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A Study on the Effects of Damage Models and Wavelet Bases for Damage Identification and Calibration in Beams

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Abstract: *Damage detection and calibration in beams by wavelet analysis involve some key factors such as the damage model, the choice of the wavelet function, the effects of windowing and the effects of masking due to the presence of noise during measurement. A numerical study has been performed in this paper addressing these issues for single and multispan beams with an open crack. The first natural modeshapes of single and multispan beams with an open crack have been simulated considering damage models of different levels of complexity and analyzed for different crack depth ratios and crack positions. Gaussian white noise has been synthetically introduced to the simulated modeshape and the effects of varying signal to noise ratio have been studied. Wavelet based damage identification technique has been found to be simple, efficient and independent of damage models and wavelet basis functions, once certain conditions regarding the modeshape and the wavelet bases are satisfied. The wavelet based damage calibration is found to be dependent on a number of factors including damage models and the basis function used in the analysis. A curvature based calibration is more sensitive than a modeshape based calibration of the extent of damage.*

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1. INTRODUCTION

The importance of structural health monitoring and damage detection of structures has increased significantly in recent times. A major focus in this field is the successful detection of the presence, location and the extent of damage present in a structure through new methodologies (Nicholson and Alnefaie, 2000; Kim and Zhao, 2004). Identification of damage in a freely vibrating beam with an open crack by observing the changes in natural frequencies is considered to be a popular method in the time domain (Christides and Barr, 1984; Narkis, 1994; Shen and Pierre, 1994; Chondros et al., 1998; Carneiro and Inman, 2002). These changes are often quite small, the damage location is not detected and the performance is poor in the presence of noise.

Non-parametric identification of structural systems has been advanced significantly incorporating wavelet analysis in conjunction with advanced computation concepts (Jiang and Adeli, 2005; Adeli and Jiang, 2006) and the efficiency of such identification have been experimentally verified (Jiang and Adeli, 2007). Efficient wavelet based denoising technique aiding non-parametric identification of structures has been achieved through the use of Bayesian wavelet packet denoising (Jiang et al., 2007).

Wavelet based vibration control of structures have also been efficiently demonstrated to be of wide application in recent times. The applications of such control systems include buildings, Tuned Liquid Column Dampers (TLCD) on buildings and cable stayed bridges for the suppression of wind and earthquake induced vibrations (Kim and Adeli, 2005a, 2005b, 2005c, 2005d; Adeli and Kim, 2004). A method for the time-

frequency analysis of earthquake records has also been proposed by Zhou and Adeli (2003).

Structural damage detection using wavelet analysis has been considered by a number of researchers (Zhang et al., 2001; Ovanesova and Suarez, 2004; Kim and Melhem, 2004; Zhu and Law, 2006). Damage detection using spatial data in conjunction with wavelet analysis has found considerable importance of late. The principles behind such wavelet based damage detection relate to the detection of singularities in a function or in any of its derivatives. The locations of the singularities are related to the local extrema of the wavelet coefficients propagating at finer scales in the neighbourhood of the same singularities. Detection of singularities in the time domain using wavelets have been previously carried out by a number of researchers (Moyo and Brownjohn, 2002; Robertson et al., 2003). The magnitude of the local extrema at the singularity locations relate to the extent of the sudden change in the signal or its derivatives due to the presence of a singularity (Mallat, 2001). Mallat emphasized that although a wavelet transform is able to locate singularities in a signal, there is no certainty of the absence of a rupture of the propagation of maxima at finest scales. In the case of Gaussian wavelets however the non-existence of a rupture can be guaranteed. Gentile and Messina (2003) carried out a study on wavelet based damage detection focussing mainly on a number of wavelet basis functions including the derivatives of a Gaussian and Symlets. The damage was modelled as an equivalent sub beam having a modified Young's modulus to cater for the sudden change at the damage location. Loutridis et al. (2004) used Symlet basis function to identify damage in a cracked cantilever beam using a rotational spring damage model. Chang and Chen (2003) and Chukwujekwu-Okafor and Dutta (2000)

have considered similar problems concentrating on a single wavelet basis function. Spatial response data from beam structures have been successfully analysed by wavelets to detect damage by Wang and Deng (1999). Advantages of wavelet analysis over the usual eigenvalue analysis for a simply supported beam with non-propagating open crack were shown by Liew and Wang (1998).

It is observed that although the effectiveness of wavelet analysis in damage detection is comparatively well dealt with, most of the works deal with the identification of the location of crack using a single basis function. Very few studies exist on the comparative performance of the wavelet basis functions, windowing and the effects of noise to detect the presence, identify the location and subsequently calibrate the damage. The effects of damage models have not been widely studied either. It is thus felt that there is a necessity of comparing the performance of different wavelet basis functions and damage models for damage detection and calibration in structures incorporating windowing and different levels of presence of noise.

This paper considers a simply supported Euler Bernoulli beam with an open crack. Three damage models of different levels of complexity and details have been considered to simulate the modeshape data. Subsequent analysis of the modeshape using different wavelet basis functions is performed and the effects of windowing and presence of noise are studied in details. Cracks of different sizes and locations have been used in the examples. An extension of the damage identification for multispan beams is also presented. The effectiveness of a wavelet based damage identification and calibration has been shown considering the variations of the factors stated above. Damage calibration of

a simply supported phenolic beam with an open crack has been carried out experimentally using this technique.

2. DAMAGE MODELS

2.1 Lumped Crack Model

The lumped crack model is popular among several researchers (Narkis, 1994; Chukwujekwu-Okafor and Dutta, 2000; Chang and Chen, 2003; Tian et al., 2003; Loutridis et al., 2004; Lam et al., 2005) looking at the problem of identification of the location of an open crack through wavelet based techniques. Figure 1 provides a general arrangement of such an open crack. The beam with an open crack is modelled as two uncracked beams connected through a rotational spring at the location of crack by assuming that the effects of the crack are applicable in the immediate neighbourhood of the crack location. The length of the beam is L with the damage located at a distance of 'a' from the left hand support of the beam. The crack depth is taken as 'c' and the overall depth of the beam is 'h'. The governing free vibration equation is

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = 0 \quad (1)$$

where E , I , A and ρ are the Young's modulus, the moment of inertia, the cross sectional area and the density of the material of the beam on either side of the crack. The displacement of the beam from its static equilibrium position is $y(x,t)$, at a distance of x from the left hand support along the length of the beam at any time t . Continuity in displacement, moment and shear are assumed at the location of crack. A slope discontinuity present at the location of the crack is modelled as

$$\Phi_R'(a) - \Phi_L'(a) = \theta L \Phi_R''(a) \quad (2)$$

where term θ is the non-dimensional crack section flexibility dependent on the crack depth ratio, $\delta(=c/h)$. The term $\Phi_{(.)}$ denotes the modeshape of the damaged beam and a prime represents differentiation with respect to x . As per Narkis (1994), the term θ is considered to be a polynomial of δ as

$$\theta = 6\pi\delta^2(h/L)(0.5033 - 0.9022\delta + 3.412\delta^2 - 3.181\delta^3 + 5.793\delta^4) \quad (3)$$

The free vibration equation (Equation 1) can be solved by the method of separation of variables and subsequently forming the eigenvalue problem from there.

2.2 Continuous Crack Model

A more detailed and complex continuous crack model is considered following Carneiro and Inman (2002). The model is derived from the stationarity of the Hu-Washizu-Barr functional (Christides and Barr, 1984) and is a refined version of the proposed model by Shen and Pierre (1994) ensuring the self-adjointness of the differential operator for a symmetric matrix representation after the discretization of the free vibration equation. For a rectangular cross section, the stress-strain and the displacement functions are assumed to be locally disturbed in the vicinity of the crack. The effect of the crack is considered maximum at the crack tip and decays exponentially away from it. The stress/strain disturbance function (f_1) and the displacement disturbance function (f_2) are

$$f_1(x, z) = [z - m_1(z + c/2)H(h/2 - c - z)]e^{-\alpha_1 \frac{|x-a|}{h/2}} \quad (4)$$

and

$$f_2(x, z) = [z - (z + c/2)H(h/2 - a - z)]e^{-\beta_1 \frac{|x-a|}{h/2}} \quad (5)$$

respectively. The term m_1 is a factor computed considering the continuity of bending moment in the cracked section and α_1 and β_1 are the stress and displacement decay parameters respectively. The term $H(\cdot)$ is the unit step function. The Cartesian coordinates x , y and z are along the length, breadth and depth of the beam respectively with the origin being at the midpoint of the extreme left hand section. The equation of motion for the free vibration of the beam considering these kinematic assumptions is given as

$$E[p_2(x)\xi(x,t)'''] + E[p_1(x)\xi(x,t)'] + \rho A \ddot{\xi}(x,t) = 0 \quad (6)$$

where $\xi(x,t)$ is the vertical displacement of the beam and the primes and overdots represent differentiation with respect to the space and time respectively. The terms $p_1(x)$ and $p_2(x)$ are given in Appendix 1. The term E is the Young's modulus of the undamaged beam. Equation 6 can be solved using Galerkin technique (Shen and Pierre, 1994) by considering the chosen solution to be a linear combination of a large number undamaged orthogonal modeshapes and subsequently satisfying the equation to arrive at an eigenvalue problem.

2.3 Smearred Crack Model

The smeared crack model is relatively simple and considers an open crack reducing the moment of inertia over an affected width. The governing free vibration equation is the same as in Equation 1. The damaged beam is analysed as an assembly of three sub-beams, the damaged sub-beam being positioned in between the two undamaged ones. Continuity in deflection, slope, moment and shear are assumed on both left (x_1) and right (x_2) ends of

the damaged zone. The width of the crack is computed according to the formula by Bovsunovsky and Matveev (2000) as

$$\Delta x = x_2 - x_1 = \frac{0.3675h(1-\delta)}{1-(1-\delta)^3} [(1-\delta)^6 - 3(1-\delta)^2 + 2] \quad (7)$$

A discontinuity in the modeshape or in any of its derivatives is present in a damaged beam for any model of crack. The first modeshape of the beam with an open crack is simulated as it is convenient to measure the fundamental modeshape for real structures.

3. WAVELET ANALYSIS

The continuous wavelet transform of a square integrable function $f(x)$ can be represented as

$$Wf(b,s) = \int_{-\infty}^{+\infty} f(x) \frac{1}{\sqrt{s}} \psi^* \left(\frac{x-b}{s} \right) dx \quad (8)$$

where the wavelet basis function $\psi(x)$ is a zero average function (Mallat, 2001) and s and b are the scale and the translation parameters respectively. The function $\psi(x)$ also ensures a weak admissibility condition

$$\int_0^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < +\infty \quad (9)$$

The identification of a discontinuity in a function or any of its derivatives can be linked with the number of vanishing moments of the wavelet basis function chosen for analysis. For a wavelet with no more than m number of vanishing moments, it can be shown that for very small values of s in the domain of interest, the continuous wavelet

transform of a function $f(x)$ can be related to the m^{th} derivative of the signal (Mallat, 2001). The relationship between the continuous wavelet transform of $f(x)$ and its m^{th} derivative can be expressed as

$$\lim_{s \rightarrow 0} \frac{Wf(b, s)}{s^{m+1/2}} = K \frac{d^m f(x)}{dx^m} \quad (10)$$

where

$$\int_{-\infty}^{+\infty} \zeta(x) dx = K \neq 0 \quad (11)$$

and $\zeta(x)$ is a function satisfying

$$\psi(x) = (-1)^m \frac{d^m \zeta(x)}{dx^m} \quad (12)$$

It has been observed in Section 2 that all the three damage models considered, when solved, introduce a singularity in the static/dynamic deflected shapes or in any of its derivatives at the location of damage. Since the wavelet analysis is capable of isolating singularities in a given signal or in any of its derivatives, the method is expected to locate the damage correctly when the damaged static/dynamic deflected shape of the structure is available from any of the damage models considered. Since the simulated first modeshape of a beam with an open crack contains discontinuity in the derivative/s independent of a physically admissible damage model at the location of damage, it is possible to identify the location of the damage through wavelet transform by incorporating a basis function having an appropriate number of vanishing moments (Gentile and Messina, 2003; Pakrashi et al., 2005).

4. DAMAGE IDENTIFICATION

Damaged modeshape data is simulated and Gaussian white noise is synthetically introduced for a beam of length 1 m. The cross sectional area (A), depth (h) and the moment of inertia (I) of the square beam are taken as 0.0001 m^2 , 0.01 m and $8.33 \times 10^{-10} \text{ m}^4$ respectively. The Young's modulus (E) and the density of the beam (ρ) are assumed to be $190 \times 10^9 \text{ N/m}^2$ and 7900 kg/m^3 respectively.

4.1 Basis Independence

The first modeshape was simulated from the lumped crack model for an open crack ($\delta=0.35$) situated at 0.4m from the left hand support and was analyzed using wavelet transform for different wavelet basis functions. Figure 2 shows the results of the analysis for some selected bases (Haar, the fourth derivative of a Gaussian, Coif4 and Morlet) considering both fine (2, 4) and coarse (16, 32) scales. It is observed that as long as the bases satisfy the criterion of the required number of vanishing moments, they can successfully detect the presence and the location of the damage consistently at both fine and coarse scales. Since Haar has only one vanishing moment, a jump of the wavelet coefficients at the location of the crack is observed instead of a local extremum, as seen in Figure 2 incorporating other wavelets with more than one vanishing moment. An analytical expression of the jump size of wavelet coefficients for the current problem using Haar basis has been provided by Pakrashi et al. (2005) considering a lumped crack model.

4.2 Model Independence

Figure 3 shows the results of analyzing the same damage using Coif4 wavelet basis function using different damage models. The location of the damage is found irrespective of the model chosen. However, it is observed that the magnitude of the local extremum at the damage location is different for different damage models.

4.3 Windowing

Windowing of the modeshape data (the multiplication of the modeshape data with a function which is significant only within a finite interval) and subsequent wavelet analysis improves the damage detection process. Bartlett, Hamming, Hanning, Gaussian and Bohman windows were considered for different damage models and crack depth ratios. The Haar basis is seen to be compatible best with a Bartlett window, while the smoother functions showed very good performance with Hanning window (Figures 4a-4b). The Bartlett window itself has a discontinuity in its first derivative at the midpoint and this leads to a problem of possible non-detection when the damage is near the midpoint. It is not possible to detect a damage if its position is exactly at the midpoint since it is not possible to distinguish between jumps resulting from the presence of a singularity in the window from a singularity present in the signal (Figure 4c).

4.4 Masking

The presence of noise in the modeshape function to be analyzed presents a major difficulty for the damage identification problem. Since the nature of the damage present in the modeshape is similar to that of the noise in terms of singularities present, it is quite difficult to identify the damage in the presence of high noise. The local extremum formed

in the wavelet coefficient plot due to the presence of an open crack can get masked partially or completely (Figures 5a-5b). A partial masking is present considering Coif4 basis and Hanning Window in Figure 4a with an edge crack ($\delta=0.35$) situated at 0.1m from the left hand support for a signal to noise ratio (SNR) 95 dB (the decibel being defined as ten times the logarithm of the ratio of the signal power to that of the noise), while a complete masking can be observed for the same crack at 75 dB. Finer scales are affected by masking at a lower SNR than coarser scales.

4.5 Multispan Beams

The effectiveness of a wavelet transform based damage detection problem was extended to symmetric and asymmetric multi-span beam structures for a lumped crack model. Selected results from the simulations and subsequent wavelet analysis of two span beams are presented. The two extreme ends of the two span beam have simply supported condition. The intermediate support is restrained against any displacement and the slopes on either side of the support are equal to each other. The cross sectional area and the moment of inertia of the square beam are 1m^2 and 0.0833m^4 respectively. The Young's modulus and the density of the beam are assumed to be $23 \times 10^9 \text{ N/m}^2$ and 2300 kg/m^3 respectively. Coif4 wavelet basis function and Hanning window have been employed for the analysis of the first modeshape.

It is observed that an edge crack situated near the intermediate support is structurally more important than the ones near the two ends of a two span simply supported beam and it is necessary for the wavelet based methodology to identify the crack near the intermediate support more effectively. Figures 6a-6b consider the first

modeshape of a beam with two equal span lengths of 10m. Figure 6a shows successful damage detection for a beam with a crack ($\delta=0.35$) situated 0.5m away from the intermediate support at 75 dB, while in Figure 6b the damage is masked for the same crack situated 0.5m from the left support for the same SNR. A slope discontinuity near the intermediate support affects the modeshape much more than a similar discontinuity near the two ends. As a result, a damage near the intermediate support is detected more efficiently. However, the intermediate support itself also renders a local extremum at its location after wavelet analysis of the modeshape and the magnitude of the extremum can be high enough to undermine the actual damage and lead to a problem of non-detection. A possible solution to this is to window each span separately and perform a wavelet analysis to each partially windowed section.

The possibility of reduced efficiency of damage identification to a considerable extent due to asymmetry in span lengths is studied. The length of the damaged left span is reduced to 5m. Both large and small cracks are considered at a distance of 0.5m from the intermediate support at 90 dB SNR. Crack depth ratios 0.05 and 0.35 are considered and it is observed from Figure 7a-7b that the effect of asymmetric span length does not contribute significantly in terms of damage identification.

5. DAMAGE CALIBRATION

The extent of a local extremum of the wavelet coefficients at the location of damage is an indicator of the extent of damage. However, the calibration does depend on the basis function, the damage model and the SNR. Examples are presented in this section for the single span beam considered before in this paper.

5.1 Basis Function Dependence and Effects of Noise

A lumped crack is considered at a distance of 0.4m and 0.1m respectively from the left hand side of the simply supported beam. The first windowed (Hanning) modeshape is analyzed using different wavelet basis functions at scale 4 and the crack depth ratios are calibrated against the wavelet coefficient maxima values at the damage location in the presence of noise (Figure 8). The calibration curves for different bases are significantly different and the consistency is affected for small edge cracks in the presence of higher levels of noise because of masking.

5.2 Damage Model Dependence

The same crack with $\delta=0.35$ is identified using Coif4 basis function and Hanning windowing for the crack models presented in this paper for scale 32 (Figures 9a-9b). The SNR is considered to be 120dB. The extent of the wavelet coefficient extrema near the damage location is significantly different. The extremum for the smeared crack model is found to be nearly an order lower. The lumped crack model is found to be numerically more efficient. It conforms to the detailed, realistic and numerically most expensive continuous crack beam model and yet retains the ease of computation.

5.3 Curvature Based Calibration

An alternative way of calibrating damage is by transforming the curvature, rather than the modeshape of the vibrating beam with an open crack. The curvature was numerically computed from the lumped crack model and a calibration was performed considering

scales 4 and 16 with the SNR being at 120 dB for cracks located at 0.1m and 0.4m from the left edge. It is observed that the absolute value and the relative change for a curvature based calibration are better than a calibration based on the modeshape, as shown in Figure 10.

5.4 Experimental Validation

Experiments were carried out on a 0.925m long phenolic beam. An open crack was notched into the lower section of the beam at a distance of 0.46m from the left hand side. The cross section of the beam is 50mm x 12mm. Three damage conditions ($\delta = 0.167, 0.33$ and 0.5) were considered for the beam. Figure 11 shows the general arrangement of the experimental setup. A static weight has been put on the beam to deflect the beam and the deflected photograph has been recorded in the spatial domain by an Olympus μ 800 digital camera. The photograph is converted and saved as a bitmap image of size 240 x 320 pixels and subsequently converted to black and white binary images by thresholding and the edges of the images have been found using the Sobel method (Sarfaraz, (2005)) incorporating MATLAB 7.0 signal processing toolbox. The lower edge of the beam was detected from the image by an intelligent pattern recognition scheme (Pakrashi et al. (2006)).

The deflected shape was multiplied by a Hanning window of length equal to that of the deflected shape and was analysed by Coif4 wavelet basis function. The damage was successfully calibrated in terms of the magnitude of the wavelet coefficient extremum at the damage location. Figure 12 shows one such calibration for a static load 7.5N.

6. CONCLUSION

A wavelet based methodology for detection and calibration of an open crack in beams has been studied in detail numerically. Damage models of various levels of complexity have been considered to model an open crack and the first natural modeshape has been simulated for both single span and multi span beams. A calibration of damage on a phenolic simply supported beam with progressively increasing open crack has been experimentally performed. This study on wavelet based damage detection and calibration can be helpful for any general open crack problem in structures and provides a guideline for the choice of basis functions and windows. The study also provides an idea for a reliable and successful calibration of damage due to the presence of noise.

It is observed that a wavelet based identification of the location of damage is independent of damage models and basis functions so long as the damage introduces a singularity in the damaged modeshape or in any of its derivatives at the location of the damage and the wavelet bases have a certain number of vanishing moments. The effects of different windows in conjunction with different wavelet bases for damage detection are found. A case of non-detection due to Bartlett windowing in conjunction with Haar basis function has been identified. The presence of noise in the modeshape data is seen to mask the identification of damage and the finer scales are affected at a higher value of SNR as compared to coarser scales. Structurally important edge crack for multispan beams has been detected successfully. The damage detection using wavelet analysis is seen to be chiefly affected by the position, SNR and the extent of damage. On the other hand, a wavelet based calibration is seen to be additionally dependent on the basis

function, scale and the damage models apart from SNR, position of crack and the extent of crack. A damage calibration based on curvature values is seen to be more sensitive in comparison with that based on modeshape. The wavelet based damage detection method is observed to have a definite potential for practical applications where the damaged modeshapes or deflected shapes are obtained with sufficient accuracy.

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- Figure 4b. Damage detection using fourth derivative of a Gaussian in conjunction with Hanning window.
- Figure 4c. A case of non-detection of damage for Haar basis in conjunction with Bartlett window.
- Figure 5a. Partial masking of damage due to the presence of low noise.
- Figure 5b. Full masking of damage due to the presence of high noise.
- Figure 6a. Damage detection in a two span symmetric beam with a crack near the intermediate support.
- Figure 6b. Inability of Damage detection in a two span symmetric beam with a crack near the left edge.
- Figure 7a. Damage detection for small crack in an asymmetric two span beam.
- Figure 7b. Damage detection for a large crack in an asymmetric two span beam.
- Figure 8. Effect of noise and Wavelet Bases on wavelet based damage calibration.
- Figure 9a. Dependence of damage models for a wavelet based damage calibration at finer (scale 4) scales.
- Figure 9b. Dependence of damage models for a wavelet based damage calibration at coarser (scale 32) scales.

Figure 10. Wavelet based damage calibration on computed curvature of damaged beam with an open crack.

Figure 11. General arrangement of experimental setup for damage detection on a phenolic beam.

Figure 12. Experimental Calibration of Damage Using Static Deflected Shape.

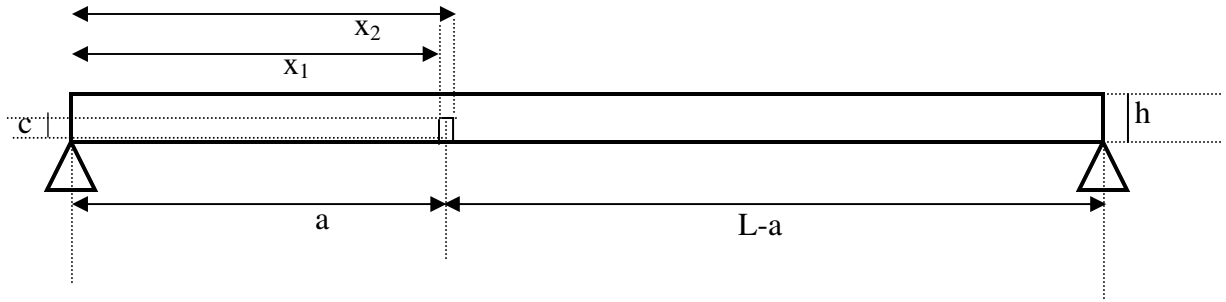


Figure 1.

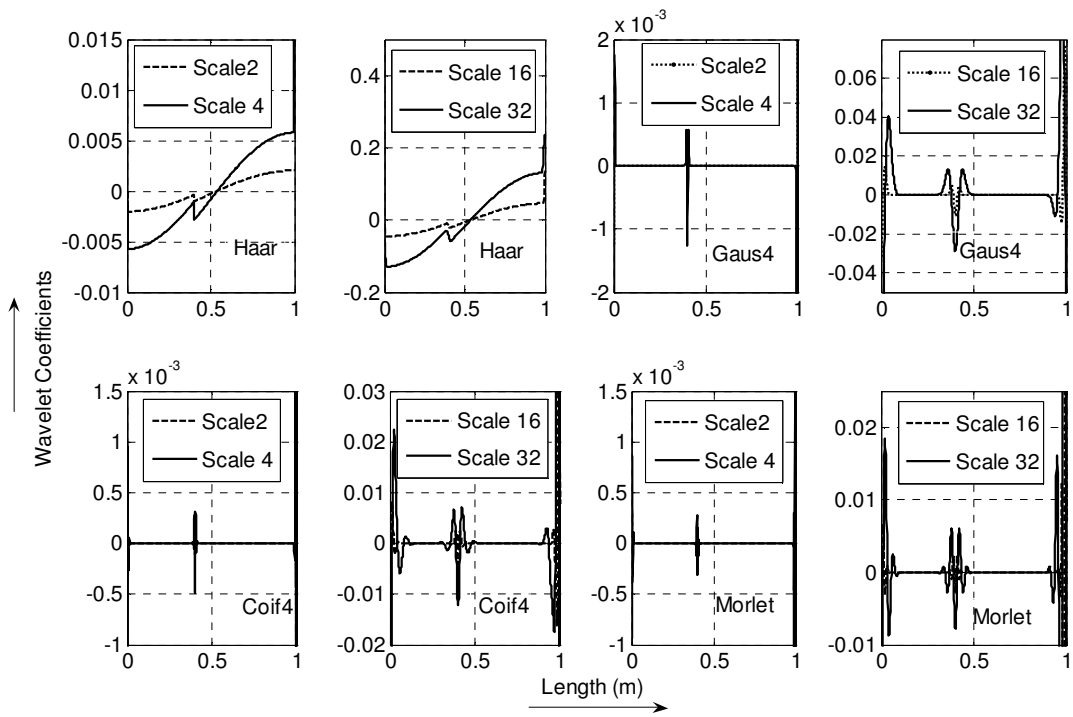


Figure 2.

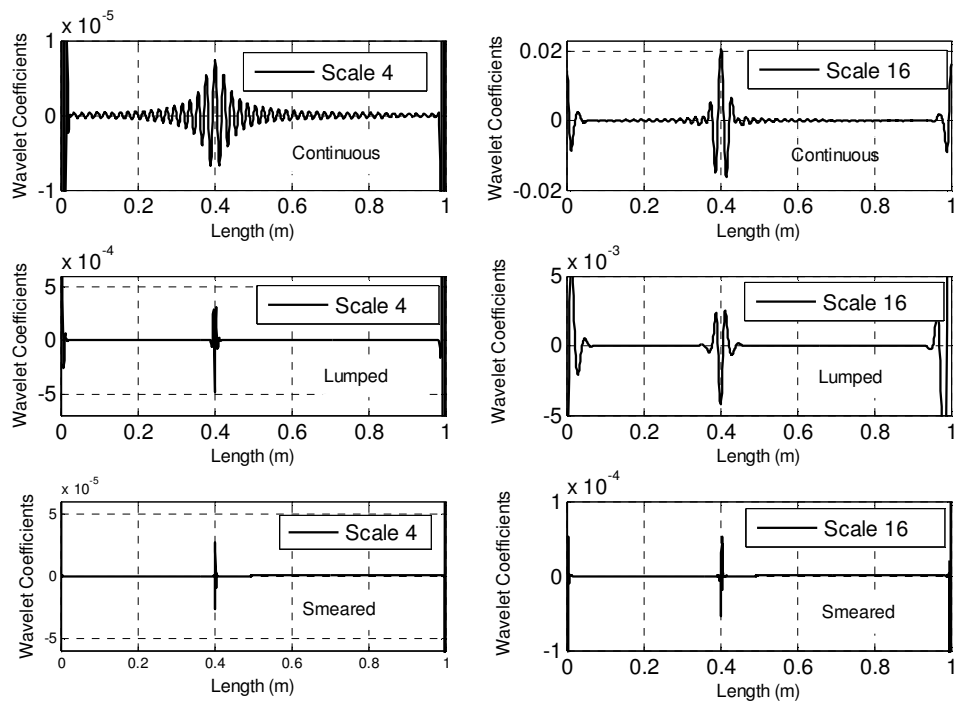


Figure 3.

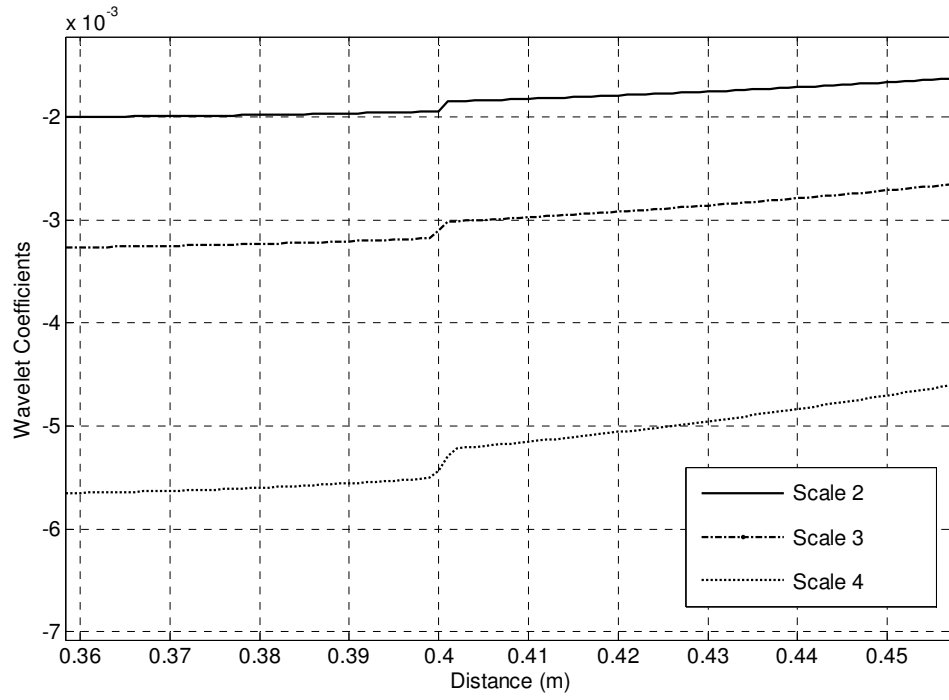


Figure 4a.

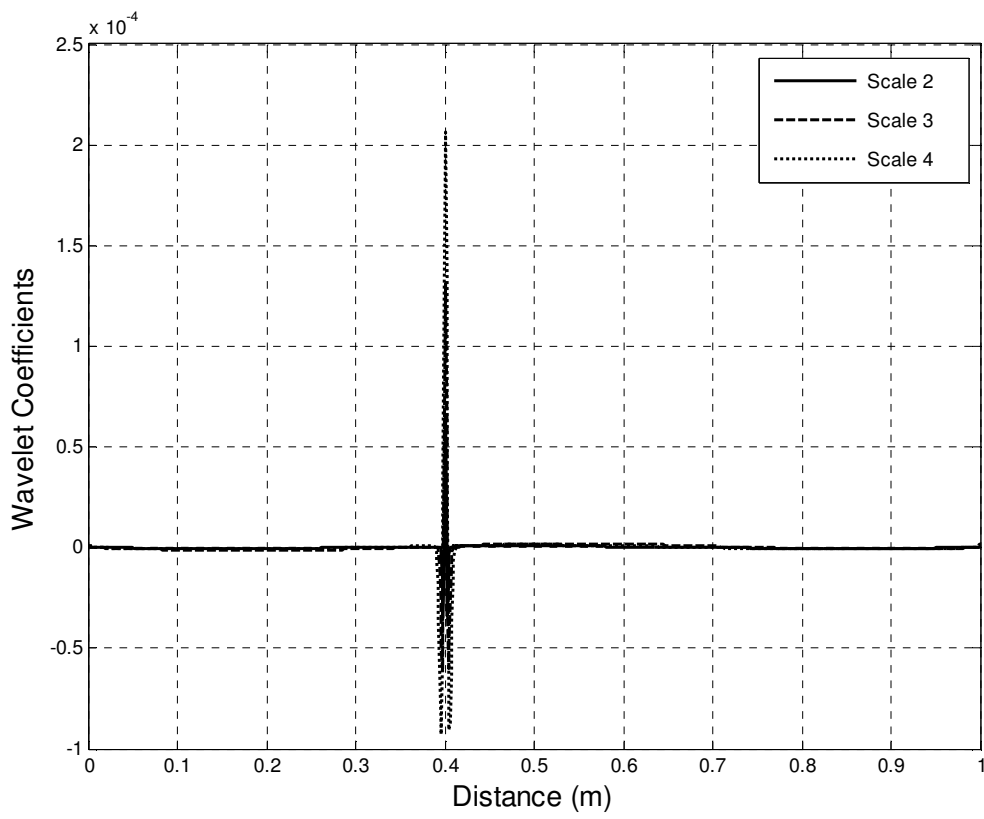


Figure 4b

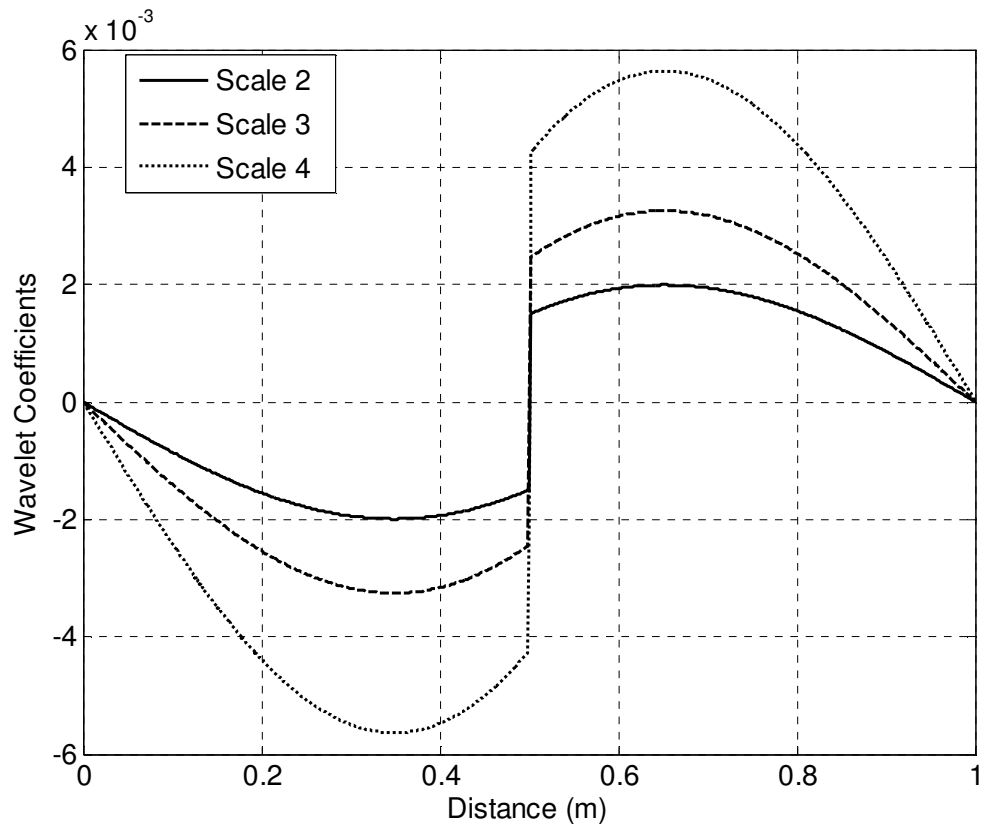


Figure 4c.

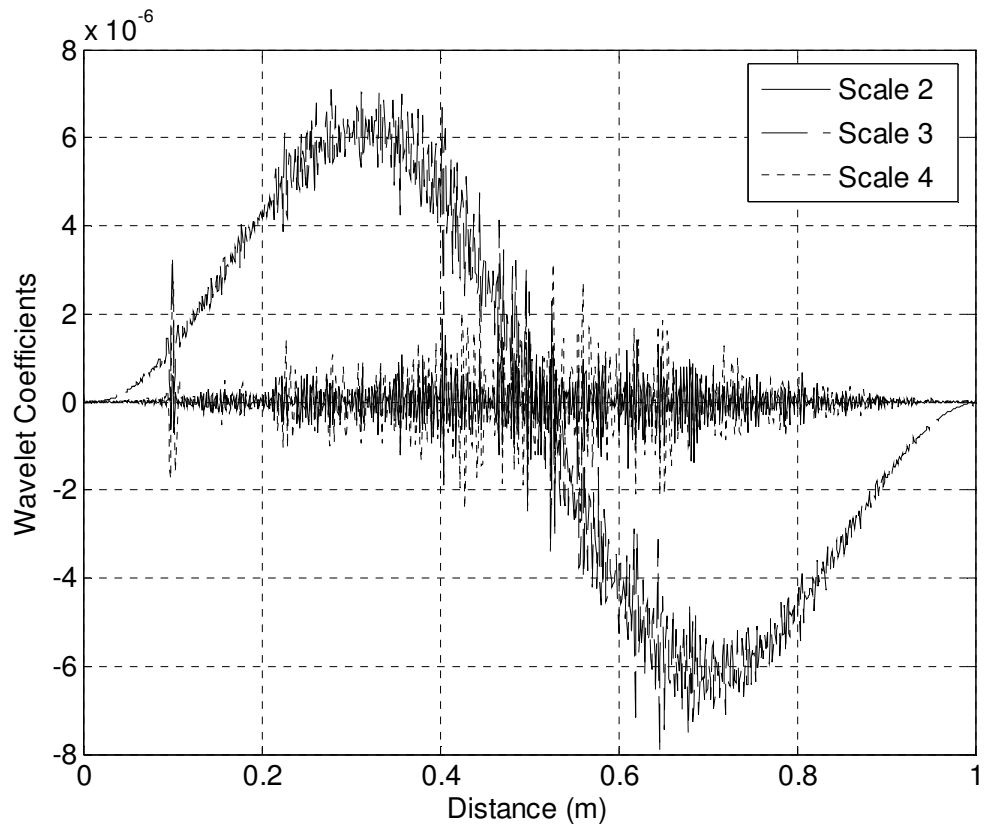


Figure 5a.

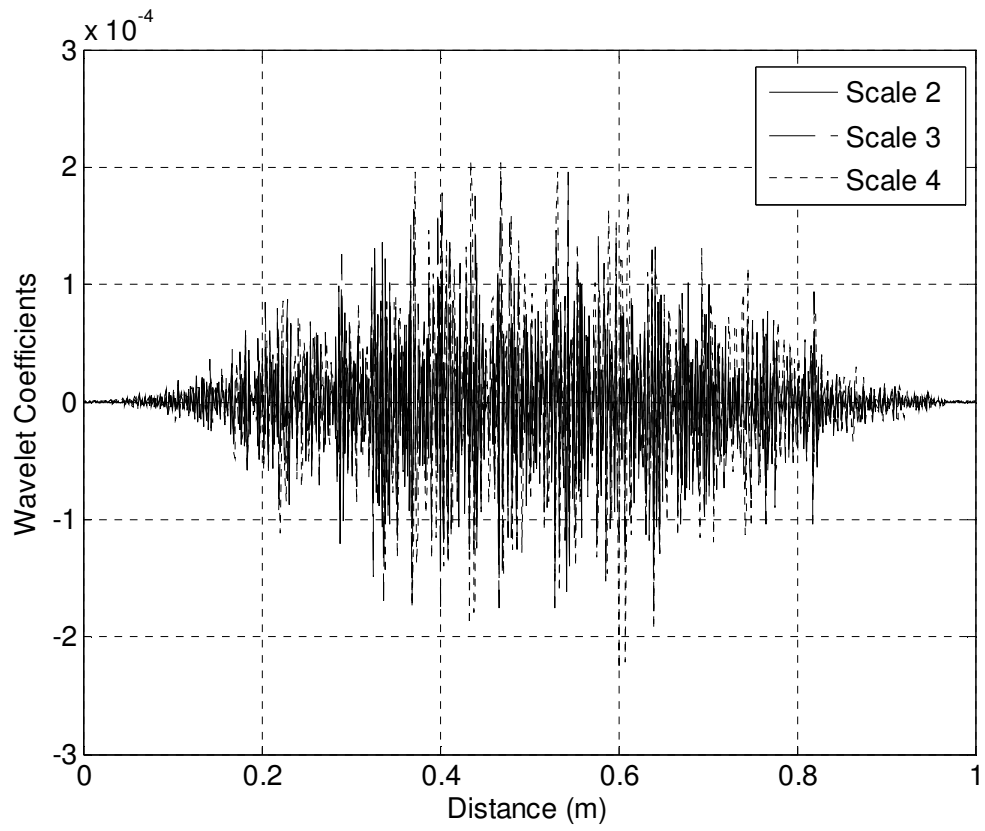


Figure 5b.

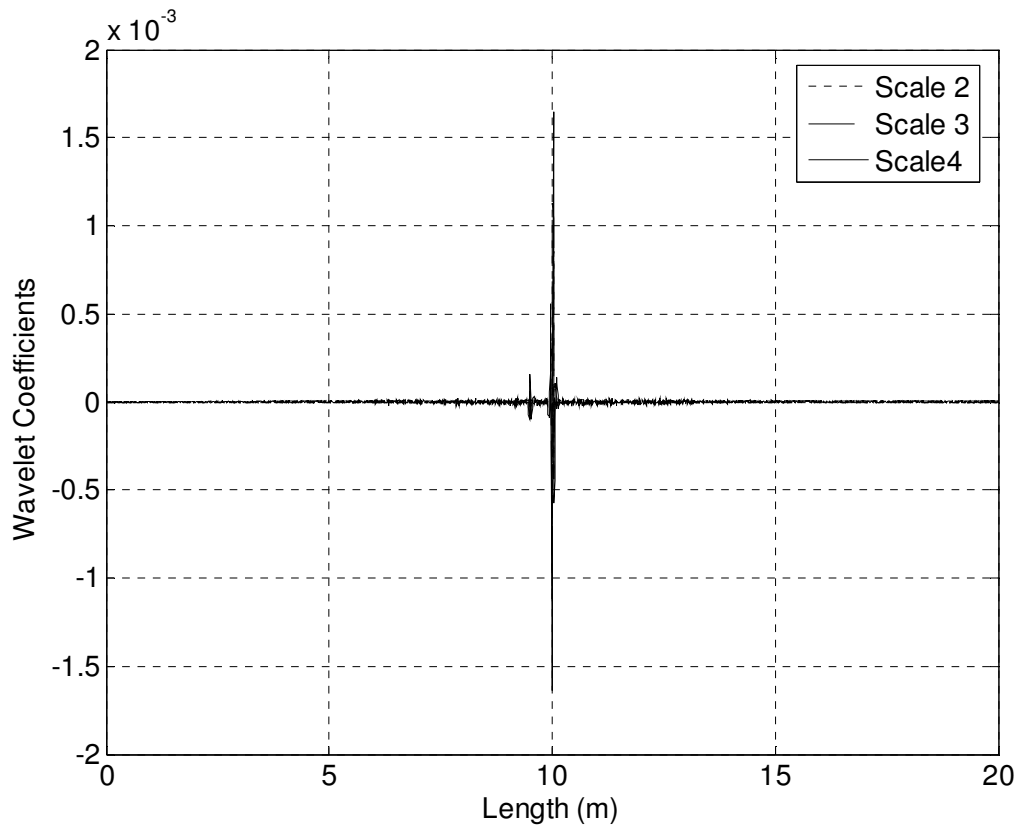


Figure 6a.

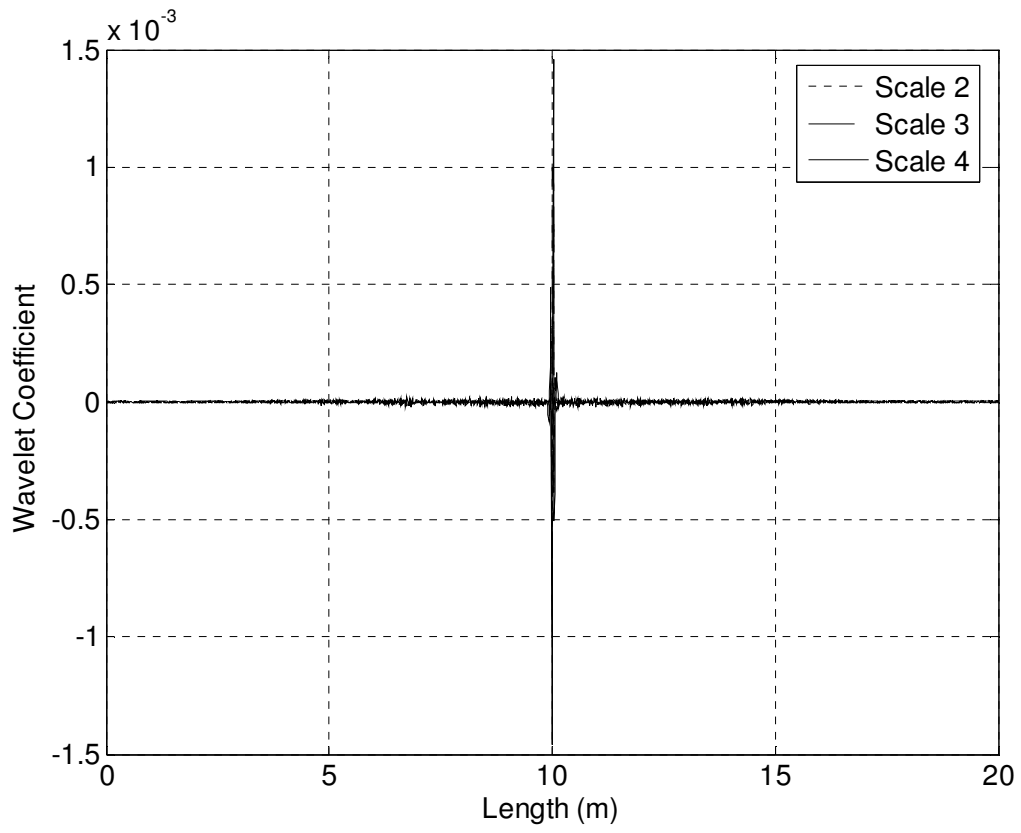


Figure 6b.

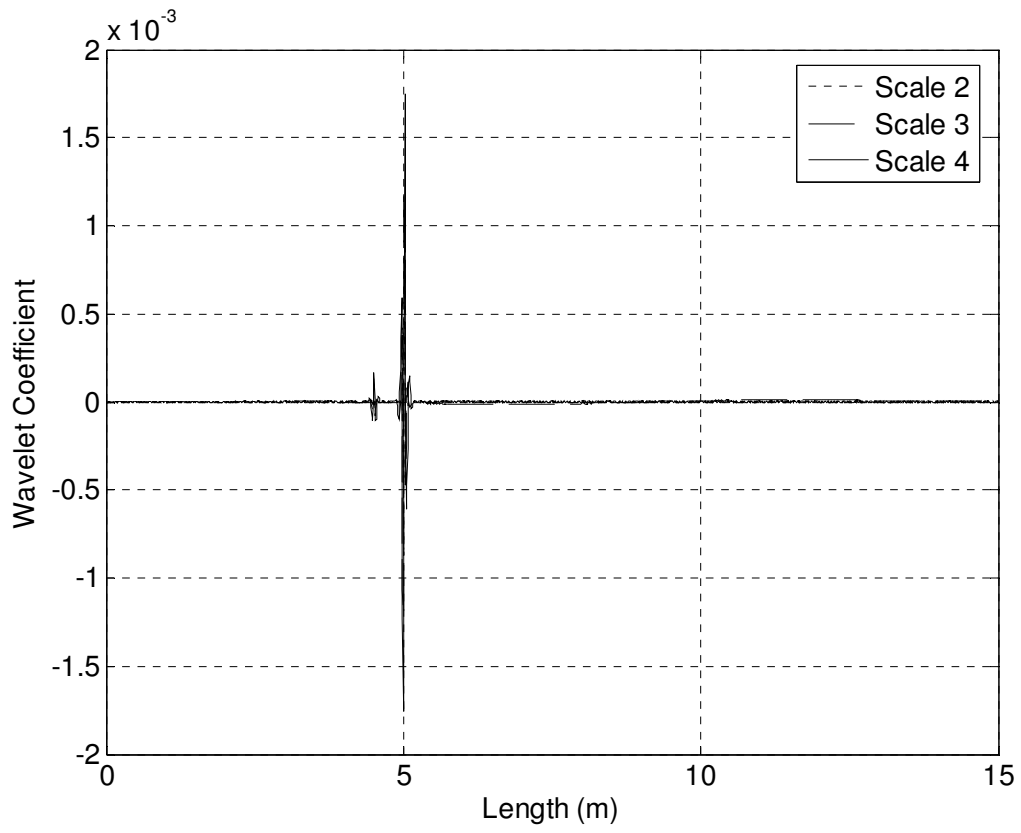


Figure 7a.

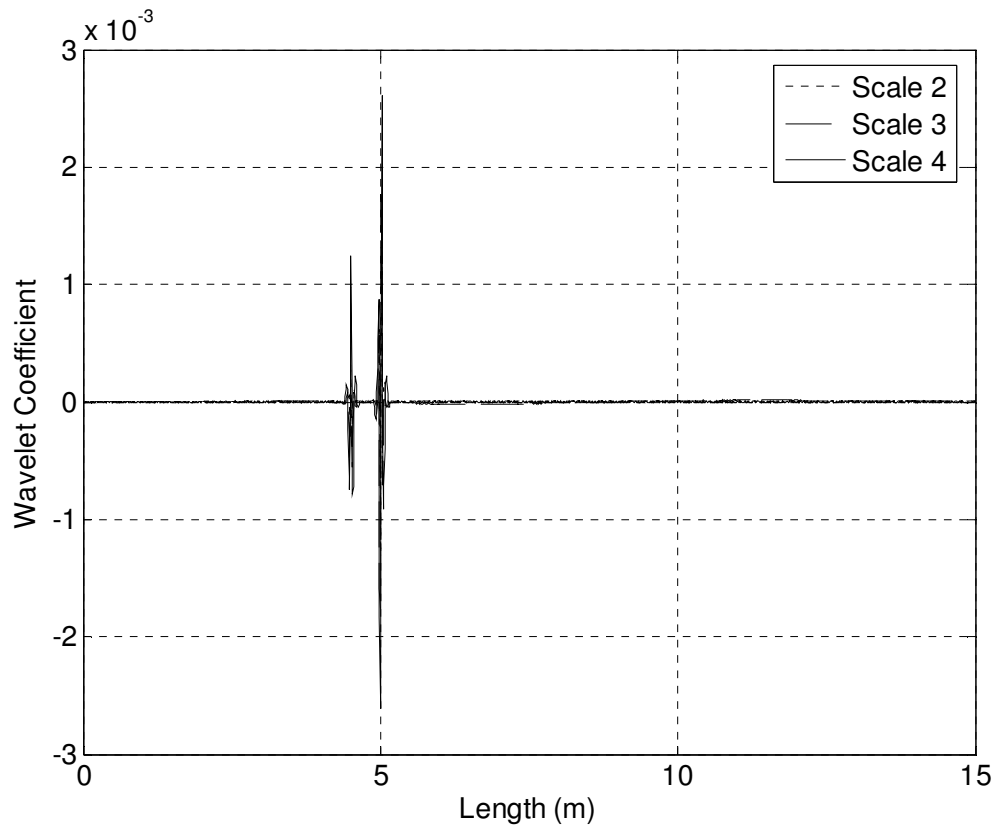


Figure 7b.

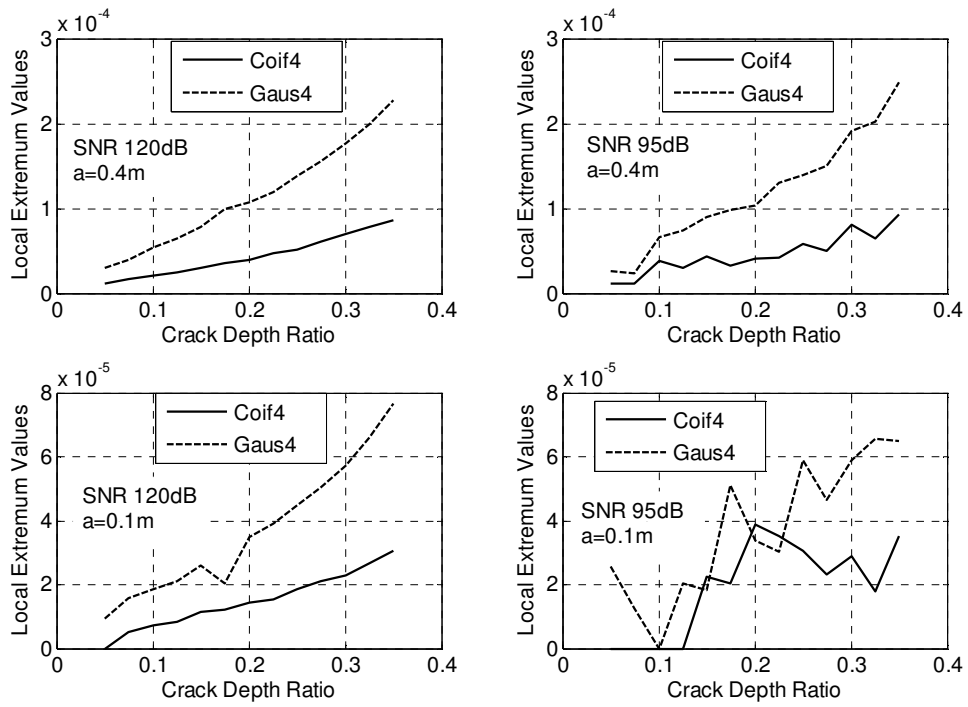


Figure 8.

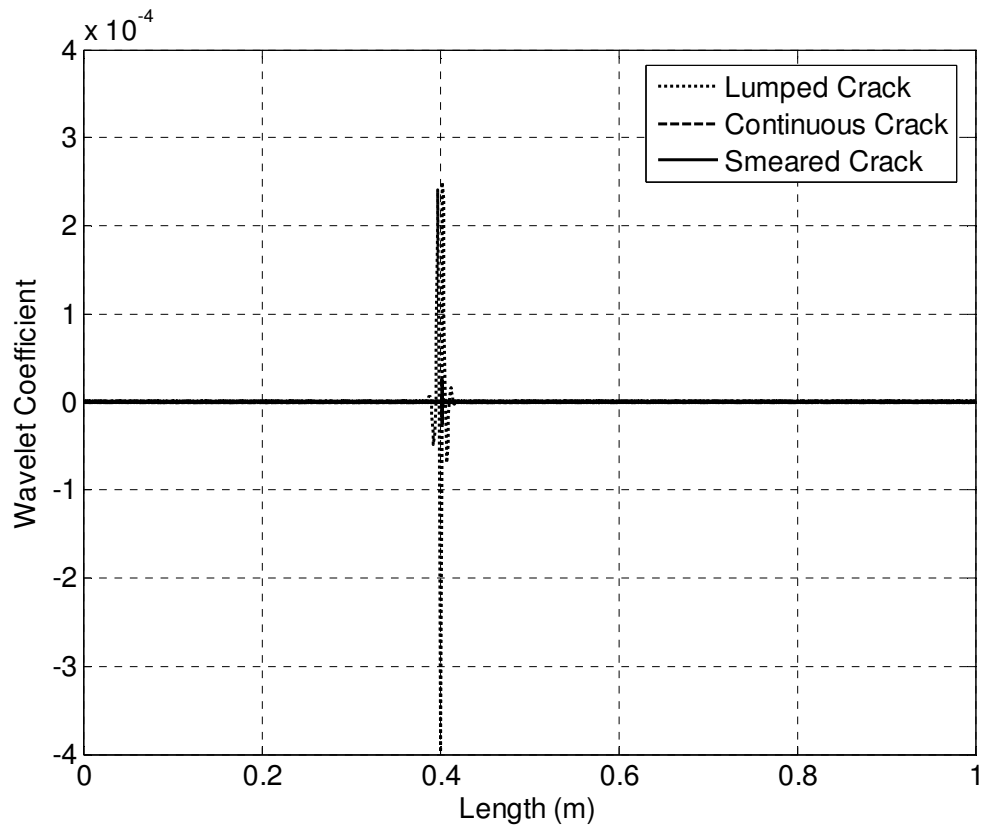


Figure 9a.

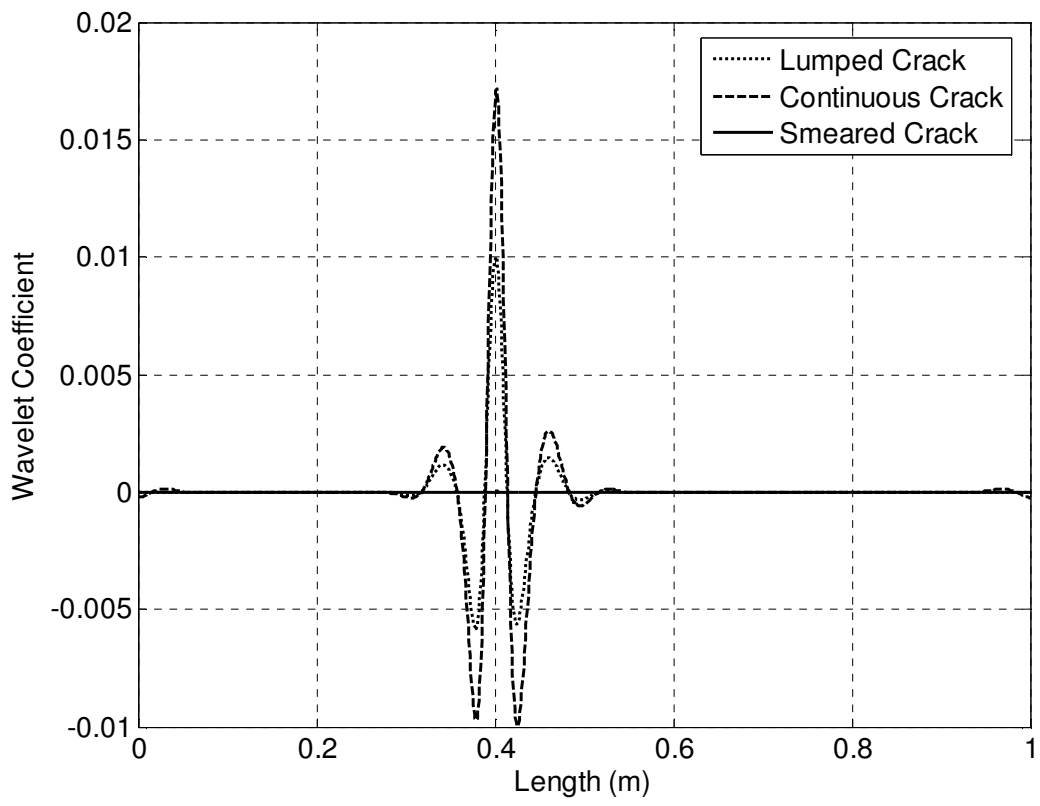


Figure 9b.

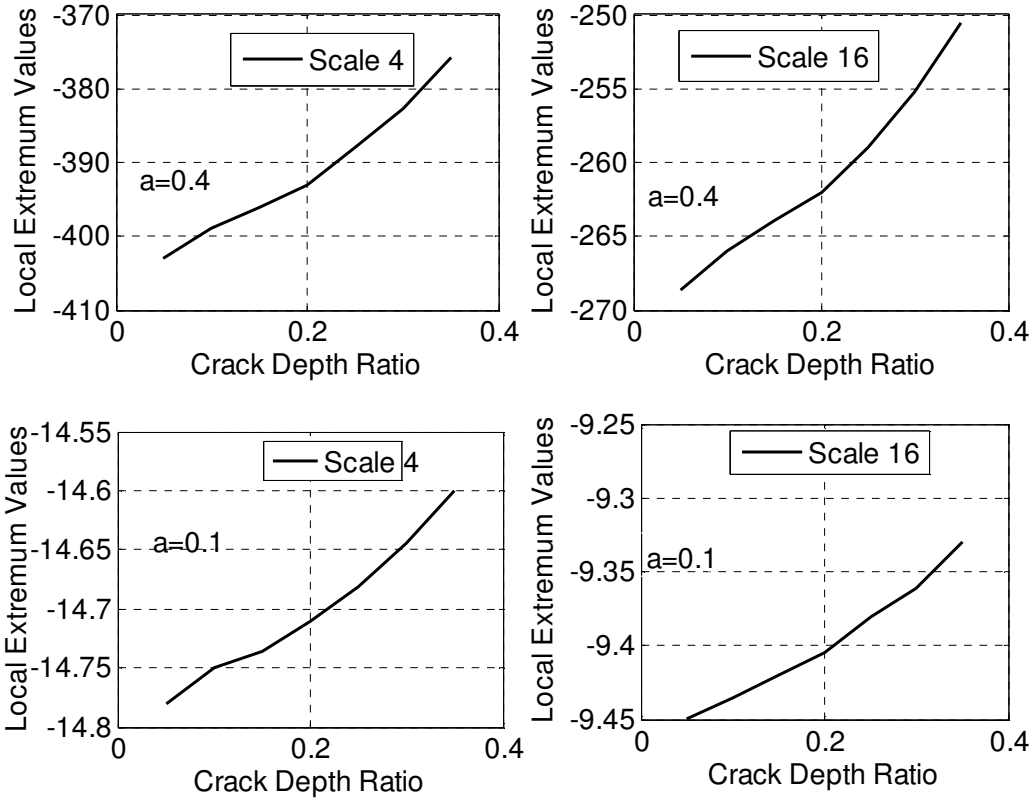


Figure 10.

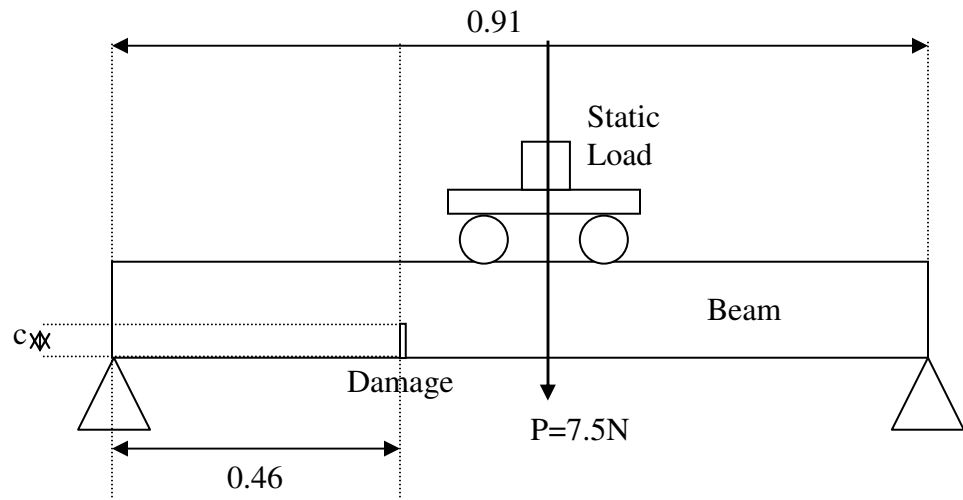


Figure 11.

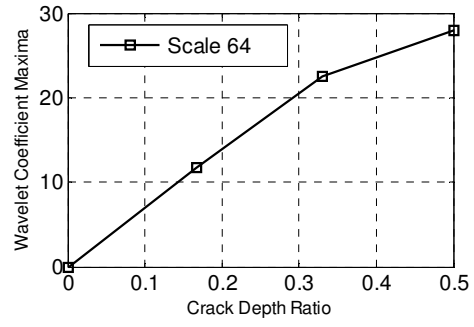
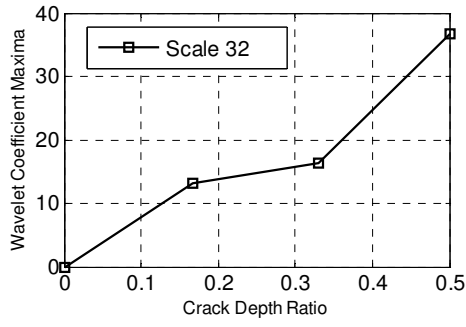
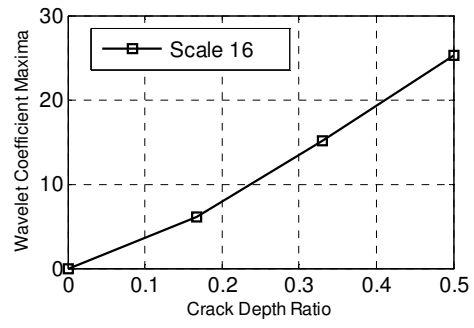
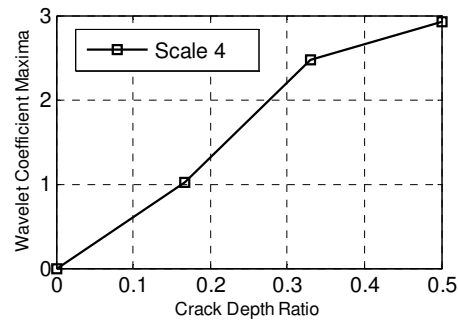


Figure 12.

APPENDIX 1. COEFFICIENTS $p_1(X)$ AND $p_2(X)$ FOR THE CONTINUOUS
CRACKED MODEL

$$\begin{aligned}
 p_1(x) = E[& \{ I - \int_A z f_1(x, z) dA - \int_A z f_2(x, z) dA + \int_A f_1(x, z) f_2(x, z) dA \} \left(\frac{\int_A f_1(x, z) f_2'(x, z) dA - (\int_A z f_2(x, z) dA)'}{I - 2 \int_A z f_1(x, z) dA + \int_A f_1^2(x, z) dA} \right)' + \\
 & \{ (\int_A f_2(x, z) f_1'(x, z) dA)' - (\int_A z f_1(x, z) dA)' \} \left(\frac{\int_A f_1(x, z) f_2'(x, z) dA - (\int_A z f_2(x, z) dA)'}{I - 2 \int_A z f_1(x, z) dA + \int_A f_1^2(x, z) dA} \right)]
 \end{aligned}
 \tag{A1}$$

$$\begin{aligned}
 p_2(x) = [& I - \int_A z f_1(x, z) dA - \int_A z f_2(x, z) dA + \int_A f_1(x, z) f_2(x, z) dA] \left(\frac{I - \int_A z f_1(x, z) dA - \int_A z f_2(x, z) dA + \int_A f_1(x, z) f_2(x, z) dA}{I - 2 \int_A z f_1(x, z) dA + \int_A f_1^2(x, z) dA} \right)
 \end{aligned}
 \tag{A2}$$