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Unit Commitment with Dynamic Cycling Costs

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***Abstract*—Increased competition in the electricity sector and the integration of variable renewable energy sources is resulting in more frequent cycling of thermal plant. Thus, the wear-and-tear to generator components and the related costs are a growing concern for plant owners and system operators alike. This paper presents a formulation that can be implemented in a MIP dispatch model to dynamically model cycling costs based on unit operation. When implemented for a test system the results show that dynamically modeling cycling costs reduces cycling operation and tends to change the merit order over time. This leads to the burden of cycling operation being more evenly distributed over the plant portfolio and reduces the total system costs relative to the case when cycling costs are not modeled.**

***Index Terms*—Thermal Power Generation, power system modeling**

NOMENCLATURE

Indices/Sets

t, T	Time step, set of time steps
g, G	Units, set of units
i, I	Interval of cycling cost function, set of intervals of cycling cost function
j, J	Level of ramp, set of all ramp levels
l, L	Segment of the piecewise linearization of the variable cost function, set of all segments of the piecewise, linearization of the variable cost function

Constants

a_g, b_g, c_g	Coefficients of the quadratic production cost function for unit g
$cost_g^S$	Cycling cost increment incurred by unit g for each additional start-up
$Th_g^S(i)$	i^{th} threshold corresponding to cumulative start-ups by unit g
$cost_g^S(i)$	Cycling cost increment incurred by unit g for each additional start-up, until cumulative start-ups ($N_g^S(t,i)$) reach a given threshold ($Th_g^S(i+1)$)
R_g	production change (MW) over time period t deemed damaging for unit g
$R_g(j)$	j^{th} production change (MW) over time period t deemed damaging for unit g
$cost_g^R$	Cycling cost increment incurred by unit g for each additional ramp $> R_g$
$Th_g^R(i)$	i^{th} threshold corresponding to cumulative ramps for unit g
$cost_g^R(i)$	Cycling cost increment incurred by unit g for each additional ramp, until cumulative ramps ($N_g^R(t,i)$) reach a given threshold ($Th_g^R(i+1)$)
I_g	Total number of intervals in cycling cost function for unit g
$\bar{j}_g$	Number of ramp levels defined for unit g
$\bar{P}_g$	Maximum capacity of unit g
$\underline{P}_g$	Minimum capacity of unit g
A_g	Fixed cost for unit g ($/h)
NL_g	Number of segments in piecewise linearization of the variable cost function of unit g
F_{lg}	Slope of segment l of the variable cost function of unit g
T_{lg}	Upper limit of block l of the piecewise linear production cost function of unit g (MW)
UT_g	Minimum up time of unit g
DT_g	Minimum down time of unit g
$\bar{T}$	Number of hours in the planning period
T_g^{cold}	Number of hours unit g must be offline, beyond its minimum downtime, before it is considered to be in a cold state
cc_g	Cold start-up cost for unit g
hc_g	Hot start-up cost for unit g
h^{up}	Number of hours unit g has been online for at start of planning period (h)
h^{down}	Number of hours unit g has been offline for at start of planning period (h)
M	Large number
$\alpha,\ \beta,\ \gamma$	Scaling factors

Binary Variables

$s_g(t)$	equal to 1 when a unit starts up at time t
$z_g(t)$	equal to 1 when a unit shuts down at time t
$v_g(t)$	equal to 1 when a unit is online at time t
$step_g^S(t,i)$	equal to 1 when $N_S(t,1) \geq Th_S(i)$ at time t
$r_g(t)$	equal to 1 when a unit undergoes ramp $> R_g$ between time $t-1$ and t
$r_g(t,j)$	equal to 1 when a unit undergoes ramp $> R_g$(j) between time $t-1$ and t

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$step_g^R(t,i)$	equal to 1 when $N_R(t,1) \geq Th_R(i)$ at time t

Positive Variables

$N_g^S(t)$	Cumulative start-ups for unit g
$N_g^S(t,i)$	Cumulative start-ups for unit g beyond threshold $Th_g^S(i)$
$C_g^S(t)$	Total cycling cost attributed to start-ups for unit g
$N_g^R(t)$	Cumulative ramps $> R_g$ for unit g
$N_g^R(t,i)$	Cumulative ramps $> R_g$ beyond threshold $Th_g^R(i)$ for unit g
$C_g^R(t)$	Total cycling cost attributed to ramping for unit g
$c_g^p(t)$	Production cost for unit g at time t
$c_g^s(t)$	Start-up fuel cost for unit g at time t
$p_g(t)$	Output (MW) for unit g at time t
$D(t)$	System demand (MW) at time t
$\delta_l(g,t)$	Power produced in block l of the piecewise linear production cost function of unit g at time t (MW)

I. Introduction

INCREASED competition in the electricity generation sector coupled with the large-scale deployment of variable renewable energy sources, particularly wind power, has led to increased plant cycling in power systems worldwide [1], [2]. Cycling may be defined as frequent start-ups or ramping of units. Some generation types (such as hydro or even open-cycle gas turbines) are more suited to frequent cycling, but for others, particularly units designed for base-load operation, cycling can accrue large levels of damage within the plant's components leading to increased maintenance requirements and forced outage rates. Thermal shock, metal fatigue, corrosion, erosion and heat decay are common damage mechanisms that result from cycling operation [3] and work done in [4] and [5] has attempted to limit such operation via the incorporation of ramping constraints in the dispatch algorithm. In the absence of such constraints, the wear-and-tear which arises due to cycling will incur increased maintenance costs for generators, and in addition to this, loss of revenue due to more frequent and longer outages, increased fuel costs due to more frequent start-ups and reduced plant efficiency, as well as additional capital costs due to component replacement can also be expected. Studies indicate that the magnitude of these cycling related costs are high, but accurately quantifying them is challenging [6], [7]. The level of wear-and-tear for a unit that undergoes cycling operation will be dependent on many factors including the operating history of the plant (i.e. how much creep damage it has accumulated), and the engineering design of the plant. It is also typical to see a time lag of several years from when cycling occurs to when the damage manifests itself [8].

Research related to the cost of generation cycling has been undertaken by EPRI (Electric Power Research Institute) and Intertek Aptech and the approaches employed can be categorized as top-down (statistical analysis) or bottom-up (component modeling). EPRI carried out a top-down study utilizing multivariate regression models to analyze the operating regimes of 158 units from NERC (North American Electric Reliability Corporation) GADS (Generating Availability Data System) and CEMS (Continuous Emission Monitoring) data in an attempt to identify patterns relating plant operation to capital expenditure. However, the inconsistency in accounting practices between the units complicated the modeling and no correlation was found [9], [10]. Intertek Aptech employ a combination of top-down models based on historical operations, forced outage and cost data as well as bottom-up methods which calculate operational stresses and the life expenditure of critical components to determine cycling costs for individual generating units [6]. Intertek Aptech have analyzed cycling costs for over 300 generating units and found that the cost of cycling a conventional fossil-fired power plant can be as much as $2,500-500,000 per start/stop cycle depending on unit age, operating history and design features, and these costs are often grossly underestimated by utilities [6], [8].

Not considering these costs, however, will result in an uneconomic plant dispatch, yet markets currently do not include specific cycling cost components in their bidding mechanisms, or at best cycling costs are bundled into a generator's start-up or operating costs. Depending on the operating regime of a plant, these cycling related costs can accumulate rapidly and are therefore dissimilar to plant characteristics such as heat rate, which typically vary over a much longer time-scale. Therefore, to examine the impact of these costs accurately, they should be modeled in a dynamic manner such that they accumulate within the optimization process based on how the unit is being operated and thereby can influence dispatch decisions.

This paper presents a novel formulation to dynamically model these cycling costs, which can be integrated into a MIP (mixed integer programming) unit commitment and economic dispatch model. This facilitates more accurate modeling of these costs and examination of how they accumulate in line with the operating regime of the plant. The formulation defines a cycling cost which increments with each additional plant start-up or ramp with the resulting cost function being linear, piecewise linear or step-shaped. A case study is included to determine how implementing dynamic cycling costs for a test system over a period of up to three years will affect the resulting dispatch, relative to a scenario where cycling costs are not considered. This new approach to modeling cycling costs is particularly suitable for long-term planning studies where it can be used to reflect the ageing effect on a plant over time. It may also have applications for real-world market models where it can discourage the same unit from being repeatedly dispatched to cycle by incurring an incremental cost to reflect the wear-and-tear to that unit, which can consequently alter its position in the merit order.

The paper is organized as follows: Section II details the formulation of dynamic cycling costs, Section III describes a unit commitment model and economic dispatch model used to implement the dynamic cycling cost formulation and also describes the test system, Section IV details the results of the case study and Section V summarizes the findings.

II. Formulation of Dynamic Cycling Costs

A detailed formulation for implementing dynamic cycling costs which increase in line with unit operation is presented. Cycling costs are subdivided into costs for (A) start-ups and (B) ramps. The formulation utilizes three main steps: (i) a binary variable is set to indicate that damaging operation has occurred at time step t, (ii) a counter tracks how much of that type of operation has occurred up to that point, and (iii) an incrementing cycling cost is incurred at that time step. Linear, piecewise linear and step-shaped cost functions for both start-ups and ramps are detailed here.

A. Cycling costs related to start-ups

1) Linear: Constraints (1)-(3) allow a dynamic, linearly incrementing cost for wear-and-tear related to start-ups to be modeled. Based on the online binary variable, $v_g(t)$, constraint (1) sets the start-up, $s_g(t)$, and shut-down, $z_g(t)$, binary variables equal to 1 appropriately, when unit g is started up or shut down at time t. Constraint (2) increments a counter, $N_g^S(t)$, to track how many start-ups have been performed by that unit. Constraint (3) determines the start-up related cycling cost, $C_g^S(t)$, with the final term ensuring that a cost is only incurred when the decision is made to start the unit at time t (i.e. $s_g(t)$ = 1). Table I and Figure 1 provide an example of this linearly increasing cost function, where the cycling cost increment $cost_g^S$ is set equal to 100. (It is also possible to initialize the counter $N_g^S(t)$ with the number of start-ups that have been carried out previously if this is known).

$$s_g(t) - z_g(t) = v_g(t) - v_g(t-1),\ \forall\ t \in T,\ \forall\ g \in G \quad (1)$$

$$N_g^S(t) \geq N_g^S(t-1) + s_g(t),\ \forall\ t \in T,\ \forall\ g \in G \quad (2)$$

$$C_g^S(t) \geq N_g^S(t) \cdot cost_g^S - M \cdot \big(1 - s_g(t)\big),\ \forall\ t \in T,\ \forall\ g \in G \quad (3)$$

TABLE I
LINEAR CYCLING COST FUNCTION

Time, t	$s_g(t)$	$N_g^S(t)$	$C_g^S(t)$
1	0	0	0
2	1	1	100
3	0	1	0
4	0	1	0
5	1	2	200
6	0	2	0
7	0	2	0
8	1	3	300
9	0	3	0

2) Piecewise Linear: By defining i thresholds, $Th_g^S(i)$, each corresponding to a cumulative number of plant start-ups, at which point the start-up related cycling cost, $C_g^S(t)$, will increase by incremental cost $cost_g^S(i)$ for each additional start, a piecewise linear incremental cost function can be modeled. Constraint (4) is a modified form of constraint (2) which counts the cumulative number of start-ups. For $i > 1$, the start-up counter, $N_g^S(t,i)$, will not have a positive value until $N_g^S(t,1)$ has reached $Th_g^S(i)$. $Th_g^S(1)$ must equal 1. Constraint (5) determines the total cycling cost. Table II and Figure 2 provide an example of a piecewise linearly increasing cost function, where $cost_g^S(1)$ is set equal to 100, $cost_g^S(2)$ is set equal to 150 and $Th_g^S(2)$ equals 4.

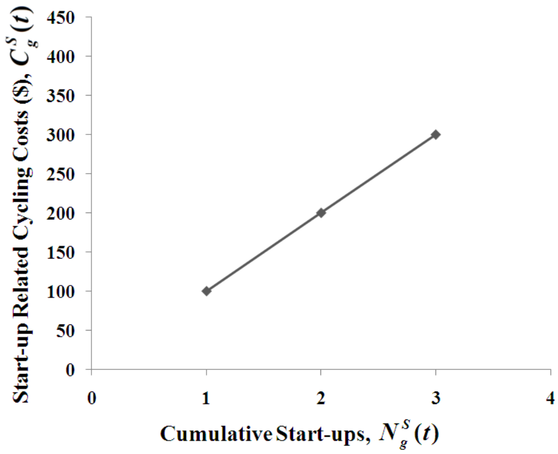

Fig. 1. Linearly increasing start-up related cycling cost

$$N_g^S(t,i) \geq \Big(N_g^S(t-1,1) + s_g(t) + 1\Big) - Th_g^S(i), \quad \forall\ t \in T,\ \forall\ g \in G,\ \forall\ i \leq I_g \quad (4)$$

$$C_g^S(t) \geq \sum_i^{I_g} \Big(N_g^S(t,i) \cdot \big(cost_g^S(i) - cost_g^S(i-1)\big)\Big) - \big(1 - s_g(t)\big) \cdot M,\ \forall\ t \in T,\ \forall\ g \in G \quad (5)$$

TABLE II
PIECEWISE LINEAR CYCLING COST FUNCTION

Time, t	$s_g(t)$	$N_g^S(t,1)$	$N_g^S(t,2)$	$C_g^S(t)$
1	0	0	0	0
2	1	1	0	100
3	0	1	0	0
4	0	1	0	0
5	1	2	0	200
6	0	2	0	0
7	0	2	0	0
8	1	3	0	300
9	0	3	0	0
10	0	3	0	0
11	1	4	1	450
12	0	4	1	0
13	0	4	1	0
14	1	5	2	600
15	0	5	2	0

3) Step Function: Alternatively, if less information is known regarding the shape of the cost function an appropriate simplification may be to define a step function, where $C_g^S(t)$ does not increment until $Th_g^S(i)$ is reached. Again, it is

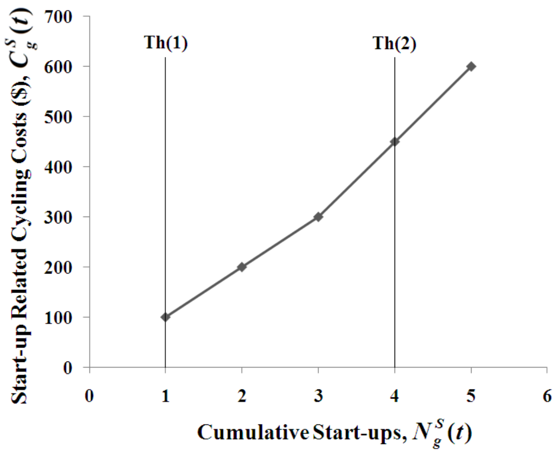

Fig. 2. Piecewise linearly increasing start-up related cycling cost

required that $Th_g^S(1)$ is equal to 1. $N_g^S(t,i)$ is determined by constraint (6) and in this case can be greater than or less than 0 (it was previously defined as a positive variable only). Constraint (7) sets the binary variable $step_g^S(t,i)$ equal to 1 when $N_g^S(t,i)$ has exceeded $Th_g^S(i)$, and constraint (8) determines the cycling cost. Table III and Figure 3 provide an example of this incrementing, step-shaped cost function, where $cost_g^S(t,1)$ is set equal to 100, $cost_g^S(t,2)$ is set equal to 150 and $Th_g^S(2)$ equals 4.

$$N_g^S(t,i) = \Big(N_g^S(t-1,1) + s_g(t) + 1\Big) \;-\; Th_g^S(i), \quad \forall\, t \in T,\ \forall\, g \in G,\ \forall\, i \leq I_g \tag{6}$$

$$N_g^S(t,i) \;-\; step_g^S(t,i){\cdot}M \;\leq\; 0, \quad \forall\, t \in T,\ \forall\, g \in G,\ \forall\, i \leq I_g \tag{7}$$

$$C_S(t) \geq cost_g^S(i){\cdot}step_g^S(t,i) \;-\; \big(1 - s_g(t)\big).M, \quad \forall\, t \in T,\ \forall\, g \in G,\ \forall\, i \leq I_g \tag{8}$$

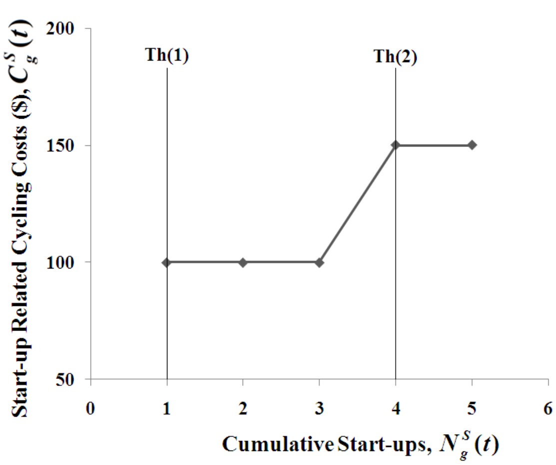

Fig. 3. Step increasing start-up related cycling cost

TABLE III
STEP COST FUNCTION

Time, t	$s_g(t)$	$N_g^S(t,1)$	$N_g^S(t,2)$	$C_g^S(t)$
1	0	0	-3	0
2	1	1	-2	100
3	0	1	-2	0
4	0	1	-2	0
5	1	2	-1	100
6	0	2	-1	0
7	0	2	-1	0
8	1	3	0	100
9	0	3	0	0
10	0	3	0	0
11	1	4	1	150
12	0	4	1	0
13	0	4	1	0
14	1	5	2	150
15	0	5	2	0

4) Hot and Cold Starts: Either the linear, piecewise linear or step formulations can be extended to differentiate between hot and cold start-ups for units. Constraint (9) will set the binary variable $s_g^{cold}(t)$ equal to 1 only if unit g is started at time t, having been offline for T_g^{cold} plus its minimum downtime, DT_g. In constraints (2), (4) and (6) '+ $s_g(t)$' is replaced with '+ $s_g(t)$ + $\alpha.s_g^{cold}(t)$'. A scaling factor, α, is chosen based on the ratio of cycling damage caused by a hot start relative to a cold start, and thus normalizes $N_g^S(t,i)$ to count in terms of hot starts.

$$s_g^{cold}(t) \geq v_g(t) \;-\; \sum_{n=1}^{T_g^{cold}+DT_g} v_g(t-n), \ \forall\, t \in T,\ \forall\, g \in G \tag{9}$$

B. Cycling costs related to ramping

1) Define one ramp level: The simplest form of incurring cycling costs related to ramping duty is to define a change in output, R_g, between consecutive time periods, greater than which, damaging transients will occur within unit g. Constraints (IX) and (11) ensure that the binary variable $r(t)$ is set to 1 when a change in output exceeding R_g occurs. To avoid double counting cycling costs when large ramps are experienced in the start-up or shut-down process, the final term ensures that the constraints are non-binding when the unit is in the start-up or shut-down process. If the ramp-related cycling costs are likely to exceed the start-up or shut-down cost, constraint (12) is needed to prevent the model setting $s(t)$ and $z(t)$ both equal to 1 in constraint (1), in order to make constraints (IX) and (11) non-binding.

$$\big(p_g(t) - p_g(t-1)\big) - M{\cdot}r_g(t) \leq R_g \;+\; M{\cdot}s_g(t), \quad \forall\, t \in T,\ \forall\, g \in G \tag{10}$$

$$\big(p_g(t-1)-p_g(t)\big)-M\cdot r_g(t)\le R_g\ +\ M\cdot z_g(t),\quad \forall\ t\in T,\ \forall\ g\in G \tag{11}$$

$$s_g(t)\ +\ z_g(t)\le 1,\forall\ t\in T,\ \forall\ g\in G \tag{12}$$

Utilizing the binary variable, $r_g(t)$, a counter $N_g^R(t)$ is defined, as before, to incur an incrementing, ramp-related cycling cost, $C_g^R(t)$. Using the formulation from Section II.A, the ramp-related cycling cost function may be linear, piecewise linear or step-shaped. Constraints (2) and (3) are replaced with the analogous ramp terms shown in Table IV to implement a linearly incrementing cost. Constraints (4) and (5), or (6) to (8), are replaced with the analogous ramp terms as shown in Table IV to define a piecewise linear, or step shaped, incrementing ramp related cycling cost respectively.

TABLE IV
ANALOGOUS TERMS

	Start-ups	Ramps
Linear	s_g(t)	r_g(t)
	cost$_g^S$(t)	cost$_g^R$(t)
	N$_g^S$(t)	N$_g^R$(t)
	C$_g^S$(t)	C$_g^R$(t)
Piecewise Linear & Step	s_g(t)	r_g(t)
	cost$_g^S$(t,i)	cost$_g^R$(t,i)
	N$_g^S$(t,i)	N$_g^R$(t,i)
	Th$_g^S$(t,i)	Th$_g^R$(t,i)
	C$_g^S$(t)	C$_g^R$(t)
	step$_g^S$(t)	step$_g^R$(t)

2) Define multiple ramp levels: The previous formulation, where one level R_g is set to define a ramp, can be expanded to incur a dynamic ramp-related cycling cost, for j ramps of different magnitudes, $R_g(j)$. Constraint (13) ensures that for a ramp less than $R_g(1)$, the binary variable $r_g(t,j)$ will equal zero for all j. A ramp greater than $R_g(1)$, but less than $R_g(2)$, will set $r_g(t,1)$ equal to one, and so forth. The final term ensures that the constraint is non-binding when the unit is starting up. A corresponding constraint is needed for down ramps, where $\big(p_g(t)-p_g(t-1)\big)$ in constraint (13) is replaced with $\big(p_g(t-1)-p_g(t)\big)$ and $M.s_g(t)$ is replaced with $M.z_g(t)$. Constraint (14) ensures that the binary variable, $r_g(t,j)$, which indicates that a ramp $\ge R_g(j)$ has occurred, can only have a value of 1 for one ramp level j, at any given time. As before, constraint (12) is required to prevent $s_g(t)$ and $z_g(t)$ both being set to 1, to make constraint (13) and its corresponding down ramping constraint non-binding.

$$\begin{aligned}\big(p_g(t)-p_g(t-1)\big) &< R_g(1)\cdot\big(1-\sum_{j=1}^{j} r_g(t,j)\big)\\ &+R_g(2)\cdot r_g(t,1)\ +...\ +\ R_g(j)\cdot r_g(t,j-1)\\ &+\bar{P}_g\cdot r_g(t,j)\ +\ M\cdot s_g(t),\\ &where\ R_g(1)<R_g(2)<R_g(j)...<\bar{P}_g,\\ &\forall\ t\in T,\ \forall\ g\in G,\ \forall\ j\le\bar{j}_g\end{aligned} \tag{13}$$

$$\sum_{j=1}^{j} r_g(t,j)\le 1,\forall\ t\in T,\ \forall\ g\in G,\ \forall\ j\le\bar{j}_g \tag{14}$$

As with hot and cold starts, scaling factors (β and γ) are used to normalize $N_g^R(t)$ to count in terms of one ramp level, as shown in constraint (15), where $r(t,j)$ is expressed in terms of $r(t,1)$. Constraint (16) determines the total ramp-related cycling cost, shown here with a constant cost increment, $cost_g^R$, with the final term ensuring that the cost is only incurred in a time period when a ramp ($> R_g(1)$) occurs.

$$\begin{aligned}N_g^R(t)&=N_g^R(t-1)\ +\ r_g(t,1)\ +\ \beta\cdot r_g(t,2)\\ &+....+\ \gamma\cdot r_g(t,j),\ \forall\ t\in T,\ \forall\ g\in G,\ \forall\ j\le\bar{j}_g\end{aligned} \tag{15}$$

$$\begin{aligned}C_g^R(t)\ge N_g^R(t)\cdot cost_g^R\ -\ \big(1-\sum_{j=1}^{j} r_g(t,j)\big)\cdot M\\ \forall\ t\in T,\ \forall\ g\in G,\ \forall\ j\le\bar{j}_g\end{aligned} \tag{16}$$

To combine this formulation of j ramp levels with i cost thresholds (i.e piecewise linear) constraints (15) and (16) are replaced by constraints (17) and (18), such that once $N_g^R(t,i)$ reaches $Th_g^R(i)$, $C_g^R(t,i)$ will begin incrementing by $cost_g^R(i)$.

$$\begin{aligned}N_g^R(t,i)=\big(N_g^R(t-1,1)\ +\ r_g(t,1)\ +\ \beta\cdot r_g(t,2)\\ +....+\ \gamma\cdot r_g(t,j)\ +\ 1\big)-Th_g^R(i)\\ \forall\ t\in T,\ \forall\ g\in G,\ \forall\ j\le\bar{j}_g,\ \forall\ i\le I_g\end{aligned} \tag{17}$$

$$\begin{aligned}C_g^R(t)\ge \sum_{i}^{I_g}\Big(N_g^R(t,i)\cdot\big(cost_g^R(i)\ -\ cost_g^R(i-1)\big)\Big)\\ -\sum_{j=1}^{j} r_g(t,j)\cdot M,\ \forall\ t\in T,\ \forall\ g\in G,\ \forall\ j\le\bar{j}_g\end{aligned} \tag{18}$$

To include a step-shaped ramp related cycling cost function, constraints (6)-(8) are replaced with the analogous terms for ramping from Table 1.

III. Dispatch Model and Test System

To examine how cycling costs, modeled dynamically, will impact plant dispatch the new formulation was implemented in a conventional MIP unit commitment model based on [11], [12]. The unit commitment problem was formulated as

$$Minimize\ \sum_{t\in T}\sum_{g\in G} c_g^p(t)\ +\ c_g^s(t)\ +\ C_g^S(t)\ +\ C_g^R(t) \tag{19}$$

subject to

$$\sum_{g\in G} p_g(t)\ =\ D(t),\forall\ t\in T \tag{20}$$

$$p_g(t)\ \le\ \bar{P}_g\cdot v_g(t),\forall\ t\in T \tag{21}$$

$$p_g(t) \;\geq\; \underline{P}_g{\cdot}v_g(t), \forall\; t \in T \tag{22}$$

As per [11], a piecewise linear approximation of a quadratic production cost function for each unit was adopted, as represented by:

$$c_g^p(t) \;=\; A_g v_g(t) \;+\; \sum_{l=1}^{NL_g} F_{lg}\delta_l g(t), \forall\; t \in T, \forall\; g \in G \tag{23}$$

$$p_g(t) \;=\; \sum_{l=1}^{NL_g} \delta_l g(t) \;+\; \underline{P}_g v_g(t), \forall\; t \in T, \forall\; g \in G \tag{24}$$

$$\delta_1(g,t) \;\leq T_{1g} \;-\; \underline{P}_g, \forall\; t \in T, \forall\; g \in G \tag{25}$$

$$\delta_l(g,t) \;\leq T_{lg} \;-\; T_{l-1g}, \forall\; t \in T, \forall\; g \in G,\; \forall\; l = 2..NL_g - 1 \tag{26}$$

$$\delta_{NL}(g,t) \;\leq \bar{P}_g \;-\; T_{NL_g-1} \;-\; T_{l-1g}, \forall\; t \in T, \forall\; g \in G \tag{27}$$

$$\delta_l(g,t) \;\geq 0, \forall\; t \in T, \forall\; g \in G,\; \forall\; l = 1..NL_g \tag{28}$$

where A_g = a_g + $b_g\underline{P}_g$ + $c_g\underline{P}_g^2$.

Start-up costs which were dependent on the period of time the unit had been offline were modeled as follows:

$$c_g^s(t) \geq \big(v_g(t) \;-\; v_g(t-1)\big){\cdot}hc_g \;\; \forall\; t \in T,\; \forall\; g \in G \tag{29}$$

$$c_g^s(t) \geq \big(v_g(t) \;-\; \sum_{n=1}^{T_g^{cold}+DT_g} v_g(t-n)\big){\cdot}cc_g, \quad \forall\; t \in T,\; \forall\; g \in G \tag{30}$$

Minimum up time constraints were formulated by constraints (31), (32) and (33). Equation (31) is only included if the number of hours a unit must remain online to satisfy its minimum up time, B_g, is greater than or equal to 1.

$$\sum_{t}^{t \leq B_g} \big(1 - v_g(t)\big) \;=\; 0, \forall\; g \in G \tag{31}$$

$$\sum_{n=t}^{t+UT_g-1} v_g(n) \;\geq\; UT_g{\cdot}s_g(t), \forall\; g \in G \quad \forall\; t \;=\; B_g \;+\; 1...\bar{T} - UT_g \;+\; 1 \tag{32}$$

$$\sum_{n=t}^{\bar{T}} \big(v_g(n) \;-\; s_g(t)\big) \;\geq 0, \forall\; g \in G \quad \forall\; t \;=\; \bar{T} \;-\; UT \;+\; 2...\bar{T} \tag{33}$$

where B_g = max$\big(0,\; v_g(\bar{T}){\cdot}UT_g$ - h_g^{up} + $v_g(\bar{T})\big)$

Minimum down time constraints were formulated using constraints (34), (35) and (36). Equation (31) is only included if $L_g \geq 1$.

$$\sum_{t}^{t \leq L_g} \big(v_g(t)\big) \;=\; 0, \forall\; g \in G \tag{34}$$

$$\sum_{n=t}^{t+DT_g-1} v_g(n) \;\geq\; DT_g{\cdot}z_g(t), \forall\; g \in G \quad \forall\; t \;=\; L_g \;+\; 1...\bar{T} - DT_g \;+\; 1 \tag{35}$$

$$\sum_{n=t}^{\bar{T}} \big(1 \;-\; v_g(n) \;-\; z_g(t)\big) \;\geq 0, \forall\; g \in G \quad \forall\; t \;=\; \bar{T} \;-\; DT \;+\; 2...\bar{T} \tag{36}$$

where L_g = max$\big(0,\; (1 - v_g(\bar{T})){\cdot}DT_g - h_g^{down}$+$(1 - v_g(\bar{T}))\big)$

The formulation was applied to the 10 unit test system used in [11], [13], which was duplicated to give a 20 unit system, thus facilitating a larger case study. The peak demand (1500 MW) was doubled (3000 MW) and a historical, three-year long, hourly demand profile for the Irish system was scaled to produce a demand profile with a 3000 MW peak. The model was run both for a single year and a three-year period, with and without cycling costs, optimizing each day at an hourly resolution. The simulations without cycling costs (which do not model any cycling costs, either dynamically or as part of the start-up costs) provide a reference case against which to compare the simulations which include cycling costs. In order to run the model for longer than one day it is necessary for the values of $v_g(t)$, $p_g(t)$, h^{up}, h^{down}, $N_g^S(t)$ or $N_g^S(t,i)$ and $N_g^R(t)$ or $N_g^R(t,i)$ to be carried over from one day to the next.

Generator cycling costs are difficult to determine and largely uncertain, as discussed in Section I. In addition, cycling costs found in the literature are “static costs”, i.e. each generator start-up is assumed to cost the same as the next. By contrast the model described in this paper utilizes a cost increment such that each start-up (or ramp) is incrementally more expensive than the previous one. Thus, the figures quoted in the literature are not directly applicable for this model so some approximations had to be made. Conservative costs were chosen such that a gradual change in plant dispatch could be observed as cycling costs accumulated, as opposed to large incremental costs which would have caused drastic changes to the merit order early in the simulation (and thereby would not have been representative of reality). The relative magnitudes of the incremental costs used here (i.e. the size of incremental cycling cost for a base-load unit versus a mid-merit unit etc.), as shown in Table V, is based on those in [14].

Piecewise linear costs for start-ups and ramps were implemented with the incremental cost ($cost_g^S(i)$ or $cost_g^R(i)$) increasing by 10% and 20% when the start counter ($N_g^S(t,1)$), or ramp counter ($N_g^R(t,1)$), exceeded 100 ($Th_g^S(2)$ or $Th_g^R(2)$) and 200 ($Th_g^S(3)$ or $Th_g^R(3)$) respectively. The scaling factor, α, was chosen to be 2 based on [15], i.e. each cold start incremented $N_g^S(t,1)$ by 2 (while a hot start incremented

$N_g^S(t,1)$ by 1). Two ramp levels, $R_g(1)$ and $R_g(2)$ corresponding to 20% and 40% of the difference between maximum and minimum output for a unit, were modeled. Scaling factors were chosen such that ramps greater than $R_g(1)$ or $R_g(2)$ incremented $N_g^R(t,1)$ by 1 or 2 respectively.

TABLE V
INCREMENTAL CYCLING COSTS $, (I=1)

Units	$cost_g^S(i)$	$cost_g^R(i)$
1-4	300	15
5-10	60	3
11-20	30	1.5

IV. RESULTS

This section examines how plant dispatches for the test system are affected when (i) a cycling cost related to start-ups is implemented, (ii) a cycling cost related to ramping is implemented, and (iii) cycling costs related to start-ups and ramping are implemented simultaneously.

A. Start-up Related Cycling Costs Results

Implementing a dynamic cycling cost for plant start-ups, as shown in Table V, over a one year period was seen to result in an overall reduction in plant start-ups. This is seen in Table VI, which reveals reducing start-ups for base-load and mid-merit units. For base-load units, the reduction in starts was correlated with increased production as, having the largest incremental cycling costs, these units avoided shut-downs and their online hours increased. This is evident through the average capacity factor shown in Table VII. Mid-merit units, however, which also had reduced start-ups, saw reduced production indicating that they were utilized less often. As these units were started up and shut down, and subsequently incurred cycling costs, it became more economical after some point to dispatch peaking units. Thus, start-ups and production increased for peaking units when a dynamic cycling cost for start-ups was modeled, as seen in Tables VI and VII. Figure 4 illustrates the cumulative start-ups for the mid-merit and peaking units over a single year when (i) cycling costs were modeled and (ii) when cycling costs were not modeled. Starts are seen to accumulate rapidly between 0 and 2000 hours and for hours greater than 7000, as these are the winter months and thus have higher demand, requiring more plant start-ups. Beyond 1000 hours the cycling costs which are accumulated by mid-merit plant begin to have an effect on their position in the merit order and consequently peaking plant are seen to be dispatched more frequently. Figure 5 shows a similar trend when cycling costs were modeled over a three-year period.

Units within the same class, i.e. base-load, mid-merit or peaking, were also seen to converge to a similar number of annual start-ups, as indicated by the reduced standard deviation of annual start-ups seen in Table VIII, when modeled for one year or three years. This indicates that once a unit has been cycled and its cycling cost is incremented, the next time a unit needs to be cycled the costs will have now changed such that a different unit (most likely the next in the merit order) may be

TABLE VI
IMPACT OF DYNAMIC CYCLING COSTS FOR START-UPS ON TOTAL ANNUAL STARTS

Units	No cycling costs modeled	Cycling cost for start-ups modeled
Base-load (Units 1-4)	34	12
Mid-merit (Units 5-10)	1372	1005
Peaking (Units 11-20)	577	838
Total	1983	1855

TABLE VII
IMPACT OF DYNAMIC CYCLING COSTS FOR START-UPS ON AVERAGE PLANT CAPACITY FACTORS (%)

Units	No cycling costs modeled	Cycling cost for start-ups modeled
Base-load (Units 1-4)	92.59	92.73
Mid-merit (Units 5-10)	27.82	25.42
Peaking (Units 11-20)	0.85	2.23

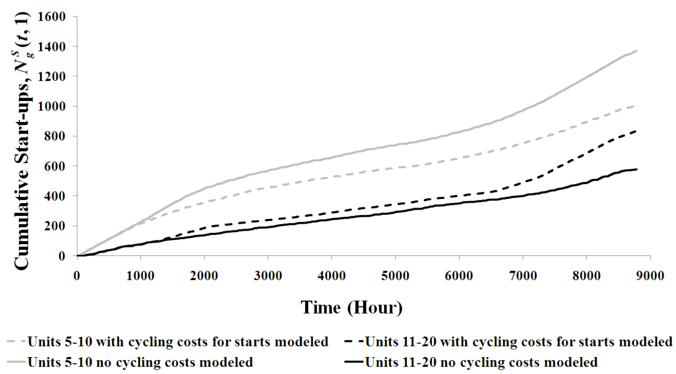

Fig. 4. Cumulative plant start-ups over one year, shown when dynamic cycling costs for starts were (i) modeled and (ii) not modeled

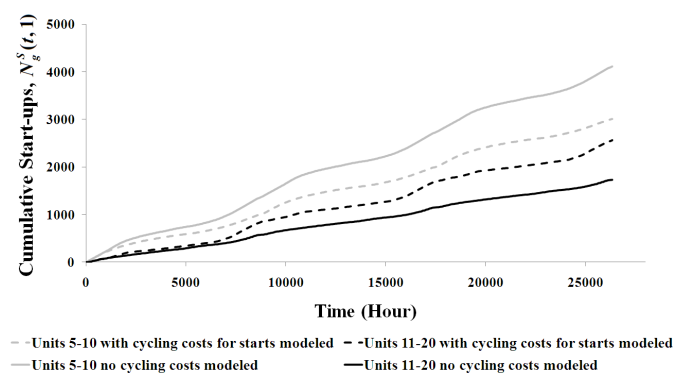

Fig. 5. Cumulative plant start-ups over three years, shown when dynamic cycling costs for starts were (i) modeled and (ii) not modeled

scheduled. This leads to the burden of cycling operation being more evenly distributed across the units. Over a long horizon this effect can lead to a shift in the merit order, a trend which can be seen in Figures 4 and 5 when dynamic cycling costs were modeled for one year or three years respectively.

Modeling dynamic cycling costs for plant start-ups was also found to result in increased generator ramping. Over one year a 22% increase in ramping ($N_g^R(t,1)$) was observed, as seen

TABLE VIII
IMPACT OF DYNAMIC CYCLING COSTS FOR START-UPS ON ANNUAL START-UPS PER UNIT GROUP

	No cycling cost modeled		Cycling cost for starts modeled	
1 Year	Avg.	Std. Dev.	Avg.	Std. Dev.
Base-load (Units 1-4)	8.5	9.9	3	3.6
Mid-merit (Units 5-10)	228.7	75.7	167.5	26.1
Peaking (Units 11-20)	57.7	73.1	83.8	27.5
3 Years	Avg.	Std. Dev.	Avg.	Std. Dev.
Base-load (Units 1-4)	25.5	29.8	9.25	10.7
Mid-merit (Units 5-10)	686	227.2	502.3	66.2
Peaking (Units 11-20)	173.1	219.2	256.1	83.1

in Table IX, relative to the case when no cycling costs were modeled. This was due to generators being more frequently ramped down to minimum output, rather than shut-down, in an effort to avoid incurring cycling costs for starting up.

TABLE IX
TOTAL GENERATOR RAMPING, $N_g^R(t,1)$, WHEN A DYNAMIC CYCLING COSTS FOR START-UPS IS MODELED FOR ONE YEAR

	Ramps
No cycling costs modeled	6726
Cycling costs for starts modeled	8214

To facilitate a sensitivity analysis, multiples of the initial incremental cycling costs, $cost_g^S(1)$, shown in Table V, were also examined for one year. As the incremental cost was increased the reduction in start-stop cycling that is achieved by modeling dynamic cycling costs quickly saturated as seen in Figure 6, thus indicating that the majority of plant cycling is unavoidable. Table X shows a breakdown of the total number of plant start-ups by unit group, which again reveals that increasing starts for peaking units are correlated with increasing incremental cycling cost, as it becomes more favorable to dispatch these units due to the relatively larger cycling costs associated with the mid-merit units. (The ripples in the curve shown in Figure 6 result from the increasing starts for peaking units, as seen in Table X.)

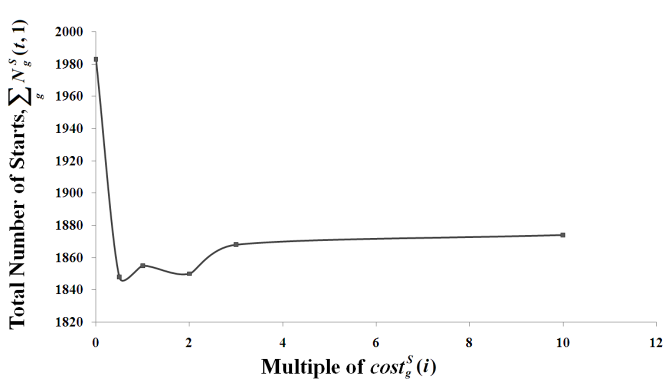

Fig. 6. Impact of dynamic cycling cost on total start-ups, shown for various multiples of $cost_g^S(i)$

A scenario where cycling costs were only modeled for a subset of the total fleet for one year was also examined. The 6 largest units on the system (units 1, 2, 3, 4, 9, 10) were chosen based on the assumption that these units would be most impacted by cycling operation and thus most likely to bid a wear-and-tear cost into the market if such an option was available. The results showed that although the number of annual start-ups was reduced for these units, the start-ups for the other units increased by a much greater amount as seen in Table XI. This would indicate the need for a uniform policy relating to the bidding of cycling costs to be implemented in markets, such that all units reflect their cycling costs, or do not, to avoid the situation where only some generators are bidding cycling costs as this leads to inefficient operation and excessive costs.

TABLE X
IMPACT OF DYNAMIC CYCLING COSTS FOR STARTS ON TOTAL PLANT START-UPS, SHOWN FOR VARIOUS MULTIPLES OF $cost_g^S(i)$

	Base-load (Units 1-4)	Mid-merit (Units 5-10)	Peaking (Units 11-20)
No cycling cost	34	1372	577
$cost_g^S(i)$*0.5	13	1104	781
$cost_g^S(i)$*1	12	1005	838
$cost_g^S(i)$*2	13	941	896
$cost_g^S(i)$*3	13	907	948
$cost_g^S(i)$*10	13	869	992

TABLE XI
CHANGE IN STARTS WHEN A SUBSET OF UNITS BID CYCLING COSTS FOR START-UPS

	Δ Starts
Units 1, 2, 3, 4, 9, 10	-86
All other units	+256

B. Ramping Related Cycling Costs Results

Implementing a dynamic cycling cost for plant ramping (shown in Table V) over one year resulted in a 90% reduction in ramping overall, as seen in Table XII. As described previously, assuming a ramp greater than 20% or 40% of the difference between a unit's maximum and minimum output increments the ramp counter, $N_g^R(t)$, by a value of 1 or 2 respectively. The total value of $N_g^R(t)$ at the end of the test year, summed for all units, is shown in Table XII. Base-load units which carried out the greatest amount of ramping when cycling costs were not modeled, saw the greatest reduction in ramping operation when cycling costs for ramps were implemented. The dramatic reduction in ramping that was achieved by implementing dynamic ramping costs, however, led to increased start-stop cycling as might be expected, although only by 3.3% over the year. The most notable change to the overall dispatch that resulted from the introduction of dynamic ramping costs was a slight reduction in production from base-load plant allowing for increased production from mid-merit and peaking units as seen in Table XIII, thereby spreading the ramping requirement over more units. Thus, including the ramping cost was also seen to result in a slightly greater number of units online (5.94 per hour on average when dynamic ramping costs were modeled, versus 5.92 when no cycling costs were modeled).

TABLE XII
IMPACT OF DYNAMIC CYCLING COSTS FOR RAMPING ON TOTAL ANNUAL RAMPING ($N_g^R(t,1)$)

Units	No cycling costs modeled	Cycling cost for ramps modeled
Base-load (Units 1-4)	3717	120
Mid-merit (Units 5-10)	2214	1224
Peaking (Units 11-20)	795	623
Total ramping	6726	1967

TABLE XIII
IMPACT OF DYNAMIC CYCLING COSTS FOR RAMPING ON AVERAGE PLANT CAPACITY FACTORS (%)

Units	No cycling costs modeled	Cycling cost for ramps modeled
Base-load (Units 1-4)	92.59	92.21
Mid merit (Units 5-10)	27.82	28.61
Peaking (Units 11-20)	0.85	1.02

C. Start-up and Ramping Cycling Costs Results

Implementing dynamic cycling costs (as shown in Table V) for starts and ramping simultaneously over a one year period, reduced both types of cycling operation relative to the case when no cycling costs were modeled, as shown in Table XIV. Base-load units, having the largest cycling costs, see the greatest reductions in cycling operation. Nonetheless, neither total starts nor total ramps were reduced in this scenario as much as starts alone or ramps alone were reduced when cycling costs for starts or ramps were modeled individually. However, when cycling costs for start-ups only were modeled, ramping operation increased and likewise when cycling costs for ramping only were modeled, starts increased. Thus when the cycling costs that would have been incurred due to both start-ups and ramping are examined (assuming the costs given in Table V), the case in which cycling costs for start-ups and ramping were modeled simultaneously had the lowest overall cycling costs, as shown in Figure 7. This would indicate that modeling cycling costs for starts and ramping simultaneously is the most cost effective way to reduce cycling and as such one should not be considered without the other.

TABLE XIV
IMPACT ON TOTAL ANNUAL STARTS AND RAMPS WHEN DYNAMIC CYCLING COSTS FOR BOTH START-UPS AND RAMPING WERE MODELED

Units	No cycling costs modeled		Cycling cost for starts and ramps modeled	
	Starts	Ramps	Starts	Ramps
Base-load (Units 1-4)	34	3717	12	144
Mid merit (Units 5-10)	1372	2214	1003	2069
Peaking (Units 11-20)	577	795	855	1456
Total	1983	6726	1870	3669

Finally, when total system costs are examined for the scenario including cycling costs and compared to the total

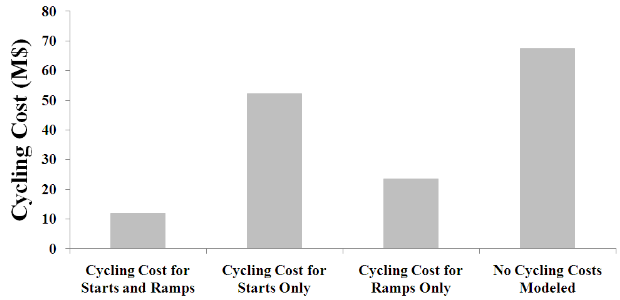

Fig. 7. Cycling costs (that would have been incurred) shown for various scenarios

system cost for the scenario in which cycling costs were not modeled, but were calculated and added afterwards, it can be seen that modeling cycling costs leads to lower system costs overall. This is shown for one year in Figure 8 and for three years in Figure 9. In these examples, the cost savings seen are considerable, i.e. 54 M$ (14%) for one year and 493 M$ (30%) over three years. Thus, it can also be concluded that the savings yielded by modeling cycling costs will increase over time.

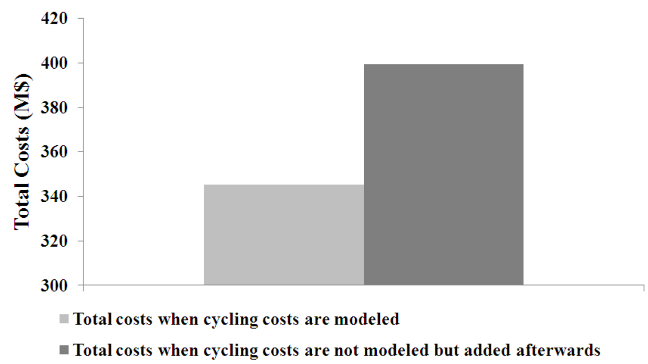

Fig. 8. Total system costs when dynamic cycling costs are modeled and not modeled over one year

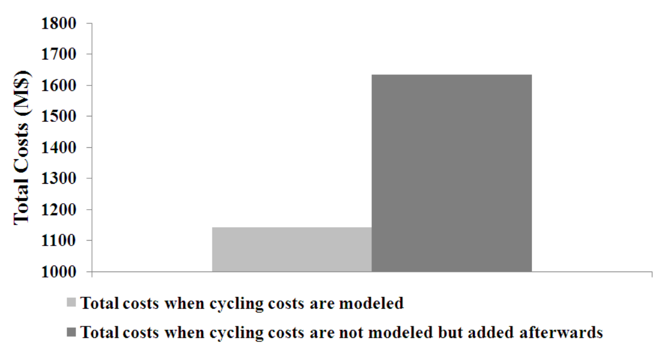

Fig. 9. Total system costs when dynamic cycling costs are modeled and not modeled over three years

V. CONCLUSIONS

Interest concerning cycling costs is growing and this paper sets out a formulation that can utilize knowledge of incremental wear-and-tear costs related to plant start-ups or ramping, to implement a dynamic incrementing cycling cost. The formulation covers linear, piecewise linear and step-shaped cycling cost functions, the appropriate choice for a user being determined by the level of knowledge of the generator's cycling costs.

The formulation for piecewise linear incremental cycling costs related to plant start-ups and ramps was implemented for a test system. Although the incremental costs chosen are approximations, the results reveal certain trends that are likely for power systems where generators undergo regular cycling and reflect the resulting wear-and-tear costs in their bids. For example, dynamically modeling cycling costs for generator starts was seen to reduce the number of starts, but caused ramping operation to be increased (and vice-versa), whilst modeling cycling costs for only a subset of the generation fleet was seen to induce much higher levels of cycling in the remaining generation. It was also seen that as cycling costs accumulated over time changes in the merit order occurred, and that modeling cycling costs led to an overall saving for the system as cycling operation was subsequently reduced.

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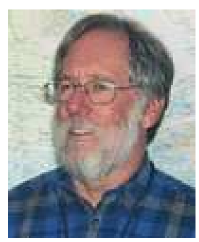

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