| | |
|---|---|
| ***Title*** | 3D Location and Orientation Estimation using Angle of Arrival |
| ***Authors(s)*** | Ballal, Tarig, Bleakley, Chris J. |
| ***Publication date*** | 2009-08-28 |
| ***Publication information*** | Ballal, Tarig, and Chris J. Bleakley. “3D Location and Orientation Estimation Using Angle of Arrival.” IEEE, August 28, 2009. https://doi.org/10.1109/WISP.2009.5286580. |
| ***Conference details*** | 6th IEEE International Symposium on Intelligent Signal Processing (WISP), Budapest, Hungary, 26 - 28 August, 2009 |
| ***Publisher*** | IEEE |
| ***Item record/more information*** | http://hdl.handle.net/10197/7229 |
| ***Publisher's statement*** | © © 2009 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. |
| ***Publisher's version (DOI)*** | 10.1109/WISP.2009.5286580 |

# 3D LOCATION AND ORIENTATION ESTIMATION USING ANGLE OF ARRIVAL

*Tarig Ballal and C. J. Bleakley*

Complex & Adaptive Systems Laboratory, School of Computer Science & Informatics,
University College Dublin, Belfield, Dublin 4, Ireland
phone: + (353) 1 716 2915, fax: + (353) 1 269 7262, email: {tarig.ballal, chris.bleakley}@ucd.ie
web: http://www.csi.ucd.ie/

**ABSTRACT**

This paper discusses the problem of joint location and orientation estimation of a receiver using only Angle Of Arrival (AOA) information. Conventional formulations of the problem consist of a number of nonlinear equations where the number of unknowns exceeds the number of equations. However, formulations presented in this paper simplifies the problem in a way that leads of efficient solutions. Two solutions are presented and their performance is compared via simulations using an indoor application as an example. Results emphasize the effectiveness of the the proposed methods.

## 1. Introduction

The problem of locating an object or a device is an important problem in many fields (e.g., [1, 2, 3, 4]). A special case is the source localization problem, where a signal emitter is to be localized utilizing a number of sensors configured as a fixed array. This problem has received much attention, with many techniques having been proposed in the literature (e.g., [1, 2, 3, 4, 5, 6, 7]). Another case, is when a mobile *receiver* device is to be localized with respect to fixed emitters. This problem has been discussed in the literature (e.g., [8, 9, 10, 11]). Recently, localization of a mobile device in an indoor environment has emerged as an important problem for many modern applications. In particular, this paper is concerned with the receiver localization problem in the context of ubiquitous computing systems, for which location awareness has been demonstrated to be one of the most important components [8, 9]. For these systems, the receiving device must determine its own location to ensure privacy.

Coupled with location, another important piece of information is the orientation of the mobile device [12]. Orientation can be thought of as the identifier that answers the question as to where the device is pointing. This is very useful for Human Computer Interaction (HCI).

In this paper, the problem of receiver location and orientation estimation using Time-Difference Of Arrival (TDOA) or Angle Of Arrival (AOA) information is considered. By TDOA we mean the difference in the delay of a signal received by two sensors that are part of the receiving device. The techniques presented in this paper, despite focusing more on indoor location-orientation for ubiquitous computing systems, are quite generic and can be applied to various situations in which receiver location and orientation is of interest.

The Authors would like to thank Science Foundation Ireland (SFI) for funding this work.

This paper is structured as follows. Section 2 presents related works. In Section 3, a general problem description is given. Solutions to the problem are proposed in Section 4. Simulation results are presented in Section 5, and Section 6 gives the conclusions of the paper.

## 2. Related Works

Navigation systems such as the GPS (Global Positioning System) [11] are related to this work as they focus on locating a receiver based on signals originating from known locations. These systems are based on measuring the distance from the receiver to at lease three transmitters with known locations using signal Time-Of-Flight (TOF), then trilateration [13] is applied to determine the receiver location. Formulations for determining the receiver orientation (referred to as *attitude* in the GPS literature) have been derived (e.g., [14, 15, 16]). The orientation determination uses the phase-difference observed by at least two sensors that are part of the receiving device.

In [17], location is determined using trilateration in much similar way to the GPS. In the same reference, orientation is determined based on AOA.

In this paper, we present methods showing that only TDOA /AOA information is needed for determining both the location and orientation of the receiver in 3D. A direct advantage of this is to reduce system complexity by removing the synchronization requirement associated with TOF determination. It should be noted here that the GPS formulations of the orientation problem are much affected by the phase-difference ambiguity problem due to the separation of the sensors being grater than the half wavelength of the received signal. In this paper we use a straightforward formulation assuming unambiguous TDOA /AOA estimates. If necessary, disambiguation can be performed using one of the methods discussed in the literature (e.g., [18]). It should also be mentioned that, a similar location-orientation problem for mobile robot was discussed in the literature (e.g., [10]), however it is limited to 2D only. Adding a third dimension increases the complexity of the problem significantly.

## 3. Problem Description

The scenario considered in this paper assumes $N_b$ beacons with fixed and known locations. The receiver contains $N_s$ sensors that are used to provide TDOA/AOA information with a certain degree of accuracy. Angles are measured relative to the device's local axes. The goal is to find the location and orientation of the receiver in 3D based on the available TDOA/AOA information only. Location of a 3D object with non trivial dimensions can be thought of as the location of a point on that object. Orientation can be represented by the global unit vectors corresponding to the directions of each of the receiver's local Cartesian axes ($\bar{X}$, $\bar{Y}$,$\bar{Z}$ as depicted in Fig. 1). The difficulty of the problem stems from the fact that with unknown receiver orientation, AOA information becomes ambiguous since the frame of reference is not fixed.

## 4. Proposed solutions

The most straightforward formulation for the problem is to use two sensors. In such a case, the location-orientation problem is transformed into a problem of locating the two sensors. TDOAs of $N_b$ signals measured between these two sensors result in the following set of equations:

$$\| \mathbf{b}_k - \mathbf{s}_1 \| - \| \mathbf{b}_k - \mathbf{s}_2 \| = c\,\delta_{12,k}, \quad k = 1,...,N_b \quad (1)$$

where $\mathbf{s}_1$ and $\mathbf{s}_2$ are the position vectors of the two sensors; $\mathbf{b}_k$ is the position vector for beacon $k$; $\delta_{12,k}$ is the TDOA between the two sensors; $c$ is the speed of propagation; and $\| \, . \, \|$ denotes the Euclidean norm. From (1), it can be seen that solving for $\mathbf{s}_1$ and $\mathbf{s}_2$ requires at least six independent equations (which requires at least six non collinear beacons). The equations are nonlinear and their solution was found to be quite involved. Numerical solutions were found to be inefficient with convergence possible only when the initial guess is very close to the true solution. Due to this complication, alternative approaches were sought.

### 4.1. Solutions Using Four Sensors

In this subsection, using four coplanar sensors on the receiver device is proposed as a solution to the location-orientation problem. Precisely, we assume two orthogonal pairs of sensors as depicted in Fig. 1. The purpose of this configuration is to provide 3D AOA information. The receiver's local coordinate system is assumed to coincide with the baselines of these two pairs of sensors (see Fig. 1). Based on the TDOA information provided by these sensors, one can formulate the problem in a similar way to Eq. (1). Four non collinear sensors typically provide three distinct TDOAs. Consequently to solve for all the sensor locations requires at least 12 equations from four non collinear beacons. Due to the difficulty already encountered in solving (1), we will consider an alternative approach.

The approach adopted in this subsection is based on converting TDOA to AOA. It is well known that TDOA can be converted to AOA based on the assumption that the emitter is sufficiently far from the sensors (the far-field assumption). In such a case, the hyperboloid defined by TDOA can be approximated by a cone by considering only the asymptotes of the hyperboloid [3]. Based on this assumption, the two TDOA equations from the two orthogonal pairs of sensors define the unit vector of the Line Of Sight (LOS) of each beacon from the centroid of the sensors positions (the origin in Fig. 1). The unit vector to each beacon can hence be expressed as

$$\bar{\mathbf{u}}_k = [\cos\alpha_k \, \cos\beta_k \, \cos\gamma_k]^T, \quad k = 1,...,N_b \qquad (2)$$

where $\alpha_k$, $\beta_k$ and $\gamma_k$ are the angles with the three local axes $\bar{X}$, $\bar{Y}$ and $\bar{Z}$, respectively (see Fig. 1). The values of $\cos\alpha_k$ and $\cos\beta_k$ are determined from the TDOAs using

$$\cos\alpha_k = \frac{c\delta_{12,k}}{\| \mathbf{s}_2 - \mathbf{s}_1 \|}$$
$$\cos\beta_k = \frac{c\delta_{34,k}}{\| \mathbf{s}_4 - \mathbf{s}_3 \|} \qquad (3)$$

where $\delta_{12,k}$ and $\delta_{34,k}$ are the TDOAs of the signal from beacon $k$, between each of the orthogonal pairs of sensors (see Fig. 1). The value of $\cos\gamma_k$ is further determined from $\cos\alpha_k$ and $\cos\beta_k$ using

$$\cos\gamma_k = \left(1 - \cos^2\alpha_k - \cos^2\beta_k\right)^{0.5}. \qquad (4)$$

In (4), only the positive root has been considered. This is motivated by the fact that the maximum value that $\gamma_k$ can take, without loss of LOS due to the panel where the sensors are mounted, is $\pi/2$, as illustrated in Fig. 2. It should be noted that $\bar{\mathbf{u}}_k$ in (2) are estimates of the unit vectors, however, for simplicity of notation, the estimation symbol " ˆ " is ignored in this case and also in subsequent cases.

**4.1.1. Solution 1.** Now, taking the unit vectors $\mathbf{u}_k$ pairwise and performing the dot-product, the following set of equations is obtained:

$$(\mathbf{b}_i - \mathbf{m}) \cdot (\mathbf{b}_j - \mathbf{m}) = \| \mathbf{b}_i - \mathbf{m} \| \| \mathbf{b}_j - \mathbf{m} \| \, \bar{\mathbf{u}}_i^T \bar{\mathbf{u}}_j, \quad \{i,j\} \subset \{1,...,N_b\}, i \neq j \qquad (5)$$

where $\mathbf{m} \triangleq [x\,y\,z]^T$ is the position vector of the centroid of the sensors positions, and is the center of the localization problem; "·" is the dot-product operator; and

$$\bar{\mathbf{u}}_i^T \bar{\mathbf{u}}_j \equiv \cos\theta_{ij} \qquad (6)$$

with $\theta_{ij}$ the angles between the two LOS lines to beacon $i$ and beacon $j$ (see $\theta_{12}$ in Fig. 1). Eq. (5) represents a system of nonlinear equations in $\mathbf{m}$. Since $\mathbf{m}$ is a triad, a minimum of three beacons (i.e three equations) are required to find the location of the receiver. Eq. (5) has the advantage that the unknowns pertaining to the receiver orientation are excluded from the formulation, and the orientation problem can hence be handled subsequently using the location information obtained from the solution of (5). This allows for location and orientation determination using only three beacons. Assuming $N_b = 3$, Eq. (5) can be written as

$$f_{ij}(\mathbf{m}) = 0, \{i, j\} \subset \{1, ..., 3\}, i \neq j \tag{7}$$

where

$$\begin{aligned} f_{ij}(\mathbf{m}) &= (\mathbf{b}_i - \mathbf{m}) \cdot (\mathbf{b}_j - \mathbf{m}) \\ &- \parallel \mathbf{b}_i - \mathbf{m} \parallel \parallel \mathbf{b}_j - \mathbf{m} \parallel \bar{\mathbf{u}}_i^T \bar{\mathbf{u}}_j. \end{aligned} \tag{8}$$

Eq. (7) represents three surfaces that are formed by rotating circle arcs around their chords. The chords in this case are the line segments between each pair of beacons. One such surface can be the internal surface of a spindle torus [19] (when $\theta_{ij} < \pi/2$), the external surface of a spindle torus (when $\theta_{ij} > \pi/2$), or a sphere (when $\theta_{ij} = \pi/2$). The intersection of three such surfaces is the location of the receiver.

It should be noted that Eq. (7), when expanded, is differentiable in in $x$, $y$ and $z$ and can hence be solved using the well known Newton-Raphson (NR) method [20]. In fact the problem is found to be well suited for solution using the NR method. The method has been found to converge in most cases when a *good* initial guess ia available. A good guess has always been found to be one with the three component values taking *sufficiently large* absolute values.

Having found the location $\mathbf{m}$ of the receiver, orientation can readily be determined. Orientation can be defined as a rotation matrix of the coordinate system. The rotation matrix can be defined as

$$\mathbf{A} = [\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3] \triangleq \begin{bmatrix} \cos\theta_{x\bar{x}} & \cos\theta_{y\bar{x}} & \cos\theta_{z\bar{x}} \\ \cos\theta_{x\bar{y}} & \cos\theta_{y\bar{y}} & \cos\theta_{z\bar{y}} \\ \cos\theta_{x\bar{z}} & \cos\theta_{y\bar{z}} & \cos\theta_{z\bar{z}} \end{bmatrix} \tag{9}$$

where $\theta_{v\bar{w}}$ is the angle that the w-axis of the receiver makes with the global v-axis. The rows of $\mathbf{A}$ are the directions of orientation of the three receiver axes respectively. A rotation of any vector from the local receiver coordinate system to the global coordinate system can be achieved using

$$\bar{\mathbf{u}} = \mathbf{A}\mathbf{u}. \tag{10}$$

The matrix $\mathbf{A}$ can be determined if at least three direction vectors are known in both the global and the local coordinate systems. Such vectors can be $\bar{\mathbf{u}}_\mathbf{k}$ (as in (2)) and $\mathbf{u}_k$, $k = 1, ..., 3$. The latter can be determined (when $\mathbf{m}$ is known) as

$$\mathbf{u}_k = \frac{\mathbf{b}_k - \mathbf{m}}{\parallel \mathbf{b}_k - \mathbf{m} \parallel}. \tag{11}$$

Now define

$$\begin{aligned} \mathbf{U} &\triangleq [\mathbf{u}_1 \; \mathbf{u}_2 \; \mathbf{u}_3]^T \\ \bar{\mathbf{U}} &\triangleq [\bar{\mathbf{u}}_1 \; \bar{\mathbf{u}}_2 \; \bar{\mathbf{u}}_3]^T. \end{aligned} \tag{12}$$

From (10), it directly follows that

$$\bar{\mathbf{U}} = \mathbf{A}\mathbf{U} \tag{13}$$

and consequently

$$\mathbf{A} = \bar{\mathbf{U}}\mathbf{U}^{-1}. \tag{14}$$

From (14), it can be observed that $\mathbf{A}$ can be determined only if $\mathbf{U}$ is nonsingular, a condition that can be satisfied only if the beacons *and* the receiver do not lie on a plane where one of the three coordinates is a constant. Nevertheless, it is found that, even in the latter case, $\mathbf{A}$ can be determined by exploiting the orthogonality of its columns. In the sequel, we determine $\mathbf{A}$ when the beacons and the receiver lie on a plane $z = c$, where $c$ is a constant. In this case, column 3 of the matrix $\mathbf{U}$ is expected to contain all zero values. The consequence of this is to eliminate $\cos\theta_{z\bar{x}}$, $\cos\theta_{z\bar{y}}$ and $\cos\theta_{z\bar{z}}$ from Eq. (13). Manipulating Eq. (13) results in the following three systems of equations:

$$\begin{aligned} \mathbf{Q}\mathbf{a}_{\bar{x}} &= \mathbf{q}_{\bar{x}} \\ \mathbf{Q}\mathbf{a}_{\bar{y}} &= \mathbf{q}_{\bar{y}} \\ \mathbf{Q}\mathbf{a}_{\bar{z}} &= \mathbf{q}_{\bar{z}} \end{aligned} \tag{15}$$

where

$$\begin{aligned} &\mathbf{a}_{\bar{x}} \triangleq \begin{bmatrix} \cos\theta_{x\bar{x}} \\ \cos\theta_{y\bar{x}} \end{bmatrix} \mathbf{a}_{\bar{y}} \triangleq \begin{bmatrix} \cos\theta_{x\bar{y}} \\ \cos\theta_{y\bar{y}} \end{bmatrix} \mathbf{a}_{\bar{z}} \triangleq \begin{bmatrix} \cos\theta_{x\bar{z}} \\ \cos\theta_{y\bar{z}} \end{bmatrix} \\ &\mathbf{q}_{\bar{x}} \triangleq \begin{bmatrix} \bar{u}_{1,x} \\ \bar{u}_{2,x} \\ \bar{u}_{3,x} \end{bmatrix} \quad \mathbf{q}_{\bar{y}} \triangleq \begin{bmatrix} \bar{u}_{1,y} \\ \bar{u}_{2,y} \\ \bar{u}_{3,y} \end{bmatrix} \quad \mathbf{q}_{\bar{z}} \triangleq \begin{bmatrix} \bar{u}_{1,z} \\ \bar{u}_{2,z} \\ \bar{u}_{3,z} \end{bmatrix} \\ &\mathbf{Q} \triangleq \begin{bmatrix} u_{1,x} & u_{1,y} \\ u_{2,x} & u_{2,y} \\ u_{3,x} & u_{3,y} \end{bmatrix} \end{aligned} \tag{16}$$

where a subscript "$k, v$" (with $k$ a numerical value and $v$ a character ) associated with $u$ and $\bar{u}$ denotes the $v$ component of the vectors $\mathbf{u}_k$ and $\bar{\mathbf{u}}_k$ respectively. Eq. (15) represents three overdetermined systems of linear equations that have Least Squares (LS) solutions

$$\begin{aligned} \mathbf{a}_{\bar{x}} &= \left(\mathbf{Q}^T\mathbf{Q}\right)^{-1}\mathbf{Q}^T\mathbf{q}_{\bar{x}} \\ \mathbf{a}_{\bar{y}} &= \left(\mathbf{Q}^T\mathbf{Q}\right)^{-1}\mathbf{Q}^T\mathbf{q}_{\bar{y}} \\ \mathbf{a}_{\bar{z}} &= \left(\mathbf{Q}^T\mathbf{Q}\right)^{-1}\mathbf{Q}^T\mathbf{q}_{\bar{z}}. \end{aligned} \tag{17}$$

Eq. (17) determines the elements of the two columns $\mathbf{a}_1$ and $\mathbf{a}_2$. The third column $\mathbf{a}_3$ is orthogonal to both $\mathbf{a}_1$ and $\mathbf{a}_2$, and can hence be determined using the vector cross-product as

$$\mathbf{a}_3 = \mathbf{a}_1 \times \mathbf{a}_2. \tag{18}$$

**4.1.2. Solution 2.** The following formulation provides an alternative approach to solve the location-orientation problem. From Fig. 1, using the cosine rule leads to the following system of equations:

$$f(r_i, r_j) = 0, i, j \subset \{1, ..., 3\}, i \neq j \tag{19}$$

where $f(r_i, r_j)$ are defined as

$$f(r_i, r_j) = r_i^2 + r_j^2 - 2r_i r_j \, \bar{\mathbf{u}}_i^T \bar{\mathbf{u}}_j - d_{ij}^2 \tag{20}$$

where $r_k = \| \mathbf{r}_k \|$ and $d_{ij} = \| \mathbf{d}_{ij} \|$. Eq. (19) represents three elliptic cylinders in the space spanned by the vector $\mathbf{r} \triangleq [r_1\, r_2\, r_3]^T$. Each cylinder is aligned along one of the coordinate axes. Again the NR method can be used to solve (19) effectively. In fact, the solution of Eq. (19) is less cumbersome compared to that of (7) in terms of both computational complexity and convergence speed. Better convergence is obtained when the initial guess contains three positive values. Solving Eq. (19), the vector $\mathbf{r}$ can be used to determine both the location and the orientation of the receiver. Location can be determined using trilateration [13]. However, with the information in hand orientation and then location can be obtained using closed-form formulae.

To determine orientation, three vectors expressed in both the local and global coordinate systems are required as described in Section (4.1.1). From Fig. 1, using $\mathbf{r}$ obtained from solving Eq. (19), we have

$$\bar{\mathbf{d}}_{ij} = r_i \bar{\mathbf{u}}_i - r_j \bar{\mathbf{u}}_j, \{i,j\} \subset \{1,...,3\}, i \neq j. \tag{21}$$

The global counterparts of the vectors in Eq. (21) can directly be calculated as

$$\mathbf{d}_{ij} = \frac{\mathbf{b}_j - \mathbf{b}_i}{\| \mathbf{b}_j - \mathbf{b}_i \|}. \tag{22}$$

Given (21) and (22) the rotation matrix can be determined in a similar way to (14) as

$$\mathbf{A} = \bar{\mathbf{D}} \mathbf{D}^{-1} \tag{23}$$

where $\mathbf{D}$ and $\bar{\mathbf{D}}$ are $3 \times 3$ matrices defined as

$$\begin{aligned} \mathbf{D} &\triangleq [\mathbf{d}_{12}\, \mathbf{d}_{23}\, \mathbf{d}_{31}]^T \\ \bar{\mathbf{D}} &\triangleq \left[\bar{\mathbf{d}}_{12}\, \bar{\mathbf{d}}_{23}\, \bar{\mathbf{d}}_{31}\right]^T. \end{aligned} \tag{24}$$

Alternatively, if $\mathbf{D}$ is singular due to the beacons lying on a plane $z = c$, the LS solution for $\mathbf{A}$ is given by (17) and (18) with $\mathbf{Q}$, $\mathbf{q}_{\bar{x}}$, $\mathbf{q}_{\bar{y}}$ and $\mathbf{q}_{\bar{z}}$ defined as

$$\mathbf{q}_{\bar{x}} \triangleq \begin{bmatrix} \bar{d}_{12,x} \\ \bar{d}_{23,x} \\ \bar{d}_{31,x} \end{bmatrix} \quad \mathbf{q}_{\bar{y}} \triangleq \begin{bmatrix} \bar{d}_{12,y} \\ \bar{d}_{23,y} \\ \bar{d}_{31,y} \end{bmatrix} \quad \mathbf{q}_{\bar{z}} \triangleq \begin{bmatrix} \bar{d}_{12,z} \\ \bar{d}_{23,z} \\ \bar{d}_{31,z} \end{bmatrix}$$

$$\mathbf{Q} \triangleq \begin{bmatrix} d_{12,x} & d_{12,y} \\ d_{23,x} & d_{23,y} \\ d_{31,x} & d_{31,y} \end{bmatrix} \tag{25}$$

The receiver location $\mathbf{m}$ can now be determined using the relationship

$$\begin{aligned} \bar{\mathbf{r}}_k = r_k \bar{\mathbf{u}}_k &= \mathbf{A}\mathbf{r}_k \\ &= \mathbf{A}\left(\mathbf{b}_k - \mathbf{m}\right), k = 1,...,3 \end{aligned} \tag{26}$$

where $\mathbf{r}_k$ and $\bar{\mathbf{r}}_k$ are the LOS vectors to beacon $k$ in the global and local coordinate system respectively. Eq. (26) can be solved for $\mathbf{m}$ for each value of $k$ resulting in three estimates for $\mathbf{m}$. Generally, in a noisy scenario, these three estimates constitute a triangle whose centroid can be taken as the final location estimate. In other words, $\mathbf{m}$ can be estimated from (26) using

$$\mathbf{m} = \frac{1}{3} \sum_{k=1}^{3} \mathbf{b}_k - \mathbf{A}^T \bar{\mathbf{r}}_k \tag{27}$$

where $\mathbf{A}^T$ has replaced $\mathbf{A}^{-1}$ due to the fact that $\mathbf{A}$ is unitary.

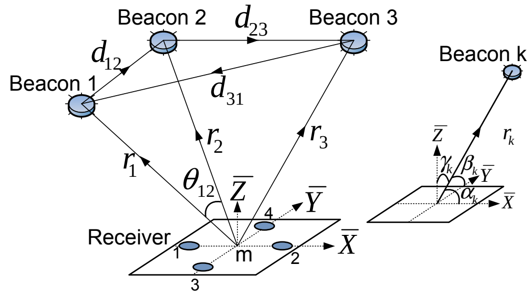


**Fig. 1**: Sensors and beacons configuration.

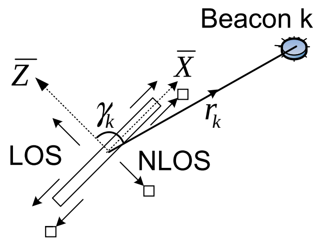


**Fig. 2**: Limitation of the angle $\gamma$. Plain arrows point to the direction of LOS while the arrows with squares point to the direction of NLOS (Non LOS).

## 5. Simulation Results

The two solutions for location-orientation presented in the previous section were simulated in Matlab. In simulation, the beacons were situated on the $z = 0$ plane with $\mathbf{b}_1 = [0\,0\,0]^T$, $\mathbf{b}_2 = [5\,0\,0]^T$ and $\mathbf{b}_3 = [0\,5\,0]^T$ (all in meters), which resembles an indoor situation. The receiver was located on the $z = -3$ plane and allowed to move on the square plane whose edge points are $[0\,0\,-3]^T$, $[5\,0\,-3]^T$, $[0\,5\,-3]^T$ and $[5\,5\,-3]^T$. The orientation of the receiver was fixed with the rows of $\mathbf{A}$ set equal to $cos[45^o\, 45^o\, 0^o]^T$, $cos[135^o\, 45^o\, 0^o]^T$ and $cos[90^o\, 90^o\, 0^o]^T$ respectively. With this setting, the true AOA values ($\alpha_k$ and $\beta_k$ in degrees) were contaminated with added error. The error followed a normal distribution with zero mean and unity variance.

Figs. 3 and 4 show the location error for solution 1 and 2 respectively. Fig. 5 and 6 plot the orientation error for these solutions respectively. The resolution of the plots is 0.2 m for both the $x$ and $y$ coordinates. The location error was calculated as the distance between the estimated location and the true location, while the orientation error for each axis was calculated as the angle between the estimated direction for that axis and the true direction. Each figure was the average of 20 simulation runs. The figures show a general trend of an increase in error at the corners with the farthermost corner from the beacons receiving the highest error, which reflects algorithmic non-convergence at that spot. However, the error is acceptable around the center of the test plane. Such a spatial distribution of the error can be attributed to the so-called Geometric Dilution Of Precision (GDOP) [11], that is the effect of the beacons' arrangement. Apparently, this effect can be

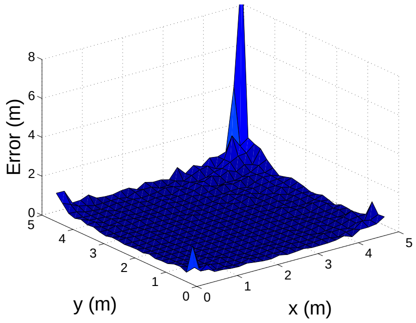


**Fig. 3**: Location error surface for solution 1.

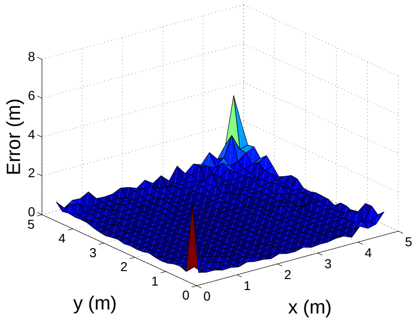


**Fig. 4**: Location error surface for solution 2.

reduced by adding a fourth beacon at the farthermost corner. However, a study of the effect of the GDOP and identifying the best configuration is left for a future publication. It can also be seen that the error in orientation of the z-axis was always found to be greater than that associated with the other two axes. It is clear that is due to error accumulation in Eq. (18).

The figures also show that solution 2 exhibits slightly better performance for location and worse performance for orientation compared to solution 1. This is despite the fact that solution 2 uses orientation estimates to find location. This contradiction can be explained by that Eq. (27) tolerates the noise associated with the orientation estimates that are affected by the error accumulation in (21). This interpretation is not conclusive and should be a subject of consideration in future works. Table 1 summarizes the results in terms of the overall mean of location and orientation errors.

**Table 1**: Mean location and orientation errors.

| | *solution* 1 | *solution* 2 |
|---|---|---|
| *Location* | $0.28\,m$ | $0.26\,m$ |
| *x orient.* | $1.3^o$ | $2.6^o$ |
| *y orient.* | $1.4^o$ | $2.1^o$ |
| *z orient.* | $1.7^o$ | $2.9^o$ |

## 6. Conclusion

In this paper, we presented formulations and two solutions for the receiver location-orientation problem using only TDOA/AOA information. It has been demonstrated that, both location and orientation can be

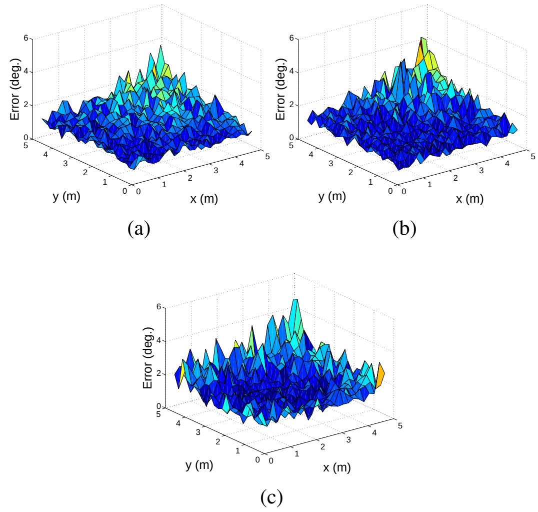


**Fig. 5**: Orientation error surface for solution 1: a) x-axis b) y-axis c) z-axis.

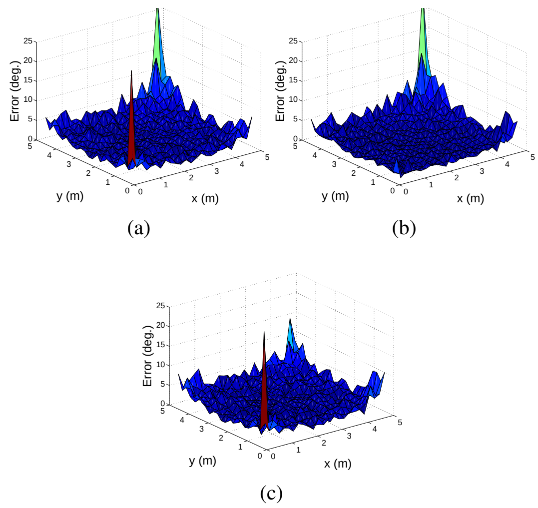


**Fig. 6**: Orientation error surface for solution 2: : a) x-axis b) y-axis c) z-axis.

determined using as few as three non collinear beacons and two pairs of orthogonal sensors. Comparison of the two proposed approaches was performed using simulations.

## 7. References


[1] H. Schau and A. Robinson, "Passive source localization employing intersecting spherical surfaces from time-of-arrival differences," *IEEE Trans. on Acoustics, Speech and Signal Processing*, vol. 35, no. 8, pp. 1223–1225, Aug 1987.

[2] Y. Chan and K. Ho, "A simple and efficient estimator for hyperbolic location," *IEEE Trans. on Signal Processing*, vol. 42, no. 8, pp. 1905–1915, Aug 1994.

[3] M. Brandstein, J. Adcock, and H. Silverman, "A closed-form method for finding source locations from microphone-array time-delay estimates," *Proc. of Int. Conf. on Acoustics, Speech, and Signal Processing, ICASSP-95*, vol. 5, pp. 3019–3022, May 1995.

[4] D. Bliss D. Young, C. Keller and K. Forsythe, "Ultra-wideband (UWB) transmitter location using time difference of arrival (TDOA) techniques," *Conf. Record of the 37th Asilomar Conf. on Signals, Systems and Computers*, vol. 2, pp. 1225–1229, Nov 2003.

[5] D. J. Torrieri, "Statistical theory of passive location systems," *IEEE Trans. on Aerospace and Electronic Systems*, vol. AES-20, no. 2, pp. 183–198, Mar 1984.

[6] J. Smith and J. Abel, "Closed-form least-squares source location estimation from range-difference measurements," *IEEE Trans. on Acoustics, Speech and Signal Processing*, vol. 35, no. 12, pp. 1661–1669, Dec 1987.

[7] S. C. Nardone and M. L. Graham, "A closed-form solution to bearings-only target motion analys," *IEEE Jour. of Oceanic Eng.*, vol. 22, no. 1, pp. 168–178, Jan 1997.

[8] C. Randell and H. Muller, "Low cost indoor positioning system," *Proc. of UbiComp2001, Atlanta, Georgia, USA*, pp. 42–48, Sep 2001.

[9] M. Hazas and A. Ward, "A high performance privacy-oriented location system," *Proc. of the 1st IEEE Int. Conf. on Pervasive Computing and Communications, Dallas-Fort Worth, Texas, USA*, pp. 216–223, Mar 2003.

[10] J. Sena Esteves, A. Carvalho, and C. Couto, "Position and orientation errors in mobile robot absolute self-localization using an improved version of the generalized geometric triangulation algorithm," *IEEE Int. Conf. on Industrial Technology, ICIT-2006*, pp. 830–835, Dec 2006.

[11] B. Parkinson and J. Spilker, *The global positioning system: theory and applications*, American Institute of Aeronautics and Astronautics, 1996.

[12] N. B. Priyantha, A. K. Miu, H. Balakrishnan, and S. Teller, "The cricket compass for context-aware mobile applications," *Proc. of the 7th Ann. ACM/IEEE Int. Conf. on Mobile Computing and Networking*, pp. 1–14, Jul 2001.

[13] D. Manolakis, "Efficient solution and performance analysis of 3-D position estimation by trilateration," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 32, no. 4, pp. 1239–1248, Oct 1996.

[14] R. Brown and P. Ward, "A GPS receiver with built-in precision pointing capability," *IEEE Position Location and Navigation Symposium*, pp. 83–93, Mar 1990.

[15] C. Tu, K. Tu, F. Chang, and L. Wang, "GPS compass: novel navigation equipment," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 33, no. 3, pp. 1063–1068, Jul 1997.

[16] K. Xia, X. Zhang, J. Gao, and L. Zhang, "Study on GPS attitude determination technology based on QPSO algorithm," *7th World Congress on Intelligent Control and Automation, WCICA 2008*, pp. 1869–1873, Jun 2008.

[17] J. R. Gonzalez and C.J. Bleakley, "High precision robust broadband ultrasonic location and orientation estimation," *IEEE Jour. of Selected Topics in Signal Processing*, Aug 2009, accepted for publication.

[18] T. Ballal and C. J. Bleakley, "Phase-difference ambiguity resolution for a single-frequency signal," *IEEE Signal Processing Letters*, vol. 15, pp. 853–856, Dec 2008.

[19] E. Weisstein, *CRC concise encyclopedia of mathematics*, CRC Press, 2nd edition, 2003.

[20] C. J. Zarowski, *An Introduction to Numerical Analysis for Electrical and Computer Engineers*, John Wiley & Sons, 2004.