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# Optimal DR and ESS Scheduling for Distribution Losses Payments Minimization Under Electricity Price Uncertainty

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**Abstract**—The distribution network operator is usually responsible for increasing the efficiency and reliability of network operation. The target of active loss minimization is in line with efficiency improvement. However, this approach may not be the best way to decrease the losses payments in an unbundled market environment. This paper investigates the differences between loss minimization and loss payment minimization strategies. It proposes an effective approach for decreasing the losses payment considering the uncertainties of electricity prices in a day ahead energy market using energy storage systems and demand response. In order to quantify the benefits of the proposed method, the evaluation of the proposed technique is carried out by applying it on a 33-bus distribution network.

**Index Terms**—Active losses, demand response, energy storage system, robust optimization, uncertainty.

## NOMENCLATURE

For quick reference, the main notation used throughout the paper is stated in this section.

### A. Sets and Indices

$i$	Index for network buses.
$\ell$	Index for network feeders.
$t$	Index for operation intervals.
$\Omega_L$	Set of lines in distribution network
$\Omega_{ESS}$	Set of nodes containing ESS
$\Omega_{DR}$	Set of nodes participating in demand response
$\Omega_n$	Set of all network nodes
$\Omega_T$	Set of time periods

### B. Parameters

$\theta_{ij}$	Angle of $ij^{th}$ element of admittance matrix.
$\Gamma$	Conservativeness degree of decision maker regarding the price uncertainty.
$\epsilon_i$	Curtail-able percent of energy of demand in node $i$ .
$\eta_{ch/dch}$	Efficiency of charging and discharging of ESS (%).
$\lambda_t^{f/a}$	Forecast/actual value of electricity price at time $t$ (\$/MWh).
$(P/Q)_{i,t}^{D0}$	Initial active/reactive demand of node $i$ at time period $t$ without demand response (MW).

$\bar{\Lambda}$	Maximum number of nodes allowed to participate in demand response.
$V_{min/max}$	Maximum/minimum voltage magnitude (pu).
$\bar{I}_\ell$	Maximum feeder capacity (A).
$\gamma_i^{max/min}$	Maximum/minimum demand flexibility at node $i$ .
$ES_i^{max/min}$	Maximum/minimum energy stored at node $i$ (MWh).
$P_i^{ch,max/min}$	Maximum/minimum power charge of ESS at node $i$ (MW).
$P_i^{dch,max/min}$	Maximum/minimum power discharge of ESS at node $i$ (MW).
$\lambda_t^{max/min}$	Maximum/minimum bounds of electricity price at time $t$ (\$/MWh).
$Y_{ij}$	Magnitude of $ij^{th}$ element of admittance matrix (pu).
$\Delta_t^\pm$	Positive/negative deviation of actual price from the forecasted price (\$/MWh).
$\tilde{\lambda}_t$	Uncertain electricity price at time $t$ (\$/MWh).

### C. Variables

$(P/Q)_{i,t}^{D/G}$	Active/reactive demand of node $i$ at time period $t$ with demand response (MW).
$\omega_t, \zeta_t, \Upsilon$	Auxiliary variables.
$\Lambda_i$	Binary decision variable indicating whether node $i$ participates in demand response or not.
$P_{i,t}^{ch/dch}$	Charge/discharge power of ESS at node $i$ at time period $t$ .
$I_{\ell,t}$	Current flowing in feeder $\ell$ at time $t$ (A)
$\gamma_{i,t}$	Demand response decision variable of node $i$ at time period $t$ .
$ES_{i,t}$	Energy stored in ESS at node $i$ at time period $t$ .
$(P/Q)_{i,t}^{net}$	Net active/reactive power injection to node $i$ at time period $t$ with demand response (MW).
$L_t^{ESS}$	Power losses in ESS at time $t$ (MW).
$\psi_t$	Total active power losses at time $t$ (MW).
$V_{i,t}$	Voltage magnitude at node $i$ at time period $t$ .
$\delta_{j,t}$	Voltage angle at node $i$ at time period $t$ .

## I. INTRODUCTION

### A. Background and Aim

**T**HE goal of the distribution network operator (DNO) is to maximize the efficiency of the network in its territory as well as monitoring and improving the technical condition of the network. The cost of electricity is directly linked to the efficiency of the transmission and particularly the distribution system. The financial treatment of losses is crucial in this regard. The role of DNO for dealing with active losses (as a measure of network efficiency) is different in each regulatory framework. In some countries like Denmark, France, Belgium, Austria and Germany the active losses are procured in wholesale market while in Ireland, Italy, UK and Portugal some incentive efficiency measure indicators are used [1]. There are different

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strategies to efficiency improvement of distribution networks such as scheduling the distributed energy resources (DER) [2], [3], capacitor switching, network reconfiguration [4], energy storage systems (ESS) [5], demand response (DR) [6], etc. The traditional strategy for DNO is to decrease active losses using the available options. In this paper, without loss of generality, among the wide range of performance improving actions, the focus is placed on ESS scheduling and DR. Demand response is referred to all actions (including energy storage devices management, energy reduction and demand shifting) to change the nominal demand pattern of the end-use consumers [6]. This paper proposes a method for optimal ESS and DR scheduling to minimize the active losses payments. This optimization has one important uncertainty source namely, electricity prices. There are different techniques to handle the uncertainties in decision making frameworks such as information gap decision theory (IGDT) [7], stochastic programming, fuzzy mathematics and robust optimization. These techniques are inherently different in nature and can't be easily compared with each other. Choosing the best technique among them depends on the uncertainty nature and available data about the uncertain parameters of the model. Using fuzzy techniques requires knowing membership functions. The stochastic models need to know the probability distribution function (PDF) of uncertain parameters and usually these techniques are computationally inefficient [8]. The IGDT framework is very conservative and may lead to over-estimated actions [8]. It is more suitable in severe uncertainty cases [9]. In this paper, robust optimization is used for handling this uncertainty. The gap that this paper tries to fill is to answer two questions:

- 1) "Loss minimization or loss payment minimization?". Which is the best strategy for efficiency maximization under price uncertainty?
- 2) How should it be done using DR and ESS?

### B. Literature Review

Different references referred to ESS and DR for increasing the efficiency and flexibility in distribution networks. The ESS are used to increase the network capacity for accepting new wind turbines [10], voltage regulation [11], maximizing revenue for non-firm distributed wind generation [12], energy management and power quality improvement [13] and loss reduction [10]. DR actions can also bring ancillary services to the grid [14], voltage control [15], active loss reduction [16] and better exploitation of renewable energy sources as well as a reduction of the customers' energy consumption costs with both economic and environmental benefits [17].

In [18], a heuristic algorithm is proposed to reduce the active losses costs reconfiguration of distribution networks. A distribution system expansion planning model which considers the construction/reinforcement of substations/feeders/capacitors banks and the radial topology modification was introduced in [19]. The optimal allocation of capacitor banks and DG units is found using the differential evolution algorithm in [20]. It is multi-objective and tries to optimize the cost of energy not supplied, reliability index, costs of energy losses and investment. The shortcoming of these models ([18]–[20]) is assuming the constant cost of energy losses as well as ignoring the

uncertainties associated with market prices. ESS and DR are not considered in them.

### C. Contributions

To the best knowledge of the authors of this paper, there is no reference addressing the impact of hourly electricity prices as well as their uncertainty on loss payment minimization actions. Given the discussed context, the contributions of this work are fourfold:

- 1) To provide a framework for economic efficiency increase for DNO.
- 2) To consider the uncertain electricity prices using robust optimization technique and converting the bi-level optimization into a single optimization problem.
- 3) To model the optimal scheduling of ESSs.
- 4) To quantify the benefits of DR for efficiency maximization.

### D. Paper Organization

The remainder of the paper is organized as follows. Section II describes the problem formulation. Section III presents the modelling features and assumptions made in the proposed decision making framework. Simulation results and discussions are presented in Section IV. Section VI concludes the paper.

## II. PROBLEM FORMULATION

### A. Assumptions

- The DNO is responsible for active loss procurement from day ahead electricity market [1]. The day ahead market mechanism is followed in many countries such as Ireland, Greece and Poland [21]. In this framework, the electricity prices are set based on market clearing mechanism one day in advance of actual operating point. The DNO is assumed to be price taker. However, in some regulatory frameworks like Nordic countries the real time and intraday balancing market [22] is used.
- The electricity prices of the day ahead market are subject to uncertainty. It is due to many different reasons like: competition between price maker generating units, contingencies of transmission network and generating units, volatile and uncertain renewable energy sources and demand uncertainty [23]. It is assumed that only limited information is available regarding the electricity prices (interval based modeling [24]). It is more explained in section III-A.
- The DNO is the owner of ESS and therefore responsible for controlling the operating schedules of ESS.
- The DNO has the authority for controlling demands in some specific nodes. This can happen using mutual agreement/contract [25] between the consumers and the DNO. The gained benefits of this agreement will be shared between the DNO and the consumers.

## B. Objective functions and constraints

In a generic active power losses minimization strategy, the following optimization problem is solved:

$$\min_{\mathbf{DV}} z = \sum_{t \in \Omega_T} \psi_t \quad (1)$$

$$\mathbf{F}(\mathbf{DV}, \Pi) \leq \mathbf{0} \quad (2)$$

$$\mathbf{G}(\mathbf{DV}, \Pi) = \mathbf{0} \quad (3)$$

$\psi_t$  in (1) is the hourly active loss.  $\mathbf{DV}$  and  $\Pi$  represent the decision variables and input parameters (price values and technical data), respectively.  $T$  denotes the operating horizon.  $\mathbf{F}$  and  $\mathbf{G}$  represent the inequality and equality constraints of the optimization framework as described in (5) to (22), respectively. In this paper, a new strategy is proposed that tries to minimize total payments related to the active power losses. Obviously, the optimal actions  $\mathbf{DV}$  directly depend on the input parameters ( $\Pi$ ) including price values for the day ahead market. The issue is that usually there is limited information about the electricity prices of the next day. The optimization problem can therefore be formulated as follows:

$$\min_{\mathbf{DV}} z = \sum_{t \in \Omega_T} (\psi_t \tilde{\lambda}_t) \quad (4)$$

$$\mathbf{F}(\mathbf{DV}, \Pi) \leq \mathbf{0}$$

$$\mathbf{G}(\mathbf{DV}, \Pi) = \mathbf{0}$$

$\tilde{\lambda}_t$  is the uncertain electricity price at time  $t$  in day ahead market.

The power flow equations to be satisfied  $\forall i \in \Omega_n, \forall t \in \Omega_T, \forall \ell \in \Omega_L$  are:

$$\psi_t = \sum_{i \in \Omega_n} P_{i,t}^{net} + L_t^{ESS} \quad (5)$$

$$P_{i,t}^{net} = P_{i,t}^G - P_{i,t}^D - P_{i,t}^{ch} + P_{i,t}^{dch} \quad (6)$$

$$Q_{i,t}^{net} = Q_{i,t}^G - Q_{i,t}^D \quad (7)$$

$$P_{i,t}^{net} = V_{i,t} \sum_{j \in \Omega_n} Y_{ij} V_{j,t} \cos(\delta_{i,t} - \delta_{j,t} - \theta_{ij}) \quad (8)$$

$$Q_{i,t}^{net} = V_{i,t} \sum_{j \in \Omega_n} Y_{ij} V_{j,t} \sin(\delta_{i,t} - \delta_{j,t} - \theta_{ij}) \quad (9)$$

$$V_{min} \leq V_{i,t} \leq V_{max} \quad (10)$$

$$I_{\ell,t} = Y_{\ell=ij} (|V_{i,t} \angle \delta_{i,t} - V_{j,t} \angle \delta_{j,t}|) \leq \bar{I}_{\ell} \quad (11)$$

where  $L_t^{ESS}$  is the power losses in ESS at time  $t$ .  $P_{i,t}^{net}, Q_{i,t}^{net}$  in (6) and (7) are the net injected active and reactive power to bus  $i$ , respectively.  $Y_{ij}, \theta_{ij}$  are the magnitude and angle of the  $i-j$ th element of admittance matrix, respectively.  $V_{i,t}, V_{min}, V_{max}$  in (10) are the voltage magnitude, min/max operating limits of each bus, respectively.  $I_{\ell}$  in (11) is the current passing through feeder  $\ell$  and  $\bar{I}_{\ell}$  in (11) is the maximum allowable current in feeder  $\ell$ .  $P_{i,t}^G, Q_{i,t}^G$  in (6) and (7) are the active and reactive power injected to the network by the DG units or grid connection.  $\Omega_n, \Omega_T, \Omega_L$  are the set of system nodes, operating hours, feeders, respectively.  $P_{i,t}^{ch/dch}$  is the charged/discharged power of ESS in (6).

The ESS technical operating constraints to be satisfied  $\forall i \in$

$\Omega_{ESS}$  &  $\forall t \in \Omega_T$  [26] are:

$$ES_{i,t} = ES_{i,t-1} + (\eta_{ch} P_{i,t}^{ch} - P_{i,t}^{dch} / \eta_{dch}) \Delta_t \quad (12)$$

$$ES_i^{min} \leq ES_{i,t} \leq ES_i^{max} \quad (13)$$

$$P_i^{ch,min} \leq P_{i,t}^{ch} \leq P_i^{ch,max} \quad (14)$$

$$P_i^{dch,min} \leq P_{i,t}^{dch} \leq P_i^{dch,max} \quad (15)$$

$$L_t^{ESS} = (1 - \eta_{ch}) P_{i,t}^{ch} + P_{i,t}^{dch} (1 / \eta_{dch} - 1) \quad (16)$$

where  $\Omega_{ESS}$  is the set of nodes which have ESS. The energy stored in ESS in time  $t$  and bus  $i$ ,  $ES_{i,t}$  depends on the energy stored in ESS in time  $t-1$  and the charging and discharging of the ESS ( $P_{i,t}^{ch}/P_{i,t}^{dch}$ ) which is described in (12).  $\eta_{ch}$  and  $\eta_{dch}$  are the charging and discharging efficiency of ESS, respectively.  $\Delta_t$  is the duration of time interval  $t$ . The stored energy in ESS should be kept between specific limits ( $ES_i^{max/min}$ ) as enforced by (13).  $ES_{i,t_0}$  is the initial value of stored energy in ESS. The charging and discharging limits of ESS are given in (14) and (15).

Demand response constraints for  $\forall i \in \Omega_{DR}$  are:

$$P_{i,t}^D = P_{i,t}^{D0} \times \gamma_{i,t} \quad (17)$$

$$Q_{i,t}^D = Q_{i,t}^{D0} \times \gamma_{i,t} \quad (18)$$

$$(1 - \gamma_i^{min} \Lambda_i) \leq \gamma_{i,t} \leq (1 + \gamma_i^{max} \Lambda_i) \quad (19)$$

$$\sum_{i \in \Omega_{DR}} \Lambda_i \leq \bar{\Lambda} \quad (20)$$

$$\sum_{t \in \Omega_T} P_{i,t}^D \Delta_t \geq (1 - \epsilon_i) \sum_{t \in \Omega_T} P_{i,t}^{D0} \Delta_t \quad (21)$$

$$\sum_{t \in \Omega_T} Q_{i,t}^D \Delta_t \geq (1 - \epsilon_i) \sum_{t \in \Omega_T} Q_{i,t}^{D0} \Delta_t \quad (22)$$

The set of demands participating in DR program is represented by  $\Omega_{DR}$ .  $(P/Q)_{i,t}^{D0}, (P/Q)_{i,t}^D$  specify the original/modified demand pattern without/with DR perturbation in (17), (18).  $\gamma_{i,t}$  denotes the decision variable for changing the demand pattern in (17),(18). The constraint (19) models the flexibility degree of the demands.  $\gamma_i^{max}$  and  $\gamma_i^{min}$  specify the maximum possible increase and decrease of demand in node  $i$ .  $\Lambda_i$  is a binary variable. If  $\Lambda_i = 0$  then the node  $i$  does not participate in a DR program and vice versa. The total number of nodes which can participate in a DR program are specified in (20) as  $\bar{\Lambda}$ . Although the demand pattern changes, the total energy consumption of the demand in node  $i$  is kept more than  $100 \times (1 - \epsilon_i)$  percent of its initial energy value (without DR) as imposed by (21) and (22). In other words,  $\epsilon_i$  is the curtail-able percent of energy of demand in node  $i$ . Without these equations ((21) and (22)), the DR decision variables ( $\gamma_{i,t}$ ) as defined in ((17) and (18)) would take their least possible values ( $\gamma_i^{min}$ ) for all time periods. It should be noted that these equations are valid for each node  $i \in \Omega_{DR}$ . This means that the energy of node  $i$  is redistributed in different time periods (not transferred to other nodes). In the current formulation, if  $\Lambda_i$  are given as constant input parameters then the model is a non-linear problem (NLP). This means the nodes participating in demand response are known in advance. It is also possible to find the optimal locations of nodes to participate in DR program. In this case, the resulting problem is a mixed integer non-linear problem (MINLP).

It is interesting to know how to determine the order of DR nodes with respect to their impact on energy losses payments. A technique to identify the merits of nodes for participating in DR is enumerating the total number of nodes ( $\bar{\Lambda}$ ) permitted to participate in DR ( $\Lambda$ ) from 1 to the number of load points. Then for the given number of permitted nodes ( $\Lambda$ ) the DR participating nodes are found using binary variables  $\Lambda_i$ . In each case, the optimal nodes (with  $\Lambda_i = 1$ ) are identified. The frequency of selection in each scheme specifies the merit of each node.

### III. PROPOSED STRATEGY

The optimization strategy is to find the optimal decision variables in such a way that the worst case cost is controlled for a given degree of conservativeness ( $\Gamma$ ). In this section, first the uncertainty modeling is introduced and finally the robust optimization based solution strategy is given.

#### A. Uncertainty modeling

There are several techniques available for modeling the uncertainty of electricity price in (4). These techniques include stochastic scenario modeling (Fig.1a ) [8], fuzzy modeling (Fig.1b ) [27] and robust optimization (Fig.1c) [28]. Using each technique requires certain information regarding the uncertain parameter. In stochastic scenario based modeling, the decision maker should be aware of probability density function of uncertain parameter. In fuzzy modeling the membership function of uncertain parameters should be known. The computational burden of these techniques are high and the obtained results are subject to risk. For example the actual realization of the uncertain parameter may deviate drastically from the expected value of the objective function. The robust optimization uses the

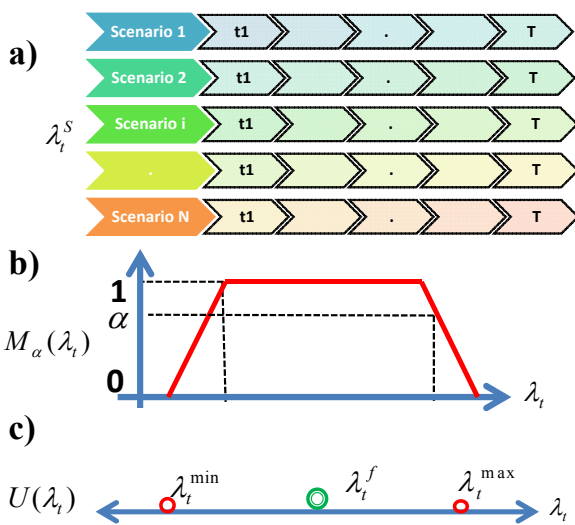


Fig. 1. a) Scenario based stochastic uncertainty modeling, b) Fuzzy based uncertainty modeling, c) Robust optimization based uncertainty modeling.

uncertainty sets for handling the uncertainties. One of the most frequently used uncertainty set is interval set. The uncertainty intervals can be found using different methods as follows:

- Using time series models (ARIMA) [29]
- Using Neural Networks

- Using expert opinion and historic data

The same technique has been used in the literature such as in [30]–[33]. It is formulated as follows:

$$\tilde{\lambda}_t \in U(\tilde{\lambda}_t) = \left\{ \tilde{\lambda}_t : \lambda_t^{\min} \leq \tilde{\lambda}_t \leq \lambda_t^{\max} \right\} \quad (23)$$

$\lambda_t^{\min}, \lambda_t^{\max}$  are the lower and upper bounds of  $\tilde{\lambda}_t$ , respectively. It is assumed that no information is available from day ahead market prices other than these bounds.

#### B. Robust optimization formulation

The idea of robust optimization is to minimize  $z$  in eq. (4) without knowing the exact values of  $\lambda_t$ . Additionally, the optimal decision making is done in a way that these actions still remain good (not optimal) even though the actual values ( $\lambda_t^a$ ) of uncertain parameters deviate (to some degree  $\Gamma$ ) from the forecasted values  $\lambda_t^f$ . Two cases may happen: first, the actual price  $\lambda_t^a$  is more than the forecasted price  $\lambda_t^f$ . The constraint for uncertainty modelling of the price can be expressed as:

$$\lambda_t^a = \lambda_t^f + \Delta_t^+ \omega_t \quad (24a)$$

$$\Delta_t^+ = \lambda_t^{\max} - \lambda_t^f \quad (24b)$$

$$0 \leq \omega_t \leq 1 \quad (24c)$$

where,  $\omega_t$  is the prediction error. The second case happens when the actual price  $\lambda_t^a$  is less than  $\lambda_t^f$  as:

$$\lambda_t^a = \lambda_t^f + \Delta_t^- \omega_t \quad (25a)$$

$$\Delta_t^- = \lambda_t^{\min} - \lambda_t^f \quad (25b)$$

As the decision maker seeks the robustness against the undesired events, the equations given in (25a), (25b) do not cause trouble. Actually the main concern of the decision maker is on the equations given in (24a), (24b) where the actual prices may be more than the forecasted values. Thus, the formulation expressed in equations (4), (23) can be replaced by the following one:

$$\min_{\mathbf{DV}} z = \sum_{t \in T} \psi_t \lambda_t^f + \psi_t \Delta_t^+ \omega_t \quad (26a)$$

$$0 \leq \omega_t \leq 1 \quad (26b)$$

$$\sum_{t \in T} \omega_t \leq \Gamma \quad (26c)$$

Subject to :

$$(5) \text{ to } (22)$$

$\Gamma$  in (26c) is a parameter specified by the decision maker which is also called the *conservativeness degree*. It denotes the maximum total deviation (robustness degree [28]) that can be tolerated. This parameter can take a value from 0 to 24 (increases with the conservativeness of the decision maker). For example, if  $\Gamma = 2$  this means that the algorithm will remain robust even though the maximum total prediction error is 100% in 2 hours or 50% in 4 hours of the day ahead market. The robust counter

part of (26) would become [24]:

$$\min_{\mathbf{DV}} z = \sum_{t \in T} \psi_t \lambda_t^f + \left\{ \begin{array}{l} \max_{\omega_t} \psi_t \Delta_t^+ \omega_t \\ \text{Subject to :} \\ (26b), (26c) \end{array} \right\} \quad (27)$$

Subject to :  
(5)to(22)

The formulation described in (27), requires to solve a bi-level optimization since the inner maximization tries to simulate the worst case realization of uncertain price (by changing  $\omega_t$ ) while the outer minimization attempts to decrease the undesired impacts of uncertain prices by controlling  $\mathbf{DV}$ . The decision variables in inner maximization is  $w_t$  and the constraints are the (26b), (26c). This is done in order to find the worst case condition of uncertainty in electricity prices that would cause the maximum increase in total payments. Once the optimal values of  $w_t$  are found, these values are passed to the outer minimization. The decision variables of this level are the network power flow, demand response and energy storage system constraints.

The complexity of the optimization block,  $\left\{ \begin{array}{l} \max_{\omega_t} \psi_t \Delta_t^+ \omega_t \\ \text{Subject to :} \\ (26b), (26c) \end{array} \right\}$  in (27), is linear with respect to  $\omega_t$  since the terms  $\psi_t \Delta_t^+$  are determined in the upper level of optimization. According to the duality gap theory [9], it is concluded that :

$$\max_{0 \leq w_t \in T} [\psi_{t_1} \Delta_{t_1}^+ \quad \dots \quad \psi_T \Delta_T^+] \begin{bmatrix} w_{t_1} \\ \vdots \\ w_T \end{bmatrix} \quad (28)$$

$$\begin{pmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \\ 1 & 1 & \dots & 1 & 1 \end{pmatrix} \begin{bmatrix} w_{t_1} \\ \vdots \\ w_T \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \\ \Gamma \end{bmatrix} \quad (29)$$

is equivalent to :

$$\min_{0 \leq \Upsilon, \zeta_t \in T} [1 \quad \dots \quad 1 \quad \Gamma] [\zeta_{t_1} \quad \dots \quad \zeta_T \quad \Upsilon]^T \quad (30)$$

$$\begin{pmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \\ 1 & 1 & \dots & 1 & 1 \end{pmatrix}^T \begin{bmatrix} \zeta_{t_1} \\ \zeta_{t_2} \\ \vdots \\ \zeta_{t_{T-1}} \\ \zeta_T \\ \Upsilon \end{bmatrix} \geq \begin{bmatrix} \psi_{t_1} \Delta_{t_1}^+ \\ \psi_{t_2} \Delta_{t_2}^+ \\ \vdots \\ \cdot \\ \cdot \\ \psi_T \Delta_T^+ \end{bmatrix} \quad (31)$$

where  $\zeta_i, \Upsilon$  are dual variables.

Using (28) to (31), the bi-level optimization described in (27) would transform into (32):

$$\min_{\mathbf{DV}} z = \sum_{t \in T} \lambda_t^f \psi_t + \sum_{t \in T} \zeta_t + \Upsilon \Gamma \quad (32a)$$

$$\Upsilon + \zeta_t \geq (\lambda_t^{max} - \lambda_t^f) \psi_t \quad (32b)$$

$$\Upsilon, \zeta_t \geq 0 \quad (32c)$$

Subject to :

$$(5)to(22)$$

The obtained single level optimization in (32) can be solved using decomposition technique [34] or Lagrange Relaxation approach [35]. It is obvious that the resulted single level optimization is easier to solve than the original bi-level optimization structure.

The decision variables ( $U$ ), parameters ( $\Pi$ ) and the sets are as follows:

$$DV = \left\{ \begin{array}{l} \psi_t, \gamma_{i,t}, \Lambda_i, \omega_t, \zeta_i, \Upsilon \\ (P/Q)_{i,t}^{D/G}, (P/Q)_{i,t}^{net}, I_{\ell,t}, V_{i,t}, \delta_{j,t} \\ P_{i,t}^{ch/dch}, ES_{i,t} \end{array} \right\} \quad (33)$$

$$\Pi = \left\{ \begin{array}{l} (P/Q)_{i,t}^{D0}, \gamma_{i,t}^{min/max}, ES_i^{min/max} \\ \eta_{ch/dch}, P_i^{ch,min/max}, P_i^{dch,min/max} \\ \lambda_t^{max/min}, \lambda_t^{f/a}, \tilde{\lambda}_t, \theta_{ij}, Y_{ij} \\ \Delta_t^\pm, \Gamma, \bar{\Lambda}, V_{min/max}, \bar{I}_\ell \end{array} \right\} \quad (34)$$

$$Sets = \{\Omega_{DR}, \Omega_T, \Omega_n, \Omega_L, \Omega_{ESS}\} \quad (35)$$

Indeed the DNOs would rather minimize the maximum costs that they may experience. This maximum cost occurs when the actual price is more than the forecast price. The reformulated single level optimization minimized the maximum regret (payments) of DNO by using duality gap theory [36] and robust optimization. This is because in deregulated environment the DNO's concern is the payments toward the losses (not the losses as in traditional distributions network management systems). It is shown that minimizing the  $z_1 = \sum_{DV} \psi_t$  does not result in minimum  $z_2 = \sum_{DV} \lambda_t \psi_t$ . Especially when  $\lambda_t$  is uncertain.

#### IV. SIMULATION RESULTS

##### A. Data

The proposed algorithm is implemented in GAMS [37] environment running on an Intel<sup>®</sup> Xeon<sup>™</sup> CPU E5-1620 3.6 GHz PC with 8 GB RAM. As far as the demand response node/nodes is/are known, the proposed framework is a NLP model which can be easily solved by commercial solvers such as Modular In-core Nonlinear Optimization (MINOS) [38]. However, if the optimal DR allocation is to be investigated the model would become MINLP and the Discrete and Continuous OPTimizer (DICOPT) [39] solver is used. In large scale networks, using the bender decomposition technique [40] would be beneficial. The non-convexity of the AC-OPF problem makes it difficult to find the global optimal solution. Some novel techniques have been proposed in the literature to address the duality gap in OPF and make it convex [41], [42]. The proposed model is applied to a 33-bus distribution network [43]. The peak demand values used in this study are higher than what is reported in [43] in order to increase the active losses in the network and can be accessed in [44].  $T$  is considered to be 24h. The predicted price values as well as price bounds are depicted in Fig. 2. These values can be found using time series models like ARIMA [45] based on historic data. The daily load curve shown in (Fig. 2) is obtained from EirGrid which is the Irish TSO (accessed 28/12/2014) [46].

The daily load curve is shown in Fig. 2 [46]. Without loss of generality it is assumed that no load curtailment can be done e.g.  $\epsilon_i = 0, \forall i \in \Omega_n$ . The technical characteristics of the considered ESS are described in Table I.

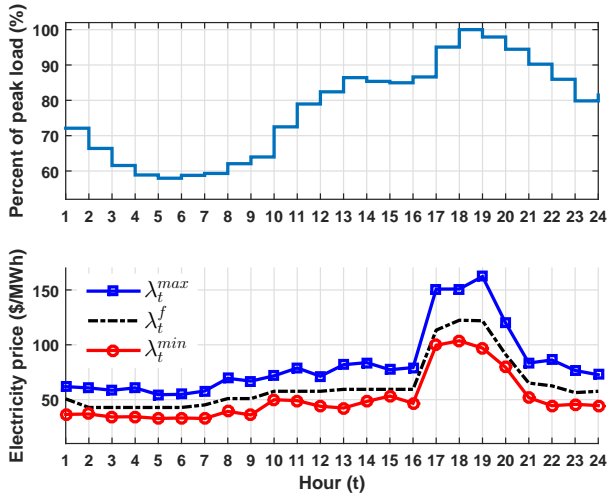


Fig. 2. Day ahead demand and price characteristics

TABLE I  
THE TECHNICAL CHARACTERISTICS OF ESS

Parameter	Value	Unit
$ES_{i,t}^{max}$	4 (8 × 500KW REDOX batteries [47])	MWh
$ES_{i,t}^{min}$	1	MWh
$ES_{i,t_0}$	2	MWh
$P_{i,t}^{ch,max} = P_{i,t}^{dch,max}$	1	MW
$P_{i,t}^{ch,min} = P_{i,t}^{dch,min}$	0	MW
$\eta_{ch} = \eta_{dch}$	95	%

## B. Considered cases

In this study, three different cases are studied:

- Case A) This case is added for the purpose of providing a basis for comparison. In this case, neither ESS nor DR is scheduled. No optimization is performed in this case. The constraints to be satisfied are (5) to (11). The decision variables of this case are limited to load flow variables and no optimization is performed. This means that  $\mathbf{DV}_a = \{V_{i,t}, \delta_{i,t}, P_{i,t}^G, Q_{i,t}^G\}$ . In this case, it is tried to satisfy the constraints (5) to (22). This is basically because there is no independent DV (DR or ESS) so the objective function can be chosen as (1) or (4).
- Case B) The active loss minimization (objective function is (1)) is achieved without considering the price uncertainties and using optimal scheduling of:
  - $B_1$  : The loss minimization is performed by optimizing the ESS schedule. The constraints to be satisfied are (5) to (16). This implies that  $\mathbf{DV}_{b_1} = \mathbf{DV}_a \cup \{ES_{i,t}, P_{i,t}^{ch}, P_{i,t}^{dch}\}$ . It is supposed that only one ESS exists in the network.
  - $B_2$  : The loss minimization is performed by optimizing the DR schedule. The constraints to be satisfied are (5) to (11) and (17) to (22). This implies that  $\mathbf{DV}_{b_2} = \mathbf{DV}_a \cup \{\gamma_{i,t}, \Lambda_i\}$ . In this case, it is assumed that load demand at only one node participates to a DR program. The flexibility degree can be adjusted by changing the  $\gamma_{i,t}^{min/max}$  in (19). It is assumed that  $\gamma_{i,t}^{min} = 0.6$  and  $\gamma_{i,t}^{max} = 0.6$ .

- $B_3$  : The loss minimization is performed by optimizing the ESS and DR schedule. The constraints to be satisfied are (5) to (22). This implies that  $\mathbf{DV}_{b_3} = \mathbf{DV}_{b_1} \cup \mathbf{DV}_{b_2}$ .

It should be noted that the electricity price uncertainties have an impact on the final payments of case A, B and C. It is assumed that the DNO is a price taker entity and its operating decisions do not influence the market price values. The difference between these cases is that case A does not have the tools (DR & ESS) to reduce the undesired price uncertainties. In case B, the tools (DR &/OR ESS) are available but not an appropriate operating strategy is chosen for reducing the payments. In fact, in case B it is tried to minimize the losses without considering the price values and their uncertainties. In contrary to case A & B, the decision maker in case C incorporates the price uncertainties in decision making process. That is why in all of these cases the impact of  $\Gamma$  values (the degree of conservativeness regarding the future prices) on final payments are assessed.

- Case C) Loss payment minimization (objective function is (32a)) is achieved by considering the price uncertainties and using optimal scheduling of corresponding decision variables which are as follows:
  - $C_1$  : The decision variables are the same as case  $B_1$ . Therefore  $\mathbf{U}_{c_1} = \mathbf{U}_{b_1}$ . The constraints to be satisfied are (5) to (11).
  - $C_2$  : The decision variables are the same as case  $B_2$ . Therefore  $\mathbf{U}_{c_2} = \mathbf{U}_{b_2}$ . The constraints to be satisfied are (5) to (11) and (17) to (22).
  - $C_3$  : The decision variables are the same as case  $B_3$ . Therefore  $\mathbf{U}_{c_3} = \mathbf{U}_{b_3}$ . The constraints to be satisfied are (5) to (22).

The value of  $\Gamma$  shows the conservativeness degree of the decision maker. It is a parameter which is set by the decision maker. It can vary from 0 (meaning no uncertainty may happen) to 24 (all uncertain parameters may take their worst value). The simulations have been done for all values of  $\Gamma = 0 \rightarrow 24$ .

## C. Results

1) Case A: The total payments are \$641.449 ( $\Gamma = 0$ ) and the total daily active energy losses are 8.669 MWh. The hourly active losses are shown in Fig. 3.

The possible reduction in loss payments vs the degree of conservativeness ( $\Gamma$ ) are depicted in Fig. 4. The numerical values of possible loss payments for different degrees of uncertainty ( $\Gamma$ ) are given in Table II. It is observed that as the uncertainty degree increases, the possible payments would increase from \$641.449 ( $\Gamma = 0$ ) to \$800.452 ( $\Gamma = 24$ ).

### 2) Case B:

- Case  $B_1$ : The connection node of the ESS can have an influence on the efficiency of the active management strategy. This is investigated by changing the connection node of ESS in the network. Based on the plots shown in Fig. 5, it is evident that the best location for ESS connection is bus #15. In this case, the active losses do not change with the change of  $\Gamma$  values. However, the possible payments

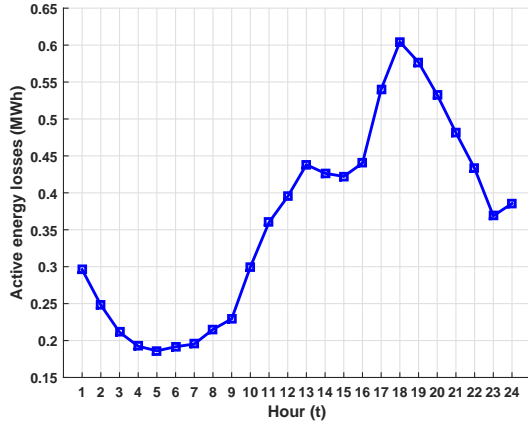


Fig. 3. The hourly active energy losses in case A, where neither ESS nor DR exists in problem formulation.

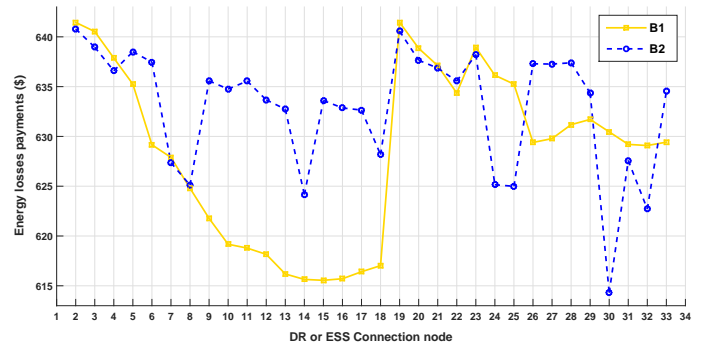


Fig. 6. The energy losses payments in loss minimization strategy vs the ESS node (case  $B_1$ ) and DR node (case  $B_2$ ).

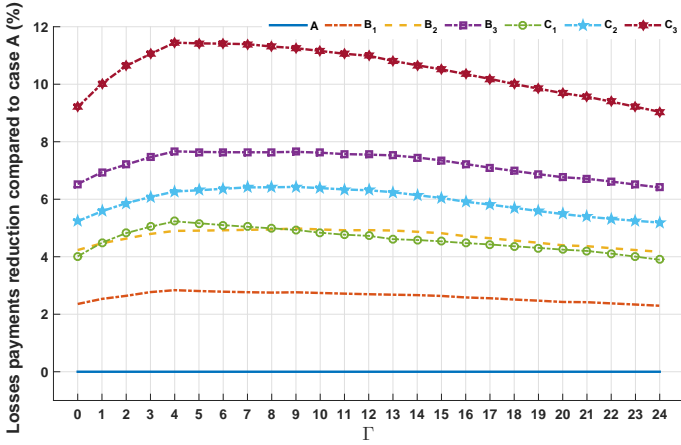


Fig. 4. The energy losses payment reductions (%) vs  $\Gamma$  in different cases. A: base case, B: loss minimization (using ESS  $B_1$ , using DR  $B_2$ , using both DR & ESS  $B_3$ ), C: loss payments minimization (using ESS  $C_1$ , using DR  $C_2$ , using both DR & ESS  $C_3$ ).

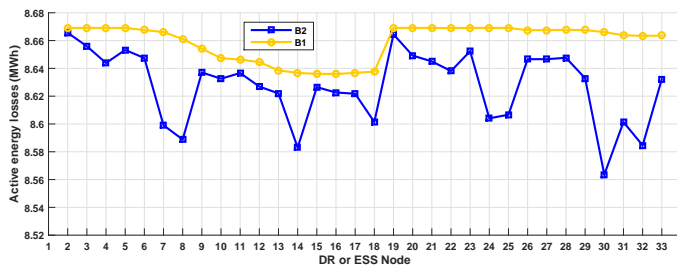


Fig. 5. a) The impact of ESS connection node on active losses (case  $B_1$ , loss minimization using ESS). b) The impact of DR connection node on active losses (case  $B_2$ , loss minimization using DR)

will change with ESS connection node as shown in Fig. 6. If the ESS is connected to node #15 then in case  $B_1$  the stored, charged and discharged energy pattern of ESS are depicted in Fig. 7. As shown in Fig. 4, this strategy can reduce the loss payments up to 2.83% compared to case A. The minimum total active losses are 8635.97 kWh.

- Case  $B_2$ : Fig. 5 shows the energy losses vs the node where a load with a demand response capability is assumed in case  $B_2$ . Node #30 is the best node for demand response

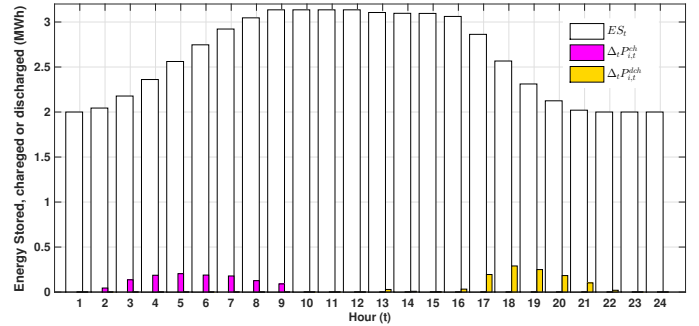


Fig. 7. The stored, charged and discharged energy schedule of the ESS connected to node 15 (case  $B_1$ , loss minimization using ESS).

participation regarding the energy losses minimization. Fig. 6 shows the energy losses payment vs the node where a load with a demand response capability is assumed. As shown in Fig. 4, this strategy can reduce the loss payments up to 4.98% compared to case A. The minimum total active losses are 8563.55 KWh. In the case that bus 30 is selected as the node with DR capability, the new demand pattern of bus 30 is depicted in Fig. 8. This new pattern is determined based on the technical characteristics of the network including the admittance matrix as well as the demand pattern of other nodes (which do not participate in DR program).

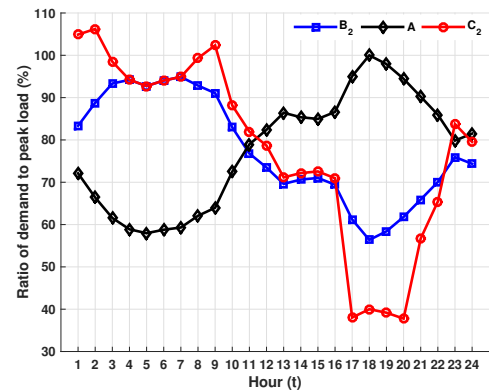


Fig. 8. The hourly demand pattern in different cases. A: no ESS/DR, loss minimization using DR ( $B_2$ ), loss payments minimization using DR ( $C_2$ ).

- Case  $B_3$ : It is assumed that the DR node is node #30 and the ESS is connected to node 15. Table II and Fig. 4 show the energy losses payment as well as total losses vs  $\Gamma$  in case  $B_3$ , respectively. As shown in Fig. 4, this strategy can reduce the loss payments up to 7.66 % compared to case A. The minimum total active losses are 8531.49 kWh.

3) Case C: In this case, the proposed algorithm tries to minimize the total daily payments due to active losses in the network using different combinations of actions as previously described:

- Case  $C_1$ : Again, the impact of ESS connection node on active losses payments in case  $C_1$  is shown in Fig. 9. This clearly shows that minimum active losses does not necessarily occur at minimum active losses payments. Node 15 is optimal for loss minimization not loss payment minimization. Fig. 10 depicts the impact of ESS connection

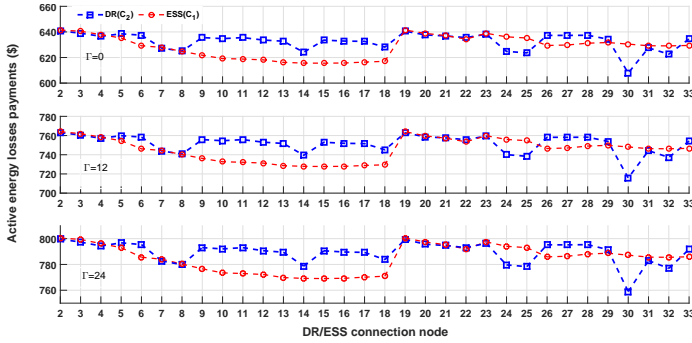


Fig. 9. The energy losses payments in losses payments minimization strategy vs the ESS node (case  $C_1$ ) and DR node (case  $C_2$ ) for three different values of  $\Gamma$ .

node on active losses in case  $C_1$ . The variations of active energy losses in Fig. 10 shows that ESS operation changes the line flows and this would increase the total active losses for different ( $\Gamma$ ) and connection nodes.

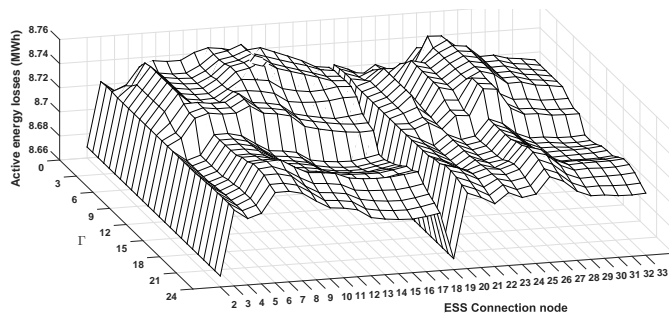


Fig. 10. The impact of ESS connection node on active energy losses (with loss payments minimization strategy using ESS ( $C_1$ )).

Fig. 11 shows the hourly energy stored in ESS vs  $\Gamma$  in case  $C_1$  if it is connected to node 11.

As shown in Fig. 4, this strategy can reduce the loss payments by up to 5.23 % compared to case A. In this case, the minimum total active losses vary from 8703.38 kWh to 8746.99 kWh (based on  $\Gamma$  values).

- Case  $C_2$ : Fig. 12 shows the energy losses vs the node where a load with a demand response capability is assumed and

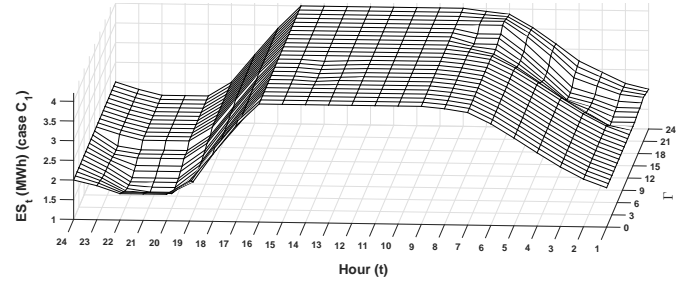


Fig. 11. The hourly energy stored in ESS vs  $\Gamma$  in case  $C_1$  (loss payments minimization using ESS).

vs  $\Gamma$  in case  $C_2$ . This implies that the node #30 is the best node for demand response participation regarding the losses payments minimization. Fig. 9 shows the energy losses

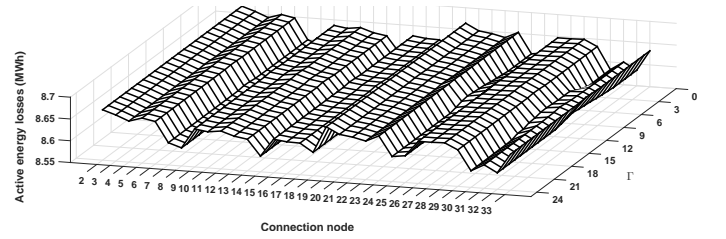


Fig. 12. The energy losses vs the DR node vs  $\Gamma$  (in loss payments minimization strategy using DR ( $C_2$ )).

payment vs the DR node and vs  $\Gamma$  (case  $C_2$ ). In this case, the minimum total active losses vary from 8582.55 kWh to 8603.97 kWh (based on the variation of  $\Gamma$  values). The new demand pattern of bus 30 is depicted in Fig. 8. This new pattern is determined based on the technical characteristics of the network (like case  $B_2$ ) as well as the electricity price variations. As shown in Fig. 4, this strategy can reduce the loss payments up to 6.43% compared to case A.

- Case  $C_3$ : It is assumed that the DR node is node #30 and the ESS is connected to node 15. Fig. 4 shows the energy losses payment vs  $\Gamma$  in case  $C_3$ . The energy losses vs  $\Gamma$  in case  $C_3$  are shown in Fig. 13. In this case, the minimum total active losses vary from 8619.76 kWh to 8672.11 kWh (based on  $\Gamma$ ). As shown in Fig. 4, this strategy can reduce the loss payments up to 11.44% compared to case A.

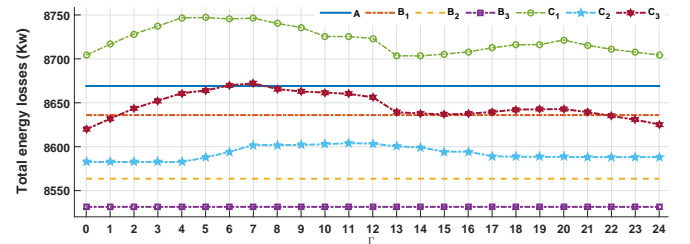


Fig. 13. The active energy losses vs  $\Gamma$  in different cases. A: no ESS/DR, B: loss minimization (using ESS  $B_1$ , using DR  $B_2$ , using both DR & ESS  $B_3$ ), C: loss payments minimization (using ESS  $C_1$ , using DR  $C_2$ , using both DR & ESS  $C_3$ ).

#### D. Comparison

The worst possible realization of electricity prices (based on the given budget of uncertainty ( $\Gamma$ )) is calculated by solving the

following optimization problem: 
$$\left\{ \begin{array}{l} \max_{\omega_t} \psi_t \Delta_t^+ \omega_t \\ \text{Subject to :} \\ (26b), (26c) \end{array} \right\}$$
. Then it

is used for loss payment calculation in all cases. This is why although the optimal decision variables in case B do not depend on price uncertainty, the payments are dependent on uncertain prices. In other words, the price uncertainty will be present in the final payments whether considered in decision variables (case C) or not (case B). The maximum reduction occurs in  $C_3$  where both ESS and DR are used to reduce the active loss payments.

The operating strategy of ESS in case  $C_1$  and  $C_3$  depends on price uncertainty so the total round trip losses will be dependent on  $\Gamma$ . The round trip losses of ESS vs time in cases  $C_1$  and  $C_3$  are shown in Fig. 14. Since the operation of ESS is not

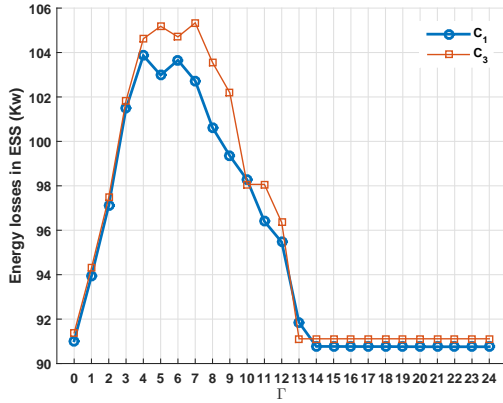


Fig. 14. The round trip losses of ESS vs  $\Gamma$  in case C: Active power losses payments minimization (using ESS  $C_1$ , using both DR & ESS  $C_3$ ).

dependent on electricity price in case B, then the  $OC_{ESS}$  is constant in this case. The round trip losses of ESS in  $B_1$  and  $B_3$  are 44.23 KWh, 43.38 KWh, respectively. However, the operating schedule of ESS changes with conservativeness degree ( $\Gamma$ ) for case C. The comparison between different cases regarding active losses and losses payments are depicted in Fig. 13 and Fig. 4, respectively.

According to Fig. 13, the best strategy for loss minimization is  $B_3$  since it focuses on loss minimization and utilizes both DR and ESS options. In Cases A,  $B_{1 \rightarrow 3}$  the total losses do not change with  $\Gamma$  values since these strategies are insensitive to price variations. The total losses in  $C_{1 \rightarrow 3}$  change with  $\Gamma$ . However, these changes in  $C_2$  are less than  $C_3$  because ESS is a more powerful tool compared to DR (with only one participating node in DR). Using the technique described in section II-B, the merits of nodes for participating in DR program are calculated and shown in Fig. 15.

The total numerical values of losses payments in different cases vs ( $\Gamma$ ) are described in Table II. It is found that the total losses payments are reduced when both DR and ESS are utilized compared to base case (A) while this value increases with the increase of conservativeness level ( $\Gamma$ ). The simulation results showed that the loss minimization and loss payment minimization strategies do not necessarily converge to the same

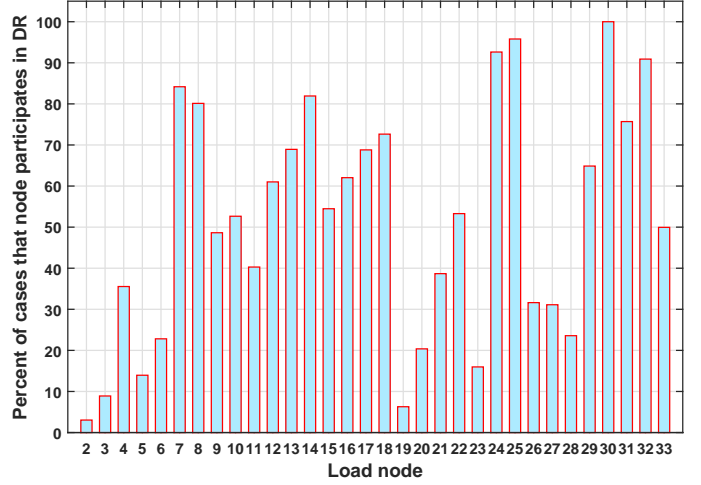


Fig. 15. The merits of nodes for participating in DR program

TABLE II  
TOTAL ACTIVE LOSSES PAYMENTS (\$) VS ( $\Gamma$ ) IN DIFFERENT CASES. A: NO ESS/DR, B: LOSS MINIMIZATION (USING ESS  $B_1$ , USING DR  $B_2$ , USING BOTH DR & ESS  $B_3$ ), C: LOSS PAYMENTS MINIMIZATION (USING ESS  $C_1$ , USING DR  $C_2$ , USING BOTH DR & ESS  $C_3$ ).

$\Gamma$	ESS & DR		DR		ESS		Base
	$C_3$	$B_3$	$C_2$	$B_2$	$C_1$	$B_1$	A
0	582.40	599.56	607.86	614.30	615.71	626.33	641.45
1	594.78	615.15	624.13	631.48	631.36	644.27	661.02
2	605.90	629.15	638.26	646.61	645.31	660.10	678.01
3	614.90	639.73	649.32	658.18	656.44	672.17	691.32
4	623.46	650.08	659.90	669.55	667.22	684.07	704.05
5	631.67	658.60	668.10	678.12	676.27	693.10	713.11
6	639.53	666.79	675.97	686.39	685.09	701.81	721.91
7	647.16	674.63	683.54	694.32	693.55	710.17	730.37
8	654.30	681.42	690.40	701.22	700.94	717.43	737.74
9	661.06	687.92	697.01	707.77	708.15	724.31	744.89
10	667.61	694.20	703.45	714.27	715.12	730.87	751.45
11	673.99	700.44	709.75	720.51	721.68	737.21	757.80
12	680.07	706.33	715.81	726.41	727.92	743.47	764.03
13	685.55	710.82	720.66	730.90	733.21	748.06	768.66
14	690.50	715.28	725.40	735.25	737.46	752.27	772.87
15	695.03	719.64	729.80	739.31	741.51	756.26	776.73
16	699.40	723.79	734.02	743.36	745.24	759.98	780.14
17	703.65	727.85	737.90	747.07	748.77	763.43	783.45
18	707.78	731.56	741.65	750.59	752.17	766.74	786.49
19	711.76	735.24	745.36	754.08	755.48	769.98	789.49
20	715.56	738.72	748.87	757.48	758.68	773.10	792.33
21	718.81	741.49	751.99	760.13	761.44	775.61	794.84
22	721.98	744.13	754.47	762.61	764.11	777.90	796.85
23	725.10	746.70	756.76	764.90	766.72	780.04	798.70
24	728.09	749.15	758.92	767.06	769.27	782.08	800.45

solution. This has several reasons as follows:

- The electricity prices are not the same in all operating periods (these values act as the weighting factors in optimization problem). If a constant cost (price) is considered for all time periods, then these strategies will converge to the same answer.
- The active losses in time period  $t$  depend on active losses values in previous and upcoming time periods. This means that the optimal decisions may increase the losses in time  $t$  (which has low price values) to decrease the losses in time  $t'$  ( $t' > t$  or  $t' < t$ ). This may increase the total active losses but it will decrease the payments. It's impossible to minimize the losses in all time periods because of the

dynamic operating constraints of DR (21) to (22) and ESS (12) to (16).

In order to check the robustness of the proposed algorithm a Monte carlo simulation has been conducted. It intends to verify the robustness of the obtained solutions. Case  $C_3$  is used for robustness verification. For this purpose, the optimal schedule of DR and ESS are obtained for a given conservativeness degree (e.g.  $\Gamma = 12$ ) and as indicated in Table II (case  $C_3$ ) the total payments are \$680.07. Next, 10000 samples of price values  $\lambda_{t_{1 \rightarrow 24}}$  are generated in a way that  $w_t$  satisfy equations (26b) and (26c). The value of total losses payments are calculated using (26a). The Monte carlo simulation results are shown in Fig. 16. The minimum, average, maximum and the standard deviation of simulated costs are \$619.06, \$649.05, \$678.13 and \$8.33, respectively. From Fig.16, it is inferred that using the decision variables found by the algorithm guarantees that the losses payments will not exceed the value specified by the algorithm (vertical line indicated in Fig.16 which is \$680.07). The Monte carlo simulation shows that applying the decision variables can ensure the DNO that the payments will not exceed the obtained results in Table II if the total electricity price uncertainties remain less than  $\Gamma = 12$  ((26b) and (26c)).

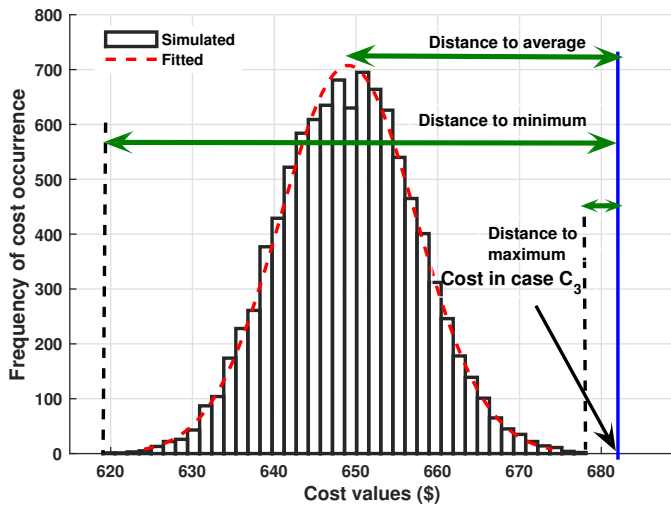


Fig. 16. The Monte carlo simulation results for robustness testing

## V. DISCUSSION

- If the exact values of uncertain electricity prices values  $\tilde{\lambda}_t$  are known ( $\lambda_t^f$ ) then solving the (4) would be an easy task. The decision maker is not able to find the optimal decision variables (because he can't be sure about the uncertain prices). The only remaining option is avoiding the high values of the price. In other words, optimal decision making is not toward minimizing the minimum costs that the decision maker may experience. It should be noted that the model is fed by some price values which some of them are the same as forecasted and some of them are more than forecasted values (worst case is calculated based on the given value of  $\Gamma$ ). Still the ESS tries to store the energy in periods where the prices are low and release them when the prices are high.

- It is assumed that the market is the only energy procurement option for DNO. In case, any renewable energy source exists in the network, the uncertainty of its generation pattern should be taken into account. On the other hand, the self owned DG units are not allowed in many regulatory frameworks.
- The maximum annual cost saving for using the strategy of case  $C_3$  is \$30645.76. The proposed framework is focused on operating strategy of DNO (using DR and ESS). This means that ESS is already installed (so investment cost are already paid). The obtained annual cost saving can be shared between the DNO and demand nodes which participate in DR program as an incentive.
- The main idea of the proposed framework is to demonstrate and quantify the effectiveness of the developed model in minimizing the losses payments. There are different frameworks for modeling the demand response such as welfare maximization on consumer side [48], [49], price elastic demand curve [50], monetary incentives [51]. The consumer welfare maximization is neglected as it is outside the scope of this paper and the DR is limited to demand shifting. The proposed model receives some inputs and provides some insights regarding the DR and ESS operation to deal with electricity price uncertainties as shown in Fig. 17. It can be used to generate the trade-off curve between consumer welfare maximization and DNO payments minimization.

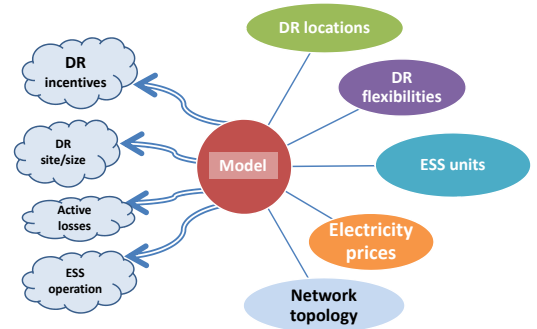


Fig. 17. The input-output interactions in the proposed model

## VI. CONCLUSION

In this paper, a general framework is presented in which the uncertain price is considered for losses payments minimization. This framework can accommodate different strategies for efficiency maximization of customers. Simulation results answered the previously posed questions regarding loss and losses payments minimization. It was demonstrated that the losses payments minimization strategy dominates the traditional losses minimization approaches in an unbundled power system environment. The ESS and DR are used as flexibility provider tools to enable the decision maker handle the uncertainties in a more efficient way. Considering the fact that robust optimization framework does not need the probability distribution of uncertain parameters, it can be used in practical cases. As evidenced by the simulation results, the proposed method offers some interesting features over traditional methods as follows:

- Modeling the uncertainty of electricity prices without knowing the probability density function using uncertainty set (with limited historic data) and robust optimization method. It is tractable and capable of controlling the conservativeness degree of decision maker.
- It can be utilized to assess the merits of nodes for participating in demand response programs based on their contributions to efficiency maximization of the network.
- Providing the optimal schedule of DR and ESS using a holistic approach immunized against the inherent operating uncertainties.
- Increasing the benefits of consumers compared to the traditional loss minimization approaches. This method by minimizing the DNO loss payments, reduces the costs of the DNO and thus provides a clear benefit to the customer.

There are three possible avenues for future work arising from this paper, namely, 1) multiple uncertainty resource modeling; e.g. renewable energy resources, demand values, component failures and 2) considering other active network management options; e.g. capacitor switching and network reconfiguration and 3) price bounds updating using forecasting tools and available data from smart grid.

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