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Novel Branch Centrality Measures for Electrical Power Systems Considering Both Load-serving and Circulating Currents

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Abstract— **Recent work on transmission usage charging has offered compelling new perspectives on how current flows in electrical power systems. One new insight is that a certain component of branch current flows solely to serve loads in the system, whereas another component arises because of voltage mismatches between generator buses. These circulating currents affect active power losses and branch congestion, and thus may be of renewed interest to system operators. This paper presents novel power system visualizations which better show the separate load-serving and circulating aspects of a network. A key insight that emerges is the dual role of the branches in a system: they carry both load serving and circulating currents, in heterogeneous proportions. This perspective permits new branch centrality metrics to be proposed, which separately gauge a branch's role in serving these distinct functions.**

I. INTRODUCTION

The present work uses recent advances in power flow tracing to discuss some new perspectives on the structure of power systems. Early work in power flow tracing was motivated by market liberalization [1, 2] and was generally notional in character (e.g [3, 4]), tending to knowingly disregard the physics of power transmission. Among the earliest contributions to frame the flow tracing question in more physical terms was Conejo's Z_{bus} approach [5, 6] This technique relies on taking the inverse of the Y_{bus} matrix, which is a very fundamental descriptor of a network's connectivity. Similarly, the present work relies on a reordering and partitioning of the Y_{bus} matrix, such that the connectivity between and among *PV* buses and *PQ* buses is quantified separately.

The equations central to this new understanding were first presented in [7], and have been applied since to various power system problems, notably by the prolific Thukuram, who has used them for, *inter alia*: voltage stability improvement [8-11]; congestion management [12]; transmission system usage charging [13, 14]; generator reactive power margin improvement [15]; siting power electronic devices [16, 17]; and generator expansion planning [18, 19]. Others have used the techniques for probabilistic voltage stability analysis [20]; power system reliability evaluation [21]; voltage stability monitoring with realistic load models [22]; improved load curtailment [23]; and voltage security monitoring [24]. Sikiru's work [25-27], using related matrix partitions and eigen decomposition, elucidates a more fundamental understanding of the inherent structural characteristics of power systems, as does work in [28].

The work of Abdelkader et al. offers a deeper conceptual understanding of the matrix partitioning enforced in [7]. In a culmination [29] of earlier research [30, 31] a compelling new perspective on how current flows in electrical networks is articulated. In a rigorous way, it is demonstrated that a separable portion of branch currents are entirely a consequence of voltage mismatches between generator nodes, whereas the remaining component of branch current necessarily arises to serve loads. The direct relationship between these circulating currents, and the theoretical minimum loss dispatch for a power system, has latterly been established in [32]. This insight on the origins of actives power losses has been applied in designing voltage control schemes for wind generators [33].

The present paper seeks to modestly extend the discussion of the new paradigm articulated in [29]. To this end, new power system centrality measures are proposed which separately gauge a branches' participation in each function, facilitating load-serving or circulation currents. Network visualizations are presented to make this separation more tangible.

These new metrics are related to other proposed measures of branch electrical centrality [34-36] which may be better suited to power system structural analysis than naïve topological measures [37, 38]. This work's new centrality metrics may open up new ways to approach such difficult problems as optimal transmission switching [39, 40], where removing a line from a system can relieve congestion and achieve substantial cost savings. Identifying such lines, however, is computationally demanding and poorly understood [41]. Likewise, identifying lines whose removal may trigger cascading failures is challenging, and simple branch centrality metrics are not always useful in this role [42].

The matrix manipulations permitting the separation of branch currents are reviewed and presented in Section II, some novel visualizations and tutorial discussions of such current separations are given in Section III and conclusions are given in Section IV.

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II. METHODOLOGY

A. Y_{bus} partitioning

The treatment begins by restating the Y_{bus} partitioning introduced by Kessel and Glavitsch in [7]. Starting with the Y_{bus} matrix, nodes are reordered such that generator buses (G index) and load buses (L) are grouped together. In block matrix form, this gives:

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix} \tag{1}$$

The motivation for this partitioning of the Y_{bus} matrix is to separately quantify the connectivity between and among generator and load buses. Symmetry in the Y_{bus} can be assumed in the absence of phase-shifting transformers [43], and so $Y_{LG}^{T} = Y_{GL}$. Note that this partitioning is particular to a specific set of online generators.

I_G and I_L are complex-valued vectors representing the nodal currents at generator and load buses, respectively, while V_G and V_L are corresponding complex nodal voltages. From (1):

$$I_G = Y_{GG}V_G + Y_{GL}V_L \tag{2}$$

$$I_L = Y_{LG}V_G + Y_{LL}V_L \tag{3}$$

Rearrange (3) to find that:

$$V_L = Z_{LL}I_L - Z_{LL}Y_{LG}V_G \tag{4}$$

Where $Z_{LL} = Y_{LL}^{-1}$. Substituting for V_L in (2):

$$I_G = (Y_{GG} - Y_{GL}Z_{LL}Y_{LG})V_G + Y_{GL}Z_{LL}I_L \tag{5}$$

Inspecting (4) and (5) indicates that they can be efficiently represented in matrix form:

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GGM} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \tag{6}$$

Where:

$$Y_{GGM} = Y_{GG} - Y_{GL}Z_{LL}Y_{LG} \tag{7}$$

$$F_{LG} = -Z_{LL}Y_{LG} = K_{GL}^{T} \tag{8}$$

B. Tracing current flows

According to the conceptual framework proposed in [29], voltage mismatches between generators in a power system cause circulating currents, I_G^{Circ}, which flow in addition to the load-serving currents in the system, I_L^{Load}. The authors of [29] show how these circulating currents components can be separately traced through the power system. Equations (9) to (13) are reproduced below from [29], and represent the central contributions of that work.

Each generator's circulating current I_G^{Circ} is given by:

$$I_G^{Circ} = Y_{GGM}V_G \tag{9}$$

The portion of this current component that flows in each branch (BR) is given by the M_{BR}^{G} matrix as follows:

$$I_{BR}^{Circ} = M_{BR}^{G}I_G^{Circ} \tag{10}$$

Where M_{BR}^{G} is calculated as:

$$M_{BR}^{G} = Y_{BR}A^{T}\begin{bmatrix} Z_{GGM} \\ -Z_{LL}Y_{LG}Z_{GGM} \end{bmatrix} \tag{11}$$

Noting that $Z_{GGM} = Y_{GGM}^{-1}$, Y_{BR} is the branch admittance matrix, and A is the system incidence matrix.

Similarly, the M_{BR}^{L} matrix describes the portion of each load node's (L) current demand that flows in each branch:

$$I_{BR}^{Load} = M_{BR}^{L}I_L \tag{12}$$

With:

$$M_{BR}^{L} = Y_{BR}A_L^{T}Z_{LL} \tag{13}$$

Where A_L is the incidence matrix partitioned to show just the connectivity of the load buses.

C. Novel centrality measures

One can readily define two novel branch centrality metrics by simply summing the rows of the M matrices, taking the absolute value of each element, denoted by the $|\cdot|$ superscript:

$$C_{BR}^{Circ} = \sum_{G} M_{BR}^{G\,|\cdot|} \tag{14}$$

And:

$$C_{BR}^{Load} = \sum_{L} M_{BR}^{L\,|\cdot|} \tag{15}$$

The motivation for the new centrality definitions in (14) and (15) is to separately quantify each branch's structural role in the power system; does a particular branch tend to redistribute circulating current between generators, or does it tend to deliver load-serving currents, or does it perform both roles? The present work conjectures that, for instance, these new centrality measures can offer insights into which branches cause congestion that affects particular market outturns. A branch with a higher centrality of C^{Circ} could be expected to have a loading that varies a lot with particular dispatch conditions, whereas a branch with low C^{Circ} centrality could be anticipated to fluctuate less.

III. RESULTS

A. Test platform

The matrix calculations were implemented in MATLAB, with Cytoscape [44] providing network visualization functions. A notional 19 bus network, `case19`, is used to show the various matrices on a small network, and the centrality measures are portrayed for the larger and well-known IEEE test network, `case118` [45]. These systems and raw results are available online in a persistent archival repository at [46].

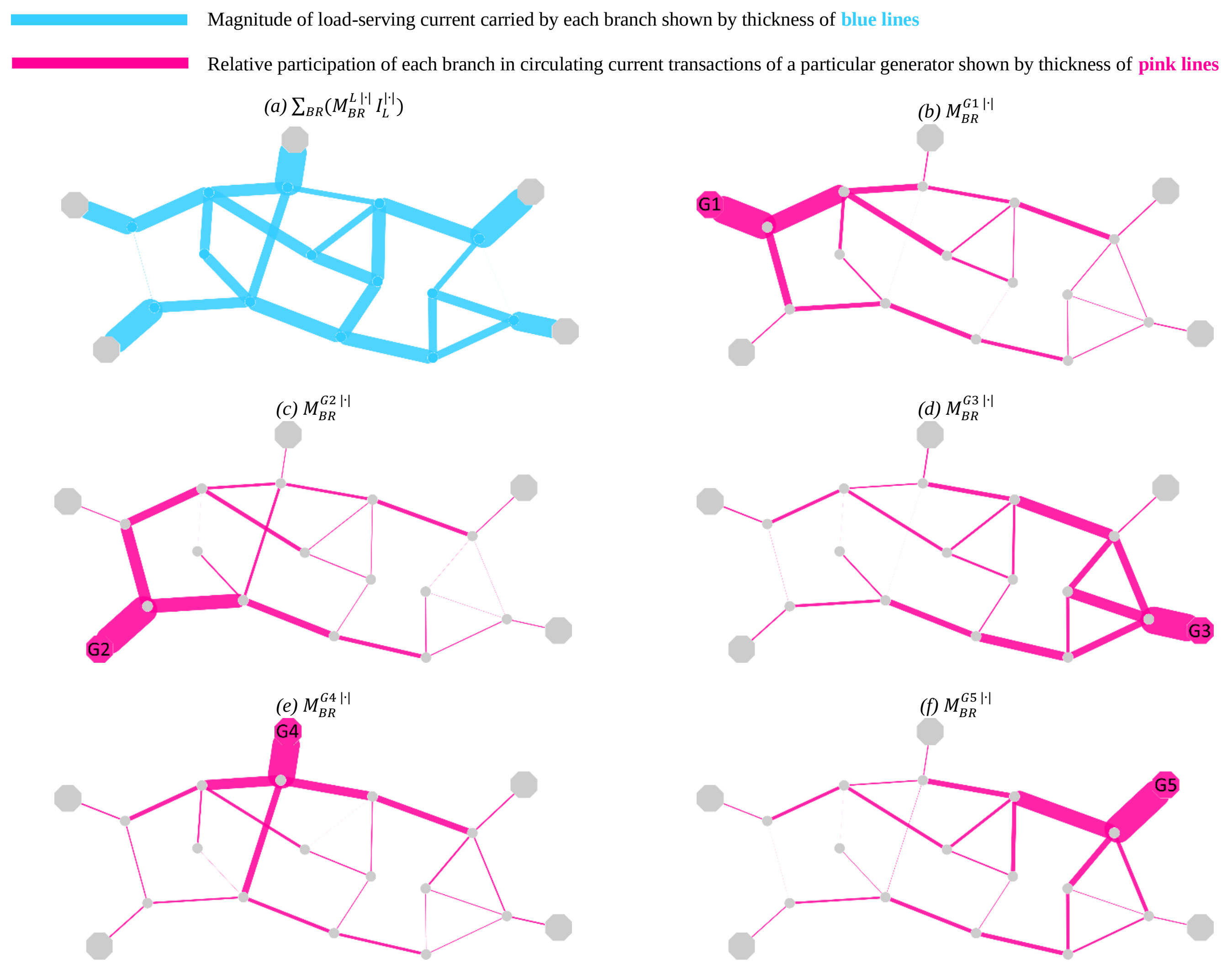

Fig. 1 Visualizations of the M_{BR}^{L} and M_{BR}^{G} matrices for the *case19* network. The load participation factors are shown in blue in the top left panel, and the remaining panels show in pink each branch's participation in a particular generator's circulating current transactions.

TABLE I
CASE19 GENERATOR DISPATCH

	G1	G2	G3	G4	G5
Optimal Dispatch (MW)	64.7	88.9	67.8	93.6	87.5
Dispatch in Fig. 2 (MW)	44.7	88.9	87.8	93.6	87.5

B. Visualizing the M^L and M^G matrices

The separation of branch currents into load and generator induced components via Abdelkader's methods [29-31] offers an interesting new perspective on power flows in electrical networks. To make this separation tangible, the M_{BR}^{L} and M_{BR}^{G} matrices of the `case19` system are represented graphically in Fig 1. The *(a)* panel shows the absolute value of the M_{BR}^{L} matrix multiplied by the prevailing load demands, as in (12). These participation factors remain static for a given load level. Note the differing thickness of the load current participation at each generator's radial feed in *(a)*: this indicates how the system's natural, loss-minimizing load-serving pathways require differing current injections at each generation site [32].

By contrast, panels *(b)* to *(f)* are more dynamic, showing which branches carry each generator's circulating current for a particular dispatch condition. A generator's circulating current contribution is proportional to its deviation from its optimal dispatch level. As such, Fig. 1 maps which branches can become congested due to the chosen dispatch levels for certain generators. As may be anticipated, these effects are most pronounced in the vicinity of the particular generator. These diagrams show the zone of influence for each generator, by explicitly identifying the set of branches a generator's dispatch can affect.

In Fig. 1 each branch is plotted with a thickness proportional to the absolute value of its corresponding entry in the *M* matrix. A scaling can be inferred by noting that the generators connect on a radial feed, and these branches have a near-unity M_{BR}^{G} participation, as, logically, they carry the full quantity of circulating current their generator exchanges.

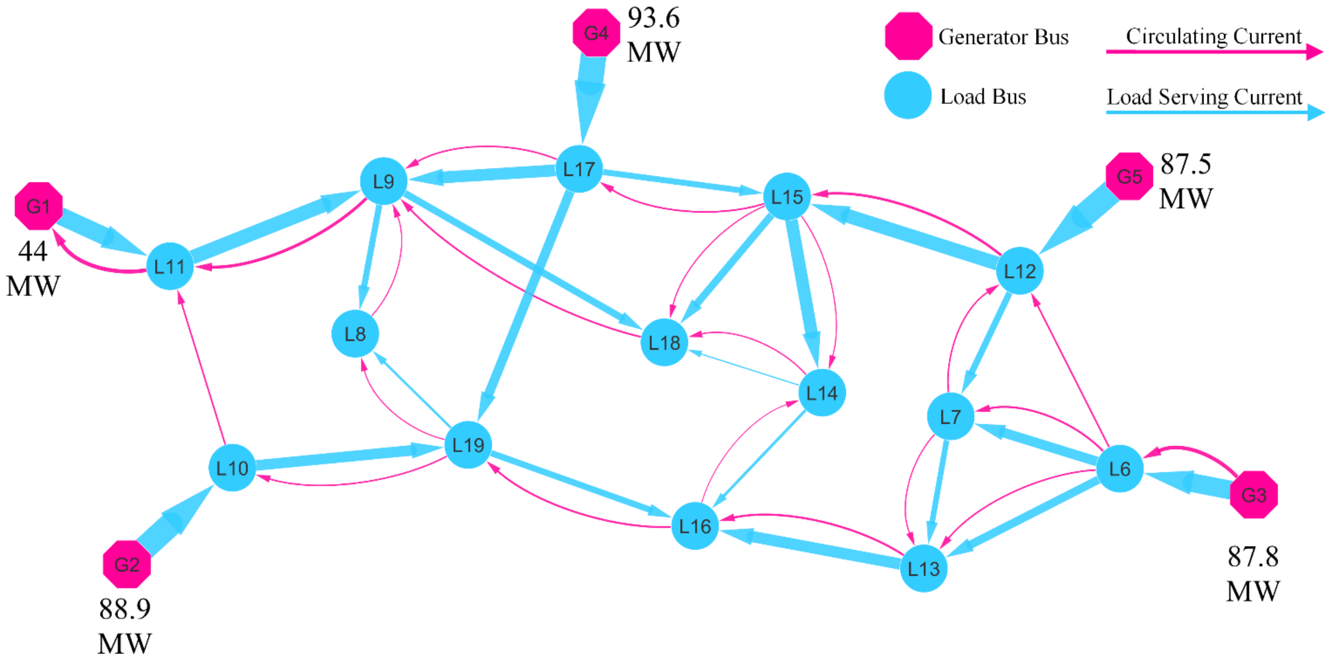

Fig. 2 The real component of the separate current components for the small `case19` test network in the dispatch of Table I. Note the transaction of circulating current between G3, which is here operating above its optimal level, to G1, which is below its ideal.

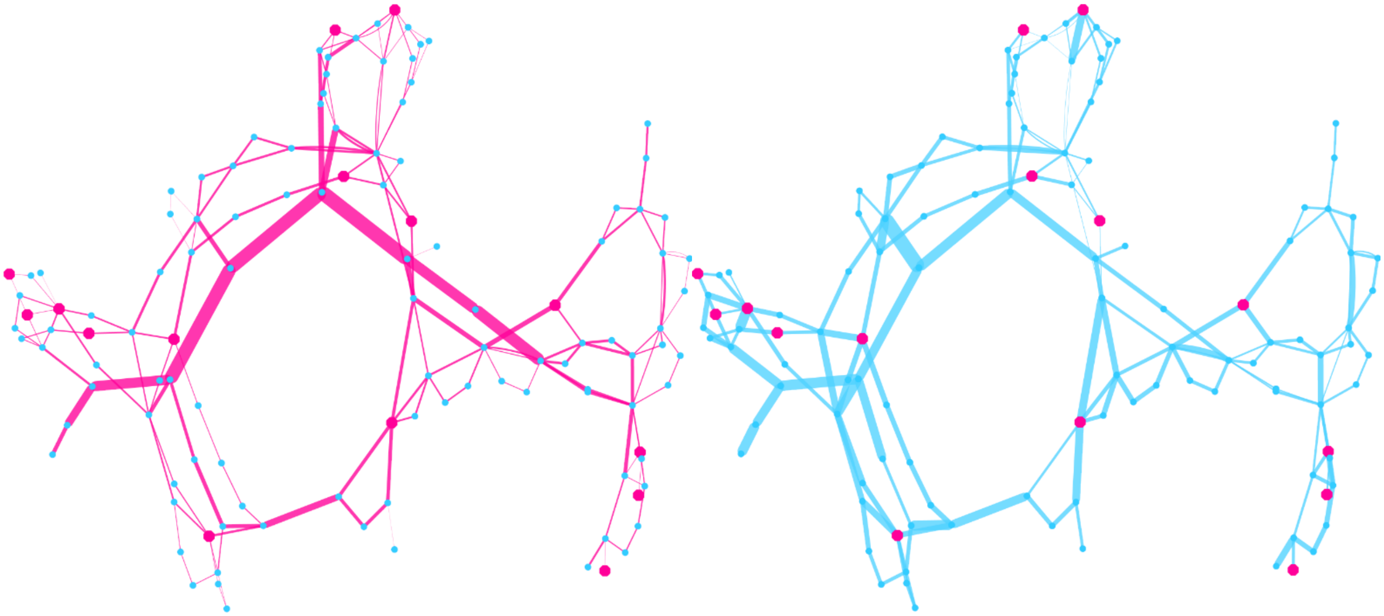

Fig. 3 The `case118` network, with branch thickness showing C^{Circ} (left) and C^{Load} (right).

C. Visualizing separable current components

As had been noted by previous authors [14], the K_{GL} matrix can be used to analytically determine an ideal loss-minimizing dispatch profile for generators [32]. This dispatch level, equal to the matrix product $K_{GL}I_L$, shows the ideal way that load-serving currents would flow through a system. When a generator's actual dispatch deviates from this theoretical optimal, this difference manifests as circulating current.

To make this idea concrete, in the specific conditions visualized in Fig. 2, *G2*, *G4* and *G5* are all producing their optimal level of active power, which manifests solely as load serving current, whereas *G3* operates 20 MW above its optimal level, and so it circulates current to *G1*, which is 20 MW below (see Table I, and compare also Fig. 1 (*a*), where the thickness of load serving currents from each generator corresponds to the optimal dispatch level) The circulating current transaction between these generators is depicted alongside the load serving currents in Fig. 2 (arrow thickness denotes the magnitude of the real component) It is clear that some branches carry only circulating current ($L6 \rightarrow L12$, $L10 \rightarrow L11$), others just load serving current ($L17 \rightarrow L19$), with most carrying varying portions of each. The circulating current transaction shown in Fig. 2 can be considered as the combined effect of the circulating current participations shown in Fig. 1 *(b)* and *(f)*, weighted by each generator's specific circulating contribution.

D. Visualizing C^{Circ} and C^{Load} centrality

The preceding discussion shows that branches in a power system may have varying importance in the load-serving and circulating current aspects of a network. To make this distinction concrete, Fig. 3 depicts both the C^{Circ} and C^{Load} centrality measures for each branch in the `case118` network [45]. While Fig. 2 shows the actual separate current flows for particular dispatch conditions on the `case19` network, the C^{Circ} and C^{Load} measures gauge the *generalized likelihood* that a branch will carry a certain class of current. Fig. 3 uses a force-directed layout to show that the `case118` system has a distinct centrality structure with respect to the load-serving and circulating dimensions of the network. A branch may appear important for load current, or for redistributing circulating generator currents, or for both. These separate centralities indicate distinct functional roles which may be useful in identifying which branches contribute to congestion, or in analyzing a system's robustness against cascading failure, however this is left for future work.

IV. CONCLUSIONS

This work has sought to extend the discussion around the seperability of load-serving and circulating currents in power systems. For instance, this approach, which does not require iterative load flow calculations, allows the direct identification of the set of branches whose loading is affected by a particular generator's dispatch level. Furthermore, the loading of these branches can be explicitly linked to how much each generator's actual dispatch deviates from its analytically calculated optimal value. Centrality measures were proposed to gauge each branches participation in each functional dimension of the network. Future work will assess the insightfulness of these centrality measures for specific power engineering problems.

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