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**A Hicksian Link Between Inflation  
and the Term Structure**

**Patrick Honohan**

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# Abstract

Hicks suggested that a "constitutional weakness" at the long end of the bond market causes long yields to exceed short yields on average. This note argues that such a weakness would be accentuated by inflation, and provides supportive empirical evidence.

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## A HICKSIAN LINK BETWEEN INFLATION AND THE TERM STRUCTURE

1. It was Hicks who suggested (1946, p.146) that the normal slope of the yield curve would be positive because of a "constitutional weakness on one side" of the market: "If no extra return is offered for long lending, most people...would prefer to lend short...but this situation would leave a large excess of demands to borrow long which would not be met." This hypothesis has never been either conclusively confirmed nor rejected. It is not compatible with the risk neutrality assumed by the pure expectations theory of the term structure, but that theory, in its strong form, has not been successful in explaining fluctuations in term premia (cf. Mankiw, 1986). The purpose of this note is to point out a link between fully anticipated inflation and the term structure of interest rates which is implied by the Hicksian approach. Preliminary empirical analysis suggests that inflation can explain a good deal of residual variation in the spread between long and short yields in the United Kingdom, which has experienced a wide range of inflation rates in the past thirty years.

2. The stream of net receipts anticipated by the typical Hicksian investor may have negative elements in the immediate future (if the investment outlays are spread over a number of periods), but it will be chiefly characterised by positive net receipts in the more or less distant future. The investor may undertake borrowing to finance immediate outlays which may be required to secure this stream. Such borrowings will also have implications for the overall stream of net receipts, inclusive of interest. In order to insulate themselves from the risk of fluctuations in the market rate of interest, there will in the Hicksian view, *ceteris paribus*, be a preference by risk-averse borrowers at fixed interest for maturities which *match* the anticipated stream of

net receipts. Matched borrowing obviates the need for any refinancing. This note argues that an increase in the fully anticipated inflation rate, which increases all nominal rates of interest, will, because of the well-known tilt which it imparts to the real stream of debt servicing costs, tend to lengthen the average maturity of such matched borrowing. This in turn alerts us to the possibility of an impact of inflation on the slope of the yield curve through this further weakening of the long end of the market.

3. To illustrate the effect, consider an agent who anticipates an inflow of £10 and £110 respectively in periods 1 and 2. Suppose the interest rate is 10 per cent. per annum for both one and two-period borrowing or lending; then these inflows will be perfectly matched by two period borrowing of £100 now. Now consider a new scenario with steady inflation of 10 per cent. per annum and a correspondingly higher interest rate of 21 per cent. per annum. The anticipated inflows now become £11 and £133, because of the inflation. It is no longer the case that the matched portfolio is simply a two period borrowing of £100: the anticipated inflow for period 1 is now insufficient to service such a debt. Instead, the matched portfolio involves lending about £10 for one period and borrowing about £110 for two periods. Notice how inflation has tilted the matched portfolio in the direction of a longer maturity of net borrowing.

4. In order to formalise these ideas, let the fixed structure of net receipts (exclusive of interest and debt repayments) be given by the vector  $y$ ,

$$y = (y_0, y_1, \dots, y_r) = (y_0, y^0) ,$$

where  $y_t$  denotes the projected net receipts in period  $t$ .

5. A portfolio of fixed interest assets and liabilities for different maturities is represented by a vector  $x$ ,

$$x = (x_1, \dots, x_T),$$

where  $x_t > 0$  indicates an amount borrowed at 0 for repayment at time  $t$ . By a fixed interest liability we mean an arrangement whereby the market rate of interest at time 0 for maturity  $t$  is paid in each year on liabilities of maturity  $i$ . This rate of interest is denoted  $r_t$ .

6. The matched portfolio for the stream  $y$  is the portfolio (of fixed interest securities) which, if acquired at period 0, will provide sufficient cash  $y_t$  through interest receipts and redemptions at  $t$  ( $>0$ ), where  $y_t < 0$ ; and will absorb any cash surplus  $y_t > 0$ . It is easily shown that the matched portfolio  $z(y)$  can be written

$$z(y) = \Omega y^0, \quad (1)$$

where,

$$\Omega = I - (I + R)^{-1}R \quad (2)$$

and

$$R = \begin{bmatrix} r_1 & r_2 & r_3 & \dots & r_T \\ 0 & r_2 & r_3 & \dots & r_T \\ 0 & 0 & r_3 & \dots & r_T \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & r_T \end{bmatrix}. \quad (3)$$

7. We now consider the implications of a fully anticipated and constant rate of inflation  $\pi$ , under the simplifying assumption of a flat yield curve, and a Fisherian relationship between real ( $\rho$ ) and nominal ( $r$ ) interest rates linearized as:

$$r = \rho + \pi \quad (4)$$

All elements of the stream  $y$  are also inflated:

$$y^m_t = (1 + \pi)^t y_t. \quad (5)$$

Writing:

$$w = (\delta z / \delta \pi) |_{\pi=0}, \quad (6)$$

some manipulation yields the impact, at  $\pi=0$ , of an increase in  $\pi$  on the matched portfolio as:

$$w_t - t y_t = - \sum_{\tau=t}^{\infty} L_{\tau} (1+p)^{-(\tau-t)} (1 - m p / (1+p)) y_{\tau}, \quad (7)$$

where  $m = \tau - t + 1$ .

8. Is the Hicksian picture of long-preferring borrowers facing lenders reluctant to depart from short maturities accentuated in times of inflation? For one answer to this question, consider a stream  $y^0$  which is nonnegative and monotonic nondecreasing; i.e.  $y_t \geq y_{t-1}$ , for  $t \geq 2$ . For small values of the real interest rate  $p$ , the marginal impact of inflation on the maturity structure of the matched portfolio of such a stream (as measured by the elements of  $w$ ), is (algebraically) greater, the greater the maturity. This is because, from equation (7),

$$w_t - w_{t-1} = t(y_t - y_{t-1}) + (1 + 1/(1+p)^2) y_{t-1} + \{p \sum_{\tau=t}^{\infty} L_{\tau} (1+p)^{-(\tau-t)} (p(\tau-t)-2) y_{\tau}\} \quad (8)$$

This is nonnegative unless the bracketed term is negative and outweighs the other two, which it will not if  $p$  is sufficiently small. So, provided  $p$  is not too large, the magnitude of the inflation effect  $w$  on the matched portfolio increases with maturity. Accordingly we may expect to find the "constitutional weakness" of the long maturities to be more pronounced in times of inflation; and this hypothesis is tested empirically below.

9. A weaker concept than monotonicity of the partial derivative vector is for inflation to increase the average period of the matched portfolio. Since  $\sum w_t = 0$ , this is

equivalent<sup>2</sup> to checking that  $\sum w_t > 0$ . Once more taking the limiting case of  $\rho=0$ , we find that

$$\sum w_t = \frac{1}{2} \sum t(t-1) y_t. \quad (11)$$

Alternative sufficient conditions for this to be positive are

- (i)  $y_t > 0$  for all  $t$ , or
- (ii)  $y_t$  monotonic nondecreasing and  $\sum y_t > 0$ .

Either condition is significantly weaker than that given in the previous paragraph to guarantee monotonicity of  $w$ .

## 10. Empirical Evidence

A typical forecasting equation<sup>3</sup> for the annual average yield gap between Treasury Bills (TB) and 20-year Gilts in the United Kingdom, 1953-84 is (with estimated t-statistics in parentheses):

$$\text{GAP} = 0.489 - 0.537 \text{ TB} + 0.542 \text{ TB}_{-1} + 0.691 \text{ GAP}_{-1}$$

(1.4)    (6.5)        (6.7)        (6.6)

$R^2=0.718$      $\text{SEE}=0.766$   
 $\text{DW}=1.45$      $\text{DH}=1.75$

Adding the rate of retail price inflation one year ahead<sup>4</sup>, representing an approximation to inflationary expectations, produces a much better fit:

$$\text{GAP} = 1.189 - 0.871 \text{ TB} + 0.637 \text{ TB}_{-1} + 0.351 \text{ GAP}_{-1} + 0.232 \text{ INFL}$$

(5.9)    (15.5)        (14.9)        (5.3)        (8.9)

$R^2=0.928$      $\text{SEE}=0.392$   
 $\text{DW}=2.32$      $\text{DH}=-1.02$



The years 1979 and 1980 are two large outliers in both equations, reflecting a dramatic change in monetary policy in 1979. We can remove most of the impact of both years by a single dummy taking the value 1 in those years, 0 elsewhere:

$$\text{GAP} = 0,901 - 0,720 \text{ TB} + 0,552 \text{ TB}_{-1} + 0,373 \text{ GAP}_{-1} + 0,206 \text{ INFL} - 1,268 \text{ D}$$

(5,0) (11,8) (13,3) (6,9) (9,2) (3,8) DW=2,14 DH=-0,43

[5,6] [12,2] [12,5] [7,6] [9,7] [5,6]  $\chi^2(2)(JB)=1,58$

(Here we have also provided t-statistics estimated by White's alternative method which is robust to heteroscedasticity, and the Jarque-Bera  $\chi^2$  test for the normality of the residuals).

Instrumental variables estimation, allowing for the endogeneity of TB and INFL, resulted in almost identical values for the coefficients.

Though these estimates are clearly preliminary (and further work is in progress), it seems clear on both empirical and theoretical grounds that inclusion of a term reflecting inflationary expectations should be seriously considered for modelling the yield gap.

### Footnotes

<sup>1</sup>Cf. Hicks pp.186-8. Note that the useful Hicksian concepts of *crescendo* and *diminuendo* cannot be used without modification for the analysis at zero real interest.

<sup>2</sup>As in Hicks p.222. It is important to note that we are not here envisaging any sensitivity to inflation of the real structure of anticipated net receipts (before interest). Hicks' own analysis included examination of the response of production plans to changes in the real rate of interest, and he concluded that the average period of production would be shortened by a rise in the real rate of interest.

<sup>3</sup>Cf. Mankiw (1986) who reports an equation using the same explanatory variables for quarterly data 1960-79.

<sup>4</sup>All data from IFS: lines 60a, 61 and 64. For years prior to 1975 the average TB yield was approximated by the yield corresponding to the average discount. Experiments with other proxies for expected inflation, such as a three-year moving average of actual inflation, were also successful.

<sup>5</sup>Unpublished empirical work by Hendry and Ericsson, based on the Friedman-Schwartz data for the UK 1868-1975, suggests a transitory effect of steady inflation on the yield curve: the present note argues for a permanent effect. The Friedman-Schwartz data set is among the sets for which empirical tests of our approach will be reported later. Preliminary indications are that both for this set, and for annual post war US data, the level of expected inflation is significant.

### References

- Hendry, D.F. and N.C. Ericsson (1986) "Prolegomenon to a Reconstruction". Washington, D.C.: Board of Governors of the Federal Reserve, mimeo.
- Hicks, J.B. (1946) *Value and Capital*, 2nd Edn., Oxford: Clarendon.
- Mankiw, N.G. (1986) "The Term Structure of Interest Rates Revisited", *Brookings Papers on Economic Activity*, 1, pp. 61-96.

## Appendix 1

Alternative measures of expected inflation also work well. The first measure which was used was actually a three-year centred average of inflation (INFL2). The relevant equations in this case are:

$$\text{GAP} = 1.42 - 0.694 \text{ TB} + 0.412 \text{ TB}_{-1} + 0.333 \text{ GAP}_{-1} + 0.249 \text{ INFL} \quad R^2=0.859 \quad \text{SEE}=0.552$$

(4.5)   (10.5)   (6.5)   (3.3)   (5.2)   DW=1.63   DH=0.85

and,

$$\text{GAP} = 1.02 - 0.515 \text{ TB} + 0.318 \text{ TB}_{-1} + 0.345 \text{ GAP}_{-1} + 0.225 \text{ INFL} - 1.83 \text{ D} \quad R^2=0.916 \quad \text{SEE}=0.435$$

(3.9)   (7.6)   (5.8)   (4.3)   (5.9)   (4.2)   DW=1.81   DH=0.37

[4.3]   [9.3]   [4.6]   [3.7]   [4.7]   [4.0]    $\chi^2(2)(\text{JB})=0.37$

Finally we have explored the possibility of correlation between the disturbance term and either TB or the inflation expectations term INFL. Using lagged values of inflation and the long and short interest rates as instruments, we estimated the following equation by instrumental variables (conventional t-statistics shown):

$$\text{GAP} = 1.014 - 0.791 \text{ TB} + 0.532 \text{ TB}_{-1} + 0.354 \text{ GAP}_{-1} + 0.225 \text{ INFL} - 1.022 \text{ D} \quad R^2=0.952 \quad \text{SEE}=0.329$$

(2.6)   (3.2)   (3.5)   (5.6)   (5.0)   (1.0)   DW=2.24

The estimated coefficients are hardly altered by the change in estimation method.

## Appendix 2

### Proof of Equation (7):

Proposition 2 of Appendix 3 states:

$$I - \Omega = rG, \quad \text{with } G_{t,\tau} = k^m,$$

where  $m = \tau - t + 1$ , if  $\tau - t + 1 > 0$ ;  $m = 0$  otherwise;  
and  $k = 1/(1+r)$ .

A:  $-(\delta\Omega/\delta\pi)|_{\pi=0} = (\delta rG/\delta\pi)|_{\pi=0} = G + \rho(\delta G/\delta\pi)|_{\pi=0}$ ;  
and,

$$(\delta G_{t,\tau}/\delta\pi)|_{\pi=0} = mk^{m-1}(\delta k/\delta\pi)|_{\pi=0} = -mk^{m-1},$$

(since  $(\delta k/\delta\pi)|_{\pi=0} = -1/(1+\rho)^2 = -k^2$ ).

Therefore,  $(\delta\Omega/\delta\pi)|_{\pi=0}$  has typical element, where  $m > 0$ ,  
(0 otherwise):

$$-k^m(1 - \rho mk).$$

B: By (5),

$$(\delta y''_t/\delta\pi)|_{\pi=0} = t(1+\pi)^{t-1}y_t$$

C: By (1),

$$w = (\delta z/\delta\pi)|_{\pi=0} = (\delta \Omega y/\delta\pi)|_{\pi=0} = [(\delta\Omega/\delta\pi)|_{\pi=0} + D] y,$$

where  $D = \text{diag}\{t\}$ .

Combining the results at A, B and C gives equation (7):

$$w_t - ty_t = -\sum_{\tau=t}^T (1+\rho)^{-m}(1-\rho/(1+\rho)) y_\tau$$

### From Equation (7) to Equation (8):

$$\begin{aligned} w_t - w_{t-1} &= ty_t - (t-1)y_{t-1} \\ &\quad + \sum_{\tau=t-1}^T (1+\rho)^{-(\tau-t+1)-1} [1 - (\tau-t+2)\rho/(1+\rho)] y_\tau \\ &\quad - \sum_{\tau=t}^T (1+\rho)^{-(\tau-t+1)} [1 - (\tau-t+1)\rho/(1+\rho)] y_\tau \\ &= t(y_t - y_{t-1}) + y_{t-1} + y_{t-1} \{ (1+\rho)^{-1} (1-\rho/(1+\rho)) \} \\ &\quad + \sum_{\tau=t}^T y_\tau (1+\rho)^{-(\tau-t+1)-2} \\ &\quad \quad [ (1+\rho) - \rho(\tau-t+2) - (1+\rho)^2 + \rho(1+\rho)(\tau-t+1) ]. \\ &= t(y_t - y_{t-1}) + [1 + (1/(1+\rho))^2] y_{t-1} \\ &\quad + \rho \sum_{\tau=t}^T y_\tau (1+\rho)^{-m-2} (\rho(\tau-t)-2) \end{aligned}$$

### Appendix 3

In this appendix are established two results which are needed in the text. The first proposition, which characterizes the matched portfolio, is used in section 6.

Definition: The matched portfolio  $z$  for a cash flow plan  $y = (y_0, y_1, \dots, y_t) = (y_0, y^0)$  is the portfolio which, if acquired at time 0, will provide cash through interest receipts and repayments at  $i$  where  $y_i < 0$ , and will absorb any cash surplus  $y_i > 0$ .

Proposition 1  $z = \Omega y$ , where  $\Omega = I - (I+R)^{-1}R$ , and

$$R = \begin{bmatrix} r_1 & r_2 & r_3 & r_t \\ 0 & r_2 & r_3 & r_t \\ 0 & 0 & r_3 & r_t \\ 0 & & & r_t \end{bmatrix}$$

Proof Let  $p_i$  be the net interest receipts at time  $i$  from the matched portfolio  $z$  ( $p_i < 0$  implies net interest payments at time  $i$ ). Then, by the definition of  $z$ ,

$$z_i = y_i + p_i \quad i = 1, \dots, T \quad (A1)$$

But since interest will be received at time  $i$  on all loans for maturity  $i$  or greater,

$$p_i = -\sum_{j=i}^T r_j z_j \quad i = 1, \dots, T \quad (A2)$$

Now substituting (A1) into (A2), and collecting the results for all  $i$ ,

$$-p = R (y^0 + p); \quad (A3)$$

Solving (A3) for  $p$  and substituting back into (A1) completes the proof.

The following proposition is used in section 3.

Proposition 2 If  $r_i = r_j$  for all  $i, j$ , then, if  $k = 1/(1+r)$

$$I - \Omega = r \begin{bmatrix} k & k^2 & k^3 & k^T \\ 0 & k & k^2 & k^{T-1} \\ 0 & 0 & \ddots & \vdots \\ 0 & \ddots & 0 & k \end{bmatrix}, \text{ where } \Omega \text{ is defined in proposition 1}$$

Lemma 1 (Murata, 1977, p. 10)

$$\begin{bmatrix} k & -1 & 0 & \dots & 0 \\ 0 & k & -1 & & 0 \\ 0 & 0 & k & & \vdots \\ & & & \ddots & \vdots \\ 0 & & & & k \end{bmatrix}^{-1} = \begin{bmatrix} k^{-1} & k^{-2} & k^{-3} & \dots & k^{-T} \\ 0 & k^{-1} & k^{-2} & \dots & k^{-T+1} \\ 0 & 0 & k^{-1} & \dots & \vdots \\ & & \ddots & \ddots & \vdots \\ 0 & & & & k \end{bmatrix}$$

Lemma 2 (Murata, 1977, p. 30)

$$(A + B)^{-1} = (B^{-1}A + I)B^{-1} \quad \text{for square } A, B,$$

Proof of proposition 2

Under the hypothesis,

$$R = r \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & & 1 \\ 0 & 0 & 1 & & 1 \\ & & & \ddots & \vdots \\ 0 & & & & 1 \end{bmatrix}$$

and by lemma 1

$$R^{-1} = (1/r) \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & & 0 \\ 0 & 0 & 1 & & \vdots \\ & & & \ddots & \vdots \\ 0 & & & & 1 \end{bmatrix}$$



By lemma 2,

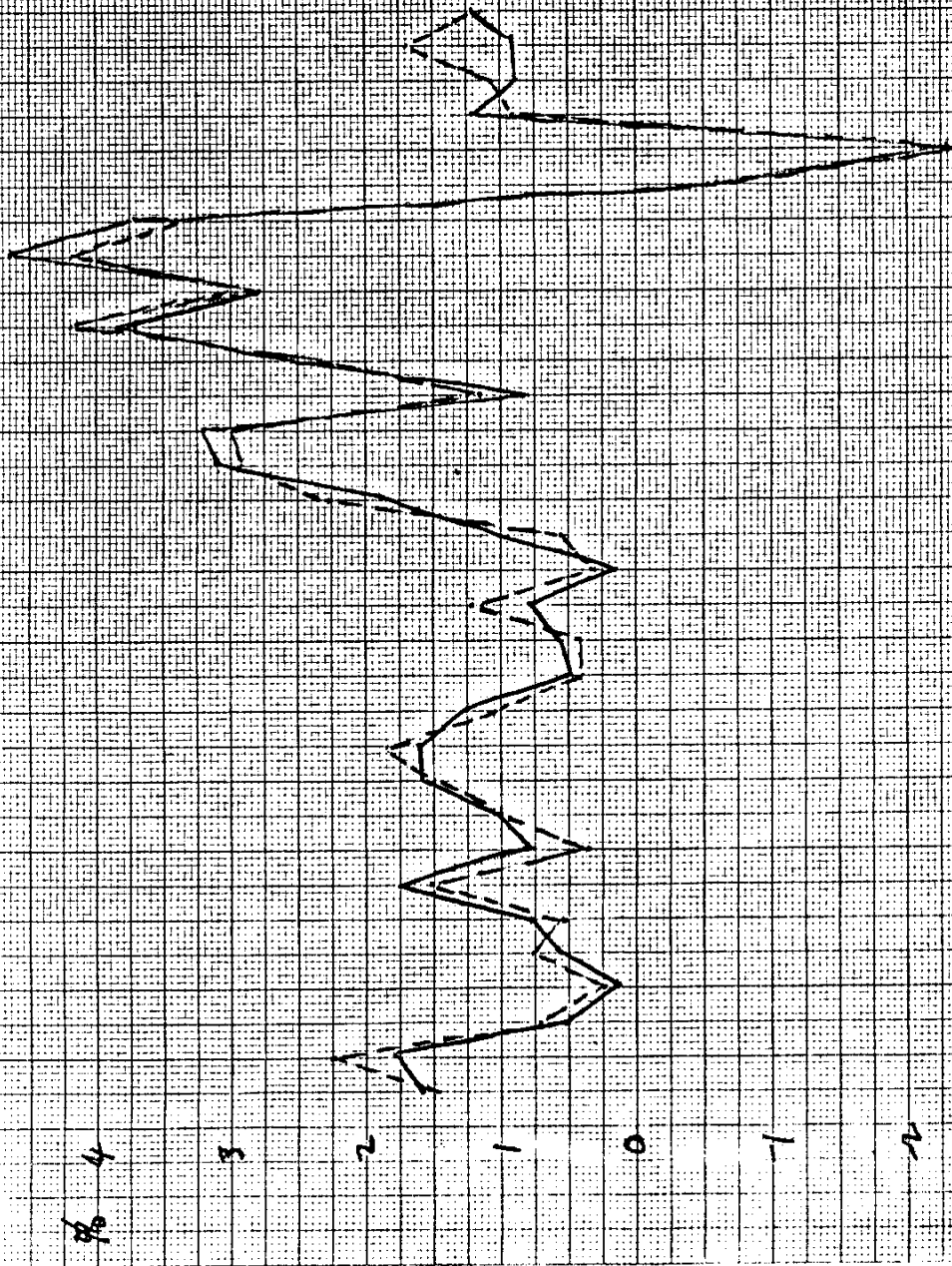
$$\Psi = I - (I + R)^{-1} R = I - (I + R^{-1})^{-1}$$

And, since  $(1/r) + 1 = (1+r)/r$

$$I - \Psi = (I + R^{-1})^{-1} = r \begin{bmatrix} 1+r & -1 & 0 & 0 \\ 0 & 1+r & -1 & 0 \\ 0 & 0 & 1+r & 0 \\ 0 & 0 & 0 & 1+r \end{bmatrix}^{-1}$$

A further application of lemma 1 completes the proof.

— ACTUAL  
--- PREDICTED



1984

1953