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On the design of cyber-physical control system for a smart pedelec (Ebike)

Andrew Mannion*, Hugo Lhachemi*, Giovanni Russo, Shaun Sweeney and Robert Shorten

Abstract— **We present a cyber-physical control system for deployment on a smart pedelec (Ebike). The goal of the control is to manage the interaction between a human and a motor intervention, for applications in which we wish to control physical aspects of the cycling behaviour (e.g. heart rate and breathing rate). The basis of the control is a pitchfork bifurcation system, augmented using ideas from gain-scheduling. Experiments have been conducted, showing the validity of the proposed control strategy. A use case dealing with the regulation of human ventilation characteristics in response to exogenous pollution measurements is presented.**

I. INTRODUCTORY REMARKS

Our objective in this paper is to discuss the design of a pedelec (electric bike) cyber-physical control system. Electric bikes are becoming very popular modes of transport in many jurisdictions [4]. An electric bike is similar to a standard push-bike, with the main difference being that it also has an electric motor that can assist the cyclist in completing journeys. From a control perspective, the electric motor can be used to reject disturbances (wind, hills), and to provide new services to the cyclist. The benefits of convincing car users to transition from automobiles to Ebikes, include reduced noise and particle emissions from road vehicles, as well as reduced road and parking congestion. Furthermore, the electrical assist provided by an Ebike to compensate for wind and topology make these bikes much more suitable for ageing populations than conventional bikes. Finally, it is worth noting that as Ebike batteries can be charged using standard power sockets, all these benefits can be realised without deployment of any special infrastructure. Consequently, Ebikes are being increasingly viewed as essential component on the general path to e-mobility across the world.

Our interest in Ebikes stems from the fact that as this mode of transport becomes more and more popular, significant opportunities will emerge to develop secondary services for cyclists, pedestrians, and municipalities alike. Ebikes are powered vehicles, and as such, can be used to deploy a multitude of services in urban environments. For example, they can be used for crowd sensing applications (for example pollution monitoring), and to provide other services to municipalities, pedestrians, and road users, in exchange for monetary reward (for example, mobile wifi hubs); see [3] and the Copenhagen Wheel Project[1] for some ideas in this direction. Further, in many of contemporary Ebikes, the battery can be large - up to 500 Whrs. Thus, when aggregated together, groups of batteries can provide energy buffering services to both infrastructure and citizens alike [5].

The specific background to this present work is the significant opportunity that exists to develop services for the Ebike users themselves. Again, there exists a multitude of such services, ranging from basic routing services (min time, safest route, low pollution) to more advanced actuation that are designed to manage the interaction between the cyclist and the electric bike. In this latter context we are currently working on a number of applications underpinned by control and dynamical systems theory. These include:

- control strategies to nudge the cyclist to obey traffic light signalling;
- camera based control strategies that help cyclists avoid pedestrians on cycle paths;
- control strategies to manage cyclist perspiration so that they do not arrive at work sweating;
- and control strategies to regulate the breathing rate of the cyclist (to, for example, mitigate the effects of pollution peaks).

These latter applications classes are challenging for a number of reasons. For example, consider the problem of regulating the ventilation rate of the cyclist in areas of elevated pollution. To do this, we judiciously apply electric assistance. However, since this reduces the load on the cyclist, their natural instinct may be to increase cycling effort, thereby possibly increasing his/her ventilation rate. A design that takes into account this interaction in a principled manner is more difficult. Clearly, in such circumstances, one wishes the motor to gradually switch-off, and only become active when the cyclist is cooperating with the motor. Thus, any design must account for situations where the cyclist cooperates with the motor, and other situations when the cyclist competes with the motor. Building on our previous work, our objective in this paper is to develop a gain-scheduling cyber-physical control system [11], [14] that manages such an interaction between the cyclist and the electrical motor.

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[1] http://senseable.mit.edu/copenhagenwheel/

A. Related work

The work presented in this paper builds on our initial work in [17]. In this work an Ebike design is presented, as well as a holistic control strategy managing the interaction between the cyclist and the Ebike motor. A similar, control based, energy management system for series hybrid electric bicycles is presented in [6], although without consideration of the competitive interaction between cyclist and the electrical motor system. The development of Ebike services are presented in a number of publications; see [15], [9], as well as the the work carried out as part of the Copenhagen-Wheel project at MIT, and the recent paper from IBM Research [1]. Our work is also related to the rich body of work on the design of cyber-physical systems, that is the subject of much interest in the control engineering community. For context, we refer the interested reader to the the recently published paper on human-in-the-loop control systems [7] and the excellent recent review paper [12]. We note briefly, that while conceptually the work presented here is related to this latter body of work, the *competitive/cooperative* nature of the control is a somewhat unique feature of the problem class considered in this paper. Perhaps, as such, our work is most related to the stabilisation of interacting unstable systems discussed by Narendra [13] and the agent coexistence problems discussed in the networking community [2]; our solution owes much inspiration to these latter two works.

B. Contribution

The work presented in this paper builds on our initial work in [17], where experimental results were obtained via a PI-type control strategy. While the basic idea of using a pitchfork bifurcation as the basis for a possible control strategy was mentioned in [17], our present work extends the previous for in a number of ways. Specifically:

- we present a full non-linear gain scheduling [11], [14] cyber-physical control design based on a pitchfork bifurcation that is designed to work over a full envelope of operating conditions;
- this design is implemented and tested on a real Ebike;
- experimental tests are conducted to confirm the efficacy of the design;
- the basic mathematical properties of the feedback control system are established;
- and a use-case based on the control of cyclist's breathing (ventilation) rate is presented.

We emphasize again the fact that, while the idea of using a pitchfork bifurcation-based control strategy was originally suggested in [17], both its detailed design and practical implementation were not investigated. This is precisely the objective of the present paper. In this context, the detailed control strategy, as well as theoretical analyses assessing the validity of the proposed control scheme, are reported in III. Results for real Ebike tests are presented in Section IV.

II. INSTRUMENTED EBIKE

We use the test set-up and repeat the following discussion from [17] for convenience. For convenience, we give a brief description of the modified Ebike here. The Ebike that we use is a modified BTwin Original 700 purchased from Decathlon (see Figure 1). The original bike is modified in our study in several ways. First, the original motor controller was replaced by a more advanced controller: a Grinfineon C4820-GR. Second, measurement sensors were added to the bike; pedal torque and speed (using a THUN X-CELL RT sensor[2]), battery voltage, motor current, wheel speed, motor temperature, brake sensor, and hand throttle sensor. Data from sensors are read either by using an Arduino (brake and hand throttle sensors) or using a commercially available computer system–the Cycle Analyst[3]–(all the other sensors), and then communicated to a smartphone app using a bespoke especially designed Arduino-controlled Bluetooth module. Control inputs are sent to the bike controller using the same Bluetooth based communication system. Full details of the hardware system are given in [16].

Fig. 1. Electric bike from BTwin.

III. CONTROL DESIGN

In this section, we describe the control strategy adopted in this work. First, we present a simple model of the Ebike power-generation process. Second, we discuss the necessity of a human-in-the-loop-based design of the control strategy for fostering a cooperative behavior of the cyclist. Third, the basic idea used to realise such a control design is presented. Finally, the fully functional gain-scheduling control design procedure is introduced and theoretical analysis of the design presented.

In the sequel, $\mathbb{R}_+$ and $\mathbb{R}_+^*$ denote the sets of non-negative real and positive real numbers, respectively. For any given connected set $S \subset \mathbb{R}^n$ and any interval $I \subset \mathbb{R}$, $\mathscr{C}^0(S;I)$ denotes the set of functions $g : S \to I$ that are continuous. Similarly, $\mathscr{C}^1(S;\mathbb{R})$ denotes the set of functions $g : S \to \mathbb{R}$ that are continuously differentiable.

A. Basic modeling

We follow the modeling approach used in the early work [17]. Specifically, we make the two following assumptions. First, we can neglect any dynamics associated with the

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[2] http://www.ebikes.ca/shop/electric-bicycle-parts/torque-sensors/thun-120l.html

[3] www.ebikes.ca

Ebike's electric motor (motor in short). Second, we consider a fixed mechanical gear setting. The first instantaneous input power to be considered is from the motor to the bike $P_{M_{\text{in}}}(t)$. The control input to the motor is denoted by $Y(t)$ and is related to the motor current $I_M(t)$ via $I_M(t) = \mu Y(t)$ where $\mu > 0$ is a constant. Introducing V_M the motor voltage, the power delivered by the motor is given by $P_{M_{\text{in}}}(t) = V_M I_M(t)$. The second instantaneous input power to be considered is from the human to the bike $P_{H_{\text{in}}}(t)$. It is expressed as $P_{H_{\text{in}}}(t) = \tau_p(t)\omega_p(t)$ where $\tau_p(t)$ represents the torque provided by the cyclist at the pedal and $\omega_p(t)$ is the angular velocity at pedals. Introducing $E_m \in (0,1)$ and $E_c \in (0,1)$ the efficiency of the motor and the crankset, respectively, the output motor power and output human power are given by $P_{M_{\text{out}}}(t) = E_m P_{M_{\text{in}}}(t)$ and $P_{H_{\text{out}}}(t) = E_c P_{H_{\text{in}}}(t)$, respectively. Thus, the total power available to move the rear wheel can be approximated as $P_w(t) = P_{M_{\text{out}}}(t) + P_{H_{\text{out}}}(t)$.

B. Control Algorithm

The control objective is to the fraction of effort delivered by the human. For example, such a control strategy is important to control the transpiration or the ventilation rate of the cyclist. To achieve such a control objective, we introduce the proportion $m \in [0,1]$ of power provided by the cyclist:

$$m(t) = \frac{P_{H_{\text{out}}}(t)}{P_{M_{\text{out}}}(t) + P_{H_{\text{out}}}(t)}. \tag{1}$$

A value of $m(t)$ close to 0 indicates that the system is essentially in electric mode while a value of $m(t)$ close to 1 means that most of the power is delivered by the human. For a given desired value $m^* \in [0,1]$, the control objective is that m tracks m^*.

Note that the configuration $m = 0$ with $P_{H_{\text{out}}} = 0$ and $P_{M_{\text{out}}} > 0$ corresponds to a full electrical mode and is undesirable as it would transform the Ebike into a motorbike, which is illegal in some jurisdictions. Therefore, we assume the existence of a constant $\eta \in (0,1)$ such that $m^*(t) \in [\eta, 1]$ for all $t \geq 0$.

In practice, the direct use of the quantities $P_{M_{\text{out}}}(t)$ and $P_{H_{\text{out}}}(t)$ is not adequate because of biases, uncertainties in the system operation, and the stochasticity of the cyclist behavior over short period of time (such as sudden speed variations over short distances). Consequently, the direct control of m defined by (1) can be counterproductive as it might lead to a very erratic control effort Y. This would result in a very erratic power delivery of the motor $P_{M_{\text{out}}}$, inducing safety issues and a lack of comfort for the cyclist. Consequently, we introduce the following filtered version of the proportion $m \in [0,1]$ of power provided by the cyclist:

$$m(t) = \frac{\overline{P}_{H_{\text{out}}}(t)}{\overline{P}_{M_{\text{out}}}(t) + \overline{P}_{H_{\text{out}}}(t)}, \tag{2}$$

where $\overline{P}_{H_{\text{out}}}$ and $\overline{P}_{M_{\text{out}}}$ are the corresponding filtered and averaged versions of $P_{H_{\text{out}}}$ and $P_{M_{\text{out}}}$ (see [16] for details). In this setting, the control objective remains that m should track a given reference value m^*. For notational simplicity, $\overline{P}_{H_{\text{out}}}$ and $\overline{P}_{M_{\text{out}}}$ are simply denoted by $P_{H_{\text{out}}}$ and $P_{M_{\text{out}}}$ in the sequel.

Denoting by ΔT the sampling time and introducing the regulation error $e_k = m^*(k\Delta T) - m(k\Delta T)$, it was proposed and implemented in [17] the following discrete time (with zero order holder) integral controller:

$$Y_{k+1} = Y_k - \gamma e_k, \tag{3}$$

where $Y_k = Y(k\Delta T)$ and $\gamma > 0$ is a proportional gain. The validity of this approach was assessed in [17] by means of experiments. However, it does not fully account for the human-in-the-loop aspect of the problem, i.e., the complex interactions between the cyclist and the motor. Specifically, a decrease of the value of the reference m^* should reduce the workload of the cyclist by decreasing accordingly the value of m. However, due to this reduction of the workload on the cyclist, their instinct may be to increase cycling effort, which is the opposite of the original objective. Consequently, the control strategy must consider the possibility of such an uncooperative behavior of the cyclist by progressively switching-off the motor.

C. Basic idea

As suggested in [17], one approach for tackling this problem consists in the following control strategy:

$$\dot{P}_{M_{\text{out}}} = f\left(P_{H_{\text{out}}}\right) P_{M_{\text{out}}} - P_{M_{\text{out}}}^3, \tag{4}$$

which is the normal form of a pitchfork bifurcation [10]. The function f is designed based on engineering considerations such that $f(0) = 0$ and $f(x) > 0$ for any $x > 0$. The motivation behind the use of (4) is to regulate the interaction between the cyclist and the motor. Indeed, in the case $P_{H_{\text{out}}} = 0$, then $P_{M_{\text{out}}}$ converge to 0 when $t \to +\infty$, whatever the initial condition $P_{M_{\text{out}}}(0) \geq 0$ might be. In the case[4] $P_{H_{\text{out}}}(t) = P_{H_{\text{out}},e} > 0$, $P_{M_{\text{out}}}$ converges to $P_{M_{\text{out}},e} = \sqrt{f(P_{H_{\text{out}},e})}$ when $t \to +\infty$, whatever the initial condition $P_{M_{\text{out}}}(0) > 0$ might be. In this context, the shape of the function f is tuned such that the motor assists the cyclist. Specifically, m tracks m^* as long as $P_{H_{\text{out}}}$ is below a given threshold $P_T > 0$, while the motor is gradually switched off when $P_{H_{\text{out}}} > P_T$. The graph of such a typical function f is depicted in Fig. 2.

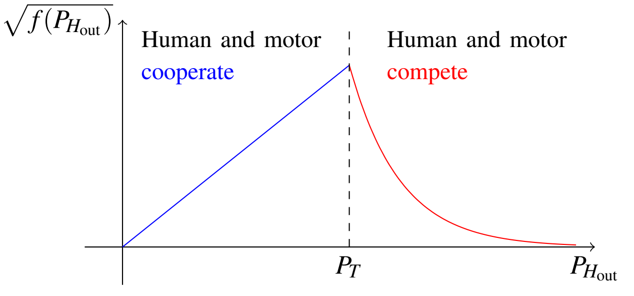

Fig. 2. Typical function f specifying cooperative and competitive behaviors

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[4] Subscript "e" stands for an equilibrium value.

D. Fine tuning of the pitchfork bifurcation by means of gain-scheduling control design

The control law adopted in this paper takes the form:

$$\dot{P}_{M_{\text{out}}} = \alpha_{m^*}(P_{H_{\text{out}}})\left[f_{m^*}(P_{H_{\text{out}}})P_{M_{\text{out}}} - P^3_{M_{\text{out}}}\right] \tag{5a}$$

$$P_{M_{\text{out}}}(0) = P_{M_{\text{out}},0} \geq 0 \tag{5b}$$

with $\alpha_{m^*}(P_{H_{\text{out}}}) > 0$, $f_{m^*}(0) = 0$, and $f_{m^*}(P_{H_{\text{out}}}) > 0$ for all $P_{H_{\text{out}}} > 0$. Functions $(P_{H_{\text{out}}}, m^*) \rightarrow f_{m^*}(P_{H_{\text{out}}})$ and $(P_{H_{\text{out}}}, m^*) \rightarrow \alpha_{m^*}(P_{H_{\text{out}}})$, that are selected in $\mathscr{C}^0(\mathbb{R}_+ \times [\eta,1];\mathbb{R}_+)$, must be tuned in function of $m^* \in [\eta,1]$. The former aims at capturing the desired trade-off between cooperative and competitive behaviors of the cyclist, while the latter is for the tuning of the speed of convergence of the control strategy.

The function $(P_{H_{\text{out}}}, m^*) \rightarrow f_{m^*}(P_{H_{\text{out}}})$ is tuned such that, for any constant reference input m^* and any equilibrium value $P_{H_{\text{out}},e} \leq P_T(m^*)$, we have at the equilibrium that

$$m^* = m_e = \frac{P_{H_{\text{out}},e}}{P_{M_{\text{out}},e} + P_{H_{\text{out}},e}} = \frac{P_{H_{\text{out}},e}}{\sqrt{f_{m^*}(P_{H_{\text{out}},e})} + P_{H_{\text{out}},e}}.$$

In this work, this is achieved by defining for $P_{H_{\text{out}}} \leq P_T(m^*)$:

$$f_{m^*}(P_{H_{\text{out}}}) = \left[\frac{1-m^*}{m^*}P_{H_{\text{out}}}\right]^2$$

and for $P_{H_{\text{out}}} > P_T(m^*)$:

$$\begin{aligned} f_{m^*}(P_{H_{\text{out}}}) = & \left[\frac{1-m^*}{m^*}P_T(m^*)\right]^2 \\ & \times \left[1 + 2\left(\gamma + \frac{1}{P_T(m^*)}\right)(P_{H_{\text{out}}} - P_T(m^*))\right] \\ & \times e^{-2\gamma\left[P_{H_{\text{out}}} - P_T(m^*)\right]}, \end{aligned}$$

where $\gamma > 0$ is the exponential decay rate for the competitive scenario. $P_T(m^*) > 0$ denotes the threshold associated with the current value of the reference signal $m^* \in [\eta,1]$. In other words, for a given value of m^*, $P_T(m^*) > 0$ stands for the maximum value of the effort provided by the cyclist that is considered as a cooperative behavior. For large values of m^* (i.e., close to 1), the cyclist should be able to modulate freely his effort. Therefore, the corresponding value of the threshold $P_T(m^*)$ should be large enough. Conversely, the effort of the cyclist should be constrained when m^* is close to the lower bound η. So, in this case, the value of the threshold $P_T(m^*)$ should be reduced. Thus, we select $m^* \rightarrow P_T(m^*)$ as an element of $\mathscr{C}^0([\eta,1];\mathbb{R}_+^*) \cap \mathscr{C}^1([\eta,1];\mathbb{R})$ satisfying $\dot{P}_T > 0$, i.e., is a (strictly) positive and increasing function of the reference value $m^* \in [\eta,1]$. Thus, it is easy to see that $(P_{H_{\text{out}}}, m^*) \rightarrow f_{m^*}(P_{H_{\text{out}}})$ is an element of $\mathscr{C}^0(\mathbb{R}_+ \times [\eta,1];\mathbb{R}_+) \cap \mathscr{C}^1(\mathbb{R}_+ \times [\eta,1];\mathbb{R})$.

We now tune the function $(P_{H_{\text{out}}}, m^*) \rightarrow \alpha_{m^*}(P_{H_{\text{out}}})$ to tune the convergence rate of the control law. To do so, assuming a constant reference signal m^*, we resort to the following Jacobian linearization of the controller dynamics (5a) in the vicinity of the equilibrium point $P_{M_{\text{out}},e} = \sqrt{f_{m^*}(P_{H_{\text{out}},e})}$:

$$\delta\dot{P}_{M_{\text{out}}} = -2\alpha_{m^*}(P_{H_{\text{out}},e})f_{m^*}(P_{H_{\text{out}},e})\delta P_{M_{\text{out}}}.$$

In order to impose a constant convergence rate $\kappa > 0$ in the vicinity of any equilibrium point, we select $\alpha_{m^*}(P_{H_{\text{out}}}) = \kappa/(2f_{m^*}(P_{H_{\text{out}}})) > 0$ which yields $\delta\dot{P}_{M_{\text{out}}} = -\kappa\delta P_{M_{\text{out}}}$. However, for either small or large values of $P_{H_{\text{out}}}$, $f_{m^*}(P_{H_{\text{out}}})$ is getting arbitrarily close to zero and thus induces the divergence of $\alpha_{m^*}(P_{H_{\text{out}}})$. To avoid this pitfall, we introduce a threshold $\varepsilon_T > 0$ and set $\alpha_{m^*} = \kappa/(2\varepsilon_T)$ for any $P_{H_{\text{out}}}$ satisfying $f_{m^*}(P_{H_{\text{out}}}) \leq \varepsilon_T$. Thus, we obtain that $\alpha_{m^*}(P_{H_{\text{out}}}) = \kappa/(2\max(f_{m^*}(P_{H_{\text{out}}}), \varepsilon_T)) > 0$ which is bounded above by the constant $\kappa/(2\varepsilon_T)$.

E. Discussion on the robustness of the control strategy

In order to validate the proposed controlled strategy, we present in this section a tracking performance of m with respect to the reference signal m^*. Due to space limitation, only the main results are presented while the associated proofs are omitted.

1) Preliminary results: We introduce[5]

$$C_f \triangleq \sup_{(m^*, P_{H_{\text{out}}}) \in [\eta,1] \times \mathbb{R}_+} f_{m^*}(P_{H_{\text{out}}}) \in \mathbb{R}_+^*$$

and, for any constants $P_{H,M} > P_{H,m} > 0$, we define[6]

$$c_f(P_{H,M}, P_{H,m}) \triangleq \inf_{m^* \in [\eta,1]\,,\, P_{H,m} \leq P_{H_{\text{out}}} \leq P_{H,M}} f_{m^*}(P_{H_{\text{out}}}) \in \mathbb{R}_+^*.$$

We can now state the following result dealing with the maximum amplitude of the power delivered by the motor for a suitable range of admissible initial conditions $P_{M_{\text{out}},0}$.

Lemma 3.1: Let $P_{M_{\text{out}},0} \in [0, \sqrt{C_f}]$, $P_{H_{\text{out}}} \in \mathscr{C}^0(\mathbb{R}_+;\mathbb{R}_+)$, and $m^* \in \mathscr{C}^0(\mathbb{R}_+;[\eta,1])$ be given. The unique maximal solution $P_{M_{\text{out}}}$ of (5a-5b) is defined over $\mathbb{R}_+$ and satisfies $P_{M_{\text{out}}}(t) \in [0, \sqrt{C_f}]$ for all $t \geq 0$.

Now, assuming that a minimal level of power is provided by the human, we obtain the existence of a lower bound on the power delivered by the motor for a suitable range of admissible initial conditions $P_{M_{\text{out}},0}$.

Lemma 3.2: Let $P_{H,M} > P_{H,m} > 0$ be given constants and define $c_f = c_f(P_{H,M}, P_{H,m}) > 0$. Let $P_{M_{\text{out}},0} \in [\sqrt{c_f}, \sqrt{C_f}]$, $P_{H_{\text{out}}} \in \mathscr{C}^0(\mathbb{R}_+;\mathbb{R}_+)$ with $P_{H,m} \leq P_{H_{\text{out}}}(t) \leq P_{H,M}$ for all $t \geq 0$, and $m^* \in \mathscr{C}^0(\mathbb{R}_+;[\eta,1])$ be given. Then, the associated trajectory $P_{M_{\text{out}}}$ of (5a-5b) satisfies $P_{M_{\text{out}}}(t) \geq \sqrt{c_f}$ for all $t \geq 0$.

2) Assessment of the robust tracking of the reference signal: We can now introduce the main theoretical result of this paper. Specifically, assuming that a minimal level of power is provided by the human, we assess for a suitable range of initial conditions and in the case of a cooperative behavior that m tracks m^*.

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[5] It can be easily seen from the definition of $f_{m^*}(P_{H_{\text{out}}})$ that $M_f < +\infty$ by using the fact that $0 < P_T(\eta) \leq P_T(m^*) \leq P_T(1) < +\infty$ for all $m^* \in [\eta,1]$ and that $xe^{-ax} \leq e^{-1}/a$ for all $x \geq 0$ and $a > 0$.

[6] $c_f(P_{H,M}, P_{H,m}) > 0$ because it is defined as the infimum over a compact set of a continuous function that is (strictly) positive.

Theorem 3.3: Let $P_{H,M} > P_{H,m} > 0$ and $p \in \{1,2\}$ be given constants and define $c_f = c_f(P_{H,M}, P_{H,m})$. There exist constants $\beta, C_1, C_2, C_3, C_4, C_5 > 0$ such that, for any initial condition $P_{M_{\text{out}},0} \in [\sqrt{c_f}, \sqrt{C_f}]$, any $P_{H_{\text{out}}} \in \mathscr{C}^0(\mathbb{R}_+;\mathbb{R}_+) \cap \mathscr{C}^1(\mathbb{R}_+;\mathbb{R})$ with $P_{H,m} \le P_{H_{\text{out}}}(t) \le P_{H,M}$ for all $t \ge 0$, and any $m^* \in \mathscr{C}^0(\mathbb{R}_+;[\eta,1]) \cap \mathscr{C}^1(\mathbb{R}_+;\mathbb{R})$, the associated trajectory $P_{M_{\text{out}}}$ of (5a-5b) satisfies the estimate:

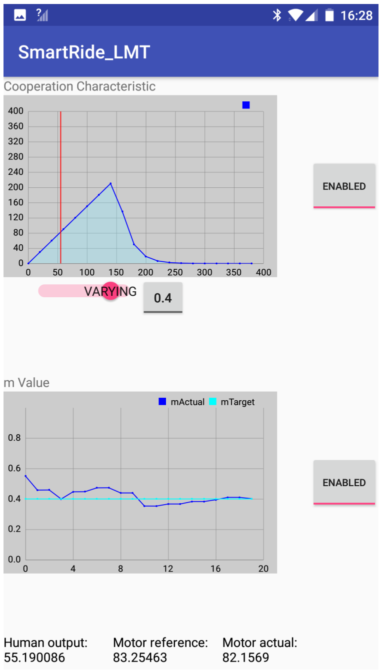
$$\begin{aligned}\left|P_{M_{\text{out}}}(t) - \sqrt{f_{m^*(t)}\left(P_{H_{\text{out}}}(t)\right)}\right| \\ \le \left|P_{M_{\text{out}},0} - \sqrt{f_{m^*(0)}\left(P_{H_{\text{out}}}(0)\right)}\right| e^{-\beta t} \\ + C_1 \sup_{\tau\in[0,t]} |\dot{m}^*(\tau)|^{1/p} + C_2 \sup_{\tau\in[0,t]} \left|\dot{P}_{H_{\text{out}}}(\tau)\right|^{1/p}\end{aligned} \tag{6}$$

with $\sqrt{c_f} \le P_{M_{\text{out}}}(t) \le \sqrt{C_f}$ for all $t \ge 0$. Furthermore, assuming a cooperative behavior of the cyclist, i.e., $P_{H_{\text{out}}}(t) \le P_T(m^*(t))$ for all $t \ge 0$, we have that

$$\begin{aligned}|m(t) - m^*(t)| \le C_3 |m(0) - m^*(0)| e^{-\beta t} \\ + C_4 \sup_{\tau\in[0,t]} |\dot{m}^*(\tau)|^{1/p} + C_5 \sup_{\tau\in[0,t]} \left|\dot{P}_{H_{\text{out}}}(\tau)\right|^{1/p}.\end{aligned} \tag{7}$$

IV. EXPERIMENTAL RESULTS

The experimental configuration involves the Ebike which is mounted securely in a stationary cycling turbo trainer: this allows for rotation of the rear wheel with minimal resistance. The control algorithm is implemented by means of the Runge-Kutta method with a sampling period of $\Delta T = 1.1$ s. It runs on an Android application, written in the Java programming language, which also displays (see Fig. 3) real-time plots of m^*, m, P_H, P_M, and the shape of the time-evolving co-operation/competition characteristic. The experiments reported in this section involve the human operating in the co-operative region of the characteristic.



Human output:
55.190086

Motor reference:
83.25463

Motor actual:
82.1569

Fig. 3. Screenshot of Android application running control algorithm in background process and plotting relevant parameters during experimentation (colors online)

A. Experimental validation of controller design

1) Disturbance rejection for a constant reference m^*: The system has been tested with a constant reference input m^* to evaluate the tracking performance of the closed-loop system in the presence of an input disturbance, namely, a sudden change in the power delivered by the human $P_{H_{\text{in}}}(t)$. The experimental result is depicted in Fig. 4. It can be seen that the constant reference, $m^* = 0.5$, can be tracked satisfactorily in the presence of small variations in $P_{H_{\text{in}}}(t)$. The tracking is also ensured in the presence of a larger change in average human power from approximately 70 W to 120 W, which occurs at time $t = 56$ s. Indeed, a sudden increase of the power delivered by the human induces first a significant increase of the ratio m. Nevertheless, the control algorithm successfully increases the power delivered by the motor to regulate the value of the ratio m to track the constant reference value $m^* = 0.5$.

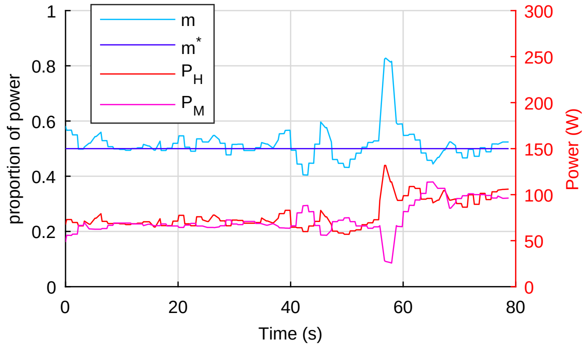


Fig. 4. Tracking of the constant reference signal $m^* = 0.5$ with disturbance rejection with respect to changes in the power delivered by the human $P_{H_{\text{in}}}(t)$ (colors online)

2) Tracking of a time-varying reference signal m^*: A simple experiment to test the tracking performance with respect to a time-varying m^* has been devised and performed. The reference signal m^* is initially set as a constant value of 0.5. Then, it is smoothly transitioned over the range of time from $t = 25$ s to $t = 50$ s to the constant final value of 0.81.

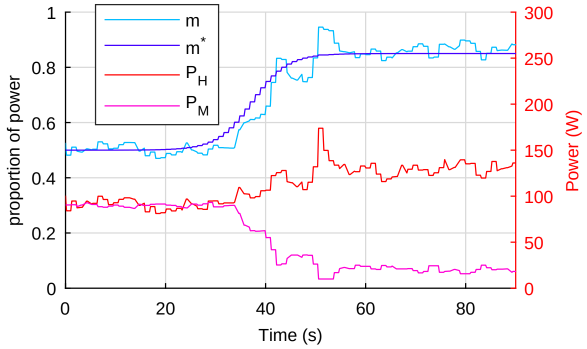
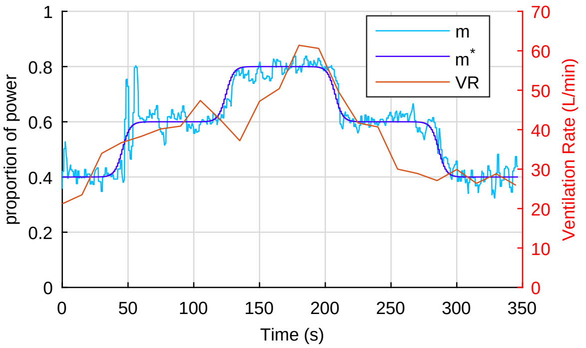


Fig. 5. Reference tracking of a time-varying reference signal m^* with $P_{H_{\text{in}}}(t) \approx 100 - 125$ W (colors online)

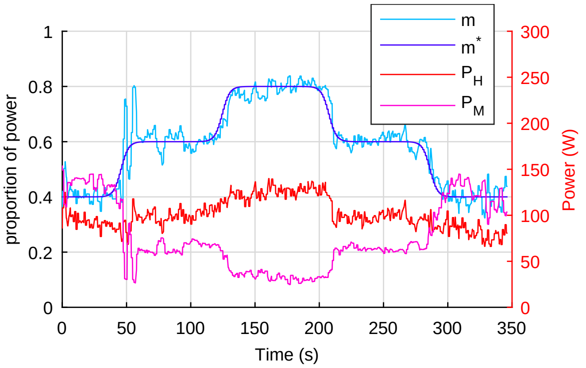
The experimental results are depicted in Fig. 5 with a cyclist trying to maintain a $P_{H_{\text{in}}}(t)$ of approximately $100 - 125\,\text{W}$. It can be seen that m successfully tracks the time-varying reference signal m^* with satisfactory accuracy, even over the range of time $t = 25\,\text{s}$ to $t = 50\,\text{s}$ corresponding to a smoothly time-varying m^*.

B. Indirect control of the ventilation rate of the cyclist

In this test case, we show that the control strategy investigated in this paper can be successfully used to perform an indirect control of the breathing rate of the cyclist. In this experiment, the ventilation rate (VR) of the cyclist is monitored by the spirometry equipment *COSMED Spiropalm 6MWT* with a *COSMED* VO_2 max digital flowmeter. In this setting, the reference signal m^* is generated depending on a time-varying artificial environmental pollution signal. Specifically, large values of m^* (i.e. close to 1) correspond to low pollution levels. In this case most of the power is delivered by the human which can freely adjust his effort. Conversely, small values of m^* (i.e. close to the lower bound $\eta > 0$) correspond to high pollution levels. In this case, the motor assists the cyclist in order to limit the effort provided by the human. The experimental results are depicted in Fig. 6. First, it can be seen that the reference signal m^* is tracked by the actual ratio m with satisfactory precision. Second, the cyclist's VR can be seen to increase and decrease accordingly. Specifically, the VR is significantly reduced for low values of m^*, i.e. for configurations associated with a high level of pollution.



(a) Tracking performance and ventilation rate



(b) Tracking performance and power inputs

Fig. 6. Tracking a time-varying m^* which is set according to an artificial pollution level signal, with effect on cyclist's ventilation rate (colors online)

V. CONCLUSION

We presented a cyber-physical control system for deployment on a smart pedelec (Ebike). The basis of the control is a pitchfork bifurcation system, augmented using ideas from gain-scheduling. After having investigated the robustness of the proposed approach, we experimentally evaluated the effectiveness of our system via a use case. The results from the use case indicate that the system is able to fraction the effort delivered from the ebike to the human in order to regulate the VR. This was used in the use case in response to exogenous pollution.

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