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Evolving Efficient Limit Order Strategy using Grammatical Evolution

Wei Cui, Anthony Brabazon, Michael O'Neill

***Abstract*— Trade execution is concerned with the actual mechanics of buying or selling the desired amount of a financial instrument of interest. A practical problem in trade execution is how to trade a large order as efficiently as possible. A trade execution strategy is designed for this task to minimize total trade cost. Grammatical Evolution (GE) is an evolutionary automatic programming methodology which can be used to evolve rule sets. It has been proved successfully to be able to evolve quality trade execution strategies in our previous work. In this paper, the previous work is extended by adopting two different limit order lifetimes and three benchmark limit order strategies. GE is used to evolve efficient limit order strategies which can determine the aggressiveness levels of limit orders. We found that GE evolved limit order strategies were highly competitive against three benchmark strategies and the limit order strategies with long-term lifetime performed better than those with short-term lifetime.**

I. Introduction

Trade execution is the process of trading a particular instrument of interest. A practical issue in trade execution is how to efficiently trade a large order, the size of which to be traded is sizeable according to prevailing market conditions. For example, an order with volume accounting for 10% of average daily volume is potentially able to change the price of that asset causing significant market impact cost. To reduce such cost, a large order is always divided into a number of smaller child orders and traded over time. However, this incurs risk of suffering opportunity cost. An efficient trade execution strategy seeks to balance out these costs in order to minimize the total trade cost.

The task in devising an efficient execution strategy is complex as it entails multiple sub-decisions including how best to split up the large order, what *style* to adopt in executing each element of the order (aggressive or passive), what type of order to use, when to submit the order, and how execution performance is to be measured. In addition, the electronic order book(s) faced by the investor are constantly changing.

Grammatical Evolution is an Evolutionary Automatic Programming (EAP) technique which allows the generation of computer programs in an arbitrary language. GE can conduct an efficient exploration of a search space, and notably permits the incorporation of existing domain knowledge in order to generate 'solutions' with a desired structure. In finance (for example), this allows the users to seed the evolutionary process with their current trading strategies in order to see what improvements the evolutionary process can uncover. Recently GE has been successfully applied to a number of financial problems. These include financial time series modelling, intraday financial asset trading, corporate credit rating, and the uncovering of technical trading rules [2], [19].

In our previous work [6], GE has been used to evolve quality trade execution strategies which determine appropriate time to change limit orders to market orders. GE evolved strategies have been proved to outperform two benchmark strategies: simple market order strategy and simple limit order strategy. This study extends our previous work. In this paper, GE is used to evolve efficient limit order strategies which determine the aggressiveness levels of limit orders with short-term lifetime and long-term lifetime respectively. And three benchmark limit order strategies are adopted, which are simple aggressive limit order strategy, simple modest limit order strategy and simple passive limit order strategy. A simulated artificial limit order market is used to test trade execution strategies. An advantage of doing this is that the strategies can interact with the changing market.

This paper is organized as follows. The next section provides a brief synopsis of the typical operation of an electronic double auction marketplace; Section III discusses trade execution strategies; Section IV explains the grammar used in this study and describes our performance evaluation approach; Section V explains agent-based modeling and describes how we implement the artificial limit order market used in this study; Section VI provides our results, with conclusions and some future work being presented in the final section of this paper.

II. Background

Today most market places operate an electronic double auction *limit order book*. Traders can either submit *a limit order* or *a market order*. A market order is an order to buy or to sell a specified number of shares. It guarantees immediate execution but provides no control on its execution price. In contrast, a limit order is an order to buy or to sell a specified number of shares at a specified price. It provides control over its execution price but does not guarantee its execution.
Table I shows a sample order book, where all the buy and sell orders are visible to traders in the market. It consists of two queues which store buy and sell limit orders, respectively. Buy limit orders are called *bids*, and sell limit orders are called *offers* or *asks*. The highest bid price on the order book is called *best bid*, and the lowest ask price on the order book

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TABLE I
ORDER BOOK 1

Bid		Ask	
Shares	Prices	Prices	Shares
300	50.19	50.22	200
200	50.18	50.23	300
400	50.17	50.24	100
500	50.16	50.25	300
300	50.15	50.26	200
100	50.14	50.27	400

TABLE II
ORDER BOOK 2

Bid		Ask	
Shares	Prices	Prices	Shares
300	50.19	50.22	**200**
500	50.18	50.23	300
400	50.17	50.24	100
500	50.16	50.25	300
300	50.15	50.26	200
100	50.14	50.27	400

TABLE III
ORDER BOOK 3

Bid		Ask	
Shares	Prices	Prices	Shares
300	50.19	50.22	**100**
500	50.18	50.23	300
400	50.17	50.24	100
500	50.16	50.25	300
300	50.15	50.26	200
100	50.14	50.27	400

is called *best ask*. The difference between best bid and best ask is called *bid-ask spread*. Prices on the order book are not continuous, but rather change in discrete quanta called *ticks*.

Limit orders on the order book are typically (depending on market rules) executed strictly according to (1) price priority and (2) time priority. Bid (ask) orders with higher (lower) prices get executed first with time of placement being used to break ties. A buy (sell) market order is executed at the best ask (bid) price. The limit order book is highly dynamic, because new limit orders will be added into the order book, and current limit orders will get executed or cancelled from the order book throughout the trading day. Table II shows the order book after a trader submits a buy limit order with 300 shares placed at price 50.18. Table III shows the order book after a trader submits a buy market order with 100 shares.

III. TRADE EXECUTION STRATEGY

A trade execution strategy is a set of rules determining a number of trade execution components designed to minimize transaction cost. These components include number of orders to be submitted, size of each order, what type each order should be and when each order should be submitted to the market.

The total trading volume of the order to be traded is often expressed as a percentage of the *average daily volume* (ADV) of the stock [12]. An order of less than 5% of ADV can generally be traded over a day without using complex strategies. On the contrary, if the target volume is larger than 15% of ADV, it may require execution over several days in order to minimize market impact. Normally, 5-15% of ADV is a reasonable order size which could expect to be tradable over a day using appropriate trade execution tactics. In this paper, the trading horizon of all strategies is one trading day and hence the order size is assumed to be of this magnitude. Formally, we assume that the order to be traded in one day consists of V shares. The order is sliced into N smaller child orders, with order size s_1, s_2, …, s_N,

$$V = \sum_{i=1}^{N} s_i$$

each of which is submitted to the market at regular intervals over the trading day. In practice, the size of each child order is determined from the intraday volume curve of the day, and the time intervals mostly adopted are fifteen-minute or half-hour intervals. For simplicity, we assume that the N child orders have the same order size in all our strategies and these orders are submitted to the market at a half-hour interval.

The simplest trade execution strategy is a pure market order strategy in which each child order is submitted as a market order at regular intervals over the trading day. The strategy in Fig. 1 is an example. This strategy takes market liquidity immediately by crossing the bid-ask spread. It guarantees order execution at the cost of market impact.

Limit order strategies are more often used in which each child order is submitted as a limit order at regular intervals over the trading day. For each child limit order, traders need to specify three parameters which are a lifetime, an amendment frequency and a limit price. For example, a child limit order to buy (sell) might have an lifetime of T minutes, an amendment frequency of Δt minutes and a limit price which is the same as the best available price in the market. This means that this buy (sell) limit order is placed at the best bid (ask) price at submission time, and if Δt minutes after submission, it is not fully executed, it will be amended to the best bid (ask) price. This amendment process continues in Δt intervals up to T minutes after submission, at which time the uncompleted order(s) are traded as market orders by crossing the bid-ask spread.

In the example just described, the limit price of the child order is set to the best available price (modest level). Traders also submit a more aggressive limit order (aggressive level) to buy (sell) of which the limit price is one tick size above (below) the best bid (ask) price, or a more passive limit order (passive level) to buy (sell) of which the limit price is one tick size below (above) the best bid (ask) price.

A more sophisticated limit order strategy would set limit price of a limit order according to dynamic market conditions. For example, some traders might submit aggressive orders in times of favorable price movement, and place passive orders in times of adverse price movement [13].

In our GE evolved strategies, the aggressiveness level of each limit order is determined by an execution rule evolved using GE. Three aggressiveness levels are adopted. At each submission time or each amendment time (an integral multiple of Δt minutes after submission), an appropriate aggressiveness level of limit price is chosen according to the execution rule based on the market conditions. The market variables representing the market condition are examined in the next section.

A. Information Indicators

There are a large number of studies in the literature analyzing the relationship between order placement and the information content of limit order books.

TABLE IV
DEFINITIONS OF MARKET VARIABLES

Variables	Definitions
BidDepth	Number of shares at the best bid
AskDepth	Number of shares at the best ask
RelativeDepth	Total number of shares at the best five ask prices divided by total number of shares at the best five bid and ask prices
Spread	Difference between the best bid price and best ask price
Volatility	Standard deviation of the most recent 20 mid-quotes
PriceChange	Number of positive price changes within the past ten minutes divided by the total number of quotes submitted within the past ten minutes

Traders are willing to place limit orders more aggressively if the depth away from the best price on the same side of the market is high, because higher depth away from the best bid (ask) price reduces the execution probability of incoming limit buy (sell) order [3], [8], [21], [26]. If the market depth on the opposite side is larger, traders prefer to submit limit orders conservatively. As the bid-ask spread narrows, the benefits of the better price available to limit order traders decrease, causing them to place more aggressive limit orders [8], [20], [21], [25], [26]. Oppositely, when the bid-ask spread widens, passive limit orders are preferable [3], [8], [21]. When the market is volatile, limit buy (sell) order traders have to post lower (higher) bid (ask) prices in order to protect themselves from trading disadvantage, because higher volatility increases the probability of trading against informed investors [26].

Hence, prior literature suggests a range of possible explanatory variables, but indicates that we have an incomplete theoretical understanding of how these factors interact. This suggests that there will be particular utility for the application of evolutionary methods to uncover a suitable model structure (trade execution strategy). Based on the explanatory factors considered in the literature, we selected six information indicators to construct a dynamic trade execution strategy (Table IV).

IV. Experimental Approach

In this study we consider a large order of 10% of ADV of the artificial market, which is to be traded over one day (5 hours in the artificial market). This order is equally divided into ten child orders which are submitted to the market at intervals of thirty minutes over the trading day. Each child order is submitted as a limit order with an amendment frequency of ten minutes. We adopt two different lifetimes, one is short-term lifetime and the other is long-term lifetime. The short-term lifetime is half an hour, and the long-term lifetime is up to the end of the trading day, which are used separately. In all trade execution strategies, any uncompleted orders are crossed over the spread at the end of trading day in order to ensure order completion. GE is used to evolve efficient trade execution strategies which determine the aggressiveness level of each limit order at submission time and at amendment time.

A. Grammar of Grammatical Evolution Algorithm

The grammar adopted in our experiments is defined as follows:

```
<lc> ::= if (<stamt>)
              class = "AggressiveLimitPrice"
        else {
              if (<stamt>)
                   class = "PassiveLimitPrice"
              else
                   class = "ModestLimitPrice"
                }
<stamt> ::= (<stamt><op><stamt>)|<cond1>|
          <cond2>|<cond3>|<cond4>|<cond5>|
          <cond6>|<cond7>|<cond8>
<op> ::= and
<cond1> ::= (BidDepth<comp>AvgBidDepth)
<cond2> ::= (AskDepth<comp>AvgAskDepth)
<cond3> ::= (RelativeDepth
                <comp>AvgRelativeDepth)
<cond4> ::= (Spread<comp>AvgSpread)
<cond5> ::= (Volatility
                <comp>AvgVolatility)
<cond6> ::= (PriceChange
                <comp>AvgPriceChange)
<cond7> ::= (PercOfTradedVolume
                <comp><threshold>)
<cond8> ::= (PercOfPastTime
                <comp><threshold>)
<comp> ::= <less>|<more>|<lessE>|<moreE>
<less> ::= "<"
<more> ::= ">"
<lessE> ::= "<="
<moreE> ::= ">="
<threshold> ::= 0.1|0.2|0.3|0.4|0.5|
                0.6|0.7|0.8|0.9
```

In the grammar, $AvgBidDepth$ represents the average bid depth of the market, $AvgAskDepth$ represents the average ask depth of the market, $AvgRelativeDepth$ represents the average relative depth of the market, $AvgSpread$ represents the average spread of the market, $AvgVolatility$ represents the average volatility of the market and $AvgPriceChange$ represents the average price change of the market. The six financial variables are observed at the time of order submission or order amendment. The other two variables $PercOfTradedVolume$ and $PercOfPastTime$ represents the percentage of the traded volume accounting for the total volume V shares and the percentage of the past time accounting for the whole trading period at the observed time respectively.

If the output is $class = "AggressiveLimitPrice"$, the limit orders to buy (sell) will be placed at one tick size above (below) the best bid (ask) price; if the output is $class = "PassiveLimitPrice"$, the limit orders to buy (sell) will be placed at one tick size below (above) the best bid (ask) price; if the output is $class = "ModestLimitPrice"$, the limit orders to buy (sell) will be placed at the best bid (ask) price.

B. Performance Evaluation

The standard industry metric for measuring trade execution performance is the *VWAP measure*, short for *Volume Weighted Average Price*. It is calculated as the ratio of the value traded and the volume traded within a specified time horizon

$$VWAP = \frac{\sum(Volume * Price)}{\sum(Volume)}$$

where $Volume$ represents each traded volume and $Price$ represents its corresponding traded price. An example is shown in Fig. 1.

In order to evaluate the performance of a trade execution strategy, its VWAP is compared against the VWAP of the overall market. The rationale here is that performance of a trade execution strategy is considered good if the VWAP of the strategy is more favorable than the VWAP of the market within the trading period and bad if the VWAP of the strategy is less favorable than the VWAP of the market within the trading period. For example, if the VWAP of a buy strategy ($VWAP_{strategy}$) is lower than the market VWAP ($VWAP_{market}$), it is considered as a good trade execution strategy. Conversely, if the $VWAP_{strategy}$ is higher than the $VWAP_{market}$, it is considered as a bad trade execution strategy. Although this is a simple metric, it largely filters out the effects of volatility, which composes market impact and price momentum during the trading period [1]. The performance evaluation functions for each trading day are as follows:

$$VWAP\ Ratio = \begin{cases} \frac{10^4 * (VWAP_{strategy} - VWAP_{market})}{VWAP_{market}} & (BuyStrategy) \\ \frac{10^4 * (VWAP_{market} - VWAP_{strategy})}{VWAP_{market}} & (SellStrategy) \end{cases}$$

where $VWAP_{market}$ is the average execution price which takes into account all the trades over the day excluding the strategy's trades. This corrects for bias, especially if the order is a large fraction of the daily volume [17]. For both buy and sell strategies, the smaller the VWAP Ratio, the better the strategy is.

V. Simulating An Artificial Market

In our experiments, the training and evaluation of all trade execution strategies are implemented in an artificial limit order market, which is simulated using an agent-based model.

Agent-based modelling is a computerized simulation consisting of a number of agents. The emergent properties of an agent-based model are the results of "bottom-up" processes, where the decisions of individual and interacting agent at a microscopic level determines the macroscopic behavior of the system. For a more detailed description of agent-based modelling in finance, please refer to [5], [14], [15], [16], [22], [23]. In this paper, our agent-based artificial limit order market is built based on the *Zero-Intelligence* (ZI) model [7] with a continuous double auction price formation mechanism. The notion of ZI agents was first mentioned in Gode and Sunder [11]. These agents randomly generate buy and sell orders. The orders are then submitted to a market agent, who manages all incoming orders according to the order matching mechanism in a real limit order market. The trading process is continuous, where unmatched orders are stored in an order book.

At each time step, an agent is equally likely to generate a buy order or a sell order. This order can be a market order, or a limit order, or a cancellation of a previous order, with probabilities λ_m, λ_l, and λ_c respectively. The sum of these probabilities is one ($\lambda_m + \lambda_l + \lambda_c = 1$). For a limit buy (sell) order, it has a probability of $\lambda_{inSpread}$ falling inside the bid-ask spread, a probability of λ_{atBest} falling at the best bid (ask) price, and a probability of λ_{inBook} falling off the best bid (ask) price in the book, ($\lambda_{inSpread} + \lambda_{atBest} + \lambda_{inBook} = 1$). The limit price inside the spread follows a uniform distribution. The limit price off the best bid (ask) price follows a power law distribution with the exponent of $(1+\mu_1)$. The log order size of a market order follows a power law distribution with the exponent of $(1 + \mu_2)$, while the log order size of a limit order follows a power law distribution with the exponent of $(1 + \mu_3)$.

TABLE V

Initial Parameters for Artificial Limit Order Market

Explanation	Value
Initial Price	$price^0 = 50$
Tick Price	$\delta = 0.01$
Probability of Order Cancellation	$\lambda_c = 0.34$
Probability of Market Order	$\lambda_m = 0.16$
Probability of Limit Order	$\lambda_l = 0.50$
Probability of Limit Order in Spread	$\lambda_{inSpread} = 0.32$
Probability of Limit Order at Best Quote	$\lambda_{atBest} = 0.33$
Probability of Limit Order off the Best Quote	$\lambda_{inBook} = 0.35$
Limit Price Power Law Exponent	$1 + \mu_1 = 2.5$
Market Order Size Power Law Exponent	$1 + \mu_2 = 2.7$
Limit Order Size Power Law Exponent	$1 + \mu_3 = 2.1$

As each incoming buy (sell) market order arrives, the market agent will match it with the best ask (bid) limit order stored in the order book. If this market order is fully filled by the first limit order, the unfilled part will be matched to the next best ask (bid) limit order until it is fully filled. As each incoming limit order arrives, the market agent will store it in the order book according to price and time priority. As each incoming cancelation order arrives, the market agent will delete the relevant limit order in the order book.

In order to ensure that the order flows generated by the artificial market are economically plausible, all the parameters in our model are derived from empirical evidence [4], [9], [10], [18], [24]. The parameters used in our simulation are presented in Table V.

	Submission Time	Shares		Traded Price		Value
Child Order 1:	t_0	400	$*$	50.15	$=$	20,060
		600	$*$	50.16	$=$	30,096
Child Order 2:	$t_1(t_0 + \Delta t)$	1,000	$*$	50.40	$=$	50,400
Child Order 3:	$t_2(t_0 + 2\Delta t)$	200	$*$	50.34	$=$	10,068
		800	$*$	50.36	$=$	40,288
Child Order 4:	$t_3(t_0 + 3\Delta t)$	1,000	$*$	50.39	$=$	50,390
Child Order 5:	$t_4(t_0 + 4\Delta t)$	1,000	$*$	50.68	$=$	50,680
Child Order 6:	$t_5(t_0 + 5\Delta t)$	1,000	$*$	51.10	$=$	51,100
Child Order 7:	$t_6(t_0 + 6\Delta t)$	1,000	$*$	50.87	$=$	50,870
Child Order 8:	$t_7(t_0 + 7\Delta t)$	700	$*$	50.98	$=$	35,686
		300	$*$	51.00	$=$	15,300
Child Order 9:	$t_8(t_0 + 8\Delta t)$	1,000	$*$	50.39	$=$	50,390
Child Order 10:	$t_9(t_0 + 9\Delta t)$	1,000	$*$	50.26	$=$	50,260
Total:		10,000				505,588
VWAP $= 505,588/10,000 = 50.5588$						

Fig. 1. VWAP Calculation of A Sample Buy Strategy

VI. Experimental Results and Discussions

Our experiments comprise of two periods (training and test periods). In the training period, GE is used to evolve trade execution strategies. Each individual is exposed to 20 continuous trading days in the artificial market and their fitness is calculated as their average VWAP ratio over the 20 trading days. The GE experiment is run for 40 generations, with variable-length, one-point crossover at a probability of 0.9, one point bit mutation at a probability of 0.01, roulette selection, steady-state replacement and a population size of 100. In the test period, the best evolved strategy in the training period is tested out of sample over 240 days in the artificial market.

We also compare our GE strategies to three benchmark limit order strategies, which are simple aggressive limit order strategy (SA), simple modest limit order strategy (SM) and simple passive limit order strategy (SP), where the aggressiveness levels of limit orders are aggressive level, modest level and passive level. These strategies adopt the same amendment frequency and the same lifetimes as GE strategies.

The results (all out of sample) of buy strategies and sell strategies are provided in Table VII. The "S-T" represents short-term lifetime and the "L-T" represents long-term lifetime. The "Mean" is the average VWAP ratio of each strategy over the 240 days, and "S.D." represents the standard deviation of the average (daily) VWAP ratio. P-values for the null hypothesis $H_1 : mean_{SA} \leq mean_{GE}$, $H_2 : mean_{SM} \leq mean_{GE}$, $H_3 : mean_{SP} \leq mean_{GE}$ are also shown in the table, to indicate the degree of statistical significance of the performance improvement of GE strategies over the two simple strategies. The figures show that the null hypotheses are rejected at the ≤ 0.01 level.

Based on the results, GE evolved strategies notably outperform the three benchmark strategies, simple aggressive limit order strategy (SA), simple modest limit order strategy (SM) and simple passive limit order strategy (SP). The negative VWAP ratios show that the GE evolved strategies achieve better execution prices than the average execution price of the market. Comparing the performance of the strategies for buy and sell orders, we observe that the performances of sell strategies are better than those of buy strategies in most cases. And L-T strategies all perform better than S-T strategies, which indicate that strategies with longer lifetime can achieve better execution prices than those with short lifetime.

VII. Conclusions and Future Work

Trade execution is concerned with the actual mechanics of trading an order. Traders wishing to trade large orders face tradeoffs in balancing market impact and opportunity costs. Trade execution strategies are designed to balance out these costs, thereby minimizing transaction cost relative to some benchmark like VWAP.

In this paper, we applied GE for the aim of evolving efficient limit order strategies which determine the aggressiveness levels of limit orders, and simulated an artificial limit order market for testing the evolved trade execution strategies. Three benchmark trade execution strategies were adopted, which were simple aggressive limit order strategy, simple modest limit order strategy and simple passive limit order strategy. While this paper extends previous our work, it again proves the ability of GE for the purposes of evolving efficient trade execution strategies. And we found that limit order strategies with long-term lifetime performed better than those with short-term lifetime.

There is notable scope for further research utilizing GE in this problem domain. One obvious route is to widen the number of market variables which can be included in the evolved execution strategies. Another route is to evolve the full structure of the trade execution strategy. In our approach, we focused on one aspect of trade execution strategy (aggressiveness level of limit order), and other components like the number of child orders, submission time, lifetime and

TABLE VI

RESULTS OF BEST EVOLVED GE STRATEGIES AND THREE BENCHMARK STRATEGIES (BUY ORDERS)

	SA		**SM**		**SP**		**GE**				
	Mean	(S.D.)	Mean	(S.D.)	Mean	(S.D.)	Mean	(S.D.)	H_1	H_2	H_3
S-T	14.7	(1.74%)	66.25	(2.53%)	13.19	(1.76%)	-2.35	(1.52%)	0.01	0.00	0.01
L-T	5.14	(1.69%)	60.7	(2.03%)	9.37	(1.48%)	-5.86	(1.3%)	0.01	0.00	0.00

TABLE VII

RESULTS OF BEST EVOLVED GE STRATEGIES AND THREE BENCHMARK STRATEGIES (SELL ORDERS)

	SA		**SM**		**SP**		**GE**				
	Mean	(S.D.)	Mean	(S.D.)	Mean	(S.D.)	Mean	(S.D.)	H_1	H_2	H_3
S-T	24.7	(1.86%)	43.6	(2.18%)	6.78	(2.03%)	-8.15	(1.87%)	0.00	0.00	0.01
L-T	5.28	(1.56%)	37.23	(2.27%)	4.91	(1.39%)	-12.07	(1.53%)	0.00	0.00	0.01

amendment frequency of each limit order are determined in advance. Future work will embrace the evolution of the full structure of trade execution strategy.

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