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## High Resolution Wind Power Models - an Irish Case Study

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### Abstract

This paper focuses on high resolution wind power statistical models fitted to meteorological data for the island of Ireland. A discrete Burr model efficiently represents the number of consecutive hours of wind power availability. The models developed in this study may be most useful at time resolutions less than 6 hours to capture zero power and short bursts of wind power potential. They could serve as a useful complement to other wind power modelling approaches such as MERRA reanalysis models.

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*Keywords:* Wind power modelling; Wind energy potential; Discrete count models;

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### 1. Introduction

The Irish Government is aiming for 40% of electricity to be generated from Renewable Energy Sources (RESs) by 2020 in response to the EU directive on renewable energy<sup>1</sup>. Ireland is rich in wind resources but integrating wind power creates new operational and planning challenges for Transmission System Operators. Many of the issues particular to the Irish situation are set out in<sup>2</sup>. Approaches to wind power forecasting are described in<sup>3</sup>.

Errors in wind power forecasts can cause the residual power to be over- or under-estimated. The residual power is the the power that has to be delivered from conventional generation after power from renewable sources has been committed. Improving wind modelling and forecasting has been the focus of several researchers, see<sup>4,5,6</sup>. A key recommendation from the European Wind Integration Study<sup>7</sup>, is the development of pan-European models which encapsulate more detailed regional and national models.

This gives the motivation for our Irish case study. We describe a one hour resolution statistical model of the potential wind power based on historical meteorological data. This model will serve as a complement to an aggregated wind power generation model using MERRA reanalysis wind speed data<sup>8</sup>. The MERRA model successfully reflects the measured wind production data for the period 2001 – 2014 at greater than 6 h resolution. The correlation of the reanalysis model with actual output from 2006 onwards is 0.95 - 0.96, with RMSE between 6.5% - 8.5%. At one-

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and three-hour time-horizons, the model tends to under-estimate the magnitude of ramps in capacity factor occurring with a particular frequency (i.e., for a given ramp magnitude, the model under-predicts the frequency of wind event occurrences). At horizons greater than or equal to six hours, the differences between the reanalysis model output and actual power are very small. This is reflective of a low spatial resolution reanalysis data inadequately representing variability at smaller time-horizons but also an effect of the temporal smoothing apparent in the data.

### 1.1. Wind Power Integration Issues in Ireland

The Irish electricity generation system comprises of circa 7,500 MW of generation capacity and two 500 MW HVDC interconnect lines with the UK. Mains supply is offered at 50 Hz. Currently there is an installed wind capacity of circa 3,000 MW with plans for further development. Several metrics are used to quantify the productivity of a wind turbine. The *capacity factor* compares the actual power production of a turbine over a given time with the total power the turbine would have produced if it had operated at the rated power for the time frame. However, the amount of wind power that is used on the grid is limited, there are a number of reasons why wind is “curtailed”. Curtailment can be due to system-wide curtailment and/or local network constraints.

Firstly, wind power may be curtailed to ensure security and stability since wind power is an asynchronous source. The limit for system non-synchronous penetration (SNSP) is set at 55%.  $SNSP = (\text{wind generation} + \text{HVDC imports}) / (\text{system demand} + \text{HVDC exports})$ . In order to meet EU targets, EirGrid (the Irish transmission system operator) aim to increase the SNSP limit to 75% by 2020. Ging et al.<sup>9</sup> note that such curtailment is allocated pro-rata across all wind generators with an equal bias.

As well as frequency and stability management issues, wind power may be curtailed due to operational or transmission constraints<sup>9</sup>. Two major geographical constraint areas in Ireland are identified in<sup>10</sup>: the north-west and the south-west of Ireland. These are due mainly to network congestion issues but also in some instances to system outages. They note that outages caused by storms in the south west resulted in the output of other windfarms being constrained to manage overloading lines. In addition, they note that curtailment arises mainly during the night time hours (between 11pm and 9am) due to the low overall system demand, when “must-run” conventional power plant may be able to meet the estimated residual demand.

#### 1.1.1. Wind Power Event

Section 2 gives a brief overview of wind power modelling approaches. In this study we are particularly interested in the number of hours when power is theoretically produced (i.e., ignoring market and operational curtailment). We call this phenomenon a wind power event. We say a wind power event occurs when wind speed exceeds the cut-in threshold of a wind turbine so that power is produced. The end of the event occurs when the wind speed drops below the cut-out threshold. In this study we model  $X$  the number of consecutive hours when wind power is produced. This allows us to characterise the distribution of the wind power events and the inter-event time distribution. The system of potential wind power is characterised by intermittent switching between periods of low activity and high activity bursts of potential power production.

## 2. Wind Power Modelling

Analysis of long-term data on national or regional wind power output is required to understand, in particular, the variability in the production related to inter-annual and inter-decadal climatic patterns. Thirty years is generally understood to be the minimum required period in order to properly capture such fluctuations. Wind power generation has really only become a widespread contributor to power production in the last 5-10 years, with very few instances of wind farms older than 20 years, so historical records of output are not sufficient for long-term analyses, and thus simulations of such data are required.

Two main approaches are taken in the literature to developing simulations, both using wind speed data as input. The first uses *historical records* of measured wind speeds, typically held by meteorological institutions and measured at 10m above ground level. The second relies on wind speeds as represented in “*reanalysis*” datasets. Reanalysis data is that produced by running some form of a numerical weather prediction (NWP) model in *hindcast* mode, i.e. back

in time. These models usually assimilate historical observations in some form, and are thereby expected to produce a homogeneous temporal sequence of historical weather.

The wind speed resulting from either historical measurement or reanalysis is then converted to wind power using manufacturers' turbine power curves. Figure 1 shows an example of a reference power curve. Power is not generated below the cut-in speed  $v_c$ . The turbine reaches its rated power at  $v_r$  and the turbine is shut down above its cut-out limit  $v_f$  to prevent damage in high winds. Change et al.<sup>11</sup> compare several models to describe wind turbine power output and conclude that quadratic models show better agreement between empirical performance and manufacturers' power curves.

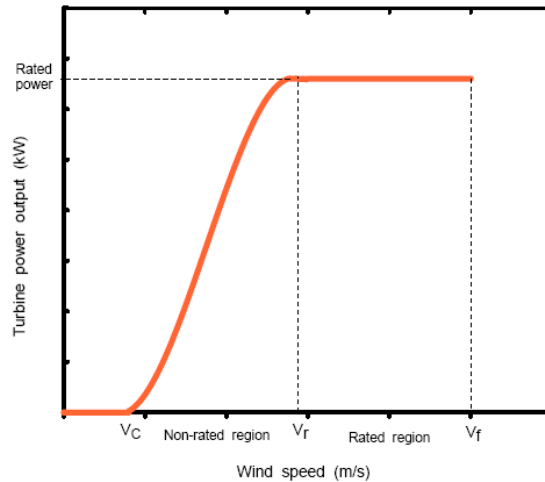


Fig. 1: Sample Reference Power transformation curve,<sup>11</sup>

Wind speeds either measured or modelled at 10 m need to be extrapolated to higher levels representing turbine hub height using a standard method, usually a power or log law. Models often produce output at multiple heights, so can be interpolated between levels to the hub height, or a relevant height output used directly. Hub-height wind speeds are then transformed to MW of power production using a power curve which gives the expected output for a given speed and height. These are often published by turbine manufacturers for their particular device, but can also be adapted to reflect either farm or regional aggregate production, see for example<sup>12</sup>.

Since power is only produced when the wind speed is within the operating range, we can consider the resulting wind power events as discrete temporal events whose duration and amplitude can be characterised. A similar observation in relation to wind speed is made in<sup>13</sup>. They note Weibull models are only applicable to non-zero wind speeds and exclude calm conditions from their analysis. In this study we focus on modelling the number of consecutive hours in a wind power event.

### 3. Methodology

We extracted historical climate data for Ireland for the period 2005 - 2015 from Met Éireann, the Irish meteorological service<sup>14</sup>. The mean hourly wind speed in knots at 11 stations were extracted. These stations were chosen as almost complete data sets were available. At other stations significant amounts of data were missing, possibly due to instrument failure. While the met weather stations are not co-located with wind farms, the data are sufficient to evaluate the wind power event modelling framework, see Figure 2.

Wind speed was converted to m/s at 10 m height. The expected power at a 80 m hub height was then calculated for wind speeds between 3.5 and 25 m/s using a power curve similar to that in Figure 1 assuming a single wind turbine of

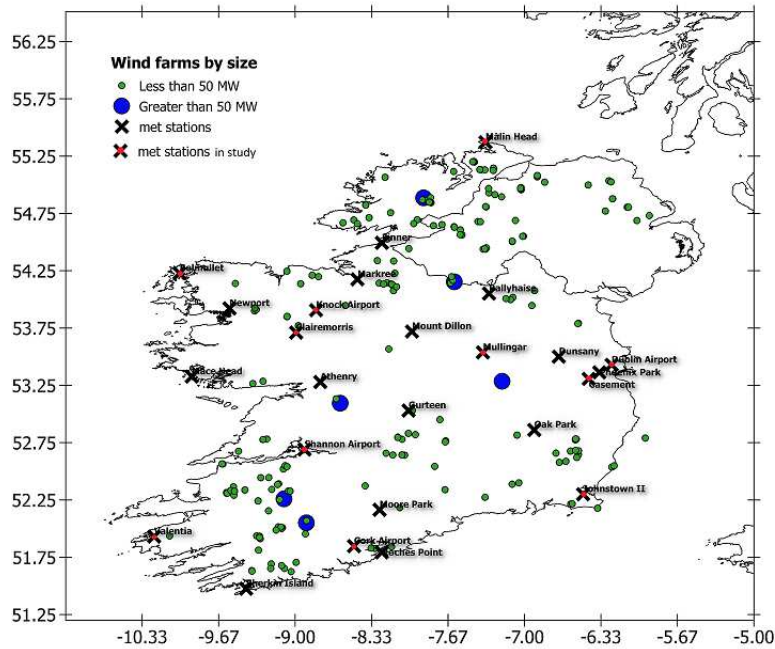


Fig. 2: Location of study met stations.

3 MW capacity. We then create a binary sequence for the period 2005 - 2015 indicating whether power is produced in each hour or not:  $W(t) = 1$  at time step  $t$  if power is produced, otherwise  $W(t) = 0$ .

We let  $X$  be the count of consecutive hours when wind power is produced. We fit statistical models for the distribution of  $X$  and evaluate their usefulness compared to the empirical data. Such models allow us to estimate how long wind power events are likely to last, and to identify the quiet periods between wind events when no wind power is likely to be available.

Poisson regression is often used as a baseline model for count data. They are sometimes used for telecommunications traffic modelling. The Poisson model generates very smooth traffic since the inter-arrival times follow an exponential distribution. The probability that an observation has count  $x$  under the Poisson distribution is given by Eq. 1.

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad (1)$$

However the assumption of equi-dispersion in the Poisson model is too restrictive for many empirical applications. In practice, the variance of observed count data often exceeds the mean i.e., over-dispersion is evident. In addition,<sup>15</sup> argue that Poisson models are too simple to capture the burstiness of real traffic, as the inter-arrival distributions have been observed to have heavier tails.

A more appropriate approach for bursty events such as wind power may be to consider hurdle or two-part models. In this case we consider that threshold conditions need to be reached. Wind must reach the cut in speed before power can be produced in our case study. This corresponds to surmounting a hurdle. Once over the hurdle, we then consider that the number of hours of wind power production may be modelled as a zero truncated Poisson distribution, i.e., the number of hours of potential power production is  $\geq 1$ , 0 values are not observed. So the model is a mixture of two components. The hurdle model keeps the zero-class disjoint from the non-zeros by modelling the non-zero as a truncated Poisson (ZTP) distribution.

We use a finite mixture model approach to combine the probability of the “no power” event with a ZTP model for the hours when power is produced. We calculate probabilities using the complete conditional hurdle model in Eq. 2.

$$P(X = x) = \begin{cases} \theta & x = 0 \\ (1 - \theta) \frac{P(X=x)}{1 - P(X=0|\lambda)} = (1 - \theta) \frac{\lambda^x e^{-\lambda}}{(1 - e^{-\lambda})x!} & x \geq 1 \end{cases} \quad (2)$$

where  $\theta$  is the probability that the hurdle will not be surmounted, i.e., the probability that power is not produced.  $\lambda$  is the conditional mean once power is produced. In this way, we increase the probability of the zero outcome (compared to a standard Poisson model, Eq 1) and scale the remaining probabilities of non-zero Poisson counts so that they sum to one. This approach is often used to model counts such as the number of bus trips taken by bus passengers<sup>16</sup>, the number of roots per successful plant root cutting<sup>17</sup> or the count of rare species<sup>18</sup>.

A Burr distribution may offer an even more tailored fit to the heavy tailed empirical data. This unimodal distribution is defined by two shape parameters (corresponding to the skewness and kurtosis),<sup>19</sup>. The continuous probability density function is given by Eq. 3 where  $c$  and  $k$  are the shape parameters. Values of  $c \leq 1$  yield  $L$ -shaped distributions.

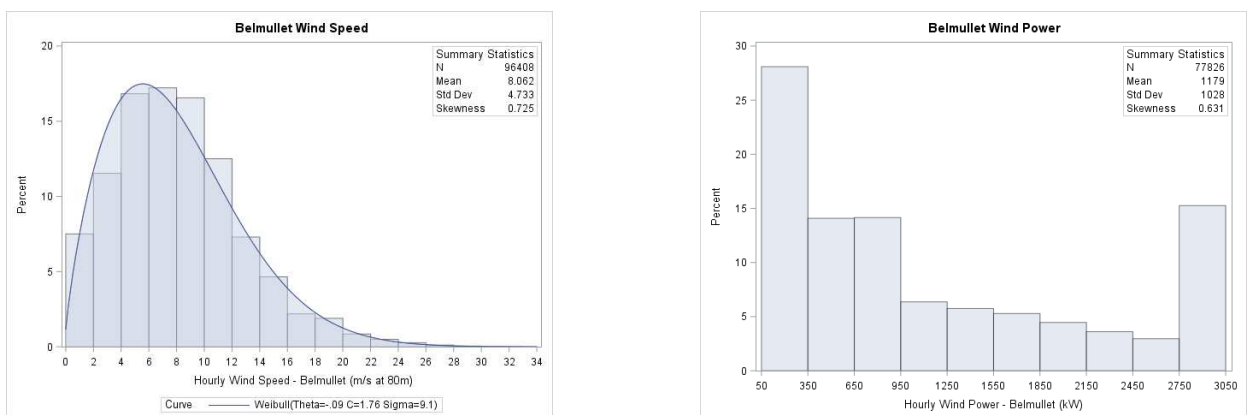
$$f(x) = ck \frac{x^{c-1}}{(1 + x^c)^{k+1}} \quad (3)$$

An approach to discretising a continuous distribution is to round down values to the nearest integer. This allows a grouping of the continuous variable e.g., discretising continuous time into units of one hour. Further details of a discrete three parameter Burr pdf are discussed in<sup>20</sup>. A discrete Burr distribution is defined by three parameters,  $B_\theta$  a scale parameter and shape parameters  $B_\alpha$  and  $B_\gamma$ . We note that as electricity markets operate in discrete time steps, it is valid to follow this approach in attempting to model the number of hours in a wind power event.

#### 4. Results and Analysis

The models were fitted using SAS 9.4. Figure 3 shows the distributions of wind speed and resulting power for a sample weather station, Belmullet. Other stations show similar distributions with some westerly locations exhibiting more consistently windy weather. We see a Weibull distribution fitted to the wind speed data. This distribution is often used to model wind speed as it captures the right hand tail. Goodness of fit statistics indicate the Weibull distribution is a good fit to the wind speed data with Kolmogorov-Smirnov D, Anderson-Darling and Cramer-von Mises W-Sq distance values all returning  $p < 0.001$ .

Approximately 19% of the time no wind power is available at Belmullet as the recorded wind speed is below the cut-in speed  $v_c = 3.5$ . Figure 3.b shows the wind power availability disregarding the calm hours of zero power output for a typical 3 MW wind turbine.



(a) Wind speed  $m/s$  at 80 m

(b) Wind Power (kW)

Fig. 3: Sample wind speed and power distributions

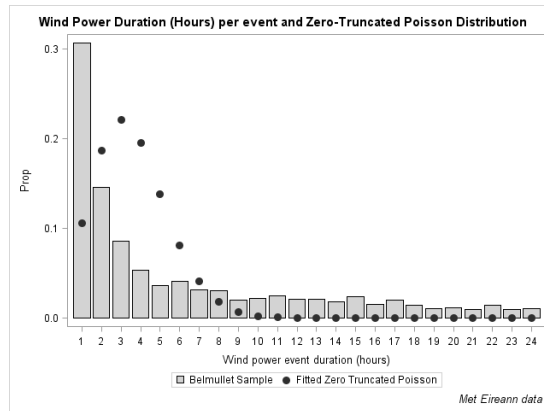


Fig. 4: Sample Zero Truncated Poisson.

Table 1: Poisson and ZTP Hurdle Model

	Belmullet	Casement A'drome	Claremorris	Cork A'port	Dublin A'port	Johnstown	Knock	Malin Head	Mullingar	Shannon	Valentia
$AIC_1$	440520	435180	380170	400629	430573	363838	412294	439411	332227	398604	412190
$\lambda_1$	3.54	2.32	1.88	2.90	4.18	1.52	2.88	6.57	1.03	2.23	2.20
$AIC_2$	169703	176568	151755	156778	176077	136217	157290	188792	122261	161077	156238
$\lambda_2$	22.77	17.33	13.11	17.25	22.68	12.49	18.51	30.48	10.81	14.79	16.71
$\theta$	0.16	0.13	0.14	0.17	0.18	0.12	0.16	0.22	0.09	0.15	0.13

Figure 4 shows the early part of the zero truncated Poisson distribution i.e., events which last up to 24 hours for the sample weather station.

We see in Table 1 that the ZTP hurdle models offer a better fit than standard Poisson models. The likelihood of no power or a non-zero number of hours of power are more closely approximated.  $AIC_1$  is the AIC for a Poisson model,  $AIC_2$  for a ZTP model. Lower AIC values were achieved e.g., the Belmullet ZTP hurdle model AIC of 169,703 compares to 440,520 for the Poisson model.  $\lambda_1$  and  $\lambda_2$  are the estimated mean values for the Poisson and ZTP models.  $\theta$  is the estimated likelihood of no power in the ZTP model. For example  $\theta = 0.16$  for Belmullet compares to the estimated value of 19%, i.e., 19% of the time no wind power is available due to low wind speeds.

The lower AIC scores indicate a better model fit of the ZTP. However, we still have not accounted fully for the over dispersion that is apparent in the empirical data.

Many of the observed wind power events are of long duration. In fact  $P(X > 24) \approx 0.65$  for the bulk of the sample sites. The wind power event duration distribution has a long tail, a small number of events can run for several hundred hours when the weather is persistently windy. The single parameter Poisson ( $\lambda = \mu$ ) is too restrictive compared to other distributions that specify additional parameters such as the variance. The Zero Truncated Poisson (ZTP) model does not help to fully explain the long tail, i.e., there are a small number of very long sequences evident in the empirical data. Fitting a generalised linear Poisson regression model also yielded a poor fit. In addition the basic assumptions such as constant decay do not hold. The observed wind power events are more bursty in nature.

In further testing of finite mixture models, a zero truncated discrete Burr model is selected as better fitting the zero-truncated count data, compared to Exponential, Gamma, Igauss, Lognormal, Pareto, Gpd, or Weibull distributions. For example, the three parameters of a zero truncated discrete Burr model are  $B_\theta = 0.96633$ ,  $B_\alpha = 0.00429$ ,  $B_\gamma = 115.45141$ , the fitted and empirical distributions (EDF) for a sample station (Belmullet) are shown in Figure 5. Details of the model fit metrics for Belmullet are shown in Table 2 including Kolmogorov-Smirnov D (KS), Anderson-Darling (AD) and Cramer-von Mises W-Sq (CvM) values. The best model under each metric is indicated by an \*. While the Burr model offers a better fit for the zero truncated count data under most headings, the wind power modelling community may be more familiar with the Weibull model so this may offer a useful alternative.

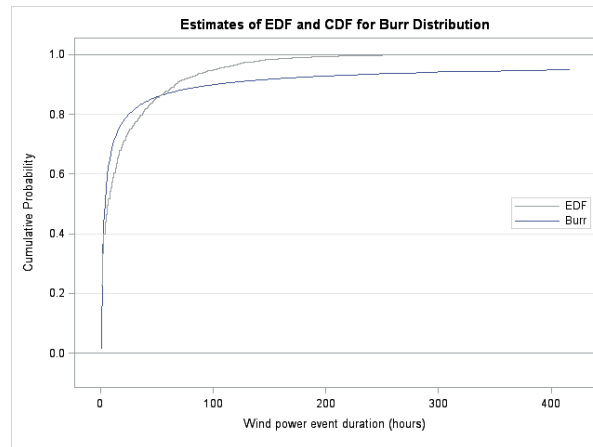


Fig. 5: Sample Burr model fit, Belmullet

Table 2: Sample Finite Mixture Model, Belmullet

Distribution	$-2$ Likelihood	Log AIC	AICC	BIC	KS	AD	CvM
Burr	25239*	25245*	25245*	25263*	12.29	1046	46.24
Exp	28201	28203	28203	28210	16.21	1247	120.93
Gamma	27146	27150	27150	27163	8.44	497.41	27.90
Igauss	25983	25987	25987	25999	8.58	568.38	27.38
Logn	26379	26383	26383	26395	7.53	512.65	23.42
Pareto	26688	26692	26692	26704	7.12	473.18	18.70
Gpd	26688	26692	26692	26704	7.12*	473.18	18.70
Weibull	26878	26882	26882	26894	8.64	450.72*	18.10*

## 5. Conclusions

One of the major concerns of renewable energy sources is their intermittency. The fact that the tails observed in the empirical data are so long is indicative of persistent availability of wind power in geographic locations like Ireland. The overall appearance of the empirical distribution of the wind event duration is an aggregation of correlated bursty trains, some of which are very long.

We conclude that Poisson models do not adequately capture the long tail of the empirical distribution. The Zero truncated Poisson models offer a better fit but still over estimate the early part of the distribution and under estimate the tail area.

The discretised Burr model offers an adequate description and could provide a useful complement to other wind power modelling approaches such as the MERRA reanalysis models in<sup>8</sup>. In particular, the models developed in this study may be most useful for early part of the distribution (at time resolutions  $\leq 6$  hours) to capture zero power and short bursts of wind power potential.

A limitation of our work is the assumption of independence of the events. Over-dispersion in count data may be due to failure of the assumption of independence of events which is implicit for example in the Poisson process. In the case of wind power, it is likely that the count of consecutive hours of power production are correlated and not independent. Statistical models such as Poisson rely on assumptions that there exists a well defined mean and variance of the distribution. Some types of events have no natural sequence length. The activity is clustered in sequences of self-similar events or “bursts”. Geophysical time series are frequently autocorrelated because of inertia or carryover processes in the physical system. The moving low pressure systems in the atmosphere might impart persistence to weather effects such as wind or rain. Positive autocorrelation might be considered a specific form of persistence. For example, the likelihood of the next hour being rainy is greater if it is rainy now than if it is currently dry. This is of

particular concern at high resolution, i.e., the weather this hour is more strongly correlated to the weather in the last hour. Therefore, in further work we aim to quantify the persistence effect and to evaluate models for bursty natural phenomena that could be incorporated with the MERRA reanalysis models.

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