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Application of value of information theory in adaptive metamodeling for reliability assessment

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ABSTRACT: The present paper discusses the application principles of value of information theory in adaptive metamodeling for reliability analysis. Metamodeling for reliability purposes has become particularly relevant in recent years. The usage of metamodels allows surrogating the, costly to evaluate, performance functions of engineering structures. Adaptive Kriging procedures are examples of the successful application of metamodeling in reliability analysis. Efficient adaptive Kriging involves the usage of some notion of improvement in what ultimately is an unsupervised decision making scheme that selects points to enrich the model. Therefore, the decision to select a point to enrich the experimental design should consider the utility of each candidate in the expectation of improvement of the metamodeling accuracy. Within this context, a comprehensive discussion on the application of value of information for reliability metamodeling is presented. Since the candidate points and surrogate are jointly built in a virtually costless model, it is possible to know the virtual outcome of the enrichment decisions. In many circumstances, points in the experimental design may provide redundant information. Furthermore, *a priori* knowledge on the performance function may be applied to weight the expected outcome of exploration and exploitation. Value of information considerations adds value to reliability metamodeling that uses adaptive methods, and is of interest for efficient design and optimization of complex structures, such as bridge structures.

1 INTRODUCTION

The present paper discusses the application of value of information (VoI) theory in metamodeling for reliability analysis. Interest on VoI has been substantial in structural health monitoring. For monitoring purposes the goal is to optimize the utility of the information obtained from a structure within a limited amount of effort or cost (Thöns 2018). However, it is a fact that every engineering decision has a utility gain and a cost or consequence associated (Raiffa & Schlaifer 1961, Thöns & Kapoor 2019), and hence VoI may appear in many contexts in engineering problems. This rationale of utility has transversal interest to all engineering fields.

The purpose of the present paper is then to research on the application of VoI principles in the field of adaptive metamodeling for reliability analysis. Due to the characteristic of reliability analyses, they can be costly by nature. In its most fundamental form of calculation, due to the low probabilities of failure (P_f) that are usually involved in the problem of reliability, multiple evaluations of a limit-state or performance function need to be assessed. Metamodeling is one of the most efficient alternatives to make the problem of reliability analysis manageable. Metamodels are surrogates of the problem of reliability that allow a fast calculation of the P_f. As a result, significant research interest has been directed to the application of these as a mean to decrease the effort required to perform reliability analysis, and without significant loss of accuracy. Adaptive metamodels are of particular interest in this context (Echard, Gayton, & Lemaire 2011). Adaptive metamodeling techniques use a notion of improvement to sequentially improve the metamodel approximation to the problem of interest (Teixeira, Nogal, O'Connor, Nichols, & Dumas 2019).

Adaptive metamodeling consists on iteratively improving the surrogate approximation considering *a priori* knowledge about the problem. In each step a decision should be made in regard to whether the surrogate approximation is improved or not. This is weighted by some measure of accuracy and cost. Decisions on improvement and stopping conditions use a learning function, or enrichment functions, and a stopping criterion that weights on further learning. In its fundamental rationale there is some resemblance with the principles that rule the theory of VoI, however, commonly no analysis is performed in relation to the output beyond of the stopping condition, e.g. multiple consequences of each decision. The present work exploits this idea of treating adaptive metamodeling with the approach of the problems of VoI, in order to further enhance the efficiency and comprehensiveness of adaptive implementations. It merges knowledge of VoI in the sequential construction of surrogates for limit-state functions in reliability.

For such, Section 2 discusses metamodeling in the context of reliability implementation, Section 3 discusses the theory behind decision analyses that measure the value of information, Section 4 presents the framework developed that uses the rationale of the VoI to explore and exploit the experimental design in metamodeling for reliability analysis, Section 5 presents and discusses results from two examples, a series system with four branches and a virtual bridge system. Finally, the main conclusion of the work developed is presented in Section 6.

2 RELIABILITY ANALYSIS

In time-invariant reliability analysis (Bourinet, Deheeger, & Lemaire 2011), the P_f is expressed as the probability $P[.]$ of the performance function having values smaller than a threshold of failure (t_f) (in reliability t_f=0) is given by

$$P_f = P[g(x) \leq t_f] = \int_{g(x) \leq t_f} f_x(x)dx \qquad (1)$$

where $f_x(x)$ is the continuous joint distribution of x input variables. $g(x)$ is the performance function of the system that is being evaluated.

It is noted that this integral calculation is complex and usually solved by using approximate methods. A common technique to solve the problem of reliability analysis is then to numerically approximate this integral calculation by defining discrete x classification, in failure or non-failure events. In the form of classification, the reliability analysis becomes divided in two discrete domains of

$$I_f(x) = \begin{cases} 0, & if \quad g(x) > 0 \\ 1, & if \quad g(x) \leq 0 \end{cases} \qquad (2)$$

where I_f is a binary classification that defines the x domains of failure ($I_F(x) = 1$) and non-failure ($I_F(x) = 0$) accordingly to $g(x)$ and the t_f defined.

With this classification scheme P_f can be approximated with,

$$\hat{P}_f = \frac{\sum_{\widehat{x}} I_f(\hat{x})}{N} \qquad (3)$$

by using a random vector $\widehat{x} = [x_1, x_2, \ldots, x_N]$ with N being the size of the vector x. This random vector is representative of the random conditions of operation of the system or structure being analysed. The prediction accuracy for the estimated P_f improves as N increases, with

$$\lim_{N \to \infty} \hat{P}_f = P_f \qquad (4)$$

This is the principle beyond the most primitive technique to solve the problem of reliability, that is the Monte Carlo Sampling (MCS) technique. It is straightforward to understand how this calculation can become cumbersome, in particular when increasing N is large and involves the usage of complex $g(x)$ that are costly to evaluate.

The principle behind the application of metamodeling to reliability is that of solving this same problem, but relying only to a limited extent in the performance function $g(x)$. A metamodel is then a surrogate $G(x)$ of $g(x)$ that is expected to provide an accurate approximation of the performance function.

2.1 *Adaptive Kriging in Reliability*

In the present paper, the adaptive Kriging is applied to discuss VoI in the context of metamodeling. The works that use Kriging pursuit to facilitate this classification procedure by using it as a metamodel. Kriging models, in addition to reducing the cost of the reliability analysis by surrogating $g(x)$, have gained relevant interest in the field of reliability analysis due to their capability to perform as self-improving functions. Because Kriging models have the capability to surrogate the limit-state function and at the same time enclose a measure of uncertainty in the approximation, they have been widely applied in active adaptive procedures (Teixeira, O'Connor, & Nogal 2019).

It is not difficult to understand that these models enclose a significant amount of information that can be used in a decision-making procedure. The adaptive Kriging stopping criteria already rely to some extent on this fact.

The Kriging model, used to create a surrogate such that $\mathbb{E}[G(x)] = g(x)$, is defined by,

$$G(x) = f(\beta_p; x) + z(x) \qquad (5)$$

where $f(\beta_p; x)$ is a deterministic function defined by a regression model with p ($p \in \mathcal{N}^+$) basis functions, and $z(x)$ a Gaussian stochastic process with zero mean. These models are defined using a likelihood search on a set of θ hyperparameters. A set of support points ($\hat{x} \in x$) is required to define $G(x)$, and these are designated as the Experimental Design (ED). Every prediction of $G(x)$ at a generic x point is defined by a mean value $\mu(x)$ and a standard deviation $\sigma(x)$, which provides a measure of uncertainty in the approximation (with $\sigma = 0$ for the ED).

To define new ED points to improve the metamodeling approximation, multiple functions and algorithms have been presented before, e.g., (Echard, Gayton, & Lemaire 2011, Bichon, Eldred, Swiler, Mahadevan, & McFarland 2008, Teixeira, O'Connor, & Nogal 2019). The U function (Echard, Gayton, & Lemaire 2011) and Expected Feasibility Function (EFF) (Bichon, Eldred, Swiler, Mahadevan, & McFarland 2008) are of particular interest in this context. These two functions have set the early benchmarks for the AK methods, and are still widely applied.

3 VALUE OF INFORMATION (VOI)

The VoI can be measured in different forms, depending on the context of the problem in-hand. This is due to the dependence of the VoI on the concept of utility. Thöns & Kapoor (2019) define the expected utility gain as the difference between a pre-posterior (for predicted) or a *posterior* decision analysis (for obtained information) and a *prior* decision analysis, which implies that the information acquirement states (predicted, obtained, not considered) define the decision analysis and thus the VoI conceptual definition. In the same work, the authors introduce a general consistent formulation to apply and consider VoI in decision processes.

Three main aspects can be highlighted in VoI implementations, (1) the consideration of both information acquirement and action implementation states; (2) consideration of action implementation uncertainty; and (3) system state trade-off analysis in regard to further information. Within this context, the decision of considering additional and yet unknown information in a engineering problem can be based upon an objective quantitative analysis that represents the value of that information (commonly this appears in the form of utility (u) or benefit).

Section 1 highlighted the resemblance of these aspects with the generic problem of adaptive metamodeling that implicitly has considerations on the utility of new information in the surrogate approach. A challenge that is posed in this context is that of defining a value of u of relevance in the particular field of metamodeling for reliability. Generally, this value of utility should enclose comparative measures of two or more scenarios of implementation, such as, a base scenario and enhancement scenarios; which allows the quantification of the value of information in the analysis. Because metamodeling in reliability analysis is based on using a limited knowledge about a certain performance function $g(x)$ to characterize the complete problem of reliability estimation, a conceptual idea of information and utility can be proposed. The rationale of the present paper is to research on how to create such concepts of information and utility. In the present case, the maximization of the value of information can be calculated by finding the maximum difference between the expected utilities that may arise from increasing the "true" information about $g(x)$, in relation to the case of no additional information.

The following section discusses the application of decision analyses that use notional ideas of VoI in metamodeling, with particular emphasis on adaptive implementations. Adaptive implementations are of interest. In adaptive approaches the goal is to find the most relevant information that is expected to further improve the metamodel approximation.

4 METAMODELING BASED ON VALUE OF INFORMATION CONSIDERATIONS

The common approach to metamodeling that considers adaptive techniques is to start with an ED (or set of defined $\hat{x}$ values) and then to sequentially enlarge this $\hat{x}$ using some criterion that contributes to incrementally improve the approximation of the created surrogate. This involves a decision process that is solved with an unsupervised decision procedure, and by setting some halting criterion on the decision. Figure 1 resumes how this procedure is developed. If a metamodel is to be adaptively updated (in order to pursuit further accuracy) it uses knowledge on the present model in order to estimate the benefit of adding further points to the ED. In this context, as depicted in the figure, the analysis involves the choice of information from a candidate update scheme, the utility of each candidate is measured in terms of the benefit that it is expected to bring to current i metamodel (explicitly or implicitly), and an update choice is performed based on it. When the decision involves the selection of a new candidate, the outcome is to evaluate it in the $g(x)$. This coincides with the point (1) in the previous section. Furthermore, in the present case uncertainty is considered by means of expectation, such as discussed in (2) in the previous section. Any decision on the choice and chance of information is uncertain, and the effective utility outcome is only addressed after choice on action. Finally the point (3) concerns measuring the trade-off of further analysis that can be obtained by using the choice and chance of information, and the model state. Initially, if no information is pursued the model is not updated and the metamodel state is not modified. If the expected benefit is small, the choice decision is to not update the model, whereas if the expected benefit is large, the decision is to update $G(x)$ at i+1 with the candidate strategy. In a fully adaptive implementation this analysis is repeated until no relevant utility is expected.

The major question that arises in the context discussed is on how to define utility in metamodeling. To create a surrogate $G(x)$ of a function $g(x)$ that enables efficient reliability analysis, commonly two character-

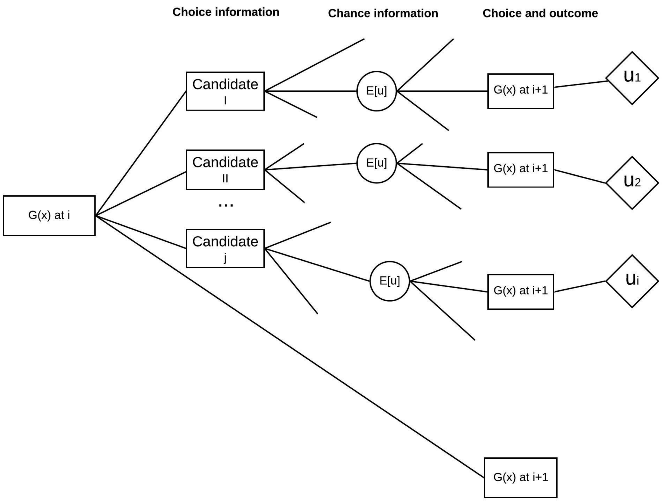

Figure 1: Example of adaptive metamodeling decision tree configuration.

istics are pursued, accuracy and limited analysis effort. Utility in metamodeling needs then to enclose these two characteristics. It was seen that, even considering that adaptive metamodeling is not commonly discussed in a context of decision theory that uses *a priori* information, most of the techniques implicitly consider provisions of it. Most of the metamodeling techniques that have become prominent in the field of metamodeling for reliability analysis use a learning function that considers a notion of improvement (value of further information) and a stopping criterion that halts the methodology when the expectation of further information is small. While the learning function evaluates candidates and their potential for improvement, the stopping criterion applies the trade-off of pursuing further information. This rationale of using information and to some extent expectation of utility (or expectation of improvement) appears in different forms in reliability and structural analysis, such as, classification error (Echard, Gayton, & Lemaire 2011), analysis budget (Teixeira 2019), or accuracy estimation (Sun, Wang, Li, & Tong 2017).

The present application of VoI concepts intends to foment discussion on this topic, by creating the formal basis of the problem of adaptive metamodeling as a problem of measuring the VoI of further improving the surrogate approximation. In parallel to setting this discussion, a new methodology to define surrogates of $g(x)$ for reliability analysis is proposed, where the adaptive metamodeling is discussed under the consideration of utility.

4.1 *Methodology to build a metamodel using VoI principles*

In the present study we construct the problem of adaptive metamodeling and merge it with VoI and decision analysis by considering three aspects of u; accuracy, uncertainty, and cost.

The goal is to use this rationale to evaluate the metamodel in each step of its definition, and in order to decide where to further improve the metamodel. Hence, the sequential or iterative approach should be continued only if the expected utility is large enough for the metamodel to harness the contribution from further information. For this purpose, a measure of utility u is required. In the present case, the utility u is measured as a posterior expectation of contribution to the accuracy of the estimation from the metamodel enclosing in a potential candidate.

As a result, in order to measure the posterior, the expected effect of adding a point j of the candidates to the i iteration model surrogate of $g(x)$ is measured by:

$$u_j = \mathbb{E}[\Delta\epsilon | G(x|x^*_{ED})] \tag{6}$$

with

$$\mathbb{E}[\Delta\epsilon | G(x|x^*_{ED})] = P_f \pm \mathbb{E}[\Delta P_f | G(x|x^*_{ED})] \tag{7}$$

where $G(x|x^*_{ED})$ is a metamodel with ED updated based on the consideration that the expected value of the j candidate is true. That is to say, that the current metamodel approximation is updated with a candidate, and the utility u_j of this candidate is given by the expected VoI of having it in the ED.

Ideally, assuming that the definition of $G(x)$ is stable for all the sizes of x_{ED}, the limit case when the size of the $x_{ED} \to \infty$, the approximation of $G(x)$ tends to the original function $g(x)$. In implementation conditions of metamodeling for reliability problems where the size of x_{ED} is relatively small, the accuracy of $g(x)$ is only approximated by $G(x)$ with a certain level of confidence, that in the case of Kriging can be estimated by a Gaussian $\mathcal{N}(\mu_G, \sigma_G)$.

Therefore, in the problem of reliability as described, this expectation can be used to determine a component of error. The current estimation of P_f would be negatively biased if the points classified as failures are in fact non-failure occurrences. An expectation of the probability content associated with this is given by

$$\Delta P_f^- = \int_x 1 - \Phi\left(\frac{\mu_G}{\sigma_G} | G(x) \leq 0\right) f(x) dx \tag{8}$$

and positively biased if the expected non-failure are in fact failures, with

$$\Delta P_f^+ = \int_x \Phi\left(-\frac{\mu_G}{\sigma_G} | G(x) > 0\right) f(x) dx \tag{9}$$

This means that a P_f prediction with $G(x)$ has an uncertain component that can be estimated in averaged terms. Φ Refers to the standard Gaussian cumulative distribution function.

An estimation of P_f has therefore an uncertain component, that relates to the amount of probability enclosed in both of the previous components,

$$\Delta P_f = \Delta P_f^+ + \Delta P_f^- \tag{10}$$

which is a range of deviation of P_f estimation with the Kriging surrogate, under the assumption of $G(x)$ being the best current representation of $g(x)$.

The expected value of adding a candidate j to the ED can be calculated by using this uncertain component in a virtually enriched surrogate $G(x|x^*_{ED})$ with the x_j candidate,

$$\nu_j = \int_x \Phi\left(-\frac{|\mu_G|}{\sigma_G}|G(x|x^*_{ED})\right) f(x)dx \tag{11}$$

which can be approximated for a finite-sample r by

$$\nu_j = \sum_{o=1}^{r} \Phi\left(-\frac{|\mu_{G(x(o)|x^*_{ED})}|}{\sigma_{G(x(o)|x^*_{ED})}}\right) \tag{12}$$

with r being the approximation sample to estimate $\hat{P}_f$, and that encloses the expected value of content in P_f of misclassifying points in the ED.

The utility u_j of enclosing the j point in the ED, is the inverse of ν_j,

$$u_j = \frac{1}{\nu_j} \tag{13}$$

which means that a point has maximum utility u when its effect is that it minimizes the value of the expected $\Delta_{P_f^{\pm}}$. That is to say, the candidate that is expected to minimize the value of ΔP_f, is the one that encloses the most relevant (or amount of) information to make an accurate surrogate. Or, the one that is expected to contribute the most to minimize the uncertain component.

In summary, the value of information of adding a new point to the ED is compared with the value of not adding that point by the relative comparison of its effect on the accuracy of the P_f estimation, which is a measure of accuracy of $G(x)$. The point that maximizes the VoI (of being enclosed in the ED), is the one that induces larger expectation of gains in accuracy, *i.e.* P_f prediction.

The new candidate selected to improve $g(x)$, x_s, is the one that has maximum utility.

$$x_s = \max\left[\mathbf{u}\right] \tag{14}$$

The problem here defined can be understood as the problem of finding the next candidate x_s to update the x_{ED} that has the largest expectation of reducing the uncertainty in the $\hat{P}_f$ estimate.

To note that by construction $v_j > 0 \, \forall \, j$, and hence, the maximum will be independent of the sign of the expectations $\Delta^{\pm}$.

Two things need to be considered in this phase, that is, interest in the region of failure and cost of updating. Because evaluating the utility of r candidates (note that r is a very large sample) has a large computational cost, it is of interest to use only subset candidates in the evaluation that are of significance for the problem in-hand. In the case of reliability analysis, some points are more relevant than others, and the points that are close to the failure region are expected to be more relevant for the problem of classification (they will contribute the most to classify the neighborhood that encloses large $\Delta^{\pm}_{P_f}$).

A subset of n_C points can be constructed from the large candidate sample in order to analyse the x^*_{ED} effects. So the subset x_C of candidates to evaluate u is defined by,

$$x_C = x_j \subseteq r_{u_j}, \;\; j = 1, \ldots, N_C \tag{15}$$

with r_u being the vector r of all candidates sorted by the value of uncertainty of classification of the response $G(r)$.

An alternative approach to select subset candidates that is of interest is that of using clustering techniques in r, such as implemented in Jiang, Qiu, Yang, Chen, Gao, & Li (2019) (Voronoi cells). A candidate quasi-random sample is also an alternative of interest for a better exploration of the x space. Despite not implemented in the present application, both approaches are expected to improve the information that may be obtained from x_C, in particular in order to provide efficient exploration of the x space.

It was seen that in each iteration a decision is to be made on whether to continue to improve the surrogate approximation or not. This is achieved by using a halting condition that stops the algorithm when fulfilled. One of the aspects that has been accounted only in a limited way in adaptive metamodeling for reliability is that of weighting the cost of taking new evaluations as a degree of freedom of the decision criteria.

It is noted that enclosing the cost is a common practice in the VoI theory and that the cost of evaluating $g(x)$ is not always the same. In some circumstances, extending the search for 10 more points is not costly, whereas, in other, it may demand few more hours of analysis time.

It is not uncommon for metamodeling techniques to consider a limit to the computational budget (Teixeira 2019), however, it is more interesting to have a cost (τ) associated to new $g(x)$ evaluations that adjusts the utility criterion to proceed or not with acquiring information. In this sense, the u_s of the selected more adequate x_s candidate accordingly to what was described in the present section is evaluated in comparison with a weighted cost of new evaluation and the current estimation of P_f,

$$u_s^{-1} < P_f \Xi \epsilon \tag{16}$$

where Ξ is the cost of new $g(x)$ evaluations that is bounded in [0,1]. If the model has a relative low cost to be evaluated, decreasing Ξ means that the decision on utility is progressively more conservative (the method is not concerned on computing a few extra points to increase the accuracy of the approximation to $g(x)$). ϵ is a measure of accuracy that weights the present expectation of error and that should be upper bounded at 0.01, or 1% of the estimated $\hat{P}_f$, in order to guarantee an accurate approximation (this upper bound will be used in the present implementations).

Under the described theoretical background, the adaptive metamodeling algorithm for reliability calculations based on VoI principles uses the following sequence:

1 Initiate the ED and the sample r, which should be large enough to allow accurate estimation of P_f. In the present application using Latin Hypercube Sampling and MCS respectively. Evaluate the ED in $g(x)$.

2 Transform all variables to the standard normal space before fitting. If r is relatively small in comparison to $\hat{P}_f$, then r can be enlarged without loss of generality of the procedure;

3 Define $G(x)$, surrogate of $g(x)$ and estimate $\hat{P}_f$;

4 Set $x_C \subseteq r_{sort}$, $x_C = [x_{j=1}, x_{j=2}, \ldots, x_{j=n_C}]$ using predictions $\mu_G(r)$ and $\sigma_G(r)$;

5 Predict ν_j and u_j by using a virtual x^*_{ED} and $G^*(x)$ built on expectation that x_j is true for the $G(x_j)$;

6 Find the x_j where u_j is maximum, this point is the selected most adequate candidate x_s and has utility u_s;

7 Evaluate the u_s of adding x_s with equation (16);

8 If the condition of equation (16) is not true or $P_f = 0$ enrich the present ED and return to 2. In this case, the utility surpasses cost, or not enough exploration of the space was promoted. If the condition of equation (16) is true and $P_f \neq 0$ stop the procedure, and proceed to 9.

9 Estimate $\hat{P}_f$. $G(x)$ is expected to be an accurate estimator of the problem of reliability for $g(x)$

Two examples of implementation of the proposed methodology are presented in the following section.

5 IMPLEMENTATION OF THE FRAMEWORK PRESENTED

Two examples are used to study the technique proposed. The first is a series system that has been widely researched in the field of adaptive metamodeling. The second is a bridge system based on the results presented in Akgül & Frangopol (2004).

5.1 *Example I - Four branches series system*

The series system, in its k, m, dependent form, has been widely researched in adaptive modeling implementations. The performance function of the series system is evaluated as the minimum of $g(x)$ in four branches,

$$g(x) = \min \begin{cases} k + 0.1(x_1 - x_2)^2 - \frac{x_1+x_2}{\sqrt{2}} \\ k + 0.1(x_1 - x_2)^2 + \frac{x_1+x_2}{\sqrt{2}} \\ (x_1 - x_2) + \frac{m}{\sqrt{2}} \\ (x_2 - x_1) + \frac{m}{\sqrt{2}} \end{cases} \tag{17}$$

Results for the reliability calculations of the series system in comparison to benchmarked results from Jiang, Qiu, Yang, Chen, Gao, & Li (2019) are presented in Table 1. Reference comparative results use the implementation of Jiang, Qiu, Yang, Chen, Gao, & Li (2019)

Table 1: Comparative results for distinct techniques to perform reliability calculations in the present example. In the present example values of k = 3 and m =6 are considered. Results presented for the framework proposed are averaged from 25 implementations. g_{eval} is the number of true function $g(x)$ evaluations in the procedure. LIF - Least Improvement Function.

Method	$P_f(10^{-3})$	g_{eval}	$e_r(\%)$
MCS	4.454	10^6	-
U	4.435	106.1	0.43
EFF	4.475	114.0	0.47
H	4.456	97.5	0.05
LIF	4.471	64.8	0.38
Results from the framework of (Jiang, Qiu, Yang, Chen, Gao, & Li 2019)			
U	4.423	64.7	0.70
EFF	4.456	64.3	0.05
H	4.411	69.7	0.97
LIF	4.497	56.5	0.97
Results from the framework presented			
n_C = 20, c =1	4.355	54.1	2.2
n_C = 40, c =1	4.447	54.7	0.16
n_C = 100, c =1	4.446	54.8	0.25

The results of the series system implementation show that using the idea of utility it is possible to achieve robust predictions for P_f with a relatively low number of performance function evaluations. The results obtained are comparable with the ones of the FPS framework, and occasionally with a slightly lower number of $g(x)$ evaluations. The gains relative to the original works of AKMCS that use the EFF and U learning functions are more evident in the present case.

Figure 2 presents the ED evolution for the cases of iterations i, 20 and 40. It is possible to infer that using information on the expected utility of the candidate in the surrogate model, an efficient balance of exploitation and explorations is achieved. After only 20 iterations, the four branches of the $g(x)$ function were found in the present example. Moreover, the ED of i = 40 shows that the sequential improvement of the surrogate approximation maintains a balance of exploitation and exploration. Points that are close in

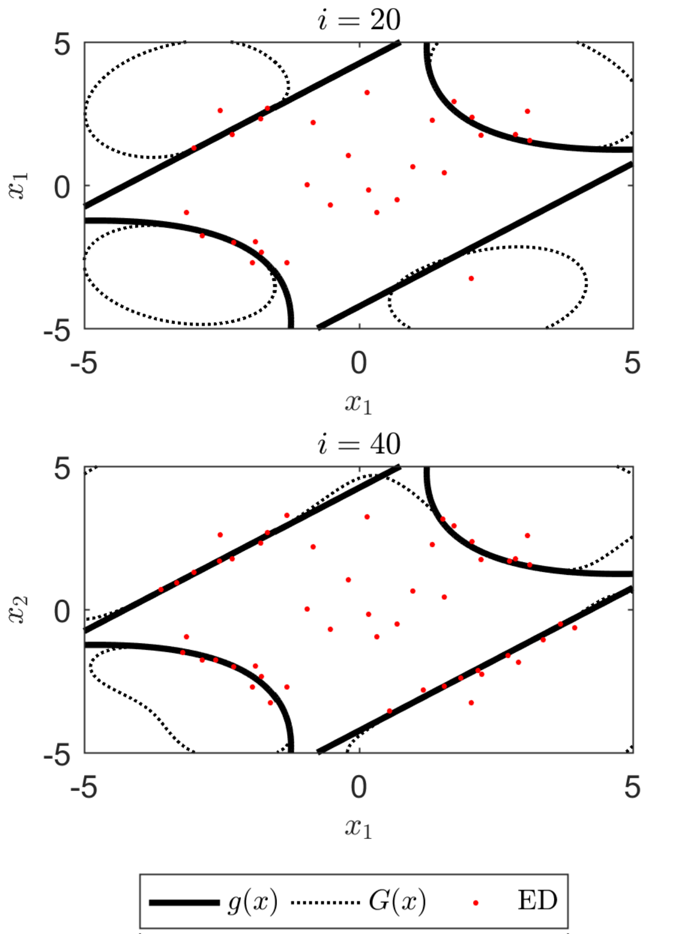

Figure 2: Example of adaptive Kriging implementations using the methodology presented. i refers to the iteration number. n_c = 40.

the ED, and that may provide redundant information in the surrogate approximation, are not added to the ED. This is due to the notional improvement based on the expectation of improvement for a neighborhood (reduction of content in misclassification) given by ν.

It is noted, however, that the implementation of a measure of utility u that is characterized on a "dummy" $G(x)$ increases the analysis time as more metamodels need to be created. Hence, in a context of implementation the *trade-off* of additional analysis time and effort should be weighted in relation to the gains in decreasing the number of g_{eval}, and hence also weighted when defining the value of Ξ. The algorithm used to build the metamodel is also expected to have large influence on the implementation. In the present case the ooDACE toolbox was used (Couckuyt, Dhaene, & Demeester 2014). An alternative to consider in future implementations is the UQLAB Kriging algorithm of (Marelli & Sudret 2014).

5.2 *Example II - Bridge reliability*

A bridge system example that is an approximation of the bridge presented in (Akgül & Frangopol 2004) is applied as a representative example of reliability analysis with the methodology proposed. The limit-state considered refers to flexure failure of the bridge.

The bridge is considered to be of compact section type. A single mode of failure is considered from the ones presented in the reference, ultimate limit state with failure by flexure. The flexure limit state (LS) function at the critical section is given by

$$g_F = M_u - (M_{DLNC} + M_{LL+I} + M_{DLC}) \quad (18)$$

$$M_u = KF_yS_p\lambda_{mfg} \quad (19)$$

$$M_{DLNC} = 32.07\lambda_s \quad (20)$$

$$M_{DLC} = C_1\lambda_c + C_2\lambda_a + C_3\lambda_s + C_4 \quad (21)$$

$$M_{LL+I} = M_{trk}I_fD_f \quad (22)$$

with C_1= 142.5; C_2 =54.72; C_3 = 4.39 and C_4 = 1.49. In this problem there are 9 random variables. Their probabilistic characterization is presented in Table 2. A tunable variable K is considered to have the value of 0.5.

Results are shown in Table 3. Similarly to the previous example, using a measure of utility allows gains in the number of evaluations of $g(x)$. When the evaluating $g(x)$ is relatively cheap $\Xi = 0.1$, the performance is comparable to the performance of the AKMCS of (Echard, Gayton, & Lemaire 2011). However, in the present example the best results were obtained with the First Order Reliability Method (FORM) (it is noted that Equation (18) indicates that $g(x)$ is expected to be relatively simple and capable of being approximated with FORM). Despite this fact, the g_{eval} needed to complete FORM in comparison with the methodology presented indicates that further benefits can be attained with a hybrid technique that combines the two.

It is noted that despite the decrease in the number of g_{eval}, with performance relatively close to the framework introduced in (Jiang, Qiu, Yang, Chen, Gao, & Li 2019), further improvements are expected from exploiting the relation between the VoI and the sample candidate in the space. One of the main future developments in the present method is related to the selection of the n_C in the space, in order to foment exploration with a relatively low number of candidates.

In order to use relatively small values of n_C two recommendations are given; the first is to decrease the cost of further iterations, and the second is to avoid strong candidates that are very close to each other in the x space (such as discussed for clustering techniques). Alternatively, uniform designs may be applied to create the initial ED for the case of the Kriging, which is particularly efficient for local interpolation, and less efficient to approximate the global trends (Schobi, Sudret, & Wiart 2015).

Table 2: Generic properties of the bridge considered in the current assessment (all variables are assumed to be lognormal). The bridge is of steel I-beam type. m and v are the lognormal mean and standard deviation, and μ and σ are mean and standard deviation of the associated normal distribution.

Variable	Specification	m	v	μ	σ
Structural resistance variables					
F_y(MPa)	Yeld strength of the material	252.56	30.31	5.5245	0.1196
$S_p(cm^3)$	Plastic modulus	9053	226.33	9.1106	0.025
λ_{mfg}	Model uncertainty factor	1.11	0.128	0.0978	0.1146
λ_s	Uncertainty steel weight	1.03	0.0824	0.0264	0.0799
λ_a	Uncertainty asphalt weight	-	-	1	0.1
λ_c	Uncertainty concrete weight	-	-	1	0.1
Structural loading variables					
M_{trk}(kNm)	Moment truck load	245.67	87.26	5.4446	0.3447
$I_f(-)$	Impact factor	1.122	0.1122	0.1098	0.0998
$D_f(-)$	Distribution factor	1.44	0.1785	0.3568	0.1235

Table 3: Comparative results for distinct AK implementations for the adapted bridge system.

Method	$P_f(10^{-3})$	g_{eval}	$e_r(\%)$
MCS	1.636	10^6	-
AKMCS	1.661	93.50	1.5
FORM	1.627	55.00	0.6
Results from the framework presented			
n_C = 20, Ξ =1	1.619	71.59	1.0
n_C = 50, Ξ =1	1.611	70.80	1.5
n_C = 20, Ξ =0.1	1.661	90.40	1.5
n_C = 50, Ξ =0.1	1.659	91.60	1.4

6 CONCLUSIONS

A methodology was presented that combines knowledge on decision theory, using value of information, in order to create an adaptive metamodeling technique for efficient reliability analysis. The proposed approach uses an expectation of utility measured on the i iteration metamodel in order to define the expected gains of adding a point to the experimental design. The gains are measured in terms of the expectation of reducing the uncertainty in the probability of failure prediction. A point is considered to have large utility when it contributes the most to approximate the surrogate, based on the expectation that its prediction is true. A measure of cost was also introduced in order to calibrate the decision on utility as a function of the performance function expense. Two examples of implementation were researched. Results showed that using *a priori* establishment of utility for *posterior* decision analysis in the adaptive approach produces efficient implementations for reliability analysis. In both of the examples studied the number of evaluations of the performance function were reduced to a comparable, and with a slight edge over, values found for efficient implementations in the literature. However, it is noted that the gains in efficiency come at the expense of additional effort in establishing utility measures for the adaptive improvement, and also inducing metamodel-algorithm dependency. Therefore, to conclude, it is important to highlight that despite the relatively large efficiency of the methodology presented, there is still room to further improve efficiency by reducing the number of performance function evaluations and procedure cost. This may be achieved by a slight increase of the exploration capability of the subset candidate sample (in the case of the Kriging).

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