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# Decomposed Vector Rotation-Based Behavioral Modeling for Digital Predistortion of RF Power Amplifiers

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**Abstract**—A new behavioral model for digital predistortion of radio frequency (RF) power amplifiers (PAs) is proposed in this paper. It is derived from a modified form of the canonical piecewise-linear functions (CPWL) using a decomposed vector rotation (DVR) technique. In this model, the nonlinear basis function is constructed from piecewise vector decomposition which is completely different from that used in the conventional Volterra series. Theoretical analysis has shown that this model is much more flexible in modeling RF PAs with non-Volterra-like behavior and experimental results confirm that the new model can produce excellent performance with a relatively small number of coefficients compared to conventional models.

**Index Terms**— Behavioral modeling, canonical piecewise-linear, digital predistortion, power amplifiers, radio frequency, Volterra series, wireless.

## I. INTRODUCTION

DIGITAL predistortion (DPD) is an approach that uses digital signal processing techniques to compensate for nonlinear distortion induced by radio frequency (RF) power amplifiers (PAs) in wireless transmitters. DPD allows PAs to be operated at higher drive levels for higher power efficiency without losing linearity. It has been widely deployed in modern wireless systems, especially in high power cellular base stations [1]. The principle of DPD is that a nonlinear function is built up within the digital domain that is the inverse of the distortion function exhibited by the PA. An accurate PA model must be developed first since it is only when the nonlinear characteristics are correctly modeled and thus reversed, that the overall system response to a signal flowing serially through the cascade of DPD-PA can become linear. In the past decades, many advanced behavioral models for RF power amplifiers have been proposed, such as memory polynomial (MP) [2], envelope-memory polynomial (EMP) [3], generalized memory polynomial (GMP) [4], dynamic deviation reduction (DDR) Volterra model [5][6], and so on. The readers can find extensive information from various review papers [7]-[9] and books [1][10].

From the signal processing point of view, if the PA is considered as a “black” box, modeling the PA can be simply treated as a general nonlinear system identification issue. One might think that there should be plenty of models available for use in DPD because nonlinear system identification is a very active and large field of research where many models and methodologies have been developed over the years [11]. Looking back, one may be surprised that, except for a few neural network papers [12]-[15], the majority of DPD models used today are either simplified or modified from the Volterra series [1]. One question may be raised that what reason has made the Volterra series so special in this particular application.

It is true that the PA modeling is indeed a nonlinear system identification problem, but there are certain constraints and conditions that we must consider when selecting a model for DPD: (i) the model must be in the discrete time domain in order to facilitate implementation in digital circuits; (ii) it should simultaneously take into account both static nonlinearity and memory effects; (iii) it must be able to handle complex-valued signals because DPD is usually conducted in baseband; (iv) the model is linear-in-parameters, meaning that the output of the model should be linear in relation to its coefficients, which is preferable so that linear system identification algorithms, such as least squares (LS), can be directly employed in model extraction. Even if there are many models available in the general nonlinear system identification field; the models that can simultaneously satisfy all the above requirements are very few.

The Volterra series is a combination of linear convolution and nonlinear power series [16]. It satisfies all the DPD conditions and it has a closed form with easy implementation in digital circuits. Furthermore, it perfectly fits the nonlinear behavior of conventional RF power amplifiers, such as class-AB and Doherty, where the PA is linear in the small signal region and tends to become nonlinear when the amplitude of the signal increases. Because its basis functions are polynomial-based, the Volterra series has some inherent limitations. For instance, it is only well-suited to modeling weakly-nonlinear and continuously-smooth systems. With stringent efficiency requirement, many advanced PA architectures have been developed in the past years, such as envelope tracking (ET), outphasing, multi-way/multi-stage Doherty, and various switch-mode PAs [17]. These PAs are to

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a greater or lesser extent using multiple transistors or building blocks in various combinations. The behavior of these PAs becomes very different from conventional single-ended versions. As shown in [18], due to dynamic changes of the supply voltage, the ET PA exhibits very distinct characteristics in different power regions while in the case of outphasing, two nonlinear switch-mode PAs are employed [19]. In multi-transistor, e.g., multi-way/ multi-stage, Doherty, an ‘‘S’’ shape of AM/AM characteristics and strong nonlinear memory effects may appear due to internal control of multiple amplifiers [20]. With the continuous push towards wider bandwidth and higher efficiency, more and more complex PA architectures will be deployed in future systems. In these complex systems, the existing Volterra series-based models are facing significant challenges. However, current on-going DPD development is still more or less concentrating on the Volterra series and seems stuck in this paradigm. Very little progress has been made to further improve model performance to meet the new requirements.

The purpose of this paper is intended to jump out of the ‘‘Volterra’’ box and to introduce a new behavioral model whose class of basis function is completely different from that of the Volterra series. This is achieved by modifying the canonical piecewise-linear functions using a decomposed vector rotation technique. Theoretical analysis will show that this new model is much more flexible and efficient in modeling highly-nonlinear and ‘‘unusual’’ power amplifiers. Experimental results will demonstrate its excellent performance compared to conventional models.

The rest of the paper is organized as follows. The proposed model is introduced in Section II and the experimental results are given in Section III, with a brief conclusion in Section IV.

## II. PROPOSED MODEL

Originally proposed by Chua in the 1970s [21], the canonical piecewise-linear function (CPWL) has a very simple structure and it has been proved that it can be used to represent a wide range of continuous nonlinear functions with a high precision [22]. In CPWL representation, the nonlinear function is approximated by a summation of a series of linear functions defined in multiple hyperplanes (partitions) using the ‘‘absolute’’ value operation. If we use CPWL to model a finite-memory nonlinear digital system, the function can be expressed as

$$y(n) = \sum_{i=0}^M a_i x(n-i) + b + \sum_{k=1}^K c_k \left| \sum_{i=0}^M \alpha_{ki} x(n-i) - \beta_k \right| \quad (1)$$

where  $x(n)$  and  $y(n)$  is the input and the output, respectively.  $|\cdot|$  denotes ‘‘absolute’’ (ABS) operation.  $K$  is the number of partition and  $\beta_k$  is the threshold that defines the boundary of the partition.  $M$  represents the memory length and  $a_i$ ,  $b$ ,  $c_k$  and  $\alpha_{ki}$  are the coefficients [23]. A block diagram of the model structure is shown in Fig. 1, where we can see that the nonlinear function can be implemented by using linear filters instead of high order polynomials. The nonlinear operation is

simply achieved by using the ‘‘absolute’’ value operation, which only involves changing the sign of the input. To date, this model has been widely used in nonlinear circuit modeling and control, but it has received very little attention in the behavioral modeling of RF power amplifiers except that Optichron have implemented this model in their DPD products [24] and a few preliminary studies have appeared in the literature [25][26]. In the author’s opinion, the main reason is that the existing model only satisfies two of the DPD modeling conditions: in the discrete time domain and simultaneously taking into account both static nonlinearity and memory effects. The two other requirements, linear-in-parameters and dealing with complex-valued signals, have not been properly addressed.

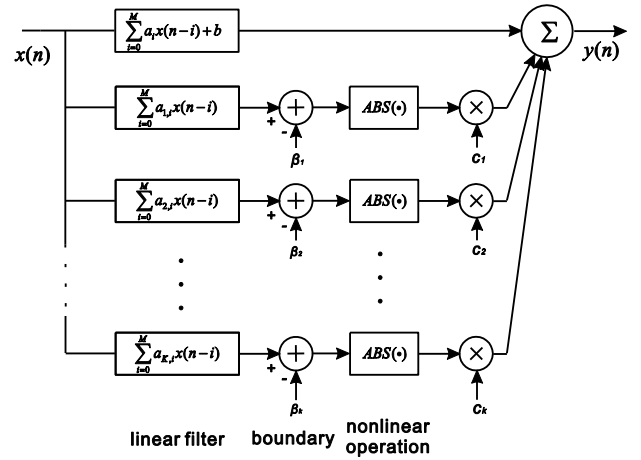


Fig. 1. CPWL model structure.

### • Linear-in-parameters

In (1), the hyperplane partition is defined by using linear filters with thresholds, i.e.,  $\sum_{i=0}^M \alpha_{ki} x(n-i) = \beta_k$ , which provides great flexibilities in modeling nonlinear systems. It, however, creates difficulties in finding proper values for  $\alpha_{ik}$  and  $\beta_k$  in practice because they are not linear in relation to the output and thus complex nonlinear optimization process must be conducted. In [27], the authors showed that it is possible to divide the partition in lattice format, i.e., using  $x(n-i) = \beta_k$  for  $i=0, 1, \dots, M$ . The absolute operation can thus be applied on each delayed sample instead of a full filter. In this case, for each given threshold, the nonlinear approximation is composed of multiple parallel branches, as shown in Fig. 2. The nonlinear basis function in (1) can then be changed from

$$\sum_{k=1}^K c_k \left| \sum_{i=0}^M \alpha_{ki} x(n-i) - \beta_k \right| \quad (2)$$

to

$$\sum_{k=1}^K \sum_{i=0}^M c_{ki} |x(n-i) - \beta_k| \quad (3)$$

and finally (1) can be converted to

$$y(n) = \sum_{i=0}^M a_i x(n-i) + b + \sum_{k=1}^K \sum_{i=0}^M c_{ki} |x(n-i) - \beta_k| \quad (4)$$

As shown in [28], (4) can be treated as a special case of (1). Because the nonlinear ABS operation is only applied on each individual samples, the nonlinear interactions between the present and past samples are not taken into account, and thus the model constructed from (4) is not capable of approximating an arbitrarily given function to a desired precision as (1) does. But importantly, this modification has a major advantage: all the coefficients,  $a_i$ ,  $b$  and  $c_{ki}$ , are now linear in relation to the output, if the thresholds,  $\beta_k$ , are pre-selected, e.g.,  $\beta_k = k/K$  for  $k=1, 2, \dots, K$ . It makes the model extraction process much simpler since general linear system identification algorithms, such as least squares, can now be employed.

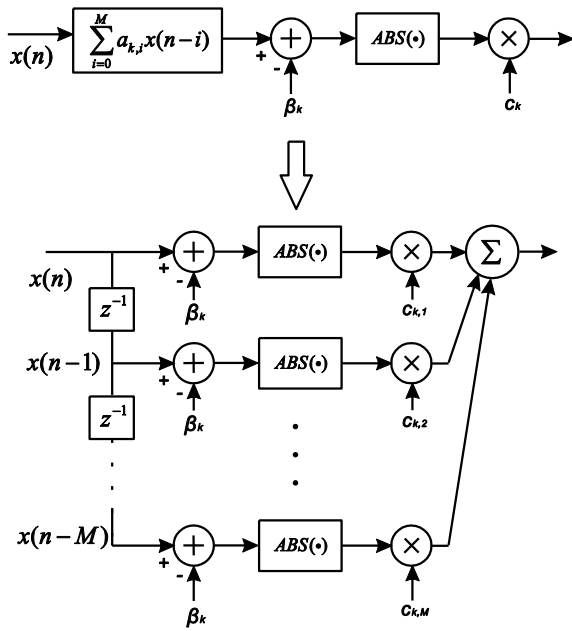


Fig. 2. Modification of the basis function.

- *Ability to deal with complex-valued signals*

To deal with complex-valued signals, it is straightforward for the linear terms in (4),

$$\sum_{i=0}^M a_i x(n-i) \Rightarrow \sum_{i=0}^M \tilde{a}_i \tilde{x}(n-i) \quad (5)$$

where the real-valued input  $x(n-i)$  and the coefficients  $a_i$  can be replaced by the complex baseband signal  $\tilde{x}(n-i)$  and the complex-valued coefficients  $\tilde{a}_i$ , respectively. The constant term  $b$  in (4) is normally used to represent DC offset which

can be eliminated in a PA or DPD model. To create the hyperplane partitions, the absolute operation in (4) cannot be directly applied because both magnitude and phase must be considered in the complex signals. To resolve this problem, in this work, we propose to conduct this operation in four steps: (i) calculate the magnitude value of the signal; (ii) subtract away the threshold; (iii) apply an “absolute” operation; (iv) restore the phase, namely, the real-valued absolute operation is changed from

$$|x(n-i) - \beta_k| \quad (6)$$

to

$$\|\tilde{x}(n-i) - \beta_k\| e^{j\theta(n-i)} \quad (7)$$

where the inner  $|\cdot|$  returns the magnitude of  $\tilde{x}(n-i)$  while the outer  $|\cdot|$  is the normal real-valued absolute operation.  $\theta(n-i)$  represents the phase of  $\tilde{x}(n-i)$ . As illustrated in Fig. 3, this operation becomes a vector decomposition and phase restoration process. The magnitude of the complex signal is first subtracted by the threshold,  $\beta_k$ , and then passed to the “ABS” block. The output from the “ABS” operation is used as the magnitude of the new vector. The phase of the new vector is finally restored to that of the original signal. For instance, for  $\beta_k=0.3$ , a complex signal  $0.6\angle 45^\circ$  passing the operation of (7) will result in  $0.3\angle 45^\circ$  while  $0.1\angle 170^\circ$  will result in  $0.2\angle 170^\circ$ . This vector decomposition process is similar to the piecewise Volterra model proposed in [18] but the signal operation is different. In [18], the input signal vector is decomposed into multiple data streams and different nonlinear functions are applied on each data stream before being combined together to form the final nonlinear transfer function, while here the original signal is directly used in the transfer function and only one data stream is required and one nonlinear function is employed.

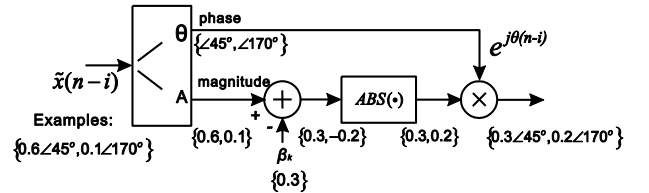


Fig. 3. Vector decomposition and phase restoration.

If we express  $\beta_k$  in a vector format, (7) can be re-written as

$$\begin{aligned} & \|\tilde{x}(n-i) - \beta_k\| e^{j\theta(n-i)} \\ &= \begin{cases} \tilde{x}(n-i) - \beta_k e^{j\theta(n-i)} & |\tilde{x}(n-i)| \geq \beta_k \\ \left[ \tilde{x}(n-i) - \beta_k e^{j\theta(n-i)} \right] e^{j180^\circ} & |\tilde{x}(n-i)| < \beta_k \end{cases} \quad (8) \end{aligned}$$

where we can see that the “absolute” operation on the complex signal here is equivalent to phase rotation on a vector: if the

original signal magnitude is greater than  $\beta_k$ , no phase change is required on the new (“difference”) vector while if the signal magnitude is smaller than  $\beta_k$ , the vector is rotated by 180 degrees. This is where the “core” nonlinear process is introduced in the model. We call this process *Decomposed Vector Rotation* (DVR) and the proposed behavioral model based on this process, *the DVR model*, as discussed below.

To facilitate the follow-on derivation, we define the term  $\|\tilde{x}(n-i) - \beta_k\| e^{j\theta(n-i)}$  as the  $1^{\text{st}}$ -order basis function term,

$$B_{ki,1}(n) = \|\tilde{x}(n-i) - \beta_k\| e^{j\theta(n-i)} \quad (9)$$

Note that “ $1^{\text{st}}$ -order” here only indicates that one signal term is used in the function. It does not mean the first-order nonlinearity. In fact, each such basis function can generate very high order nonlinearities in the model. Equation (4) is then converted into a complex format as,

$$\begin{aligned} \tilde{y}(n) = & \sum_{i=0}^M \tilde{a}_i \tilde{x}(n-i) \\ & + \sum_{k=1}^K \sum_{i=0}^M \tilde{c}_{ki} \|\tilde{x}(n-i) - \beta_k\| e^{j\theta(n-i)} \end{aligned} \quad (10)$$

where the model now satisfies all four conditions of the PA modeling for DPD.

- *Higher-order extension and variations*

In principle, (10) can now be used to form a nonlinear function to model the power amplifiers, but it might not be able to reach the desired accuracy because the model cannot take into account the interactions of the present and past samples which are often very important in the PA modeling. To improve the performance, we should explore further options and make the representation more suitable for power amplifiers.

As we know, in a real system, nonlinear distortion in a power amplifier is mainly caused by amplitude variations, e.g., AM/AM and AM/PM conversions. To effectively model this amplitude dependent nonlinearity, we propose to extend the  $1^{\text{st}}$ -order basis function to include more amplitude dependent information. For the complex-valued baseband signal, it can be achieved via multiplying the  $1^{\text{st}}$ -order basis term with the magnitude of  $\tilde{x}(n)$ , that is,

$$B_{ki,21}(n) = \|\tilde{x}(n-i) - \beta_k\| e^{j\theta(n-i)} \cdot |\tilde{x}(n)| \quad (11)$$

This makes the nonlinear basis function similar to the cross terms in the Volterra model, which can enhance the nonlinear fitting capability [29]. We define (11) as the  $2^{\text{nd}}$ -order type-1 term. Note that again the “ $2^{\text{nd}}$ -order” here only indicates two terms are multiplied and it does not mean the  $2^{\text{nd}}$ -order nonlinearity.

In the literature [1][4], we have seen that different combinations of cross terms can have different effects in terms of model accuracy for different types of PAs. Similarly, we

could consider other variations in the new model when adding the magnitude terms. For instance, if we make the phase aligning with  $\tilde{x}(n)$  rather than  $\tilde{x}(n-i)$  in (11), the basis function then becomes

$$B_{ki,22}(n) = \|\tilde{x}(n-i) - \beta_k\| \cdot \tilde{x}(n) \quad (12)$$

This is another cross-term, which is similar to the envelope memory polynomial [3] or the gain polynomial model [30]. We define it as the  $2^{\text{nd}}$ -order type-2 term.

If we change the  $|\tilde{x}(n)|$  to  $|\tilde{x}(n-i)|$  in (11), the function then becomes

$$B_{ki,23}(n) = \|\tilde{x}(n-i) - \beta_k\| \cdot \tilde{x}(n-i) \quad (13)$$

This term is similar to the memory polynomial term [2]. We define it as the  $2^{\text{nd}}$ -order type-3 term.

If we treat the ABS operation in (11) as a nonlinear curve fitting, the proposed modeling terms can be used to form a model that is equivalent to the DDR-Volterra model [5]. For instance, the first-order DDR terms can be generated from

$$B_{DDR,term1}(n) = \|\tilde{x}(n) - \beta_k\| \cdot \tilde{x}(n-i) \quad (14)$$

and

$$B_{DDR,term2}(n) = \|\tilde{x}(n) - \beta_k\| \cdot \tilde{x}^2(n) \tilde{x}^*(n-i) \quad (15)$$

Equation (11) also can be extended to higher orders, for instance,

$$B_{ki,p1}(n) = \|\tilde{x}(n-i) - \beta_k\| e^{j\theta(n-i)} \cdot |\tilde{x}(n)|^p \quad (16)$$

where  $p$  is an integral number.

It is worth mentioning that, when we construct nonlinear low-pass equivalent behavioral models for DPD, odd-parity and unitary phase constraints must be satisfied, as discussed in [31][32].

- *The complete model*

The linear and nonlinear terms discussed above can be combined together to form the complete DVR model equation, that is,

$$\begin{aligned} \tilde{y}(n)|_{DVR} = & \sum_{i=0}^M \tilde{a}_i \tilde{x}(n-i) && \text{linear} \\ & + \sum_{k=1}^K \sum_{i=0}^M \tilde{c}_{ki,1} \|\tilde{x}(n-i) - \beta_k\| e^{j\theta(n-i)} && \text{1st-order basis} \\ & + \sum_{k=1}^K \sum_{i=0}^M \tilde{c}_{ki,21} \|\tilde{x}(n-i) - \beta_k\| e^{j\theta(n-i)} \cdot |\tilde{x}(n)| && \text{2nd-order type-1} \\ & + \sum_{k=1}^K \sum_{i=1}^M \tilde{c}_{ki,22} \|\tilde{x}(n-i) - \beta_k\| \cdot \tilde{x}(n) && \text{2nd-order type-2} \\ & + \sum_{k=1}^K \sum_{i=1}^M \tilde{c}_{ki,23} \|\tilde{x}(n-i) - \beta_k\| \cdot \tilde{x}(n-i) && \text{2nd-order type-3} \\ & + \sum_{k=1}^K \sum_{i=1}^M \tilde{c}_{ki,24} \|\tilde{x}(n) - \beta_k\| \cdot \tilde{x}(n-i) && \text{DDR term-1} \\ & + \dots && \end{aligned} \quad (17)$$

where again  $\tilde{x}(n)$  and  $\tilde{y}(n)$  is the input and the output, respectively.  $\beta_k$  is the threshold,  $K$  is the number of threshold and  $M$  is the memory length. In real applications, it is not necessary to include all the terms in the model. The model terms can be selected and configured according to the PA characteristics and the real system requirements. For instance, in our test, the *linear* and the *2<sup>nd</sup>-order type-1* terms are often dominant in most cases.

We can see that the basis functions of the new model are built up from the “absolute” operations, which do not have limitations on the selections of nonlinearity orders. In other words, the new model can be used to characterize very high order nonlinearities with a small number of terms. Since the nonlinear functions are composed in piecewise manner, it does not have any restrictions on the shapes of the nonlinear curves. Therefore, this model is much more flexible and capable in modeling highly nonlinear and “unusual” power amplifiers compared to the Volterra models. Furthermore, the model is linear-in-parameters: all the coefficients can be extracted in one matrix by employing linear optimization algorithms. It has a very simple model structure that is very easy to implement in digital circuits.

### III. EXPERIMENTAL RESULTS

In this section, we present some experimental results to validate the model.

#### A. Envelope Tracking PA with LTE Signal

The first test was conducted on an envelope tracking power amplifier with an LTE signal. The test bench set up is the same as that used in [33], shown in Fig. 4, which includes a PC with MATLAB software, a baseband and RF board, an in-house designed high power GaN PA, and a commercially available envelope modulator. The main PA was operated at 2.14 GHz and was biased at Class-AB mode with  $V_{GS} = -1.21$  V. The drain voltage  $V_{DD}$  was controlled by the envelope modulator. The tracking voltage was varied from 20 to 60 V. A 20 MHz LTE signal with 6.5 dB PAPR was used for the test and the average output power of the PA was 45 dBm. Around 16,000 I/Q samples were recorded with a sampling rate at 122.88 MS/PS. After time alignment and normalization, 4,000 samples were used for model extraction, while the remaining different samples were used for system performance evaluation in separate measurements. The standard least squares algorithm was used in model extraction and in-direct learning was employed in the DPD test. ACPR (adjacent channel power ratio) and NRMSE (normalized root mean square error) were used for performance evaluation.

The AM/AM and AM/PM characteristics of the PA are shown in Fig. 5, where we can see that without DPD, there was a large gain reduction when the amplitude is low, which leads to strong nonlinearity in the low power region as usually occurs in ET PAs. As discussed in [18], it is difficult to use a single function based Volterra model to linearize this PA because the single Volterra function is only best suitable for a system that is linear in the small signal region and tends to

become nonlinear when the amplitude of the signal increases. To resolve this problem, the piecewise Volterra model proposed in [18] can be employed where the nonlinear function is divided into multiple pieces and different function can be selected to fit each piece separately. The piecewise Volterra model works reasonably well in most cases but it is still limited to the Volterra format and the system complexity can significantly increase if multiple segments are required. In this work, we proposed to use the newly proposed DVR model to linearize this PA.

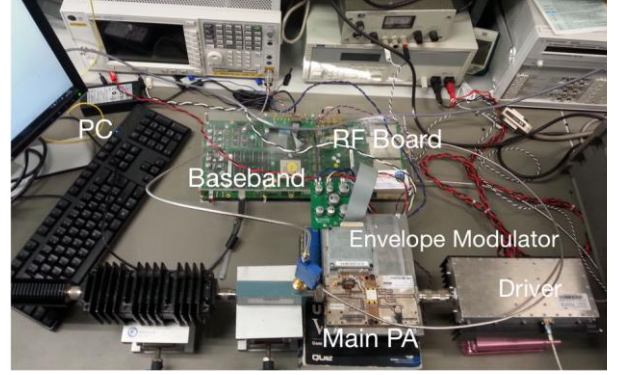


Fig. 4. The envelope tracking PA test bench.

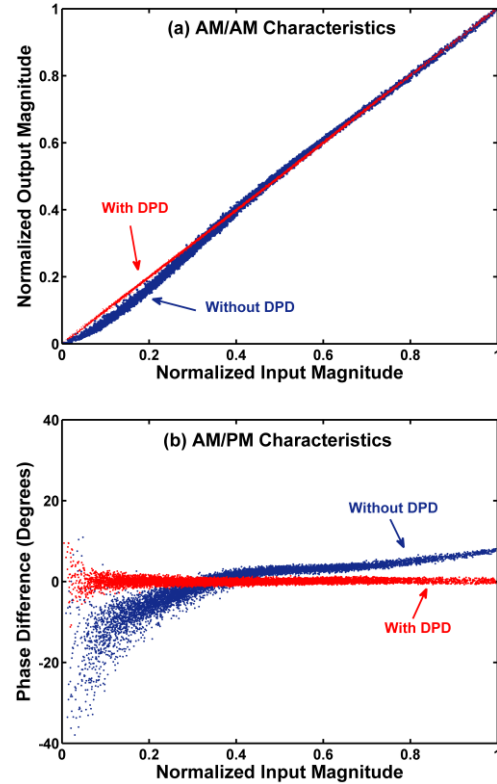


Fig. 5. (a) AM/AM and (b) AM/PM plots of the ET PA with and without DPD.

In this test, the DVR model with  $K=8$  and  $M=3$  was employed. The total number of coefficients was 84. The AM/AM and AM/PM with DPD are shown in Fig. 5, where

we can see that the AM/AM is now a straight line and the phase difference is reduced to within 5 degrees. The spectra plots are shown in Fig. 6 and the performance is summarized in Table I. The ACPR has been improved by over 22 dB at  $\pm 20$  MHz offset and 14 dB at  $\pm 40$  MHz. The NRMSE is reduced from 8.42% to 0.98%. In comparison, a piecewise Volterra model [18] was also employed. The best performance was achieved when a simplified 2<sup>nd</sup>-order DDR-Volterra model [34] with two segments used. The threshold was set at 0.5, and nonlinear order at {9, 9} and memory length {4, 4}. The total number of coefficients was 146. The performance is also summarized in Table I. The NRMSE is close to that of the DVR model, but the ACPRs are 2 to 3 dB worse.

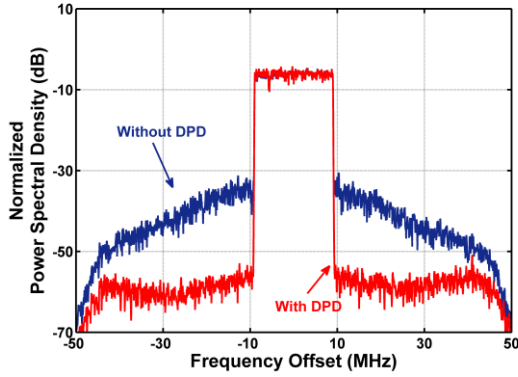


Fig. 6. Output spectra plots of the ET PA with and without DPD.

TABLE I System Performance of the ET PA

	No. of coefficients	ACPR (-dBc)		NRMSE (%)
		$\pm 20$ MHz	$\pm 40$ MHz	
Without DPD		30.8/32.1	40.3/40.9	8.42
DVR DPD	84	54.4/54.0	54.0/53.8	0.98
Volterra DPD	146	51.5/52.1	51.7/51.4	1.10

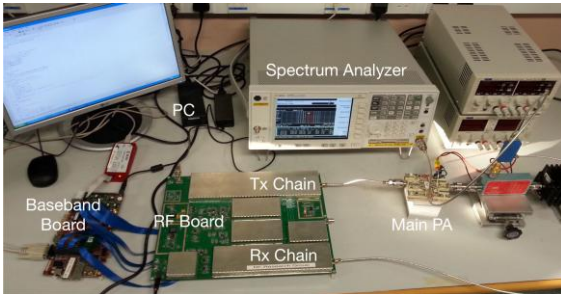


Fig. 7. The Doherty PA test bench.

### B. Doherty PA with Mixed-mode Signal

In the second test, an in-house designed LDMOS Doherty PA was operated at 2.14 GHz with an average output power at 47 dBm, excited by a 60 MHz mixed-mode signal composed

of an 18 MHz 4-carrier GSM and a 20 MHz LTE signal. The test bench set up is the same as that used in [35], shown in Fig. 7. The sampling rate was set at 368.64 MSPS.

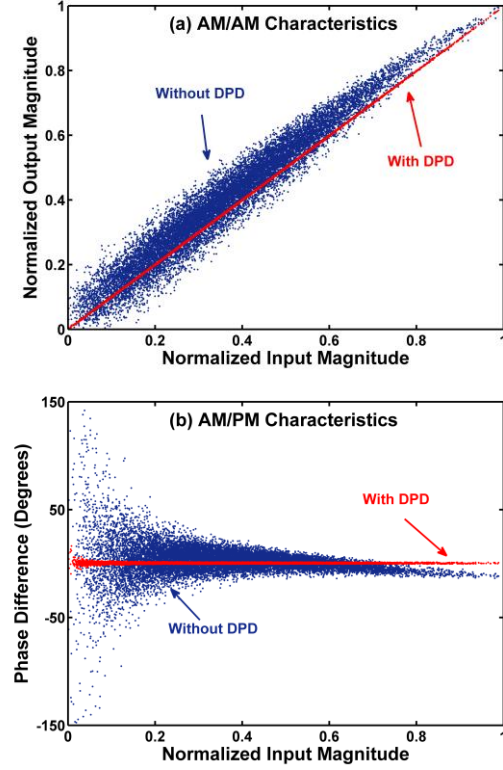


Fig. 8. (a) AM/AM and (b) AM/PM plots of the Doherty PA with and without DPD.

Without DPD, the AM/AM and AM/PM characteristics are shown in Fig. 8. From these plots, there seems not much gain compression or expansion, but it does not necessarily mean that the PA is linear. There is strong nonlinear distortion induced by the PA, which can be observed by checking the ACPR and IMD (intermodulation distortion) values, e.g., -25 dBc ACPR for the LTE signal and -29 dBc IMD3 for the GSM, shown in Fig. 9. The reason why it looks like a linear system is because AM/AM and AM/PM plots are only showing the “total” or average effects from the input to the output mapping but not the relationships between the individual samples. Inside the PA, due to the fact that multiple transistors are involved, multiple nonlinearities can occur and these nonlinearities are mixed together. For instance, in a two-transistor Doherty PA, the signals at high amplitude are amplified by two amplifiers: the main amplifier and the auxiliary amplifier. The total gain, mapping from the instantaneous input amplitude to the instantaneous output amplitude, may look linear because the two amplifiers can compensate for each other. However, both amplifiers are operated in strong nonlinear regions; the signal is nonlinearly processed by two PAs. If strong memory effects exist in both PAs, the output signal will nonlinearly depends on the previous samples. This nonlinear behavior cannot be treated as that in a normal linear system. These types of PAs are thus

usually very difficult to be linearized.

In this test, a DVR model with  $K=4$  and  $M=6$  was employed. The total number of coefficients was 95. The AM/AM and AM/PM with DPD are shown in Fig. 8, where we can see that the nonlinear distortion is almost completely removed with DPD. The spectra plots are shown in Fig. 9 and the performance is summarized in Table II. The ACPR for the LTE signal is improved by more than 28 dB and IMD3 for the GSM signal is improved by 37 dB. The NRMSE is reduced from 15.8% to 0.69%. In comparison, a piecewise Volterra model was also employed. The best performance was achieved when a simplified 2<sup>nd</sup>-order DDR-Volterra model with two segments used. The threshold was set at 0.6, and nonlinear order at {7, 7} and memory length {5, 5}. The total number of coefficients was 138. The performance is also summarized in Table II.

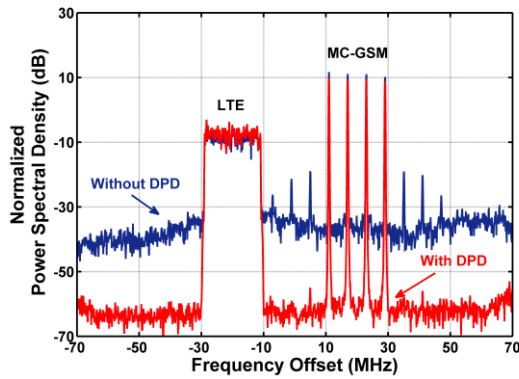


Fig. 9. Output spectra plots of the Doherty PA with and without DPD.

TABLE II System Performance of the Doherty PA

	No of coefficients	LTE ACPR (-dBc) -20MHz	MC-GSM IMD3 (-dBc)	NRMSE (%)
Without DPD		26.5	29.2	15.80
DVR DPD	95	54.7	66.3	0.69
Volterra DPD	138	52.0	62.1	1.08

#### IV. CONCLUSION

In this paper, a new behavioral model has been presented. The core idea of this model is that the polynomial-type of basis function in the Volterra series is replaced by vector decomposition and phase rotation, which makes the new model much more flexible and capable in modeling “unusual” nonlinear power amplifiers than that of the conventional models. Experimental tests have confirmed that the new model can produce excellent performance in linearizing ET and Doherty PAs with a relatively small number of coefficients.

The author would like to point out that this work is not intended to produce a “superior” model to replace the existing

solutions. In most conventional cases, one would expect that the Volterra models may still be one of the most favorable choices because of their simplicity, high performance and maturity. The proposed DVR model provides an alternative way to model RF PAs under various existing and future challenging conditions. Although excellent performance has already been demonstrated, the DVR model is only at its early stage. Further development must be conducted to expand its applications and to further enhance its performance.

#### REFERENCES

- [1] J. Wood, *Behavioral Modeling and Linearization of RF Power Amplifiers*, Norwood, MA: Artech House, 2014.
- [2] J. Kim and K. Konstantinou, “Digital predistortion of wideband signals based on power amplifier model with memory,” *Electron. Lett.*, vol. 37, no. 23, pp. 1417–1418, Nov. 2001.
- [3] O. Hammi, F. M. Ghannouchi, and B. Vassilakis, “A compact envelope-memory polynomial for RF transmitters modeling with application to baseband and RF-digital predistortion,” *IEEE Microw. Wireless Compon. Lett.*, Vol. 18, No. 5, 359–361, May 2008.
- [4] D. R. Morgan, Z. Ma, J. Kim, M. G. Zierdt, and J. Pastalan, “A generalized memory polynomial model for digital predistortion of RF power amplifiers,” *IEEE Trans. Signal Process.*, vol. 54, no. 10, pp. 3852–3860, Oct. 2006.
- [5] A. Zhu, J. C. Pedro, and T. J. Brazil, “Dynamic deviation reduction-based Volterra behavioral modeling of RF power amplifiers,” *IEEE Trans. Microw. Theory Techn.*, vol. 54, no. 12, pp. 4323–4332, Dec. 2006.
- [6] A. Zhu, P. J. Draxler, J. J. Yan, T. J. Brazil, D. F. Kimball, and P. M. Asbeck, “Open-loop digital predistorter for RF power amplifiers using dynamic deviation reduction-based Volterra series,” *IEEE Trans. Microw. Theory Techn.*, vol. 56, no. 7, pp. 1524–1534, Jul. 2008.
- [7] J. C. Pedro and S. A. Maas, “A comparative overview of microwave and wireless power-amplifier behavioral modeling approaches,” *IEEE Trans. Microw. Theory Techn.*, vol. 53, no. 4, pp. 1150–1163, Apr. 2005.
- [8] M. Isaksson, D. Wisell, and D. Ronnow, “A comparative analysis of behavioral models for RF power amplifiers,” *IEEE Trans. Microw. Theory Techn.*, vol. 54, no. 1, pp. 348–359, Jan. 2006.
- [9] F. M. Ghannouchi and O. Hammi, “Behavioral modeling and predistortion,” *IEEE Microw. Mag.*, vol. 10, no. 7, pp. 52–64, Dec. 2009.
- [10] F.-L. Luo, *Digital Front-End in Wireless Communications and Broadcasting: Circuits and Signal Processing*, Cambridge, UK: Cambridge University Press, 2011.
- [11] O. Nelles, *Nonlinear System Identification: From Classical Approaches to Neural Networks and Fuzzy Models*, Springer-Verlag Berlin Heidelberg 2001.
- [12] T. Liu, S. Boumaiza, and M. Ghannouchi, “Dynamic behavioral modeling of 3G power amplifiers using real-valued time-delay neural networks,” *IEEE Trans. Microw. Theory Techn.*, vol. 52, no. 3, pp. 1025–1033, Mar. 2004.
- [13] J. Zhai, J. Zhou, L. Zhang, J. Zhao, and W. Hong, “The dynamic behavioral model of RF power amplifiers with the modified ANFIS,” *IEEE Trans. Microw. Theory Techn.*, vol. 57, no. 1, pp. 27–35, Jan. 2009.
- [14] J. Zhai, J. Zhou, L. Zhang, and W. Hong, “Behavioral modeling of power amplifiers with dynamic fuzzy neural networks,” *IEEE Microw. Wireless Compon. Lett.*, vol. 20, no. 9, pp. 528–530, Sept. 2010.
- [15] F. M. Kadem and S. Boumaiza, “Physically inspired neural network model for RF power amplifier behavioral modeling and digital predistortion,” *IEEE Trans. Microw. Theory Techn.*, vol. 59, no. 4, pp. 913–923, Apr. 2011.
- [16] M. Schetzen, *The Volterra and Wiener Theories of Nonlinear Systems*, Malabar, FL: Krieger, 2006.
- [17] S. C.ripps, *RF Power Amplifiers for Wireless Communications*, 2<sup>nd</sup> ed. Norwood, MA: Artech House, 2006.
- [18] A. Zhu, P. J. Draxler, H. Chin, T. J. Brazil, D. F. Kimball, and P. M. Asbeck, “Digital predistortion for envelope-tracking power amplifiers using decomposed piecewise Volterra series,” *IEEE Trans. Microw. Theory Techn.*, vol. 56, no. 10, pp. 2237–2247, Oct. 2008.
- [19] P. A. Godoy, S. Chung, T. W. Barton, D. J. Perreault, and J. L. Dawson, “A 2.4-GHz, 27-dBm asymmetric multilevel outphasing power amplifier

- in 65-nm CMOS," *IEEE J. Solid-State Circuits*, vol.47, no.10, pp.2372-2384, Oct. 2012.
- [20] A. Grebennikov and S. Bulja, "High-efficiency Doherty power amplifiers: historical aspect and modern trends," *Proc. IEEE*, vol. 100, no. 12 pp. 3190-3219, Dec. 2012.
- [21] L. O. Chua and S. M. Kang, "Section-wise piecewise-linear functions: Canonical representation, properties, and applications," *Proc. IEEE*, vol. 65, pp. 915-929, 1977.
- [22] L. O. Chua and A. C. Deng, "Canonical piecewise-linear representation," *IEEE Trans. Circuits Syst.*, vol. 35, no. 1, pp. 101-111, Jan. 1988.
- [23] J. N. Lin and R. Unbehauen, "Adaptive nonlinear digital filter with canonical piecewise-linear structure," *IEEE Trans. Circuits Syst.*, vol. 37, pp. 347-353, 1990.
- [24] R. G. Batruni, "Model based distortion reduction for power amplifiers" U.S. 20090021304A1, Jan. 22, 2009.
- [25] M. Y. Cheong, S. Werner, M. Bruno, J. Figueroa, J. Cousseau, and R. Wichman, "Adaptive piecewise linear predistorters for nonlinear power amplifiers with memory," *IEEE Trans. Circuits Syst. I: Reg. Pap.*, vol. 59, no. 7, pp. 1519-1532, Jul. 2012.
- [26] J. Zhai, L. Zhang, J. Zhou, X. Zhu, and W. Hong, "A nonlinear filter-based Volterra model with low complexity for wideband power amplifiers," *IEEE Microw. Wireless Compon. Lett.*, vol. 24, no. 3, pp. 203-205, Mar. 2014.
- [27] J.-N. Lin and R. Unbehauen, "Canonical piecewise-linear approximations," *IEEE Trans. Circuits Syst. I*, vol. 39, no. 8, pp. 697-699, Aug. 1992.
- [28] H.-Q. Xu, J.-N. Lin, and R. Unbehauen, "Piecewise-linear characteristics of MSLMS filter," in *Proc. of the 1995 URSI International Symposium on Signals, Systems, and Electronics*, pp.435-438, San Francisco, Oct. 1995.
- [29] A. Zhu and T. J. Brazil, "Behavioral modeling of RF power amplifiers based on pruned Volterra series," *IEEE Microw. Wireless Compon. Lett.*, vol. 14, no. 12, pp. 563-565, Dec. 2004.
- [30] R. N. Braithwaite, "General principles and design overview of digital predistortion," in *Digital Processing for Front End in Wireless Communication and Broadcasting*, Fa- Long Luo (ed.), Cambridge University Press, pp. 143-191, 2011.
- [31] E. G. Lima, T. R. Cunha, H. M. Teixeira, M. Pirola, and J. C. Pedro, "Base-band derived Volterra series for power amplifier modeling," in *IEEE MTT-S Int. Microw. Symp. Dig.*, Boston, MA, Jun. 2009, pp. 1361-1364.
- [32] E. G. Lima, T. R. Cunha, and J. C. Pedro, "A physically meaningful neural network behavioral model for wireless transmitters exhibiting PM-AM/PM-PM distortions," *IEEE Trans. Microw. Theory Techn.*, vol. 59, no. 12, pp. 3512-3521, Dec. 2011.
- [33] C. Yu and A. Zhu, "A single envelope modulator-based envelope-tracking structure for multiple-input and multiple-output wireless transmitters," *IEEE Trans. Microw. Theory Techn.*, vol. 60, no. 10, pp. 3317-3327, Oct. 2012.
- [34] L. Guan and A. Zhu, "Simplified dynamic deviation reduction-based Volterra model for Doherty power amplifiers," in *Proc. IEEE Int. Integr. Nonlin. Microw. Millimeter-Wave Circuits Workshop*, Vienna, Austria, Apr. 2011, pp. 1-4.
- [35] C. Yu, L. Guan, E. Zhu, and A. Zhu, "Band-limited Volterra series-based digital predistortion for wideband RF power amplifiers," *IEEE Trans. Microw. Theory Techn.*, vol. 60, no. 12, pp. 4198-4208, Dec. 2012.

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