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Mathematical model as a standard procedure to analyze small and large water distribution networks

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HIGHLIGHTS

A model was developed to simulate domestic water flow> The model was applied to analyze selected domestic areas in Sligo, Ireland> Reliable parameters are generated allowing the analysis of water consumption> The results confirmed the capabilities of this approach identifying water trends> The study provides rigorous criteria to compare and predict water usage.

NOMENCLATURE

Main parameter meanings:

t	Time (*time*).
u	Interval for the data provision (*time*).
$W(t_u)$	Water consumption at any given time interval and is the (*volume/time*).
A	Amplitude of the oscillation (*volume*).
ω	Angular frequency (*angular.time*$^{-1}$).
φ	Phase (*volume*).
$\bar{w}$	Offset value or average water consumption (*volume.time*$^{-1}$).
T	Wavelength or period (*time*)
n	Number of harmonic waves.
FSS	Fourier Sinusoidal Series.
A_F	Amplitude of the sum of different sine curves.
X	Unknown water (*volume*).
A_h	Water used due to human activity (*volume.time*$^{-1}$).
L_T	Total water lost (*volume.time*$^{-1}$).
L_n	Water lost in the pipe network (*volume.time*$^{-1}$).
S	Length of the pipe network under analysis (distance)
μ_L	Specific water lost (*volume.distance*$^{-1}$*.time*$^{-1}$).
W_P	Average of water use per person per time (*volume.person*$^{-1}$*.time*$^{-1}$).
W_H	Average of water use per household per time (*volume.household*$^{-1}$*.time*$^{-1}$).
Φ	User category (domestic, hospital, etc.).
W_Φ	Water use per category per time (*volume.category*$^{-1}$*.time*$^{-1}$).

Area locations of the case studies:

Cartron Bay (area 1 of domestic case studies).
Farnacardy (area 2 of domestic case studies).
Cliffoney (area 3 of domestic case studies).
Mullaghneane (area 4 of domestic case studies).
Foxes Den (area 1 of non-domestic case studies).
Medical Center (area 1 of non-domestic case studies).
Sligo Town Center (area 1 of non-domestic case studies).

Others:

α	Significance level when testing the statistical estimations of parameters.
r^2	Correlation coefficient.

ABSTRACT

Currently, more research to implement and monitor cleaner production practices for distribution and sustainable management of natural and alternative water sources to comply with the demands of the different users while preserving water levels are needed. In this paper, a periodic hourly-based model with meaningful parameters has been developed to analyze and forecast water demand as a function of time, thus enabling a better understanding of the consumption pattern and the condition of the pipe network. The model was tested by investigating the daily water consumption from selected categories of users which were isolated from different distribution networks in Sligo, Ireland. The flow data used was obtained in 15-min intervals and averaged in different time periods for analysis. In all cases, the model fittings obtained were highly consistent and all the parameters showed satisfactory confidence intervals (α=0.05), thus demonstrating the reliability of this approach. The model provides a quick analysis revealing the regularities of water demand that could benefit water utility managers and researchers: to obtain optimal regulation and pumping schemes; for planning and design purposes; to control unexpected scenarios that can take place during the distribution of water; the performance of water distribution systems; and to locate possible network failures. In addition, the model parameters can be used as standard criteria for water utilities to compare precisely the water demand between different areas, identify complex trends and analyze the pipe network for managing, auditing and monitoring purposes.

Keywords: water management, mathematical modeling, water losses, water supply, behavioral water consumption, forecasting water demand.

1. Introduction

The population in developed countries has increased by more than 72 million per year since 1960 with growth rates being positive in almost all countries (Berrittella et al., 2007; Emelko et al., 2011). Changes have occurred in consumption and production patterns, with the endless increasing need for goods for millions of people (France, 2013; Jegatheesan et al., 2009; Makki et al., 2011). There have been many changes in lifestyle, as an example, longer and more frequent baths and showers, increasing use of washing machines and dishwashers, (Lake and Bond, 2007; Schleich, 2009).

These changes and others are key factors that exacerbate the imbalance between the demand and availability of fresh water resources, with a growing recognition that the current situation is unsustainable (Almeida et al., 2013; Arbués et al., 2003; Nataraj and Hanemann, 2011). Additionally, water losses can be significant; nevertheless, progress is being made to reduce leakage losses, although this is irregular within different countries (Liu and Kleiner, 2013). Water infrastructure, especially in cities, can be outdated or reach the end of the service life, causing leakage problems and therefore contributing to increased levels of water abstraction (Goulet et al., 2013; Wan Alwi et al., 2014). Losses of water (or non-revenue water) in the distribution network can reach high percentages, between 10 to 70 % of the water distributed (Xu et al., 2014). Northern European countries (Denmark, Sweden and Germany) are at the lower end of ranges for water loss (< 30 %). Countries like Chile, Ireland, Italy, France and Spain show figures between 30 and 50 % water loss. Mexico, Armenia, Brazil are found to have up to 70 % water loss. An effective reduction in leakage rates to an acceptable level depends on a number of factors. The most common critical issue is the poor condition of the pipe network, but others such as the pipe pressure, local climate and topography, local value of water, age of the system, the pipe material and type of soil can play an important role (Mahmoud et al., 2010). In this context, common indicators of water use efficiency, such as percentage urban leakage or specific loss, are crucial in order to know the condition of the pipe system, but unfortunately they are not directly assessed (Bentes et al., 2011; Liu and Kleiner, 2013).

In this complex scenario, water services in many cities in industrialized countries are moving towards systems capable of monitoring flows, pressures, and reservoir levels, and transmitting this information to a central station via text messaging or similar system in short time intervals (Castelletti and Soncinisessa, 2007). This data provides benefits for water utility managers and researchers for planning and operating a water distribution network improving the overview of water demand and trends, determining the location and scale of water stress, determine the uncertainties of consumption and identifying leaks with the overall intention of satisfying consumer demand by maintaining reasonable pipe pressure (Jegatheesan et al., 2009; Lake and Bond, 2007; Portnov and Meir, 2008; Velázquez, 2006; Widén et al., 2009; Wong and Mui, 2007; Yurdusev and Firat, 2009).

Sophisticated and expensive treatment techniques have become a requirement for all major cities, leading to a continuous increase of water prices (Olmstead et al., 2007; Rogers and Silva, 2002). Although, most countries use tariffs with fixed and volumetric components, many are now changing to water pricing systems that encourage economic efficiency and more sustainable use of water resources, using water prices as an eco-tool to promote "awareness" for conservation

(Jegatheesan et al., 2009). At the present time, water utilities periodically have to provide estimates of water demand for the main categories of users and overall water losses in the pipe network (Baumgartner, 2011; Bentes et al., 2011). Among other uses, these statistical outputs are used by governments (normally locally) to establish the marginal cost of drinking water and consequently the charges to be applied (Maidment and Miaou, 1986).

The lack of set criteria to analyze both water demand and losses is the main reason for many variations in the key data provided to governments (Wang et al., 2013). In this context, the application of a standard procedure to analyze historical and real-time data using a simple mathematical model with meaningful parameters is a crucial task for water utilities (Beal et al., 2013). An established quantification method could become a reliable tool for decision-makers to analyze water consumption (historical trends and predictions) and even determining an appropriate price, thus providing more realistic information to regulatory authorities in a standardized format and facilitating water management operations (Firat et al., 2009; Olmstead et al., 2007; Yurdusev and Firat, 2009).

Decision support systems are required to support the water utilities to manage their pipe networks and improve their work efficiency. A normalized criteria to audit water, would allow an investigation into the behavioral aspects of water management and would aid to balance the supply in regular circumstances and meet the normal demands in a sustainable and manageable way. The application of simple tools for analyzing the water distribution can determine whether significant losses are occurring within a predefined system boundary rapidly. Sustainable urban development will be benefited not only by the water savings, but also by the associated reduction of energy consumption and greenhouse gas emissions.

In this paper, a periodic mathematical model is developed based on historical water consumption data. Subsequently, its applicability is illustrated by applying it to daily water consumption data from selected categories of users on water distribution networks in Sligo, Ireland.

2. Material and methods

2.1. Current strategies for modeling water and energy demands

Although the focus of this paper is on water demand, the advances in the energy field are closely related, thus analysis of the available techniques for both areas are important. Only recently, more sophisticated modeling tools have emerged with more realistic parameter assessment and model uncertainties for analyzing and predicting water and energy demand (Cutore et al., 2008). Most common approaches are based on: artificial neural networks (Ghiassi et al., 2008); adaptive neuro-fuzzy inferences system (ANFIS), autoregressive (AR) and autoregressive integrated moving average (ARIMA) based models; M5 model trees in hydrological applications (Solomatine and Xue, 2004); and explicit mathematical models taking into account the periodic variables or not for the analysis of the demand (Maidment and Miaou, 1986; Zhou et al., 2000, 2002).

Researchers in the water field (Zhou et al., 2000) have examined past water demand by analyzing the daily, weekly, seasonal (monthly or yearly) periodicity (Cutore et al., 2008; Gato et al., 2007) and other variables like climatic indexes (Adrian et al., 1994; Arbués et al., 2003), population range (Chen et al., 2005), pipe network distribution size (Herrera et al., 2010; Mohamed and Al-Mualla, 2010), price (Wang et al., 2009a, 2009b), and others.

However, many models are used in a deterministic context expecting to correlate accurately with those variables, a goal that on many occasions is awkward to accomplish due to the unpredictability of the approaches. On the other hand, the lack of simple and practical tools frequently forces managers to choose some of the most unsatisfactory approaches like linear regressions and general time series analysis based on calendar periods.

2.2. Applying periodic series to model the hourly water demand

When researchers try to simulate water usage as a function of time, they need solutions that oscillate continuously. Periodic functions are those that repeat values in regular intervals or periods (Habibi and Lewis, 1996; Manera and Marzullo, 2005). The sine curve is more frequently used to simulate periodic cycles. Examples are number of hours of daylight per year, musical tones, human voice, respiration rates or monthly energy bills (Dhar and Reddy, 1993).

When modeling periodic phenomena like hourly water consumption, the most basic form of the sine equation as a function of time is:

$$W\left(t_u\right) = \sin\left(t\right) \qquad [1]$$

where $W(t_u)$ is the water consumption (volume or v) at any given time t and u is the interval for the data provision (time units).

This simple approach can be modified with several additional factors, such as: the amplitude of the oscillation (A), which is the peak deviation of the function from its center position (v units); the angular frequency (ω), that specifies how many oscillations occur in a unit time interval (t_u) in $rad\ t_u^{-1}$; the phase (φ), that indicates where in its cycle the oscillation begins at $t=0$ (v *units*). Additionally, the function should include a non-zero center amplitude, also called the offset value ($\overline{w}$) which actually corresponds to the average water consumption per cycle ($v.t^{-1}$). The equation can be rewritten as:

$$W\left(t_u\right) = \overline{w} + A\sin\left(\omega t + \varphi\right) \qquad [2]$$

Since the angular frequency (ω) is:

$$\omega = 2\pi/T \qquad [3]$$

where T is the wavelength or period (measured in time units) and inserting equation [3] into [2] gives:

$$W\left(t_u\right) = \overline{w} + A\sin\left(\left(2\pi/T\right)t + \varphi\right) \qquad [4]$$

Therefore, equation [4] can be used to describe hourly water usage as a function of time in any distribution network with one oscillation per period. If more oscillations appear per period, it can be generalized to the sum of *n* harmonic waves, by just adding the second part of the equation (*n* times as needed) proportionally to the number of cycles present in the daily flow. The general solution for *n* harmonic waves in its explicit or abbreviated form is as follows:

$$W\left(t_u\right) = \bar{w} + \sum_{i=1}^{n} \left[A_i \sin\left(\left(2\pi/T_i\right) t + \varphi_i\right) \right]_i \qquad [5]$$

Using the Fourier Sinusoidal Series (FSS; n=i; u=interval of time), practically all daily flow possibilities can be described. However, the physical meaning and usefulness of the parameters obtained decreases as more *n* cycles are added to the equation.

The amplitude of the sum of different sine curves (A_F) is not a straight-forward calculation, and it depends if the *n* periods and the phases are equal to each other or not. Therefore, in order to be computed, there are three different possibilities:

Case 1: when the periods and the phases are equal:

$$if \quad T_1 = T_2 = \ldots = T_n \quad and \quad if \quad \varphi_1 = \varphi_2 = \ldots = \varphi_n \qquad A_F = \sum_{i=1}^{n} A_i \qquad [6]$$

Case 2: when the periods are equal and the phases are different:

$$\text{if} \quad T_1 = T_2 = \ldots = T_n \quad \text{and} \quad \text{if} \quad \varphi_1 \neq \varphi_2 \neq \ldots \neq \varphi_n \qquad A_F = \sqrt{\sum_{i=1}^{n} A_i^2 + 2\sum_{j=1}^{n} A_i A_j \cos\left(\varphi_i - \varphi_j\right)} \qquad [7]$$

Case 3: when the periods and the phases are different:

$$\text{if} \quad T_1 \neq T_2 \neq \ldots \neq T_n \quad \text{and} \quad \text{if} \quad \varphi_1 \neq \varphi_2 \neq \ldots \neq \varphi_n \qquad A_F = \max\left[\left|W\left(t_i\right) - \bar{w}\right|\right] \qquad [8]$$

Because the daily fractionation behavior of domestic water consumption periods are not equal, the most probable case, number 3, has no direct solution and has to be calculated numerically. By simulating different scenarios, using equation [6] or [7] to compute A_F for case 3, it can be found that the maximum error of the water consumption produced would be 23.2 %; however, this is an acceptable figure.

2.3. Data used to test the model

County Sligo, Ireland, was selected as the study area to test the model due to the availability of data in support of this project. The county has a population of 57,341 people and an area of 1,827 km^2. It has six public water supplies, with a pipe length of 1,322 km, and a combined daily usage of 38,500 m^3.day^{-1}. There are 83 district metering areas with data loggers monitoring water pressure and flow in defined areas in the county. A logger is located beside each meter, which stores the data locally in 15-min intervals. A communications module transfers the data to a central computer.

2.4. Selected distribution networks

As a demonstration of the applicability of the model, it was tested with data of various types of users from different categories of distribution networks (domestic, business, agricultural, medical centre and complete water treatment plant outputs). These sectors were analyzed and fully modeled, obtaining in all cases high correlation coefficients. Since there could be many areas representing these user categories within County Sligo, the selected case study areas were classified in domestic and non-domestic sectors. To consider an area as a domestic sector, a minimum of 85 % of the properties must be residential. In order to extend the analysis to four domestic networks, two were selected in rural areas and two in urban ones. On the other hand, the non-domestic sectors are those areas in which the category of users is mixed.

2.4.1. Domestic case studies

1) Cartron Bay: This is an urban area of Sligo town with 568 properties of which 98 % are residences. It is the most populated domestic area selected for this study with a population of 1,158 on 4.61 km of pipe network; *2) Farnacardy*: This is an urban area of Sligo town with 66 properties of which 98 % are residences. It is the smallest area studied with a population of 170 on 6.92 km of pipe network; *3) Cliffoney*: This is a small rural village in north County Sligo with a population of 494 persons in 254 properties (85 % domestic) on 11.62 km of pipe network; and *4) Mullaghneane*: This is an agricultural rural area in north County Sligo with a population of 708 persons in 310 properties (96 % domestic) on 17.62 km of pipe network.

2.4.2. Non-domestic or mixed water use case studies

1) Foxes Den: The drinking water from Foxes Den water treatment plant is used to supply the western side of Sligo town, which has 14,254 properties and a population of 9,245 persons; *2) Medical Center*: This large medical center is supplied by 2.5 km of pipe network; and *3) Sligo Town Center*: This area incorporates a combination of shops, residential buildings, restaurants, pubs and other business activities on a pipe network of 5.2 km.

2.5. Numerical methods

The experimental results were fitted to equations by minimizing the sum of quadratic differences between the observed and model-predicted values, using the nonlinear least-squares method provided by the Excel 2003 macro *Solver*. The parametric estimates and confidence intervals were calculated using the *'SolverAid'* macro as described by other researchers (Prieto et al., 2012; Prikler, 2009). Furthermore, the *'SolverStat'* macro was used for the assessment of parameter and model prediction uncertainties (Comuzzi et al., 2003). The traditional methods such as the evaluation of likelihood ratios, Monte Carlo simulations for parameter distribution, parametric Bootstrapping, Beale MCMC (Markov Chain Monte Carlo), uncertainty propagation and Monte Carlo cross-validation were used for the analysis of different solutions in the parameter space.

3. Results

3.1. Mechanism of Fourier Sinusoidal Series model and application for analysis of hydrological sectors

Using the raw data from the Cartron Bay area in Sligo town (the most populated urban area presented here) two full years 2009-10 were used as an example. In *Figure 1*, an illustration of the working mechanism of the FSS ($n=2; u=$15-min) is shown. On the left side of the diagram, the different parts of the equation are presented: *1)* the value around which the periodic functions oscillate, which is exactly the average water consumption ($\bar{w}$), in this case 4.0 m^3.15-min^{-1}; *2)* the first sine equation (S_1) with T_1=24 h, A_1=1 m^3 and φ_1=-0.50 m^3, which corresponds to the daily cycle. In the night time, less water is used than during the day; and *3)* S_2, with T_2=12 *h*, A_2=0.5 m^3 and φ_2=0.25 m^3, which represents the fluctuations due to human actions over the previous cycle.

On the right side, a figure with the sum of those parts is displayed. The simulation shows two main peaks during the day, one in the morning and the other in the evening, dropping down sharply at night time. It is possible to distinguish three different categories of water: *a) water used by human actions*, represented by the area situated between the profile formed by the FSS (n=2), and the minimum water consumed, which should be at one point at night; *b) the unknown water (X)*, the water used at night plus the water lost at the user side; and *c) the water lost in the pipe network*. These areas can be used to analyze the performance of the pipe network, identifying leaks, assess the water consumption by human activity and even to forecast the demand.

3.1.1. Water efficiency analysis and leak identification

Water losses during transport have a negative impact on the environment. Leakage losses are still significant in many cities, generally due to the poor condition of water mains. Typical indicators of water use efficiency, such as percentage leakage, are essential values to assess the pipe network condition. The quantification measures of water efficiency are highly variable. However, the FSS analysis can be used to provide a consistent quantification procedure.

Assuming that, at one point at night, when the water used on the customer side is minimum, it can be defined that the amplitude of the new wave formed (A_F) in equation [5], is the water used due to human activity (A_h) in $m^3.t_i^{-1}$ (t_i=15-min *interval*). As discussed earlier, to find the amplitude of the sum of sine waves, if the phase and the period of both waves are not equal (the majority of cases), it has to be computed numerically by equation [8].

Therefore, it can be assumed that the water lost (L_T) in the pipe network is a constant rate and does not vary as a function of the pipe pressure. At one point at night, when the water used at the customer side is at a minimum, the total lost in the pipe network can be calculated as follows:

$$L_T = \bar{w} - A_h \qquad [9]$$

The total water lost can be split into water lost in the pipe network (L_n) (responsibility of the utility provider) and water lost on the user (L_h) side (property owner):

$$L_T = L_n + L_h \tag{10}$$

In an attempt to obtain further analysis, if one considers that the amount of water lost on the customer side would not exceed *X* percentage of the water used by the user side, it can be described as follows:

$$L_h = \left[\frac{A_h \cdot X}{100} \right] \tag{11}$$

Then the water lost in the pipe network is:

$$L_n = L_T - L_h \tag{12}$$

Using equation [11] and [13] into equation [14], another equation is derived [15]:

$$L_n = \bar{w} - \frac{A_h \cdot (100 - X)}{100} \tag{13}$$

Thus, the amount of water lost per length of the pipe (*S*) per unit time (*t*) can be determined; this parameter of specific water lost (μ_L), in units of m^3.Km^{-1}.min^{-1}, is commonly used by many engineers to audit the conditions of the pipe network and can be calculated as:

$$\mu_L = \frac{L_n}{S \cdot t} \Rightarrow \frac{\bar{w} - A_h \cdot \left(1 - X/_{100}\right)}{S \cdot t} \tag{14}$$

By giving an approximate value of X in equation [14], an estimated value for μ_L will be obtained. However, knowing that the amount of water lost through the distribution network will be much greater than that lost on the user side, it is possible to neglect X in order to calculate μ_L. Therefore, the equation can be simplified as follows:

$$\mu_L = \frac{\bar{w} - A_h}{S \cdot t} \qquad [15]$$

Consistent measures of water efficiency for each major water sector can be obtained by using either equation [14] or [15]. These values could be easily applied to create new efficiency indices to take into account the expenditure on pipe replacement programs, investment in water conservation measures, or other efforts to reduce water loss.

By calculating the specific water loss on a daily basis and quantifying the variations found with respect to the past data, reliable criteria to identify the efficiency of the pipe network and quantify the magnitude of the losses (mainly leaks) can be achieved. This routine can be easily implemented in any computerized system.

3.1.2. Water demand analysis

Some large-scale studies (Arbués et al., 2003) have demonstrated that demand for water is relatively inelastic. Nonetheless, evidence suggests that all users adjust their water consumption patterns in response to factors such as price, metering penetration, and conservation programs.

Water managers need standard user-friendly tools to control and analyze consumption patterns in order to make important decisions.

If the FSS approach can be used to determine the water losses in the network, it can also be used to estimate other crucial aspects related to demand. By knowing the relevant category that needs to be studied in a defined area or distribution network, it is possible to estimate the amount of water demanded by any category Φ per unit of time (t_u) as follows:

$$W_\Phi = \frac{\left(\bar{w} - L_n\right) \cdot {}^{24}\!/_{t_u}}{\Phi} \qquad [16]$$

Consequently, by selecting areas in which a category Φ can be generally isolated (e.g. domestic), indicators such as water used per person per day (W_P) or per household per day (W_H) can be easily calculated.

The water utility needs to apply frequent adjustments to the system in order to supply the water demanded by consumers and minimize the costs. In order to overcome these changes, managers and operators normally combine the records of different factors (independent variables) of previous years versus the water demand (the dependent variable) relationships with their own experience and assume that those relationships continue in the future. In this context, the FSS hourly model and the calculated parameters can be utilized to predict short- and long-term water demands, thus enhancing the decision making process.

3.2. Application of the Fourier Sinusoidal Series model as a standard method to analyze the water demand for domestic users in both rural and urban residential areas

In part A of *Figure 2*, the raw data from two full years 2009-10 of *Cartron Bay* area in Sligo town are displayed. For the 2009 period, the consumption shows a very stable profile with daily peak changes between 3 to 7 $m^3.15\text{-}min^{-1}$ and some random peaks and troughs which are related to some maintenance on the network. For the 2010 period, similar circumstances are shown, but with the difference that in the winter months the consumption increases due to unusual leaks caused by frozen pipes.

Using this data as an illustrative example, the typical analysis applied to extract useful information (that can be considered as the "simple way") is compared with the FSS hourly model (complex). Additionally, for both methods, different temporal formats based on calendar periods have been applied to evaluate the consumption behavior and identify trends. Afterwards, the FSS hourly model was applied in the rest of the study areas. Finally, all areas were compared.

3.2.1. Simple analysis - averaging the data in general terms

Some simple formats of averaging the data in general terms are presented in *Figure 2 B1-3*. In *Figure 2 B1*, the consumption throughout the week (~51) is found to be steady. High variations in consumption on weekends could be expected, but if it exists it seems to be very small and only an increase of the error deviation (α=0.05) towards the weekend is found. The monthly averaging format in *Figure 2 B2* may indicate a correlation with daylight and temperature. *Figure 2 B3* shows the daily water use (the grey background bars) and the cumulative use (dotted line) adjusted to a straight line (continuous line) with null intercept, like:

$$W(t) = m \cdot t \qquad [17]$$

the parameter m being the slope of the curve, which represents the average daily water use ($m^3.day^{-1}$) of the period analyzed (a year).

In general, these common formats cannot be used to identify any complex trend in the water consumption or identify problems in the pipe network. However, it is possible to provide data that can be used by various organizations to establish the price of water, for example.

3.2.2. Complex analysis using the FSS model - averaging the data in short time intervals

By plotting the water used (m^3) versus the time interval (u=15-min), a periodic cycle with several peak flows occurring at certain times of the day can be visually identified (*Figure A1* in appendix A). However, due to the small size of the distribution network, some peak flows as a result of particular situations such as community events may impede the overall intention of finding parameters to characterize the water demand of this particular area. Thus, the raw data used for the analysis and modeling need to be previously averaged in order to avoid those unexpected flows that can occur in short periods and to easily identify the majority of the cycles during the day.

Using the two years of comprehensive data from the Cartron Bay area (part A of *Figure 2*), several interval periods have been deliberately chosen, averaged in different temporal sequences and represented in graphs in *Figure 2* part *C* as follows: *C1* and *C2* show a 10-day period, averaged daily per hour from the 1st-10th of April in 2009 and 2010 (black dots), respectively; *C3* displays the consumption pattern between the annual average flow per 15-min during week days (black dots) and weekends (green dots); *C4* illustrates a monthly interval (February) averaged daily per 15-min in 2009 (black dots) and 2010 (green dots); and *C5* shows an annual period, averaging the water demand on a daily basis per 15-min for each year. All these formats of averaging the data can provide useful information to identify different trends in small, medium or large time periods.

Once the data is averaged in one of these time intervals, the water flow shows two different cycles. By combining two sine waves (FSS; n=2; u=15-min) it will be appropriate to model and predict the water usage at least for this location:

$$W\left(t_{1/4hr}\right)=\bar{w}+A_1\sin\left(\frac{2\pi}{T_1}t+\varphi_1\right)+A_2\sin\left(\frac{2\pi}{T_2}t+\varphi_2\right) \qquad [18]$$

The FSS (n=2) successfully simulates these different formats of averaging the data (*Figure 2C* and *Table 1*). In all cases, the dots (green and black) represent the average data per hour over a year and the red lines show the fitted data with the FSS (n=2; u=15-min).

As expected, due to many events that can occur, when the model is used to fit the data for the non-averaged analysis (10 days temporal format), in both cases those trends can be described (*C1* and *C2*), but most of the parameters (*Table 1*) obtained are inconsistent (α=0.05). On the other hand, the averaged temporal formats also describe the demand accurately but provide (α=0.05) consistent parameters (*Table 1*).

Therefore, in order to compare and analyze different urban areas, the monthly and yearly averaged temporal sequences seem to be more appropriate solutions. Next, all the domestic areas will be subjected to analysis and compared.

3.2.3. Applying FSS to identify trends and compare different residential areas

Researchers have a number of alternatives available for developing estimates of water consumption in residential and commercial areas (Manzardo et al., 2014). The common one, end-use metering, can provide good estimates of hourly consumption, but metering projects are also usually conducted for small sample sizes that produce low estimates of the bigger picture and are very expensive (Blaney and Inglis, 1980). In this context, the alternative developed in this study is a simple approach to assist in the study of total residential water consumption and could be applied to many different sample sizes.

If the data obtained from the loggers installed in the other selected domestic areas are averaged in any format described above, it shows the same main peaks during the day (Britton et al., 2013), one in the morning and the other in the evening, dropping down sharply at night time (*Figure 1-2*). Similar peaks were also identified by (Dhar and Reddy, 1993) for water demand and are logically similar to those analyzed by other researchers for domestic energy demand (Blaney and Inglis, 1980; Manera and Marzullo, 2005; Stokes et al., 2004).

In *Tables 2* and *3*, the parameters obtained after applying FSS (n=2) to simulate the monthly and yearly averages in 2009 are presented for all areas. In all cases, the parameters were consistent (α=0.05) and the correlation coefficient (r^2) was higher than 0.96. For example, the monthly adjusted data for the *Cartron Bay* area for 2009 can be seen in *Figure 3*; the dots represent the average data per hour for the year and the lines show the fitted data with the FSS (n=2).

In *Figure 4*, the most relevant parameters obtained ($\overline{w}$, μ_L, A_h, L_n, W_P and W_H) are presented in a monthly format for all the locations assessed in 2009. For the *Cartron Bay*, *Farnacardy* and *Cliffony* areas, it can be seen that the parameters $\overline{w}$ and μ_L decrease during 2009, indicating a constant reduction of water loss in the system, but W_P and W_H remain constant. In the *Mullaghneane* area, the $\overline{w}$ and μ_L parameters remain constant while W_P and W_H fluctuate during the year. As the daylight increases, the water consumed increases and vice versa. Finally in *Table 3*, the most relevant parameters obtained ($\overline{w}$, μ_L, A_h, L_n, W_P and W_H) are presented for 2009.

As *Tables 2* and *3* show, all the experimental data were satisfactorily modeled by the FSS equation with a good predictive capacity (adjusted coefficient of determination), statistical consistence (Fisher´s test), adequate parametric sensitivity, narrow parametric confidence intervals (Student’s test), unbiased residuals, and accuracy and bias factors close to 1.

3.3. Using the Fourier Sinusoidal Series model to analyze data from non-domestic sector

The FSS can be applied to analyze other types of sectors such as industrial areas, businesses, restaurants, schools, hospitals, among others, serving as a general tool to summarize the data available for comparison purposes. As stated before, using the FSS with different cycles, most daily flow profiles can be described, but the usefulness of the parameters obtained decreases as more cycles are added.

In *Figure 4* and *Table 4*, a set of fittings to different sectors are shown. The fitting of results was always satisfactory. The mathematical equations were robust and consistent (p-values < 0.001 from Fisher's *F* test), the residuals were randomly distributed and autocorrelations were not observed by *Durbin-Watso*n test (data not shown). The statistical analysis, parameter assessment tools and model prediction uncertainties provided by the '*SolverStat*' macro agreed accordingly. Furthermore, all the adjusted coefficients of determination between predicted and observed values were always higher than 0.95, with a majority at 0.99. Bias and accuracy factors also indicated high accuracy and the lack of bias of the FSS model (data not shown).

4. Discussion

Prediction of water consumption can help to improve the performance of water distribution systems by anticipating the corresponding system operation. The previous results demonstrate the capability of this model to describe water flow in long and short time intervals and to identify different trends. Other major benefits are discussed below.

4.1. A standard procedure to audit, monitor and analyze the water demand and losses in a consistent manner

Water utilities often have to provide estimates of water demand for various categories of users as well as an overview of water loss throughout a region. Computerized monitoring systems of water flows in the main pipe network are used, thus providing nearly real time accurate data (Nguyen et al., 2013). In some cases, water managers apply basic numerical methods, in other cases high order polynomial functions or other empirical equations to analyze the data, but there is not a common agreed procedure. The output of these calculations are compared rigorously by the regulatory authorities and detailed conclusions of the current status of each region are published. Afterwards, the national averages are used by many worldwide institutions to create reports, comparing the results between countries. However, the lack of a standard procedure to analyze the water demand and losses causes high variations in the data provided to government bodies. This paper provides a user-friendly mathematical tool (the FSS) as a solution to analyze the water demand and losses accurately as it has been proven in small or large distribution networks with different categories of users. Similar approaches are commonly used for assessing the performance of other sectors such as energy consumption. If applied to water distribution, it would aid utilities and regulatory authorities in gathering reliable information in a uniform format worldwide.

4.2. Implementation of pricing policies for peak water demand periods

In terms of electricity consumption, the concept of daily and seasonal peak pricing is well established and considered as fair by users (Baker and Rylatt, 2008; Filik et al., 2011). The consistency and periodicity of electricity as a function of the seasons of the year, the day of the week, the hour of the day over many years makes it possible to simultaneously model and forecast its demand, to analyze and interpret the human interactions and to adjust the pricing policy (Emelko et al., 2011; Nataraj and Hanemann, 2011). Additionally, some authors have also included other variables such as temperature, rainfall, geographical factors, and sun-light period (Abdel-Aal and Al-Garni, 1997; Blaney and Inglis, 1980; Paatero and Lund, 2006; Stokes et al., 2004).

However, when applying these approaches to water demand analysis, these types of models and predictions cannot be replicated and in some cases can be considered unrealistic. Water demand is highly variable with underlying regularities that change in a daily cycle as a function of the hour of the day. Proper analysis of the data would provide many advantages for water system managers and researchers for planning and implementing new strategies such as the daily peak water pricing policy. It would be desirable to have new approaches that would also benefit the end user and not only just the operation of the water distribution network.

4.3. Determining optimal plans for pumping schemes to supply the predicted demand

Another major benefit of the developed model would be for water utilities to adjust the water treatment and pumping system during operation and maintenance periods to meet the water demand and therefore, this should result in reduced costs. Water demand is highly variable with underlying regularities depending on: unpredictable factors like leaks on the pipe system, community events, holiday periods, fire and other unexpected scenarios; and on predictable factors such as the time (hour of day, day of the week, month of the year), the size of region, the characteristics of the population, the type of commercial and industrial establishments, climatic conditions (rainfall, air temperature, evaporation), among many others (Herrera et al., 2010; Zhou et al., 2000, 2002). The water utility services need to apply frequent adjustments to the system in order to supply the water demanded by consumers and minimize the costs. To overcome these changes, managers and operators normally combine the records of different factors (independent variables) of previous years versus the water demand (the dependent variable) relationships with their own experience and assume that those relationships will continue in the future. Such an approach has its drawbacks.

5. Conclusions

There has been a growing scientific interest in the development and use of models in water distribution with daily and hourly time scales. Predicting short- and long-term demand is required for planning and design purposes. Furthermore, understanding the regularities of water demands enables: *a)* from an operator's point of view, the determination of optimal regulation and pumping schemes to supply the predicted demand, thereby reducing energy consumption by lower pumping; *b)* from the quality point of view, a more suitable combination of water sources to obtain a given standard; and *c)* from the vulnerability point of view, the comparison between the predicted and the real flow measurements helping to locate possible leaks in the pipe network. In order to move water efficiently from reservoirs to users, accurate estimates of consumer demands are required. Uniform analysis of water consumption and losses can help to improve the performance of water distribution systems by anticipating the corresponding system operation.

The results of this study clearly demonstrate the capabilities of periodic sine equations to model the water flow in long and short time intervals and to identify different trends. For example, the model (FSS; n=2) was applied to predict daily water consumption in selected distribution networks. The results were obtained by using a double sinusoidal approach. Given the high level of consistency in the parameters that were identified, it can be concluded that this approach can produce rigorous criteria to compare and predict water usage as an alternative to existing models.

It has been demonstrated that by analyzing the obtained model parameters, it is possible to characterize and compare the water distribution system for different categories of users. The model provides a quick analysis of the data and could benefit water utility managers and researchers for planning, auditing and operating water distribution systems. It can also assist in improving the overview of water demand and trends, location and scale of water stress or leak identification, with the overall intention of satisfying the consumer demand by keeping the pipe system at a reasonable pressure.

ACKNOWLEDGMENTS

The authors wish to gratefully acknowledge financial support and assistance from Sligo County Council.

FIGURE CAPTIONS

Fig. 1: An illustration of the mechanism of the Fourier Sinusoidal Series (FSS; n=2) is shown.

Fig. 2: In part ***A*** the raw data from Cartron Bay area shows the average daily water consumption ($m^3.h^{-1}$) over two full-years (2009-10). In part ***B***, the data is averaged for both years (2009-10) in general terms in three common different formats as: ***B1***, the water consumption per day of the week per annum; ***B2***, the water use in a monthly basis over the entire year; and ***B3*** (2009) and ***B4*** (2010) shows the daily average water (black dotted-line) consumption ($m^3.day^{-1}$) adjusted to a straight line (red line) and in another scale the cumulative daily water (thick black line) demand (m^3). Part ***C*** illustrates the application of FSS (red lines) in four different interval periods randomly selected from the period 2009-10 water demand: ***C1*** and ***C2*** a 10-day period, averaged daily per h, from the 1st-10th of April in 2009 and 2010 data (black dots) respectively; ***C2*** consumption pattern between the annual average daily per 15-min of week days (black dots) and weekends (green dots); ***C4*** a monthly (February) interval averaged daily per 15-min in 2009 (black dots) and 2010 (green dots); ***C5*** and finally an annual period analysis, averaging the water demand on a daily basis per 15-min for each year.

Fig. 3: Parametric analysis and trends of four different domestic areas in County Sligo. Part ***A*** shows the monthly analysis of Cartron Bay area for the full year of 2009, in which the adjusted data (red lines) versus the 15-min average water (m^3) consumption (confidence intervals α=0.05). Part ***B*** illustrates the relevant parameters ($\overline{w}$, μ_L, A_h, L_n, W_P and W_H) obtained in a monthly interval for all the locations assessed for 2009 (as showned for Cartron Bay in part ***A***).

Fig. 4: Averaged daily per 15-min yearly period is applied to different categories of users in County Sligo. In all cases, the dots represent the average data and the red lines shows the fitted data with the FSS (*n*=2).

TABLE CAPTIONS

Table 1: Parameters obtained by adjusting the water demand of the Cartron Bay residential area for the data used to illustrate the capabilities of FSS (n=2) in *Figure 1*C in four temporal sequences (ten days, weekly, monthly and annually variation analysis).

Table 2: Adjusted parameters for the daily flow average of water production-demand per month to the FSS (for all cases the best fittings occurs when n=2) for the rural and urban domestic areas.

Table 3: Key statistics for case study areas in Co. Sligo. The parameters obtained by FSS (for all cases the best fittings occurs when n=2) to fit the yearly average interval period of all the areas in 2009 are presented. Relevant parameters obtained ($\bar{w}$, μ_L, A_h, L_n, W_P and W_H) are shown in a yearly format interval for all the locations assessed for 2009.

Table 4: Adjusted parameters for the daily flow average of water demand per month to the FSS (for all cases the best fittings occurs when n=2) for other type of category of users in Sligo County.

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03th of ADecember of 2014

Dear Dr. Lozano,

Please find attached our paper titled ***"Mathematical model as a standard procedure to analyze small and large water distribution networks"*** to be submitted to the *Journal Clearner Production.*

We want to apologize for submitting the revised manuscript after the end of the deadline. Since the editorial team of the Journal tagged us with a minor revision, there was confusion between the authors about who should make the final changes and none of us did anything for the last two months. I am sorry for that mistake and delay on responding to the comments of editors and reviewers.

We believe the entire journal requirements have been addressed. In general, the "reviewer's comments" have induced us to reformulate parts of our manuscript, which is to a great extent an explicit response to the questions proposed. We are grateful for your incitement and enclose next the revised text. We hope this new version has meant a substantial improvement of the old one.

Sincerely,

Miguel Ángel Prieto Lage

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FIGURES

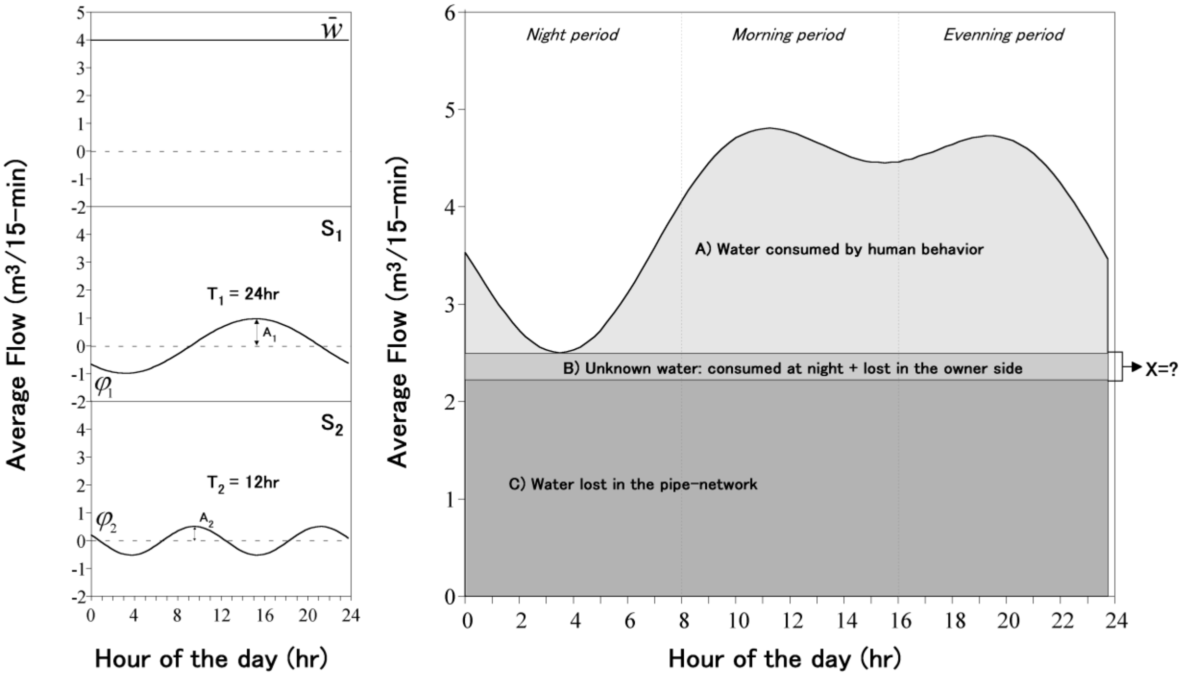


Fig. 1: An illustration of the mechanism of the Fourier Sinusoidal Series (FSS; n=2) is shown.

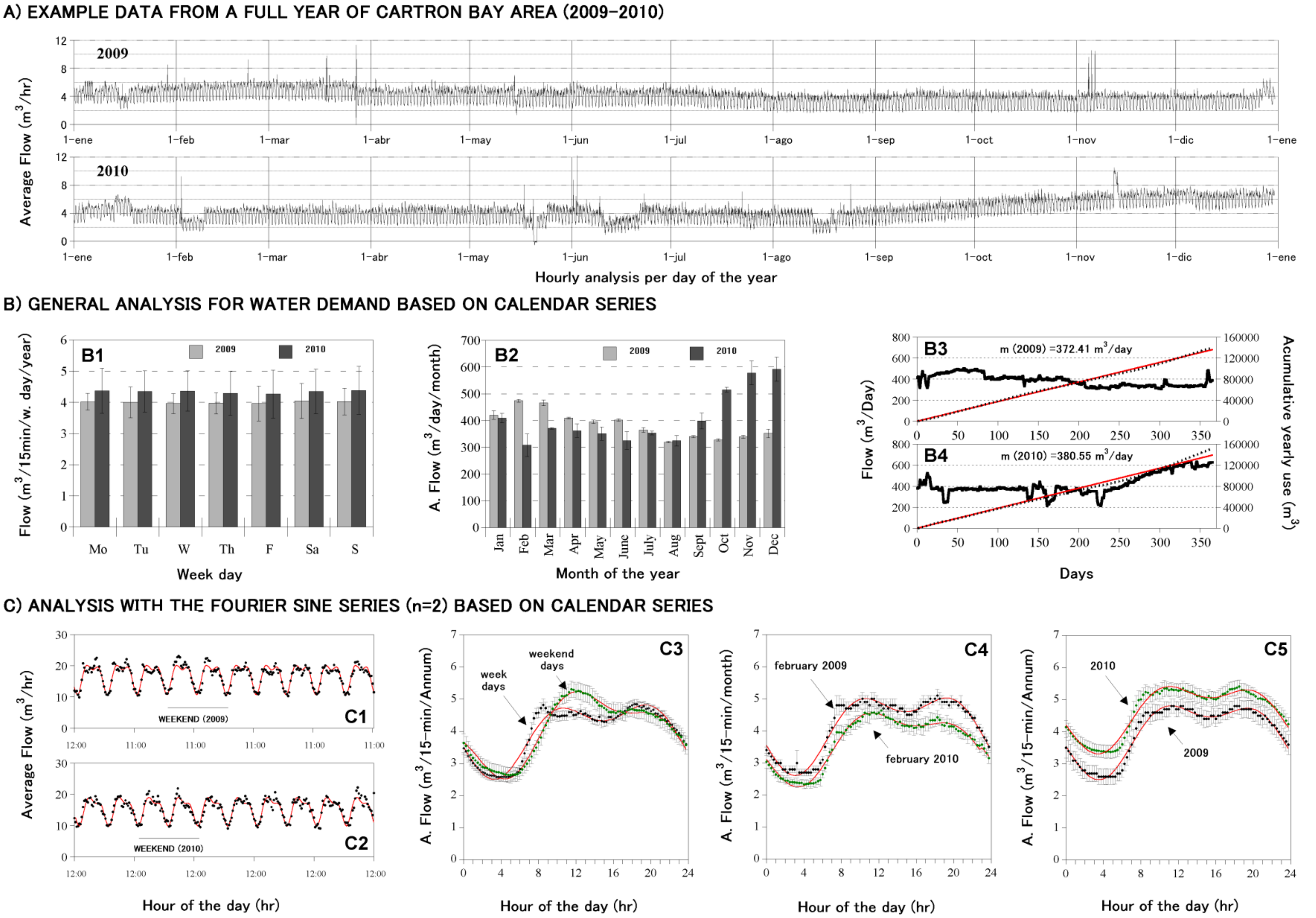
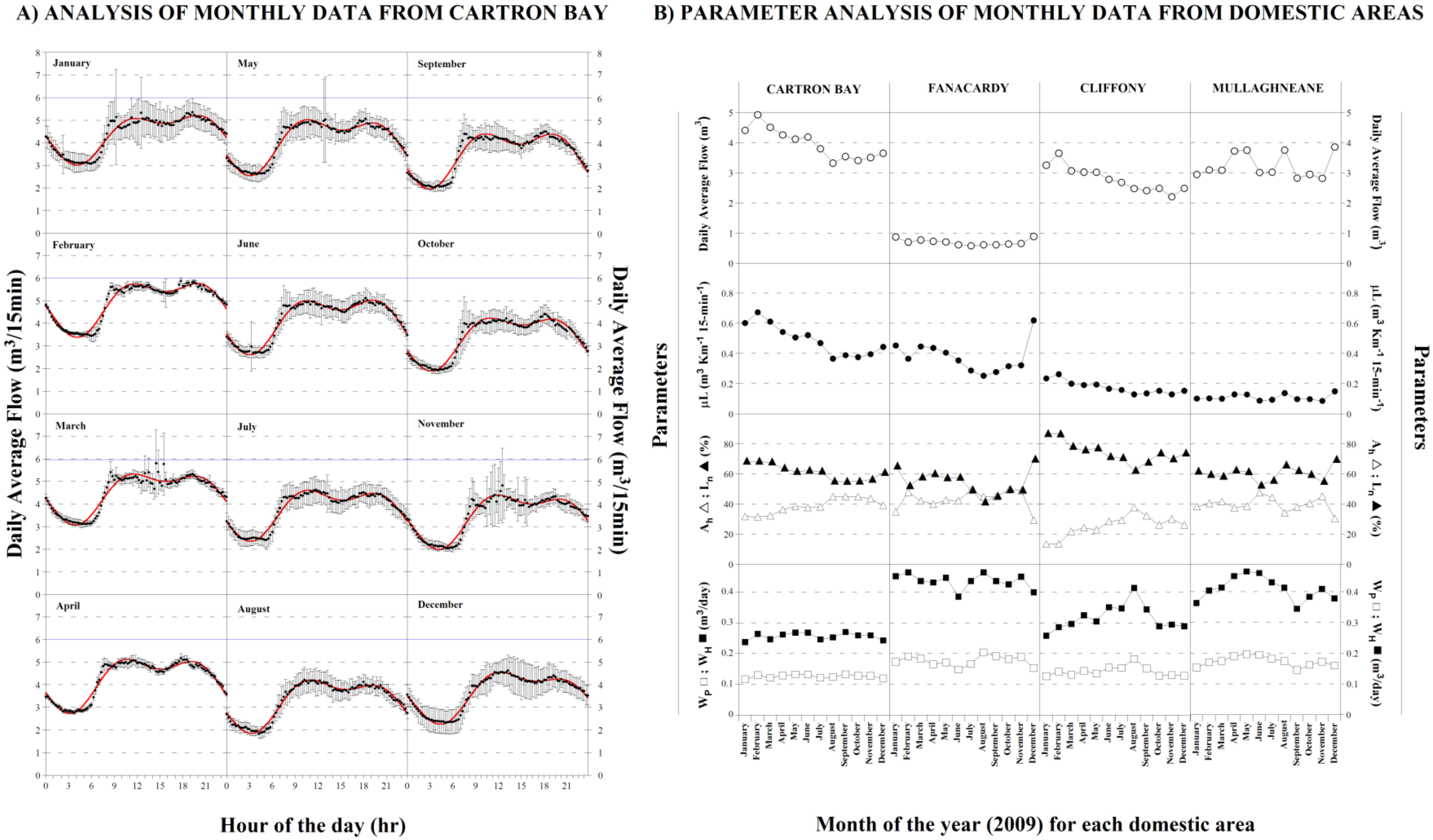


Fig. 2: In part ***A*** the raw data from Cartron Bay area shows the average daily water consumption ($m^3.h^{-1}$) over two full-years (2009-10). In part ***B***, the data is averaged for both years (2009-10) in general terms in three common different formats as: ***B1***, the water consumption per day of the week per annum; ***B2***, the water use in a monthly basis over the entire year; and ***B3*** (2009) and ***B4*** (2010) shows the daily average water (black dotted-line) consumption ($m^3.day^{-1}$) adjusted to a straight line (red line) and in another scale the cumulative daily water (thick black line) demand (m^3). Part ***C*** illustrates the application of FSS (red lines) in four different interval periods randomly selected from the period 2009-10 water demand: ***C1*** and ***C2*** a 10-day period, averaged daily per h, from the 1st-10th of April in 2009 and 2010 data (black dots) respectively; ***C2*** consumption pattern between the annual average daily per 15-min of week days (black dots) and weekends (green dots); ***C4*** a monthly (February) interval averaged daily per 15-min in 2009 (black dots) and 2010 (green dots); ***C5*** and finally an annual period analysis, averaging the water demand on a daily basis per 15-min for each year.



A) ANALYSIS OF MONTHLY DATA FROM CARTRON BAY

B) PARAMETER ANALYSIS OF MONTHLY DATA FROM DOMESTIC AREAS

Fig. 3: Parametric analysis and trends of four different domestic areas in County Sligo. Part ***A*** shows the monthly analysis of Cartron Bay area for the full year of 2009, in which the adjusted data (red lines) versus the 15-min average water (m^3) consumption (confidence intervals α=0.05). Part ***B*** illustrates the relevant parameters ($\overline{w}$, μ_L, A_h, L_n, W_P and W_H) obtained in a monthly interval for all the locations assessed for 2009 (as shown for Cartron Bay in part ***A***).

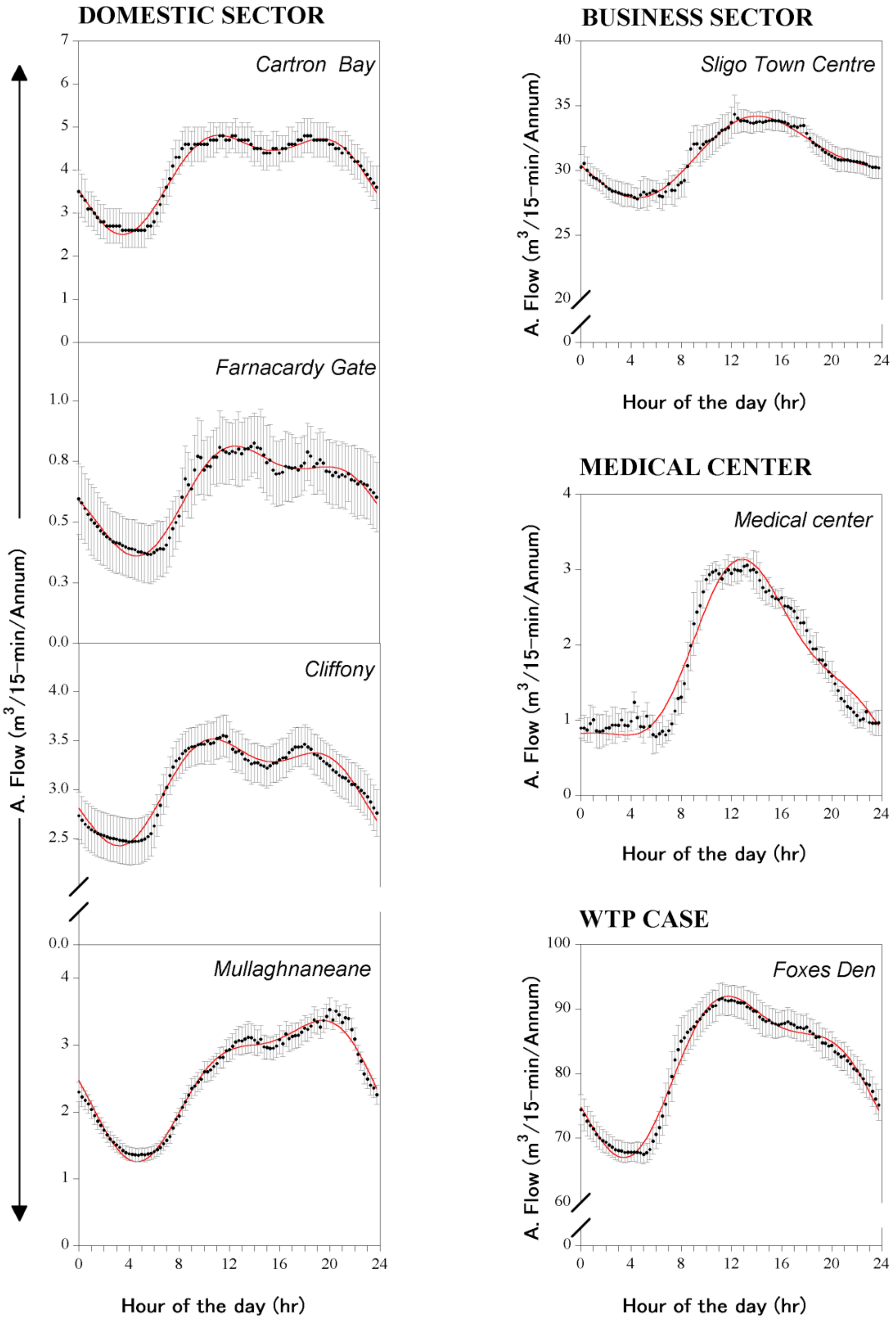


Fig. 4: Averaged daily per 15-min yearly period is applied to different categories of users in County Sligo. In all cases, the dots represent the average data and the red lines shows the fitted data with the FSS (*n*=2).

TABLES

Table 1: Parameters obtained by adjusting the water demand of the Cartron Bay residential area for the data used to illustrate the capabilities of FSS (*n*=2) in *Figure 1*C in four temporal sequences (ten days, weekly, monthly and annually variation analysis).

PARAMETER	**Daily (1st to 10th Apr.)**		**Weekly (C3)**		**Monthly (C4)**		**Annually (C5)**	
	C1 (2009)	**C2 (2010)**	**Week**	**Weekends**	**Feb. 2009**	**Feb. 2010**	**2008**	**2009**
$\hat{w}$ (m^3.15-min^{-1})	4.19±*0.13*	3.83±*0.09*	3.99±*0.04*	4.03±*0.02*	4.19±*0.03*	3.66±*0.02*	4.65±*0.02*	3.99±*0.02*
A_1 (m^3.15-min^{-1})	0.53 *(NS)*	0.45 *(NS)*	0.57±*0.05*	0.49±*0.04*	0.59±*0.05*	0.45±*0.03*	0.47±*0.04*	0.52±*0.04*
φ_1 (m^3.15-min^{-1})	2.22 *(NS)*	2.96 *(NS)*	2.95±*0.20*	1.75±*0.20*	3.05±*0.15*	2.12±*0.15*	2.76±*0.20*	2.74±*0.20*
T_1 (h)	12.00±*0.23*	12.00±*0.12*	11.71±*0.28*	11.04±*0.22*	11.78±*0.24*	11.55±*0.26*	11.63±*0.22*	11.66±*0.22*
A_2 (m^3.15-min^{-1})	1.01 *(NS)*	0.96 *(NS)*	0.92±*0.20*	1.17±*0.20*	0.99±*0.05*	0.98±*0.03*	0.98±*0.20*	0.98±*0.20*
φ_2 (m^3.15-min^{-1})	2.81 *(NS)*	4.11 *(NS)*	3.87±*0.06*	3.95±*0.04*	3.94±*0.05*	3.77±*0.04*	3.93±*0.04*	3.88±*0.04*
T_2 (h)	24--	24--	24--	24--	24--	24--	24--	24--
r^2	*0.8838*	*0.8712*	*0.9541*	*0.9897*	*0.9676*	*0.9820*	*0.9736*	*0.9780*
Nº of Days	10	10	*260*	*105*	30	30	365	365

Table 2: Adjusted parameters for the daily flow average of water production-demand per month to the FSS (for all cases the best fittings occurs when *n*=2) for the rural and urban domestic areas.

	January	February	March	April	May	June	July	August	September	October	November	December
CARTRON BAY												
$\hat{w}$ (m^3/15-min)	4.41 ±0.04	4.93 ±0.04	4.51 ±0.21	4.26 ±0.03	4.12 ±0.11	4.19 ±0.03	3.80 ±0.03	3.32 ±0.02	3.54 ±0.04	3.41 ±0.04	3.51 ±0.04	3.66 ±0.02
A_1 (m^3/15-min)	0.48 ±0.05	0.55 ±0.05	0.49 ±0.51	0.56 ±0.05	0.58 ±0.10	0.59 ±0.05	0.54 ±0.04	0.51 ±0.03	0.60 ±0.06	0.51 ±0.05	0.52 ±0.05	0.45 ±0.03
φ_1 (m^3/15-min)	2.48 ±0.20	2.39 ±0.20	2.50 ±0.04	2.92 ±0.16	2.97 ±0.51	3.05 ±0.15	2.96 ±0.16	2.82 ±0.13	3.05 ±0.19	2.92 ±0.20	2.35 ±0.22	2.12 ±0.15
T_1 (h)	11.65 ±0.38	11.51 ±0.32	11.55 ±0.12	11.61 ±0.25	11.73 ±0.72	11.78 ±0.24	11.81 ±0.27	11.75 ±0.22	11.57 ±0.29	11.65 ±0.31	11.66 ±0.37	11.55 ±0.26
A_2 (m^3/15-min)	0.92 ±0.20	1.01 ±0.05	0.97 ±0.08	0.98 ±0.05	1.01 ±0.05	0.99 ±0.05	0.91 ±0.04	0.98 ±0.03	0.99 ±0.06	1.02 ±0.05	1.01 ±0.05	0.98 ±0.03
φ_2 (m^3/15-min)	3.64 ±0.06	3.72 ±0.06	3.82 ±0.19	4.00 ±0.05	4.01 ±0.51	3.94 ±0.05	3.98 ±0.05	3.99 ±0.04	4.03 ±0.06	3.96 ±0.05	3.76 ±0.06	3.77 ±0.04
T_2 (h)	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--
r^2	0.9985	0.9866	0.8537	0.9863	0.9899	0.9867	0.9849	0.9786	0.9940	0.9986	0.9838	0.9850
FARNACARDY												
$\hat{w}$ (m^3/15-min)	0.87±0.04	0.70 ±0.04	0.77 ±0.25	0.73 ±0.03	0.71 ±0.11	0.62 ±0.03	0.58 ±0.03	0.31 ±0.02	0.42 ±0.04	0.44 ±0.04	0.46 ±0.04	0.89 ±0.02
A_1 (m^3/15-min)	-0.09±0.05	-0.10 ±0.05	-0.11 ±0.04	-0.11 ±0.05	-0.10 ±0.10	-0.07 ±0.05	-0.09 ±0.04	-0.06 ±0.03	-0.08 ±0.06	-0.08 ±0.05	-0.11 ±0.05	-0.09 ±0.03
φ_1 (m^3/15-min)	4.76±0.20	4.76 ±0.20	4.73 ±0.11	5.44 ±0.16	5.28 ±0.51	5.36 ±0.15	5.05 ±0.16	5.10 ±0.13	5.50 ±0.19	5.36 ±0.20	4.64 ±0.22	4.81 ±0.15
T_1 (h)	12.06±0.38	11.61 ±0.32	11.20 ±0.45	11.90 ±0.25	11.39 ±0.72	11.37 ±0.24	11.25 ±0.27	11.28 ±0.22	11.14 ±0.29	11.45 ±0.31	11.25 ±0.37	11.68 ±0.26
A_2 (m^3/15-min)	0.22±0.20	0.23 ±0.05	0.22 ±0.12	0.19 ±0.05	0.21 ±0.05	0.14 ±0.05	0.20 ±0.04	0.14 ±0.03	0.19 ±0.06	0.24 ±0.05	0.22 ±0.05	0.17 ±0.03
φ_2 (m^3/15-min)	3.58±0.06	3.47 ±0.06	3.71 ±0.06	3.88 ±0.05	3.95 ±0.51	4.08 ±0.05	4.21 ±0.05	3.98 ±0.04	3.87 ±0.06	3.74 ±0.05	3.58 ±0.06	3.54 ±0.04
T_2 (h)	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--
r^2	0.9258	0.9157	0.8912	0.9173	0.9189	0.9478	0.8890	0.9155	0.9838	0.9104	0.9199	0.9223
CLIFFONY												
$\hat{w}$ (m^3/15-min)	5.55 ±0.04	4.71 ±0.04	3.07 ±0.02	3.03 ±0.03	3.02 ±0.11	2.78 ±0.03	2.68 ±0.03	2.48 ±0.02	2.41 ±0.04	2.49 ±0.04	2.21 ±0.04	2.49 ±0.02
A_1 (m^3/15-min)	0.15 ±0.05	0.23 ±0.05	0.24 ±0.03	0.27 ±0.05	0.25 ±0.10	0.28 ±0.05	0.27 ±0.04	0.36 ±0.03	0.28 ±0.06	0.19 ±0.05	0.21 ±0.05	0.19 ±0.03
φ_1 (m^3/15-min)	2.19 ±0.20	1.99 ±0.20	2.38 ±0.21	2.87 ±0.16	3.07 ±0.51	3.11 ±0.15	3.08 ±0.16	2.74 ±0.13	2.77 ±0.19	2.76 ±0.20	2.23 ±0.22	2.71 ±0.15
T_1 (h)	11.59 ±0.38	10.59 ±0.32	11.27 ±0.33	11.29 ±0.25	11.55 ±0.72	11.55 ±0.24	11.47 ±0.27	11.08 ±0.22	11.28 ±0.29	11.52 ±0.31	11.27 ±0.37	11.47 ±0.26
A_2 (m^3/15-min)	0.29 ±0.20	0.26 ±0.05	0.43 ±0.03	0.47 ±0.05	0.44 ±0.05	0.51 ±0.05	0.51 ±0.04	0.58 ±0.03	0.50 ±0.06	0.47 ±0.05	0.45 ±0.05	0.47 ±0.03
φ_2 (m^3/15-min)	3.96 ±0.06	4.17 ±0.06	4.06 ±0.05	4.16 ±0.05	4.18 ±0.51	4.05 ±0.05	3.99 ±0.05	4.11 ±0.04	4.33 ±0.06	4.25 ±0.05	4.02 ±0.06	4.25 ±0.04
T_2 (h)	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--
r^2	0.9623	0.9587	0.9498	0.9577	0.9570	0.9737	0.9629	0.9459	0.9984	0.9544	0.9509	0.9543
MULLAGHNEANE												
$\hat{w}$ (m^3/15-min)	2.95 ±0.04	3.10 ±0.04	3.09 ±0.02	3.73 ±0.03	3.76 ±0.11	2.52 ±0.03	1.02 ±0.03	3.76 ±0.02	1.26 ±0.04	0.52 ±0.04	0.17 ±0.04	3.86 ±0.02
A_1 (m^3/15-min)	0.17 ±0.05	0.30 ±0.05	0.33 ±0.03	0.56 ±0.05	0.63 ±0.10	0.47 ±0.05	0.44 ±0.04	0.56 ±0.03	0.62 ±0.06	0.20 ±0.05	0.10 ±0.05	0.37 ±0.03
φ_1 (m^3/15-min)	1.57 ±0.20	0.89 ±0.20	0.52 ±0.21	1.93 ±0.16	2.38 ±0.51	2.63 ±0.15	4.58 ±0.16	5.64 ±0.13	2.92 ±0.19	2.93 ±0.20	2.42 ±0.22	0.82 ±0.15
T_1 (h)	11.33 ±0.38	10.65 ±0.32	9.73 ±0.33	11.26 ±0.25	11.47 ±0.72	11.50 ±0.24	14.19 ±0.27	15.15 ±0.22	11.70 ±0.29	11.62 ±0.31	11.45 ±0.37	10.39 ±0.26
A_2 (m^3/15-min)	0.96 ±0.20	0.96 ±0.05	0.96 ±0.03	0.84 ±0.05	0.82 ±0.05	1.06 ±0.05	1.06 ±0.04	3.72 ±0.03	0.45 ±0.06	0.15 ±0.05	0.09 ±0.05	0.81 ±0.03
φ_2 (m^3/15-min)	3.32 ±0.06	3.38 ±0.06	3.57 ±0.05	3.72 ±0.05	3.63 ±0.51	3.20 ±0.05	3.07 ±0.05	3.68 ±0.04	3.29 ±0.06	2.98 ±0.05	2.53 ±0.06	3.48 ±0.04
T_2 (h)	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--
r^2	0.9744	0.9673	0.9791	0.9435	0.9452	0.9573	0.9989	0.9425	0.9206	0.9371	0.9599	0.9653

Table 3: Key statistics for case study areas in Co. Sligo. The parameters obtained by FSS (for all cases the best fittings occurs when n=2) to fit the yearly average interval period of all the areas in 2009 are presented. Relevant parameters obtained ($\overline{w}$, μ_L, A_h, L_n, W_P and W_H) are shown in a yearly format interval for all the locations assessed for 2009.

		AREA			
		Cartron Bay	***Farnacardy***	***Cliffoney***	***Mullaghneane***
key statistics					
Supply	*Reservoir*	Kilsellagh	Kilsellagh	North Sligo	North Sligo
T.C.	*Non-domestic*	13	1	36	11
	Domestic	555 (98%)	65 (98%)	218 (85%)	299 (96%)
S	*km*	4.61	6.92	11.62	17.87
P	*Persons*	1158	170	494	708
H	*Households*	568	65	218	299
parameters					
$\hat{w}$	$(m^3/15\text{-min})$	3.99±*0.02*	0.62±*0.10*	3.08±*0.01*	2.49±*0.01*
A_1	$(m^3/15\text{-min})$	0.52±*0.04*	-0.08±*0.14*	0.23±*0.02*	0.31±*0.02*
φ_1	$(m^3/15\text{-min})$	2.74±*0.20*	5.01±*0.20*	2.71±*0.20*	2.20±*0.20*
T_1	*(h)*	11.66±*0.22*	11.40±*0.25*	11.31±*0.24*	11.23±*0.24*
A_2	$(m^3/15\text{-min})$	0.98±*0.20*	0.19±*0.20*	0.45±*0.20*	0.93 ±*0.20*
φ_2	$(m^3/15\text{-min})$	3.88±*0.04*	3.79±*0.05*	16.70±*0.05*	16.01 ±*0.05*
T_2	*(h)*	24--	24--	24--	24--
	r^2	*0.9780*	*0.9666*	*0.9662*	*0.9579*
additional analysis					
A_h	$m^3 15\text{-min}^{-1}$	1.49	0.27	0.68	1.24
L_T	$m^3 15\text{-min}^{-1}$	2.50 (62.6%)	0.35 (56.4%)	2.40 (77.9%)	1.25 (50.2%)
μ_L	$m^3 Km^{-1} 15\text{-min}^{-1}$	0.500	0.350	0.200	0.069
W_P	$m^3 P^{-1} dia^{-1}$	0.124	0.152	0.132	0.168
W_H	$m^3 H^{-1} dia^{-1}$	0.251	0.398	0.299	0.398

Table 4: Adjusted parameters for the daily flow average of water demand per month to the FSS (for all cases the best fittings occurs when *n*=2) for other type of category of users in Sligo County.

	January	**February**	**March**	**April**	**May**	**June**	**July**	**August**	**September**	**October**	**November**	**December**
FOXES DEM WTP												
$\hat{w}$ (m^3/15-min)	84.82±0.04	84.36±0.04	82.40±0.20	84.78±0.03	81.87±0.11	82.18±0.03	82.84±0.03	79.59±0.02	76.28±0.04	78.32±0.04	78.77±0.04	78.32±0.02
A_1 (m^3/15-min)	3.70±0.05	4.34±0.05	4.35±0.32	4.85±0.05	4.19±0.10	4.68±0.05	4.84±0.04	4.93±0.03	3.68±0.06	4.31±0.05	4.20±0.05	3.63±0.03
φ_1 (m^3/15-min)	2.11±0.20	2.16±0.20	2.26±0.20	2.73±0.16	2.75±0.51	2.79±0.15	2.77±0.16	2.65±0.13	2.77±0.19	2.61±0.20	2.24±0.22	1.36±0.15
T_1 (h)	11.71±0.38	11.59±0.32	11.57±0.15	11.65±0.25	11.57±0.72	11.72±0.24	11.73±0.27	11.56±0.22	11.37±0.29	11.63±0.31	11.59±0.37	11.28±0.26
A_2 (m^3/15-min)	11.55±0.20	12.23±0.05	11.86±0.24	11.89±0.05	10.23±0.05	10.80±0.05	11.31±0.04	11.05±0.03	9.41±0.06	11.35±0.05	10.96±0.05	10.22±0.03
φ_2 (m^3/15-min)	3.91±0.06	3.96±0.06	4.05±0.05	4.24±0.05	4.21±0.51	4.19±0.05	4.24±0.05	4.23±0.04	4.26±0.06	4.21±0.05	4.00±0.06	3.69±0.04
T_2 (h)	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--
r^2	0.9835	0.9751	0.9773	0.9796	0.9680	0.9769	0.9792	0.9776	0.9565	0.9787	0.9696	0.9912
MEDICAL CENTER												
$\hat{w}$ (m^3/15-min)	2.10±0.04	1.70±0.04	1.69±0.00	1.63±0.03	1.63±0.11	1.57±0.03	2.11±0.03	1.65±0.02	1.72±0.04	1.62±0.04	1.73±0.04	1.67±0.02
A_1 (m^3/15-min)	-0.27±0.05	-0.33±0.05	-0.30±0.05	-0.30±0.05	-0.29±0.10	-0.28±0.05	-0.32±0.04	-0.21±0.03	-0.27±0.06	-0.29±0.05	-0.31±0.05	-0.33±0.03
φ_1 (m^3/15-min)	4.03±0.20	3.44±0.20	3.29±0.25	3.96±0.16	4.03±0.51	4.08±0.15	4.27±0.16	3.69±0.13	4.08±0.19	3.93±0.20	3.54±0.22	3.79±0.15
T_1 (h)	12.35±0.38	10.54±0.32	9.93±0.38	10.42±0.25	10.58±0.72	10.56±0.24	11.12±0.27	9.70±0.22	10.96±0.29	10.32±0.31	10.87±0.37	11.48±0.26
A_2 (m^3/15-min)	0.61±0.20	1.28±0.05	1.25±0.10	1.23±0.05	1.23±0.05	1.23±0.05	1.43±0.04	1.00±0.03	1.26±0.06	1.20±0.05	1.27±0.05	1.17±0.03
φ_2 (m^3/15-min)	4.12±0.06	4.09±0.06	4.16±0.20	4.39±0.05	4.39±0.51	4.40±0.05	4.32±0.05	4.48±0.04	4.34±0.06	4.38±0.05	4.09±0.06	4.13±0.04
T_2 (h)	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--
r^2	0.9258	0.9279	0.9392	0.9422	0.9439	0.9375	0.9426	0.9257	0.9500	0.9397	0.9462	0.9369
SLIGO TOWN CENTER (BUSINESS AREA)												
$\hat{w}$ (m^3/15-min)	29.31±0.04	29.41±0.04	29.39±0.05	31.28±0.03	31.18±0.11	32.22±0.03	33.74±0.03	32.66±0.02	30.86±0.04	30.91±0.04	30.89±0.04	32.03±0.02
A_1 (m^3/15-min)	0.64±0.05	0.83±0.05	0.83±0.26	1.02±0.05	0.90±0.10	1.16±0.05	0.96±0.04	0.92±0.03	0.94±0.06	0.98±0.05	0.82±0.05	0.97±0.03
φ_1 (m^3/15-min)	1.60±0.20	1.69±0.20	1.25±0.05	2.30±0.16	3.15±0.51	2.19±0.15	2.42±0.16	2.31±0.13	2.57±0.19	2.20±0.20	1.74±0.22	1.85±0.15
T_1 (h)	13.42±0.38	13.42±0.32	12.55±0.21	13.82±0.25	15.65±0.72	12.94±0.24	13.63±0.27	13.26±0.22	14.39±0.29	13.52±0.31	13.36±0.37	13.61±0.26
A_2 (m^3/15-min)	2.70±0.20	2.94±0.05	3.06±0.38	2.55±0.05	2.21±0.05	2.99±0.05	2.55±0.04	2.37±0.03	2.51±0.06	2.69±0.05	2.66±0.05	2.46±0.03
φ_2 (m^3/15-min)	3.71±0.06	3.66±0.06	3.77±0.20	3.96±0.05	3.58±0.51	3.99±0.05	3.95±0.05	3.93±0.04	3.85±0.06	3.87±0.05	3.64±0.06	3.52±0.04
T_2 (h)	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--	24--
r^2	0.9592	0.9712	0.9612	0.9221	0.8769	0.9486	0.9416	0.9291	0.9586	0.9726	0.9749	0.9670