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# Towards Embedding Network Usage Charges Within a Peer-to-Peer Electricity Marketplace

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**Abstract**—This paper proposes a novel tariff regime for peer-to-peer energy trading, with an aim to increase transmission efficiency and grid stability by penalising long distance power transactions. In this scheme a portion of the transacted energy is withheld based on the electrical distance between buying and selling parties, calculated here according to the Klein Resistance Distance. This tariff regime is simulated using a dataset of producers and consumers over a 24-hour period. First, a notional marketplace equilibrium simulation is performed, in which consumers can optimally activate demand response resources to exploit local availability of energy. Consumers are observed to move some demand away from peak times to make use of local generation availability. These simulated market out-turns are then used as inputs to a time series power flow analysis, in order to evaluate the network’s electrical performance. The regime is found to decrease grid losses and the magnitude of global voltage angle separation. However, the metric whereby taxes are calculated is found to be too skewed in the utility’s favour and may discourage adoption of the peer-to-peer system. The method also attempts to encourage regulatory adoption by existing grid operators and utilities. Some counter-intuitive allocations of tokenised energy occur, owing to specific consumers’ demand profiles and proximity to generators.

## I. INTRODUCTION

THERE is a sharp increase in renewable generation, motivating proposals for a new decentralised grid model [1]. Peer-to-peer (P2P) energy trading has consequently been proposed as new evolution of electricity markets, allowing producers and consumers to trade energy in a free market [2]. Blockchain, a secured cryptographic proof-based transaction system, has been proposed as a potential facilitator for this system [3], [4]. A nascent base of research has emerged on blockchain-based P2P energy trading [5] with some real world applications [6].

P2P electricity trading arrangements act as a regulatory dispensation that can be layered onto existing power system orientations, including microgrids or larger transmission systems. A new financial regime is put in place to allow participating parties to buy and sell from each other freely [7]. This paper proposes a novel method whereby energy traded between P2P participants is taxed based on the grid distance between parties. The taxed portion of energy could be utilised as a reimbursement to the utility operator for usage of the grid network. Buyers and sellers can thus be geographically far from

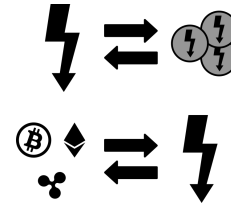


Fig. 1. Visual representation of the energy tokenisation paradigm

each other and retain the ability to buy and sell electrical energy from each other. Furthermore, consumers are encouraged to shift their demand to periods of more abundant local generation.

Since the high-profile rise of the pseudonymous Satoshi Nakamoto’s Bitcoin cryptocurrency [8] blockchain has garnered much attention. The term refers to a decentralised data ledger, which is both trustless and immutable [9]. Data is stored in *blocks* linked by hash chains [10]. A complete transaction list is maintained by all active participants. Involved parties have the option to accept or reject new transactions as they are received. The most popular consensus method is known as *proof of work*, whereby *miners* are tasked with solving increasingly difficult numerical problems. Despite their complexity, problem solutions are designed to be easily verifiable [9]. Blocks are added as they are verified [10]. Miners are rewarded for their contributions, usually by receiving a unit of cryptocurrency. A further evolution of the technology was the rise of *Smart Contracts*. These are distributed scripts that function autonomously and trustlessly [11].

A emerging base of research has been performed on blockchain’s potential use in the electrical energy industry. A majority of this research examine the role of smart contracts in facilitating P2P electricity trade on a microgrid scale [4]. P2P, in its purest form, allows generated energy to be traded between producers and consumers. This trade exists in a financial sense [12], as energy is not necessarily transmitted directly between the two parties. This system may result in a more intermediate role for electrical utilities, perhaps serving more as a liquidity providers than sole energy wholesalers.

The P2P paradigm of transactions is largely reliant on the process of energy *tokenisation*. This term, with origins in data security [13], describes the method of representing a sensitive asset or value cryptographically, enabling simple trade, analysis, or storage. Actual values are stored securely, while simpler *tokens* are manipulated and transferred [13].

For energy tokenisation, the value of a token is typically attached to a quantity of electrical energy (e.g. one kWmin or MWh). A token is created for each unit of energy that is produced [14], which can be transferred, traded or stored. A token is not bound to a physical unit of energy, so parallels to data science end here. However, the energy has now become

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Interests disclosure: the authors hold cryptographic assets.

more tangible to involved parties. Figure 1 illustrates the paradigm of energy tokenisation.

A P2P tokenised energy trading environment with an electrical distance-based tax could thus serve as a means of incentivising both demand side management and engagement from the relevant regulators, while requiring only smart metering and a financial transaction platform. This paper attempts to simulate such a market, and investigate trends in electrical export allocation and the effects on the operation of the power system. This framework forms the starting point for the investigation into these methods.

## II. METHODOLOGY

This section explains the development of a blockchain-hosted P2P energy transaction framework with electrical distance-based tax mechanism, beginning with the electrical distance metric method on which it is based. The market simulation procedure and powerflow simulations used for evaluation are discussed so as to examine the electrical effects of the regime. This exploratory study omits some elements typically used in marketplace simulations, and does not consider any form of financial transactions. Rather, local generation is assumed to be cheaper than utility-level generation. Consumers vie for this local production, with the grid operator serving as liquidity provider.

### A. Proposed Marketplace Structure

The methodology presented here proposes a P2P energy trading system, implemented on a dedicated blockchain. Parties are free to buy and sell at will, with grid-level generation serving as a liquidity provider. Energy exchanges are subject to the an electrical distance-related tax. Some form of *oracle* is required to establish these electrical distances [7].

### B. Electrical Distance Tax

This section proposes Klein resistance as a method of evaluating the electrical distance between transacting parties. The Klein resistance is a measure of Thevenin impedance distance between two nodes, developed in [15]. The method has been utilised in power systems in examples such as [16] and [17]. It has also been used in some form in [7] as a method of establishing the effect of P2P trading partners<sup>1</sup> for calculating the effect of trading with parties in other geographical locations. The Klein Resistance Distance is formulated as below.

Taking the inverse of a power system's admittance matrix  $Y_{bus}$  produces  $Z_{bus}$ . This can be used to find the Thevenin impedance between power system nodes  $i$  and  $j$  as in equation 1 [18].

$$z_{ij}^{thev} = z_{ii} + z_{jj} - z_{ij} - z_{ji} \quad (1)$$

The resulting complex value of  $z_{ij}$  is thus the parallel impedance, or *Klein Resistance Distance*, between those nodes.

<sup>1</sup>Hayes *et al* use a variation on the method that also incorporates the "shortest path".

Taking the magnitude of this complex-valued result produces the scalar as in equation 2.

$$z_{ij} = |z_{ij}^{thev}| \quad (2)$$

For the proposed method, network usage charges are applied in the form of an "energy tax". This tax consists of a portion of the generator's production when a sale is made to a specific consumer. The *energy tax* percentage imposed on each "sale" of kWh token is calculated as in equation 3. This value can be understood as the portion of the purchased energy that is supplied to the consumer, with the remainder being surrendered to the network operator. Hereafter, the term "tax" refers to this mechanism. This notional method of determining tax, which is chosen for simplicity, will be evaluated in the results of this study. It is formulated so that no more than 50% of energy can be withheld as tax.

$$B_{ij} = 1 - \frac{1}{2} \left| \frac{z_{ij}}{z_{max}} \right| \quad (3)$$

### C. Market Simulation Procedure

A marketplace simulation is performed, with the sole objective to determine how rational actors might transact energy and activate their demand response within the proposed regime. The usage of this optimisation may be counter-intuitive to the concept of blockchain, which is generally considered for decentralised platforms. It may be assumed that all transacting parties are *rational actors* i.e. make the most logical choices with regard to lowering their total consumption at any time. Participants are also assumed to have perfect foresight in their own consumption, as well as generation availability. This thus serves only as a rough estimate of a potential market equilibrium. It in no way represents a course of action for a centralised market operator, nor a set of market clearing rules.

Two optimisation variables are defined, namely  $E$  and  $H$ . The prior is a  $(I \times T)$  matrix used in defining the consumption of each consumer at each time point. The latter is a  $(I \times J \times T)$  matrix, representing the allocated generation (with its appropriate tax applied) to each individual consumer at each time point. In terms of energy tokenisation, the method assumes a 1 MWh value per token, a rate that is chosen for simplicity. These tokens can be subdivided as need be i.e. are not granular to any degree. Tokens and their attached energy value are referred to as *Enertoks* hereafter [19].

The objective function sees the optimisation minimise the total consumption of all consumers, after their appropriate Enertoks are allocated. This could be understood as the total energy consumed by the individual consumers provided by the operating utility (a *liquidity provider* role) It also serves to most effectively utilise the distributed generation in the system. This can be seen in equation 4:

$$\min \sum_{t \in T, i \in I, j \in J} (E_{i,t} - H_{i,j}) \quad (4)$$

where:

- $t$  is the time index,
- $T$  is total set of time points considered,
- $j$  is the producer index and
- $J$  is total set of producers considered.

With the objective function set, the first step of the optimisation sees load shifting implemented. A parameter  $s$  represents the percentage whereby a consumer's demand can be raised or lowered at any point. However, each consumer's total consumption across all time points needs to remain constant. This is expressed as the constraint in equation 5.

$$E_{i,t}(1-s) - H_{i,t} \leq E_{i,t} - H_{i,t} \leq E_{i,t}(1+s) - H_{i,t} \quad (5)$$

The  $H$  values in constraint equation 5 allow the optimisation to consider the allocated Enertoks when shifting the load i.e. load will be shifted to periods when more Enertoks are available.

A necessary consideration is to ensure that total consumption before and after shifting remains constant. This is accomplished by the constraint in equation 6.

$$\sum_{t=1}^T E_i = \sum_{t=1}^T C_i \quad (6)$$

where:

- $i$  is the consumer index and
- $C_i$  is the total consumption of consumer  $i$  before load shifting has occurred.

The second step sees the simulation ensure that allocated Enertoks never exceed the consumption at any point. This is shown in constraint equation 7. Similarly, equation 8 ensures that consumption is never negative.

$$E_{ti} \geq \sum_{j=1}^J H_{i,t} \quad (7)$$

$$E_{i,t} \geq 0 \quad (8)$$

The final steps sees Enertoks allocated, with their appropriate taxes applied. This is done according to the constraints in equations 9 and 10.

$$\mathbf{H} \leq \mathbf{G} \cdot \mathbf{B} \quad (9)$$

$$\sum_{t=1}^T \frac{H_{j,t}}{B_{j,t}} \leq G_{j,t} \quad (10)$$

where:

- $\mathbf{B}$  is a parameter matrix of taxes between generators and consumers, calculated according to equation 3 and
- $\mathbf{G}$  is a parameter matrix of generator outputs.

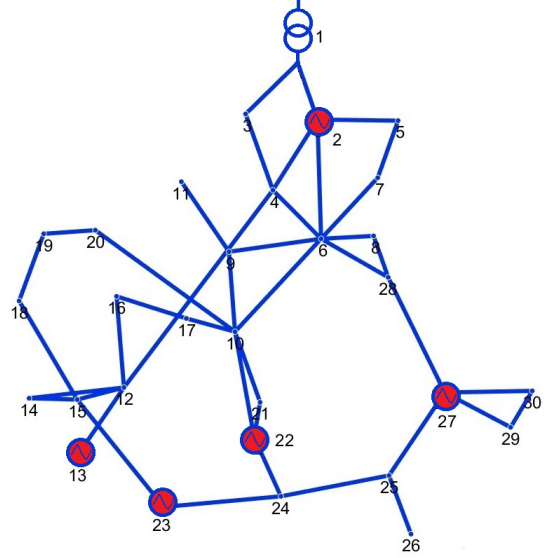


Fig. 2. Plot of IEEE Case\_30 example system [16], [21]

#### D. Powerflow Analysis

Once taxes have been established and the combined Enertok allocation/load shifting optimisation has been implemented, the outturns are applied to the powersystem in question. A powerflow analysis is performed for each dataset time point, with generation and optimised consumption profiles applied to their respective buses. A slack bus is defined, which serves as a simulated connection between this system and the larger transmission network.

### III. TEST PLATFORM

The market simulation is implemented in MATLAB using the YALMIP package [20]. With this optimisation complete, generation and consumption profiles are applied to the powerflow, using IEEE Case\_30 [16]. This is executed in the MATPOWER package [21]. The test dataset is scaled and adapted from [22].

### IV. RESULTS

IEEE Case\_30 power system is used as case study and is shown in Figure 2 [16], [21]. This system consists of 24 load buses and 6 generator buses<sup>2</sup>, namely P1, P2, P22, P13, P23 and P27. Bus 1 is the slack bus, which represents a connection to the larger grid. The grid provides *liquidity* to consumers i.e. reliably provides electricity when no local generation is available. This grid-supplied electricity is considered to be less preferable and more expensive to consumers than local generation. In the test dataset, P2, P22, P23 and P27 are large wind generation facilities, while P13 is a grid-scale PV generator. The dataset consists of hourly MWh consumption and generation values for the 24 consumers and 5 producers. Raw results data can be viewed at [23].

		Consumer																							
		C6	C10	C4	C12	C21	C28	C15	C8	C3	C9	C24	C17	C7	C16	C5	C20	C18	C14	C19	C25	C11	C29	C30	C26
Producer	P2	0.96	0.90	0.97	0.89	0.88	0.94	0.87	0.95	0.96	0.90	0.85	0.88	0.94	0.85	0.94	0.83	0.82	0.82	0.82	0.80	0.80	0.64	0.61	0.57
	P22	0.91	0.97	0.90	0.90	0.99	0.89	0.89	0.89	0.89	0.93	0.92	0.93	0.88	0.89	0.85	0.89	0.86	0.83	0.87	0.82	0.83	0.64	0.61	0.59
	P27	0.84	0.82	0.83	0.79	0.81	0.85	0.78	0.83	0.82	0.81	0.83	0.79	0.81	0.76	0.78	0.75	0.74	0.73	0.74	0.90	0.70	0.83	0.80	0.67
	P23	0.85	0.87	0.85	0.88	0.87	0.84	0.91	0.84	0.83	0.85	0.90	0.85	0.82	0.84	0.79	0.83	0.84	0.84	0.83	0.78	0.74	0.60	0.56	0.56
	P13	0.84	0.85	0.85	0.93	0.83	0.82	0.88	0.83	0.83	0.82	0.81	0.84	0.81	0.85	0.79	0.80	0.81	0.85	0.80	0.72	0.72	0.55	0.52	0.50
Average		0.88	0.88	0.88	0.88	0.87	0.87	0.87	0.87	0.86	0.86	0.86	0.86	0.85	0.84	0.83	0.82	0.81	0.81	0.81	0.80	0.76	0.65	0.62	0.58

Fig. 3. Taxed values between generators and consumers. Red indicates more taxed values, while green indicates less taxed values

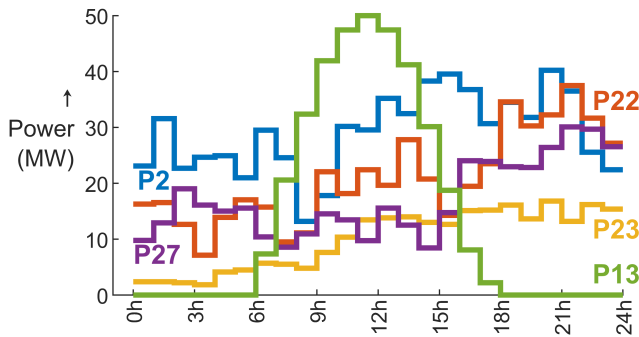


Fig. 4. Generation profiles for producers at each node. Buses 2, 22, 27 and 23 are wind farms, while bus 13 is a PV plant

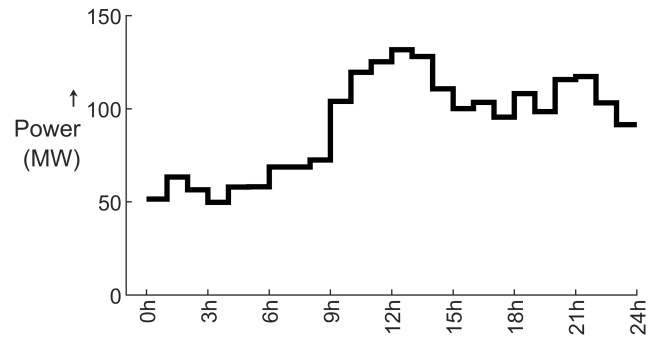


Fig. 5. Total generation by all producers

### A. Electrical Distance Tax Results

Tax values are calculated as in Section II-B, producing the results shown in figure 3. This figure shows producers on the vertical axis and consumers on the horizontal axis. It can be seen that buses P13 and C26 have the highest taxed value between them, substantiated by being on opposite sides of the network. The figure also shows the average tax value per consumer in decreasing order. Certain generators, such as P2 are more centrally located, and will thus likely have most of their generation allocated. Examining the network plot in figure 2, it can be seen that producer P13 is electrically far from most of the consumers. Thus, P13 has the highest overall burden.

Generation profiles can be seen in figure 4. Figure 5 shows the total power generated by all producers. Generation at midday is significantly higher, owing to the P13's midday peak. Afternoon/evening output is significantly higher than morning output, with wind sources becoming more active.

### B. Eneertok Allocation Results

Next, the market simulation is performed. Figure 6 shows the total energy value of Eneertoks allocated to each consumer. Interestingly, some counter-intuitive phenomena are observed.

<sup>2</sup>Hereafter producer buses will gain a *P* prefix, while consumer buses will be referred to with a *C* prefix.

The consumer that receives the most energy is C10, despite being more taxed than C6, as per figure 3. This could be a case of having the optimal relationship to all producers to ensure their unique generation profiles are best taken advantage of i.e. "being in the right place at the right time". The advantage of being centrally located in the system becomes evident in these results, as C4 and C6 both receive a considerable amount of Eneertoks. Consumers such as C29 and C30 predictably receive few Eneertoks, owing to their high tax values. However, C26, which has the most severe tax value, does not receive the least allocated Eneertoks. This is a role reserved for C24; an interesting phenomenon similar to that of C10.

### C. Market Simulation Results

For the load shifting element, each consumer is given the option to increase or decrease their consumption by 20% at any time point ( $s=0.2$ ). However, total energy consumption over the 24 hours must remain constant per consumer. Figure 7 shows the average consumption at each time point before and after the optimisation has been implemented, omitting any generation influence. It can be seen that as much load as possible is moved to the early hours of the morning, so as to make use of under-utilised generation at this time. However, with low base demand in this time period, the total load cannot be shifted dramatically. The high output of all generators, especially the solar plant, around midday means that consumption at this

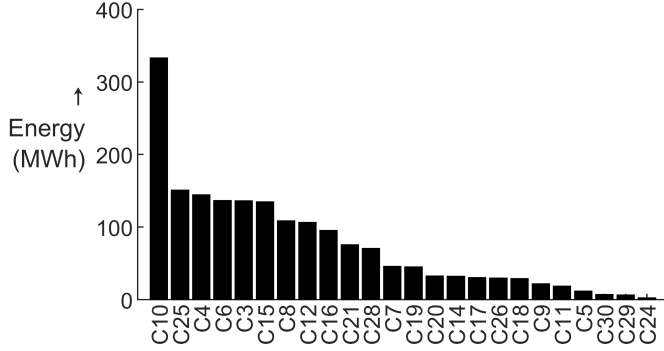


Fig. 6. Total Enertoks allocated to each consumer

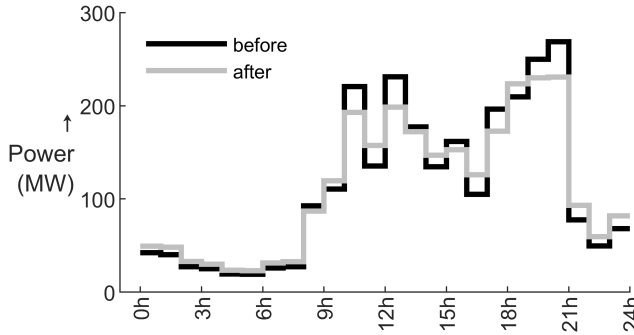


Fig. 7. Total consumption before and after load shifting implementation

point remains largely unchanged. Consumption is moved away from the evening high-demand period, especially as there is less generation during this period (when compared to midday), with solar PV production having ceased completely. Similarly to early morning period, late night consumption is increased by a small portion, to make use of available generation.

Figure 8 shows total system consumption for all consumers, with the energy value of allocated Enertoks subtracted. This can be understood as the total energy purchased by consumers from the grid utility. It can be seen that early morning and late evening consumption is decreased, even reaching zero during low-demand periods. A fairly consistent decrease during daytime and early evening hours is observed ( $\pm 90$  MW), owing to the abundance of generation, with allocation ratios being consistent.

#### D. Power Flow Results

As the Enertok values and consumption profiles have dealt solely with real power values thus far, an average power factor value is chosen to calculate reactive power values. For this study the power factor value is set to 0.90 (an approximate voltage angle of  $26^\circ$ ).

Figure 9 shows power provided by the slack bus at node 1 before and after the optimisation has been implemented. This can be understood as the power imported/exported from the broader transmission system. With a less intensive load during the early hours of the morning, a net export of energy occurs. A similar phenomenon can be observed during the late night

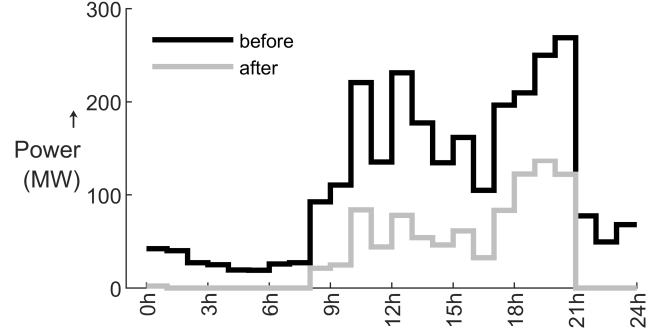


Fig. 8. Power purchased from grid operator

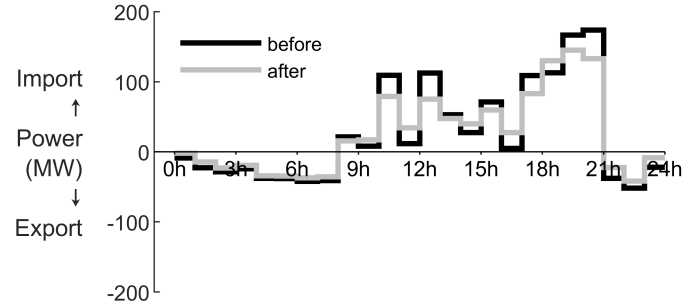


Fig. 9. Power supplied from/to broader transmission system through slack bus

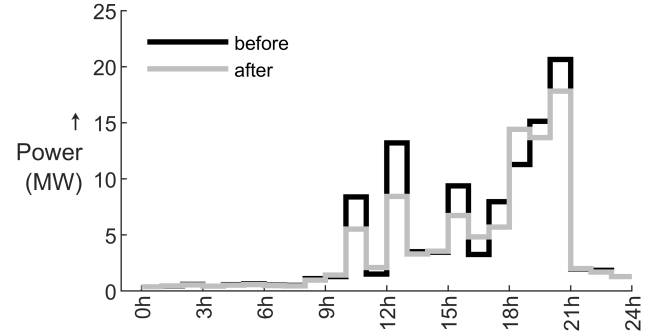


Fig. 10. Total losses in transmission system

hours, from 21h00 to 00h00. With less consumption during peak evening hours (17h00 to 21h00), net imported power is marginally decreased. These observations align with the periods of zero consumption mentioned above in figure 8. Furthermore, examining the average voltage angle of consumers shows a marginal ease, decreasing from  $-3.46^\circ$  to  $-3.44^\circ$ .

Figure 10 shows average losses across all power system branches per time point. The early morning hours see a slight increase, due to the extra load shifted to these times, mentioned above. A decrease occurs roughly around the evening peak (19h00 to 21h00). This may be due to the high supply of generation during this time, coupled with a decrease in demand due to load shifting. Ultimately, total system losses are decreased by 10.90% after the optimisation has been performed.

## V. CONCLUSIONS

The methodology developed in this paper attempts to show that an electrical tax could potentially encourage local consumption, and serves as the first step towards a complete study of this concept. The Klein Resistance method is used as an initial rough metric for tax formulation. This taxing mechanism is found to be skewed towards the utility, and may discourage P2P market participation. Despite this, results from market simulations and load flows suggest that a location-based electrical tax may encourage some measure of beneficial demand response activation. This new locational profile of demand response activations, both up and down, seems to result in reduced losses system-wide and a less stressed power system as indicated by smaller voltage angle deviations.

The first step for future studies may be implementing a more robust method of determining the taxes imposed on participating parties. In this study the magnitude of this tax is determined from a normalised value, which would result in uneven scaling if the considered system increased in size. In the tokenisation step, Enertoks could be made to be granular to some form, to more closely resemble existing cryptocurrency methods i.e. can only be subdivided multiples of some minimum value. Furthermore, by excluding any true financial metrics, electricity value estimates become approximate, and parties will need to consider the price of local vs. utility generation. Without some form of time-of-use cost mechanism, it is difficult to incentivise consumers to shift their demand when demand profiles are much higher in magnitude than local generation. Future studies thus should include such aspects.

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