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Nonlinear Effects in Electrostatic Vibration Energy Harvesters: Current Progress and Perspectives

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***Abstract*—In this review paper, we discuss the principles of electrostatic (capacitive) vibration energy harvesters and nonlinear techniques that can be applied to improve the performance of harvesters. Electrostatic vibration energy harvesters are devices that contain mechanical resonators driven by ambient vibrations and coupled with conditioning electronic circuits through a capacitive transducer. While the devices can be fabricated using MEMS technology and miniaturised, internal and external nonlinearity and complexity can lead to irregular behavior and impede the analysis of the devices. In this review we give an overview of the capacitive conversion mechanisms, discuss the basic triangular energy conversion cycle in detail and survey the nonlinear techniques that can be employed in these systems.**

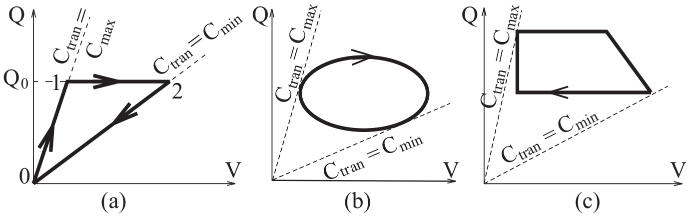

Fig. 1. Three most used QV cycles for vibration energy conversion with a capacitive transducer.

I. Introduction

A wide interest in energy harvesters during recent years is explained by the high demand for energy effective technologies and solutions. Energy harvesting systems can address the problem of extending the life cycle and supplying electrical power for low-power mobile electronic devices and wireless sensors networks [1], [2]. The motivation of energy harvesting is to create autonomous systems that derive their power supply from the environment and do not depend on resources such as batteries. The conversion of vibrations into electrical energy using electromagnetic, piezoelectric or electrostatic technologies has been discussed and demonstrated in a number of works [3]. Electrostatic vibration energy harvesters (e-VEHs) employ capacitive transducers for electromechanical conversion. They are fabricated through MEMS technologies and hence are particularly suitable for use in small-scale mobile devices [4].

While having great potential for microscale integration, electrostatic VEHs employ a nonlinear mechanism for energy conversion. This results in complex multimodal dynamic and stability issues while raising substantial difficulties for analysis and design. Understanding and mastering of these issues require a number of specific tools accounting for nonlinearity. This is especially important in light of the recent tendency in e-VEHs to use nonlinear mechanical resonators in order to improve their performance (such as bi-stable resonators experiencing stochastic resonance, Duffing resonators, etc. [5]). The multidisciplinary nature of energy harvesters raises several challenges related to the analysis and practical implementation of the devices: optimal design of the system, a comprehensive theory describing dynamics and performance, fabrication technology issues, etc.

This paper reviews some problems and difficulties related to nonlinear aspects of VEHs which justify research and development of specific design and analysis tools. We start with an overview of the operation of electrostatic VEHs, emphasing the sources and mechanisms of nonlinearity of the system. We briefly discuss the analytical and modeling approach that we have developed for the nonlinear analysis of e-VEHs on the example of a constant-charge conditioning circuit. Finally, we give an overview of some nonlinear techniques that can be applied in energy harvesting for widening the operating frequency band, allowing harvesting of the energy of wideband and noise-like ambient vibrations.

II. Electrostatic Vibrations-to-Energy Conversion Technique

A. Basic Principles of Electrostatic Conversion

A capacitive transducer is a variable capacitor with one mobile electrode whose instantaneous position determines the value of the transducer capacitance. Energy conversion from the mechanical to electrical domain occurs when the mobile electrode of a charged transducer moves in such a way that the capacitance of the transducer decreases. A mechanical force works against the electrical field, increasing the electrical energy of the transducer. In the case of VEHs, the motion of the mobile electrode is generated by a mechanical resonator driven by external vibrations, and a portion of the energy of the external vibrations is converted into electricity [6].

The process of capacitive electromechanical energy conversion is usually characterised through a charge-voltage (QV) cycle. The most common QV cycles are given in Fig. 1. For instance, Fig. 1a shows the basic *constant-charge* QV cycle (discussed in detail in the next section). Figures 1b and 1c show the examples of two other QV cycles allowing energy conversion: the former is often used with electret biased capacitive transducers [7], while the latter characterises the conditioning circuit presented in Section IIIA.

Capacitive (electrostatic) transducers are characterised by a capacitance-displacement characteristic $C_{tran}(x)$ that depends only on the geometry of the transducer. The most common types of the transducer geometry are a gap closing transducer [6], an area overlap transducer [8] and a transducer with a saw $C_{tran}(x)$ characteristic [4]. Typical plots of $C_{tran}(x)$

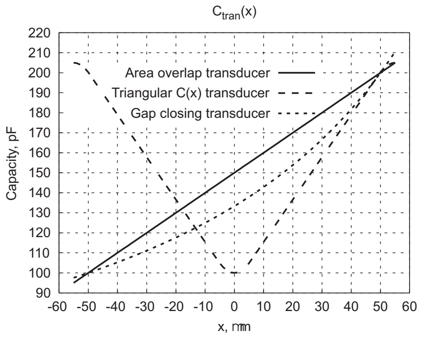

Fig. 2. Three typical $C_{tran}(x)$ characteristics of capacitive transducers.

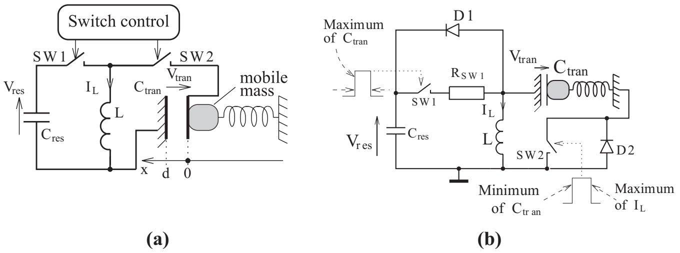

Fig. 3. (a) Schematic view of the circuit implementing the triangular QV-cycle and (b) realistic VHDL-AMS model of this circuit.

as a function of the displacement of the movable electrode x are presented in Fig. 2.

The dynamics of a mechanical resonator that generates the motion of the movable electrode in the transducer are described by a mass-spring-damper equation. This equation represents the second Newtonian law and includes all forces that affect the dynamics of the resonator displacement x:

$$\ddot{x}+(b/m)\dot{x}+\omega_0^2 x+qx^2+px^3 = A_{ext}\cos(\omega_{ext}t)+F_t/m \quad (1)$$

where x is the displacement, m is the mass of the resonator, b is the damping factor, ω_0 is the natural resonance frequency, A_{ext} and ω_{ext} are the acceleration amplitude and frequency of external vibrations. The restoring force of the resonator can be nonlinear and is given by a polynomial limited to the third order $F_{spring} = -(k_1 x + k_2 x^2 + k_3 x^3)$. The linear coefficient k_1 defines the natural frequency $\omega_0 = \sqrt{k_1/m}$, while the nonlinear coefficients define $q = k_2/m$ and $p = k_3/m$. F_t represents the force generated by the electrostatic transducer and depends on a particular energy conversion cycle implemented in the conditioning circuit and the geometry of the transducer.

B. Triangular QV-cycle of energy conversion

In this section we briefly discuss the basic and one of the most effective energy conversion cycle, the triangular charge-voltage cycle from Fig. 1a. The conditioning circuit that implements this cycle is shown in Fig. 3a. When the transducer capacitance C_{tran} is at its maximum, an external conditioning circuit charges it to a charge Q_0 (line 0-1). Then C_{tran} decreases keeping the charge Q_0 constant (line 1-2). When C_{tran} reaches the minimum value, the conditioning circuit discharges the transducer (line 2-0), taking more energy that it had used to charge the transducer (energy is taken from the ambient vibrations). The area of the QV cycle is numerically equal to the energy converted during the cycle. The transducer generates a mechanical force defined by a piecewise expression:

$$F_t(x, C_{tran}) = \begin{cases} \dfrac{Q_0^2}{2C_{tran}} \dfrac{\partial C_{tran}}{\partial x}\,, & \frac{dC_{tran}}{dt} < 0 \\ 0\,, & \text{otherwise} \end{cases} \quad (2)$$

Since $dC_{tran}/dt = \dot{x} \cdot \partial C_{tran}/\partial x$, the transducer force depends explicitly on the sign of the mobile mass velocity $\dot{x}$ and is, in general, a *nonlinear function* of x. At a local maximum C_{max}, the conditioning circuit fixes one of the three quantities, the voltage V_0, energy W_0 and charge Q_0, depending on its architecture. It should be noted that C_{max} represents a *local maximum* of C_{tran}, and hence it is a dynamic quantity that may change from one vibration cycle to another (for instance, during a transient or when vibrations are irregular).

C. Steady-State Analysis

The steady-state analysis of e-VEHs is motivated by the problem of finding the maximum converted power. In the case of periodic oscillations in an electrostatic VEH, power is expressed as [9]

$$P = W_0 \left(\frac{C_{max}}{C_{min}} - 1 \right) f_{ext} \quad (3)$$

where C_{max} and C_{min} are the capacitances corresponding to the maximal and minimal displacements in one cycle and f_{ext} is the vibration frequency. In our case, this frequency defines the frequency of the variation of the transducer capacitance.

Substantial results that we have recently obtained for the system from Fig. 3a with two configurations of the transducer, area overlap [8] and gap closing [9], are reported in [9]–[12]. We analyse the steady-state behaviour by employing the multiple scales method (MSM), a type of perturbation technique that is often applied for the analysis of weakly nonlinear systems. Figure 4 demonstrates the envelope of oscillations i.e. $x_{max} = x_{av,0} + a_0$ and $x_{min} = x_{av,0} - a_0$

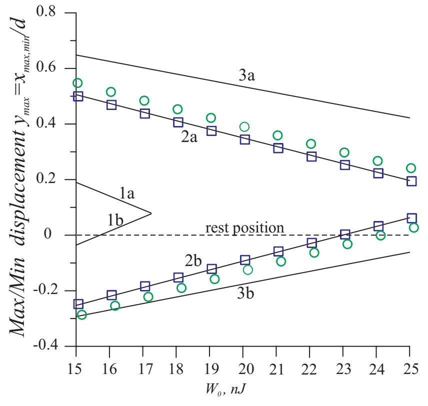

Fig. 4. Steady-state oscillations of an e-VEH with the gap-closing transducer: the envelope of oscillations as a function of W_0 at $A_{ext} = 3\ m/s^2$ (line 1), $A_{ext} = 5\ m/s^2$ (line 2) and $A_{ext} = 7\ m/s^2$ (line 3). Squares show the envelope obtained from VHDL-AMS simulations of the idealised model from fig. 3a while circles show the simulations of the realistic model from fig. 3b.

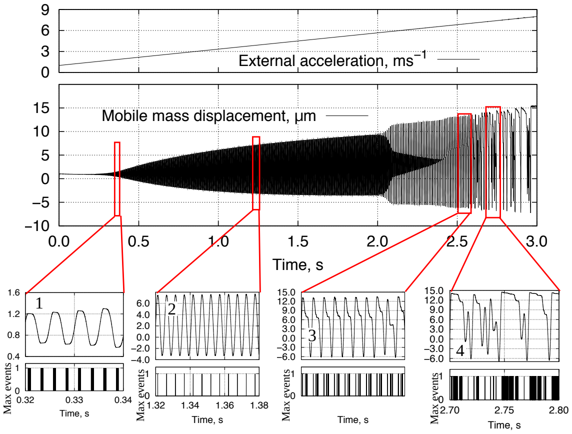

Fig. 5. A slowly growing ramp of the envelope of the external oscillations A_{ext} and the corresponding displacement of the mobile mass of a e-VEH with the gap closing transducer. Four fragments show different dynamic behaviour of the system [13].

calculated using the steady-state theory. This theory allowed us not only to calculated the converted power (3), but also to find its optimal value.

D. Behavioural Modelling

Numerical modeling of e-VEHs is a very important tool that allows one to study system properties and validate analytical approaches. For the modeling of e-VEHs, we have developed a mixed SPICE and behavioural description implemented in the VHDL-AMS/Eldo environment provided with the AdvanceMS tool of Mentor Graphics. The conditioning circuit is implemented as an electrical network described by an Eldo netlist (Eldo is a commercial variant of the SPICE simulator). A VHDL-AMS model of the transducer/resonator block can be seen as an electrical dipole behaving as a variable capacitor [13]. The capacitance variation is obtained through resolution of Newtonian equations written for the resonator which also takes into account the force f_t generated by the transducer.

An example of mixed-signal simulations is shown in Fig. 5 for the circuit from Fig. 3 with the gap closing transducer. In Fig. 5 we plot a slowly growing ramp of the envelope of the external oscillations A_{ext} and the corresponding displacement of the mobile mass of a e-VEH. Four fragments show different dynamic behaviour of the system whose characteristics have been given in [13]. From this figure, one can see that the dynamics of the e-VEH are complex due to the nonlinearity of the transducer force and the desirable harmonic oscillations are limited by a period doubling bifurcation at high accelerations and by a sliding bifurcation at low accelerations. VHDL-AMS models can be developed for various conditioning circuits, for example, for the one shown in Fig. 6.

III. Future Perspectives on Electrostatic Vibration-to-Energy Conversion

A. New Circuit Topologies

Though the triangular QV cycle is the most effective energy conversion cycle, the circuit in Fig. 3 can operate effectively only if the oscillations of the resonator are periodic. There are other circuit topologies that can work even if the resonator displays irregular motion. The circuit proposed in [14] is shown in Fig. 6. This scheme contains a charge pump and a flyback circuit. The role of the charge pump is to transfer the charge from a large capacitor C_{res} to a smaller capacitor C_{store}, making use of the variations of the transducer capacitance C_{tran}. Such transfer requires additional energy that is taken from the mechanical domain. Thus energy harvesting is achieved during charge pumping and harvested energy is stored in the system C_{res}–C_{store} as a voltage difference between these two capacitors. This circuit is less sensitive to the 'irregularity' of the oscillations of the variable capacitor C_{tran} and potentially can be used with nonlinear resonators or with resonators driven by noise-like ambient vibrations.

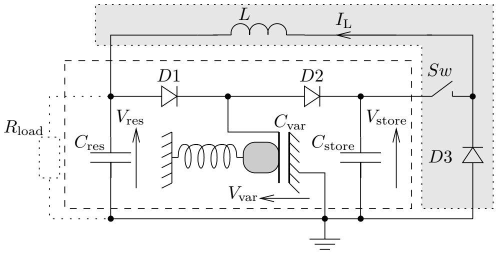

Fig. 6. Circuit containing a charge pump and a flyback for electrostatic vibration energy harvested from Ref. [14].

B. Nonlinear resonators

Conventional vibration energy harvesters are often designed using linear high-Q mechanical resonators. As a consequence, they display a very narrow frequency response and operate efficiently only when exited by vibrations whose frequency is close to the natural/resonant frequency of the resonator. However, many sources of ambient vibrations can be described as wideband or their dominant frequency can drift. Some methods to improve the frequency response include the tuning of the resonant frequency to the frequency of external oscillations, bistable structures [15] driven by noise [16], [17], resonator arrays or mechanical nonlinearities (see review [18] where many of the above techniques are discussed). In the case of mechanically nonlinear resonators employed in harvesting systems whose restoring force is expressed as

$$F_{spring} = -(k_1 x + k_2 x^2 + k_3 x^3) \tag{4}$$

one observes nonlinear resonance. (Note that k_3 can have any sign.) At some parameters, two stable coexisting solutions are possible and the width of the resonance curve increases. This is illustrated in Fig. 7 which shows the amplitude of resulting oscillations as a function of the frequency ω_{ext}. References [19]–[21] discuss and experimentally study nonlinear resonators in the context of VEHs.

C. Stochastic Resonance

Reference [16] proposes to use a bi-stable resonator driven by noise to harvest energy. The practical implementation of the system can be achieved by the use of a repulsing magnet or a bistable membrane. In both cases, the spring force can be modeled as

$$F_{spring} = k_1 x (x^2 - a^2) \tag{5}$$

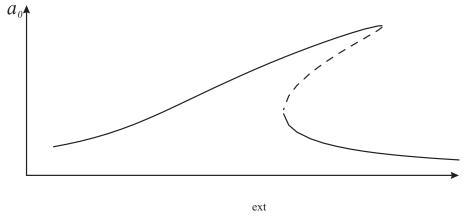

Fig. 7. Resonance curve (the frequency response) of a nonlinear resonator to external harmonic oscillations.

where k_1 is a spring constant and a is the distance between the equilibrium points. If such a system is subjected to an external driving that contains sinusoidal vibrations and weak noise, a stochastic resonance is possible: the amplitude of the resonator vibrations may be of the same order of magnitude as the distance between the maxima a, even if the amplitude of the external vibrations is small (see Fig. 8). This resonance may happen for a wide range of the frequency of external vibrations. It was demonstrated experimentally and by numerical modeling that such a resonator can harvest energy from a wide spectrum vibrations with an efficiency of up to 600% higher than that of an equivalent linear resonator.

Reference [15] proposes a detailed theoretical investigation of this harvesting system, focusing the analysis effort on the investigation of different dynamic behaviours (attractors) and of the bifurcation conditions between different modes. Stochastic behaviour was observed and qualified.

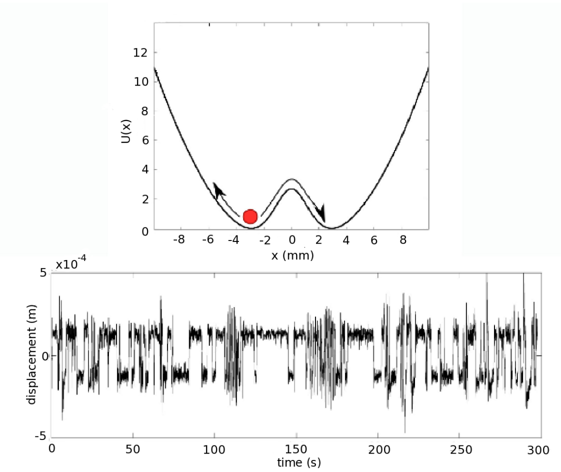

Fig. 8. Demonstration of stochastic resonance in a bistable resonator from [17]: (a) a potential well of the bi-stable resonator and (b) a sample of the signal from the bi-stable resonator driven by noise.

IV. CONCLUSIONS

We have presented a review of the electrostatic technique for vibration energy harvesting with a focus on nonlinear issues in the analysis, design and modelling of the system. The principles of electrostatic vibrations-to-energy conversion, basic triangular charge-voltage energy conversion cycle and the corresponding circuit have been discussed and studied in detail. We have given an overview of nonlinear techniques that can be employed in energy harvesting in order to improve the performance of these devices, along with future directions of research in the area of electrostatic energy harvesters. The results of research contribute not only to practical aspects of e-VEH engineering, but also to the theory of dynamical systems.

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