquad j \in N, \ t \in T \tag{5}$$

$$y_{ij}^t \leq \bar{Q}(a_{ij} + x_{ij}), \qquad (i, j) \in A \tag{6}$$

$$a_{ij} + x_{ij} + a_{ji} + x_{ji} \leq 1, \qquad i, j \in N \tag{7}$$

$$x_{ij} \in \{0, 1\}, \qquad (i, j) \in A \tag{8}$$

$$y_{ij}^t \geq 0, \qquad (i, j) \in A, \ i \in T \tag{9}$$

Let c_{ij} be the cost of installing additional arc $ij \in A$ in the upgrade network. Eq (1) is the objective function which minimises the cost of additional edges in the upgrade as well as minimising the overall flow of electricity. This will have the effect of fostering flow between geographically close neighbours, which in turn reduces transmission losses. Inequality (2) ensures all consumers are connected to the network to receive energy over an existing arc, and possibly through an additional new arc. Inequalities (3) and (4) ensure all prosumers are connected to the network by an existing arc and possibly through an additional new arc. Prosumers have the possibility of both receiving electricity, and of offering their excess to the network. Equalities (5) are the flow conservation constraints and take into account possible energy losses by a percentage factor L, $0.08 \leq L \leq 0.15$. Inequalities (6) are the variables linking constraints and limit for a maximum flow in any connection of the network. Inequalities (7) say we do not install a new arc between two locations if there is an existing link in the network, this means we restrict to one the number of connections between any two locations. Finally constraints (8) and (9) define the variables domain.

In addition, we can add clique inequalities for any subset of consumers. For any clique of size three, $C_3 = \{i, j, k\} \subseteq C$, the following is valid:

$\sum_{\{l,m\}\in C_3}(a_{lm} + x_{lm}) \leq |C_3| - 1 = 2.$

These inequalities say that for any clique $C_c \subset C$ the number of connections is restricted to $|C_c| - 1$. Restricting the number of locations in the clique avoids cycles between any set of consumers. The clique inequalities are inserted for subsets of consumers where the existing arcs are sufficient to ensure the energy flow distribution. Recall that the existing topology is a tree. New arcs are added to improve the energy flow mainly for prosumers that must have the opportunity to distribute their energy in the network. In the case of prosumers, a cycle is allowed in the solution.

4 TEST INSTANCES AND SCENARIOS

We augment the IEEE 37 node test instances with geographic information for two locations; Dublin, Ireland and Aveiro, Portugal. We simply overlay the IEEE system on geographic maps and extract GPS coordinates of the locations. This give us two test instances where we can estimate distances between nodes using the haversine formula as a proxy for c_{ij}.

Dublin, Ireland lies at latitude 53.4°C N and longitude 6.3°C W. It has a temperate climate with pronounced variation between the number of hours of daylight in winter and summer. Hence the amount of electricity generated by PV per season is quite variable [1]. In addition, the east-west orientation of some buildings offers less potential than those with southerly facing aspects. We evaluate two potential seasonal scenarios for the Dublin location; one in summer (with daylight hours 7.00 - 20.00), and the other in winter (with daylight hours 9.00 - 16.00). We create representative load profiles for Dublin using data from the Retail Market Design Service [10] allowing a proportion of the nodes to act as prosumers. Sample profiles are shown in Figure 2.

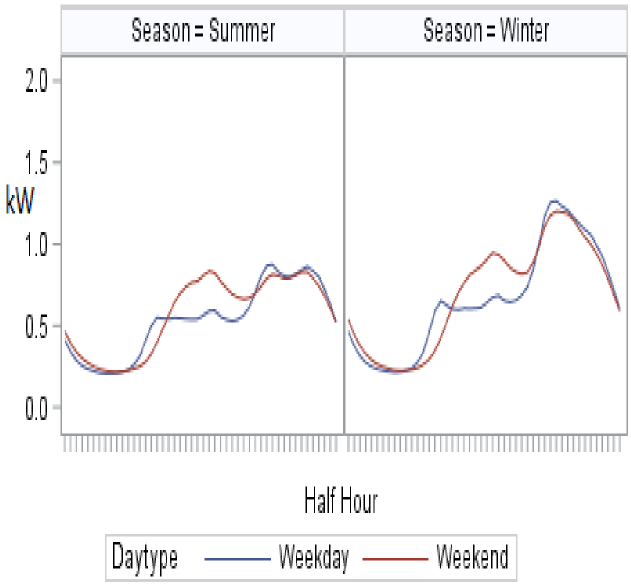


Figure 2: Reference Load Profiles for Ireland.

We follow a similar approach for Aveiro located at 40.6°C N and 8.6°C W and generate sample load profiles informed by [11]. Aveiro offers more consistent daylight hours and sunlight than Dublin, so we test just one reference load profile.

5 RESULTS AND ANALYSIS

The MIP model was implemented in Mosel and computational tests were run using XpressMP 8.5 on a Dell 64 bit Windows 8 machine with Intel i5 3.2 GHz processor and 8GB of Ram. We test the instances varying the estimated transmissions losses L, and the proportion of prosumers in the community. We assume a 2kW PV panel for each prosumer with generation during daylight hours of diminishing output depending on the prosumer's orientation.

We performed computational experiments to assess the performance of the compact model and the quality of the obtained solutions with and without the clique constraints for sets of three consumers. The use of these valid inequalities greatly improves the solution quality, but at a slight expense in computational time. For example, for the instance Aveiro with $L = 8\%$ and $P = 25\%$ the $Gap = (BestMIP - BestBound)/BestMIP$ improves from 0.39 to 0.17. Therefore we use the IP model with the clique valid inequalities for all sets of consumers of size three. We allowed a maximum run time of 3 hours for the more challenging instances.

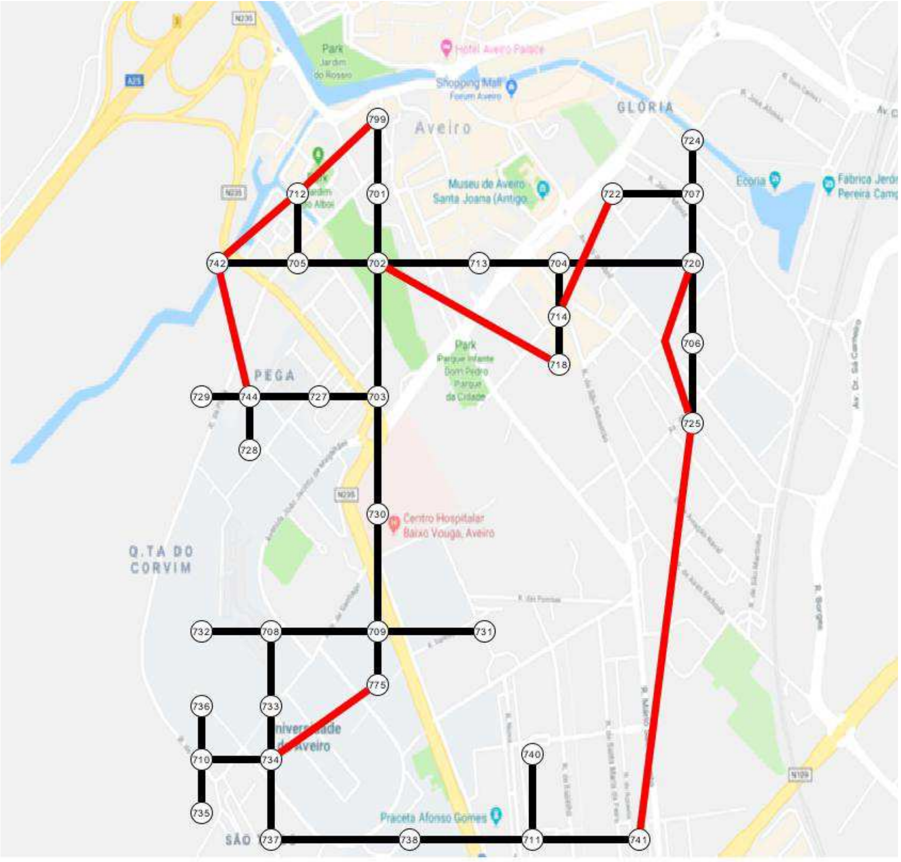
Table 1 shows sample results. From left to right we show the information about the problem instance (Name of the instance, Loss percentage, Prosumers percentage), followed by details of the IP model B&B search obtained for a time limit of three hours (problem status, BestBound value corresponds to the best lower bound obtained, BestMIP value corresponds to the best feasible integer solution, the corresponding Gap value, the number of the nodes in the B&B search procedure, the computational time in seconds and the number of new arc flows to be installed determined by the best MIP solution).

Figure 3 shows sample solution topologies. Existing edges are show in black, and proposed additional arcs in red. We see the evolution from tree to more resilient meshed networks. Figure 3a shows the best solution found for Aveiro with 30% Prosumers, and a loss factor of 15%. Figure 3b shows the best solution found for Dublin with 25% Prosumers, and a loss factor of 8%.

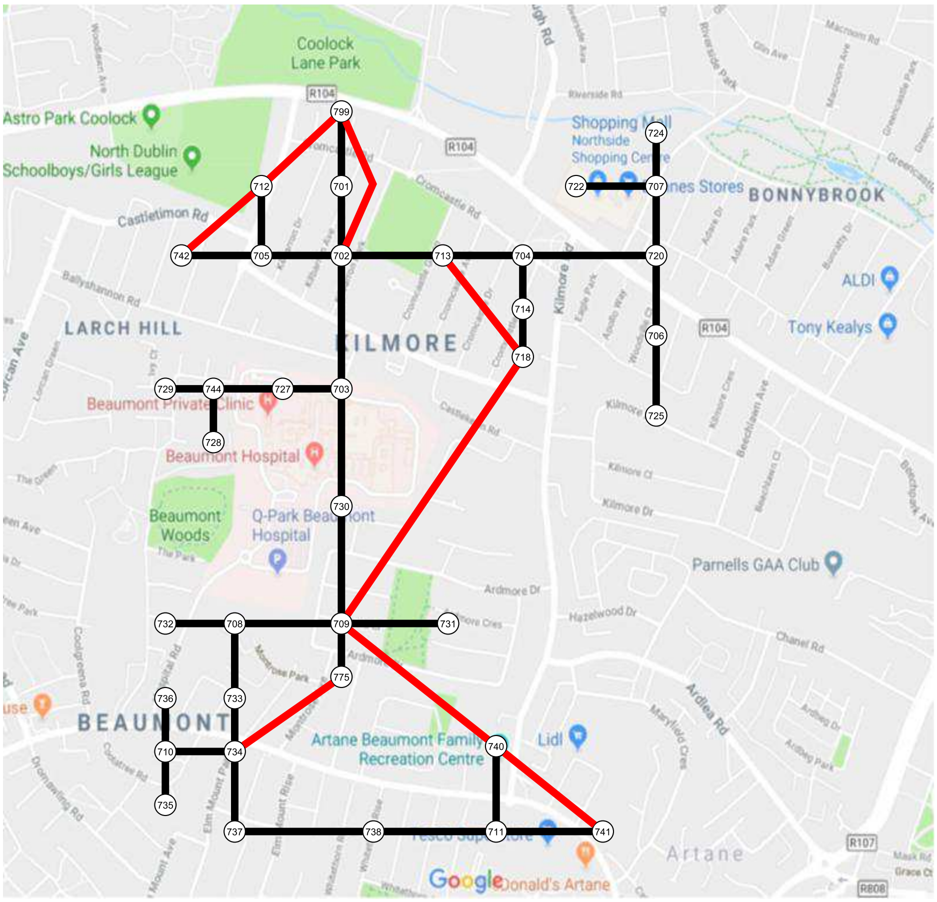
We see that problem instances with a low percentage of prosumers are solved to optimality in relatively short run times. As the proportion of prosumers increases the test instances become more difficult to solve. There is an increase in the solution values as the loss factor increases. The initial LP relaxation is quite weak. The Dublin Winter instances are solved to optimality, and reflect the low availability of excess electricity from prosumers. In contrast, the Aveiro and Dublin Summer instances are more challenging problems when excess electricity from prosumers is availably to satisfy consumers in the community, or to return to the grid via the substation root node.

6 CONCLUSIONS

In this paper we have described a smart grid topology problem which focuses on augmenting the grid topology in order to take into account new demands as some energy consumers become energy producers: prosumers. We propose a MIP model to augment the existent grid topology and identify which potential arcs could be added to support new electricity flows. The computational results show that the run times are short when the



(a) Aveiro, 30% Prosumers, loss 15%.



(b) Dublin, 25% Prosumers, loss 8%

Figure 3: Sample Smart Grid Solutions.

Table 1: Smart Grid Computational Results

Name	Loss%	%Prosumer	Status	Bestbound	BestMIP	Gap	Nodes	Time (s)	♯New Flows
Aveiro	8	20	Optimum	2115	2115	0.00	1	69	2
Aveiro	15	20	Optimum	2219	2219	0.00	0	74	2
Aveiro	8	25	Unfinished	3575	4333	0.17	7548	10885	7
Aveiro	15	25	Unfinished	3673	4320	0.15	9271	10876	5
Aveiro	8	30	Unfinished	3927	5263	0.25	5532	10838	7
Aveiro	15	30	Unfinished	4034	4883	0.17	6700	11200	8
DublinSummer	8	20	Optimum	2305	2305	0.00	265	250	3
DublinSummer	15	20	Optimum	2416	2416	0.00	217	279	3
DublinSummer	8	25	Unfinished	3520	3983	0.12	16750	10836	8
DublinSummer	15	25	Unfinished	3592	4085	0.12	19103	10872	8
DublinSummer	8	30	Unfinished	3731	4367	0.15	14798	10982	13
DublinSummer	15	30	Unfinished	3813	4684	0.19	10216	11095	10
DublinWinter	8	20	Optimum	2994	2994	0.00	1157	956	3
DublinWinter	15	20	Optimum	3159	3159	0.00	2213	1223	3
DublinWinter	08	25	Optimum	4141	4141	0.00	1815	1579	7
DublinWinter	15	25	Optimum	4298	4298	0.00	1929	1786	7
DublinWinter	8	30	Optimum	4351	4351	0.00	3803	5025	8
DublinWinter	15	30	Optimum	4506	4506	0.00	5231	6291	8

percentage of prosumers and Loss factor are low. The problem instances get harder as these parameter values are increased.

Our MIP model yields interesting results that could be used by distribution system operators and energy collectives to explore the potential of solar PV to meet RES targets and sustainability objectives. Our models could be used to perform cost benefit analysis of upgrades, or to understand potential electricity exchange flows in the network, and to understand where devices to control and record the electricity flows may need to be added.

There is potential for future work to improve the MIP model with additional valid inequalities. Other variants of the model could focus on rewarding prosumers with a higher price for excess electricity shared among the energy community, compared with excess returned to the grid via the substation root node, or alternatively new reverse arcs from prosumers to the substation may not be considered. In our computational experiments, we used a distance measure as a proxy for the arc installation costs c_{ij} in Eq (1), and no financial penalty or reward for flows within the community network. Further evaluation of the model could test weightings and alternative costs of the objective function components. In addition, since the arc installation costs substantially exceed network flow costs, a hierarchical model could provide a useful alternative to evaluate potential scenarios.

In our computational tests, we allowed a certain proportion of the nodes to act as prosumers and assumed a 2kW panel/prosumer. In further testing we may choose to only allow those nodes with high potential for solar PV to act as prosumers, i.e., those nodes with south or west facing orientations should be selected to serve the energy community, rather than those with east-west orientations, and decisions on the size of the PV panel could be considered. Such choices are of interest to policy makers and give rise to questions on the social acceptance of energy community designs.

Finally, as noted, the resulting meshed networks are more resilient, but give rise to more complex management problems such as those seen in works on bounded rings in telecommunications networks. A research agenda in the network optimisation community to share and exploit its learnings on network design and evolution could help advance the energy transformation and provides many interesting research opportunities.

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