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# Remote voltage estimation in LV feeders with local monitoring at transformer level

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**Abstract**—On-load tap changer-fitted transformers have been proposed to solve voltage problems in low voltage (LV) networks with rich penetration of distributed generation. However, knowledge of the voltage level at customers' point of connection is key for the performance of potential control strategies. This work proposes a generic methodology to estimate voltages in LV feeders without the need of remote monitoring. The methodology relies on local flow measurements at the transformer level and residential load models. Maximum Likelihood Estimation (MLE) is applied to obtain the most likely residential loads' power consumptions which are later used to calculate the associated nodal voltages. A novel formulation of power flow equations based on sensitivity analysis is used to simplify the MLE problem. The methodology is tested on a real unbalanced feeder with unique presence of residential loads. The high accuracy of results promotes it as a potential alternative to monitoring investments.

**Index Terms**—Voltage control, OLTC, State estimation, Smart grids, Distributed power generation.

## I. INTRODUCTION

The installation of photovoltaic panels (PVs) in residential low voltage (LV) feeders is incentivized by many governments in order to reduce carbon emissions. Nevertheless, several studies like the one performed in [1] have shown that high penetrations levels of such technologies can produce voltage fluctuations that violate upper statutory limits [2]. Therefore, due to the need of a more active management of LV feeders, it has been suggested [3]–[7] to Distribution System Operators (DSOs) to extend the use of on-load tap changer (OLTC)-fitted transformers from primary to secondary substations. Traditionally, three phase two-winding transformers installed on secondary substations (i.e. 10kV/0.4kV and 20kV/0.4kV in Ireland) are not capable of changing their turns ratio under on-load conditions. However, the use of OLTCs would allow DSOs to have greater control flexibility at the LV distribution level by modifying transformers ratio, whether limits are violated, without the need of interrupting service.

To archive a secure integration of PVs, one possibility is to focus on the capabilities of centralized control schemes like the one in [3]. These strategies formulate an optimization problem subject to equality constraints derived from power flow methods. In presence of OLTC-fitted transformers, feasible tap

positions are considered as decision variable and the problem solved according to real time measurements and forecast signals. Another solution can come from “decision-making” approaches like the one proposed in [4]. In this work, the operation of the OLTC is done under a pre-defined set of rules or based on a remote monitoring-based architecture. In both works, observability of remote points is either imperative or ends in outperforming results. Nonetheless, monitoring-/communication equipment is generally inexistent at the LV level and its installation would represent a costly investment.

As alternative to remote monitoring, recent works [5]–[7] have proposed ways of increasing network observability under the premise of limiting measurement resources. The authors in [5] apply traditional distribution system state estimation (DSSE) and replace real-time measurements by pseudo-measurements obtained from statistical analysis. However, it is assumed that loads and PVs are three-phase and balanced. On a different approach, pre-computed fitted second degree polynomial equations are used in [6] to accurately estimate remote variables in medium voltage distribution networks. Again, under the assumption of a balanced system. Finally, the methodology introduced in [7] allows to estimate voltages at the end point of unbalanced LV feeders based on active and reactive power flow measurements at the transformer level. It is assumed that all customers have equal power demand and are evenly distributed along the main cable path. However, these assumptions may result in lower performances in strongly ramified feeders. Furthermore, the simplified voltage calculations consider no voltage dependent loads and negligible power losses.

The objective of this work is to introduce a methodology able to accurately estimate remote voltages in unbalanced LV feeders by only means of power flows measurements at the head of the feeder. This approach intends to improve traditional line-drop compensation in OLTCs and its limitations in ramified networks with high penetration of DG and possible variant power factor [8]. The proposed methodology combines traditional maximum likelihood estimation with a simplified formulation of power flow constraints based on the derivatives of second-degree polynomials describing the evolution of specific variables. This results in a quadratic programming problem that due to its simplicity, can be easily solved and applied in real-time operation. Section II outlines the proposed voltage estimation methodology. The simulation environment, test system under investigation and results are provided in Section III. Finally, conclusions are drawn in Section IV.

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## II. METHODOLOGY

In the proposed approach, maximum likelihood estimation (MLE) is used to determine the most likely consumption of residential loads, based on their statistical model and flow measurements at the head of the feeder. The constraint functions considered in the MLE formulation diverge from those used in traditional State Estimation and derive from an off-line sensitivity analysis. Once load consumptions are estimated, voltages are obtained from direct calculations.

### A. Maximum Likelihood Estimation

The parameters of a statistical model can be estimated using maximum likelihood estimation given a finite set of measurements or observations. This well-known method is widely used in statistics and for traditional power system's state estimation [9]. In MLE, the deviation or error of each individual measurement from its expected or real value is characterized by a Gaussian (normal) distribution function:

$$f(z_i) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{z_i - E(z_i)}{\sigma_i}\right)^2} \quad (1)$$

where  $z_i$  represents the  $i$ th measurement,  $E(z_i)$  is the expected value of  $z_i$  and  $\sigma_i$  is the standard deviation of  $z_i$ . The joint probability density function or "likelihood function" represents the probability of having  $m$  independent measurements, each of them with their corresponding normal distribution.

$$f_m(z) = f(z_1)f(z_2) \dots f(z_m) \quad (2)$$

$$z = [z_1, z_2, \dots, z_m]$$

As formulated in this work, MLE maximizes the likelihood function (2) of a vector  $z$  of active and reactive power consumption pseudo-measurements obtained from a statistical model of residential loads. This means obtaining the most likely expected value for each  $i$ th pseudo-measurement. As distributions are normal, the joint probability function can be replaced by its logarithm and express the problem as:

$$\text{minimize} \quad \sum_{i=1}^m \left( \frac{z_i - E(z_i)}{\sigma_i} \right)^2 \quad (3)$$

This problem is subject to a series of constraints that will be described later in this paper.

### B. Role of Statistical Residential Loads Model

In order to estimate remote system variables it is needed to expand the available data (set of local flow measurements at the head of the feeder) and increase observability. However, due to the lack of remote monitoring equipment, the vector  $z$  is constructed with pseudo-measurements obtained from a statistical residential load model. The model used for this work is based on the tool developed in [10]. This tool creates 1 minute resolution daily time-series profiles for single-phase residential customers taking into account house occupancy, activity, electrical appliances and thermal operation.

Fig. 1 shows the probability distribution of active power demand for residential loads' with a time step of 60' (for visualization purposes) as boxplots. Medium horizontal bars

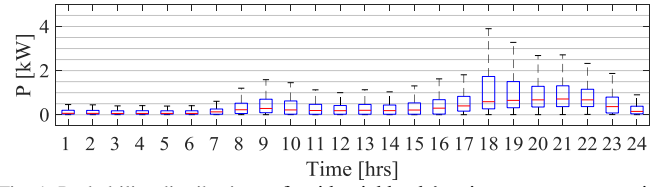


Fig. 1. Probability distributions of residential loads' active power consumption for a typical winter weekday.

reveal median values and rectangles and slim black lines 50% and 99.3% of results distribution respectively. The values at a specific time step of each pseudo-measurement  $z_i$  and its standard deviation  $\sigma_i$  will derive from these distributions. As this is done for both active and reactive power demand, the size of  $z$  will be twice the number  $l$  of houses (i.e.,  $m = 2l$ ).

### C. Power Flow Equations and MLE set of constraints

#### 1) Sensitivity analysis

In the context of distribution networks, for any system scenario, the evolution of any variable with respect to the variations of nodal active and reactive power injections can be evaluated from the point of view of any node by performing power flow calculations under different conditions [6], [11]. Each possible scenario is defined by one load modifying its power consumption while all others remain constant as their state influences the evolution of flows and voltages.

Building on the algorithm introduced in [6] for balanced medium voltage distribution networks state estimation and the consequent patent [11], fitted polynomials are employed here for sensitivity analysis in unbalanced LV residential feeders. Using non-linear regression analysis, a surface of best fit is formulated to the trend of any desired variable (e.g. nodal voltage, branch current, power flows, etc.) against any possible combination of active power  $P_i$  and reactive power  $Q_i$  consumption for the  $i$ th residential load. This analysis results in a second-order, two-variable polynomial equation in active and reactive power consumption at a customer point of connection (CPOC). A total of six shaping coefficients  $b_j$  needs to be determined for the polynomial parametric equation for each possible dependent single-phase variable  $X_p$ :

$$X_p = b_1 P_i^2 + b_2 P_i + b_3 P_i Q_i + b_4 Q_i^2 + b_5 Q_i + b_6 \quad (4)$$

$$\text{where } b_j = g_{p,j}(V_k), \quad (j=1, \dots, 6), \quad (p=1, 2, 3)$$

The  $b_j$  coefficients are not constant and depend on the current system scenario. However, they are adjusted with a linear function  $g_{p,j}$  of the phase-to-neutral voltage at a reference node  $k$ . Index  $p$  counts for phases  $a$ ,  $b$  and  $c$ .

A typical polynomial characterizing a variable's evolution is illustrated in Fig. 2 where the magnitude of phase  $a$  current flow through a feeder's branch is plotted against the active and reactive power consumptions  $P_i$  and  $Q_i$  of a customer connected to the same phase and located somewhere in the feeder. It can be observed how the polynomial obtained from the regression analysis produces a surface that fits the black dots symbolizing current values for different  $P_i$ - $Q_i$  combinations. These polynomials allow to represent the correlation of variables magnitudes leaving aside their corresponding phasor angles.

The variation of any  $X_p$  with respect to any  $P_i$  and  $Q_i$

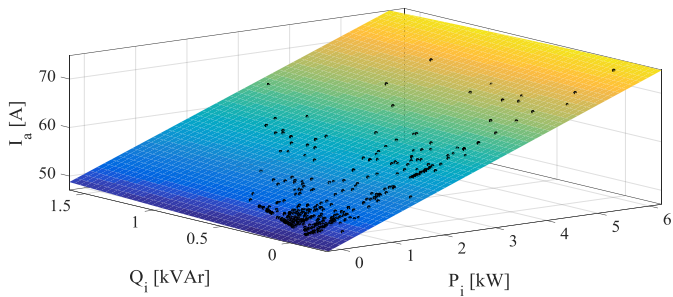


Fig. 2. Branch phase  $a$  current magnitude as function of active and reactive power demand at a remote CPOC.

can be obtained from the definite integrals of the partial derivatives of (4) evaluated for active power injection variations  $\Delta P_i$  and reactive power injection variations  $\Delta Q_i$ :

$$\int_{P_i^{initial}}^{P_i^{final}} \frac{dX_p}{dP} dP = b_1 \Delta P_i^2 + b_2 \Delta P_i + b_3 \Delta P_i Q_i^{initial} \quad (5)$$

$$\Delta P_i = P_i^{final} - P_i^{initial}$$

$$\int_{Q_i^{initial}}^{Q_i^{final}} \frac{dX_p}{dQ} dQ = b_3 P_i^{initial} \Delta Q_i + b_4 \Delta Q_i^2 + b_5 \Delta Q_i \quad (6)$$

$$\Delta Q_i = Q_i^{final} - Q_i^{initial}$$

Here, sensitivities are calculated exclusively from 2 very simple parametric equations for any network state.

### 2) Power flow closed-loop calculation

Applying the sensitivities presented in the previous subsection, it is proposed to replace traditional unbalanced three-phase Newton Rapson power flow method by a cumulative close-loop calculation as the one described by the flow chart in Fig. 3. This figure shows the process of obtaining the phase-to-neutral voltage magnitudes of phases  $a$ ,  $b$  and  $c$  for the  $k$ th node where  $k$  belongs to the network set of buses excluding the slack. If all network voltages want to be obtained, the process must be repeated for each node on the set.

At the beginning of the calculation for each phase  $p$ , the voltage at node  $k$ ,  $V_{k,p}$ , is set to be equal to its voltage level at no-load conditions. Then, the effect on  $V_{k,p}$  of each individual load  $i$  is added to the voltage calculated from the previous step according to (5)-(6). For this iterative addition, loads are ordered according their electric distance to the head of the feeder. For the  $i$ th calculation, each polynomial coefficient  $b_i$  is obtained from the linear equation  $g_{v,p,j}$  using as reference the voltage obtained from the previous stage  $V_{k,p,i-1}$ . Furthermore, as the initial values for  $\Delta P_i$  and  $\Delta Q_i$  are considered to be zero, the polynomial term corresponding to  $b_3$  disappears.

### 3) Final MLE formulation and voltage estimation

In order to proceed with the closed-loop voltage calculation,  $P_i^{final}$  and  $Q_i^{final}$  for all loads are required as input. However, as remote monitoring is inexistent, this values are not directly available. Nevertheless, maximum likelihood estimation (MLE) combined with the pseudo-measurements introduced in Section II.B can provide the most likely values for active and reactive power demands based on a suitable set of constraints.

In traditional State Estimation (SE), the expected value  $E(z_i)$  for the measurement  $z_i$  is expressed as a non-linear

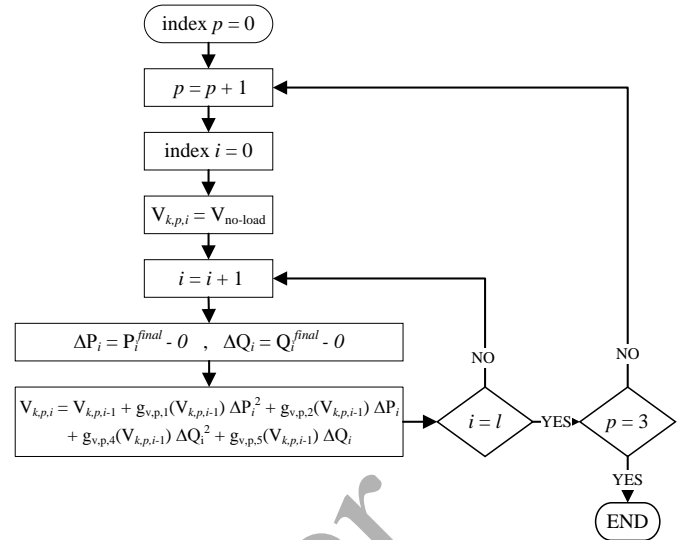


Fig. 3. “Closed-loop” power flow calculation

function relating the system state vector to the  $i$ th measurement [9]. The MLE problem is then formulated as the objective function (3) subject to a set of  $m$  equality constraints linking measurements with nodal voltages modules and phase angles. For the particular case of an unbalanced three-phase network with explicit representation of the neutral, this means a numerous number of state variables (eight variables per node) coupled through complex non-linear equations [3]. This would result on a difficult problem that if managed to be solved would require considerable computational time.

Due to the need of a platform that provides a solution in real-time control applications, the non-linear constraints used in SE must be replaced by a simpler set of equations. As explained in Subsection II.C.1), the integrals (5) and (6) are not only limited to voltage magnitudes and can be used for any kind of variable. Therefore, the available active power, reactive power and current flow three-phase measurements at the transformer level can be express as a function of every  $\Delta P_i$  and  $\Delta Q_i$  as long as the  $b_j$  coefficients are those relating flow variations at the head of the feeder with power demand variations at each CPOC. Then, a cumulative close-loop calculation of flows could be included in the procedure from Fig. 3. However, such calculation would result on an equation having the coefficients  $b_j$  for the  $i$ th load as complex functions of the voltages obtained from previous stages. Nonetheless, if node  $k$  is considered as the transformer’s secondary busbar it can be assumed that  $V_{k,p}$  will suffer very small variations during the cumulative addition of loads. Then, express every  $b_j$  as a linear function of node  $k$ ’s no-load voltage. Therefore, with every coefficient considered as a constant parameter, measured flows could be expressed as:

$$X_{flow_p} = \sum_{i=1}^l b_1^* P_i^2 + b_2^* P_i + b_4^* Q_i^2 + b_5^* Q_i \quad (7)$$

where the star \*, indicates that coefficients correspond to no-load conditions. Even if it is not indicated for simplicity, every  $b_j$  is linked to the specific measured phase  $p$ .

It is possible now to formulate the optimization problem as the MLE objective function subject to equality constraints of the

form of (7). Where the number of constraints will be equal to the number of measured variables multiplied by the number of phases:

$$\text{minimize } \sum_{i=1}^m \left( \frac{z_i - E(z_i)}{\sigma_i} \right)^2 \quad (8)$$

$$\text{s. t. : } P_{flow_p} = \sum_{i=1}^l (b_{p,1,p}^* P_i^2 + b_{p,2,p}^* P_i + b_{p,A,p}^* Q_i^2 + b_{p,5,p}^* Q_i)$$

$$Q_{flow_p} = \sum_{i=1}^l (b_{Q,1,p}^* P_i^2 + b_{Q,2,p}^* P_i + b_{Q,A,p}^* Q_i^2 + b_{Q,5,p}^* Q_i)$$

$$P_i \geq 0$$

$$(p = 1,2,3)$$

An inequality constraint restricting active power demand is added to prevent it from taking negative values (residential demand distributions have mean values closed to zero). It can be seen that current flows are not included in the set of constraints. As this formulations are simplified versions of power flow equations they will have an associated error. Considering active and reactive power flows together with current flows using non-exact equations would lead to infeasibility.

The results of this quadratic optimization problem are the most likely consumptions of residential loads, based on their statistic model and flow measurements at the head of the feeder. Once obtained, they are used as input for the closed-loop calculation to estimate voltages. As not all nodes are of interest, those at the end of the feeder will be more suitable to voltage limits violation, the close-loop method is only run a very limited number of times.

### III. CASE STUDY

#### A. Test Feeder

A real LV feeder selected from a set of open networks published by Electricity North West (ENWL) [12] is used as test network. This selection is done according to the results obtained in [1] where cluster 7, with a representative feeder of similar features, englobes the most common purely residential feeders that are expected to present technical problems due to high penetrations of PVs. The feeder itself consist of 83 single-phase houses and 1,684 meters of 4-wire cable (3 phases and neutral). It is depicted in Fig. 4 where single-phase customers are identified by dots and the head of the feeder represented by a black triangle. In compliance with EN50160 [2], a nominal line-to-neutral voltage of 230V is adopted. The network is modelled in OpenDSS which is an open-source electric power distribution system simulator [13].

#### B. Simulation Platform

A com link is established between MATLAB [14], OpenDSS and GAMS [15] in a computer with an Intel Core i7-4790 CPU. OpenDSS acts as the real network and provides flow values to MATLAB through a com interface. The values for each active and reactive power flow are send to GAMS which contains the quadratic optimization problem that is solved using Knitro [16]. Finally, MATLAB retrieves the most likely power demands and obtain remote voltages using the close-loop voltage calculation.

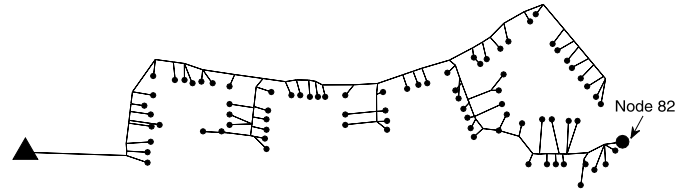


Fig. 4. Real residential LV feeder

#### C. Validation and Results

In order to assess the accuracy of the methodology and the validity of the statements introduced in Section II, a Monte Carlo based analysis was pursued. A total of 10,000 scenarios were created by randomly gathering load profiles from a pool generated with [10] and allocating them to each house. Simulations were tested at different time steps for a typical week-day. Here, we concentrate particularly on estimations based on local measurements taken at 6pm. This moment of the day is of special interest as it matches the highest standard deviation of load's parametric distributions. Therefore, corresponds to a worst case test scenario.

Fig. 5 to Fig. 9 present the percent error between the values derived from the methods described in Section II.C and those obtained from the power flow calculation performed by OpenDSS. The new set of boxplots used to represent data include outlier values (small crosses). For instance, Fig. 5 shows the case of phase-to-neutral voltage magnitudes calculated with the close-loop voltage method from Fig. 3. For each Monte Carlo scenario, the maximum error of each phase for the total set of 83 nodes was stored to account for the worst case of each simulation. As it can be observed, errors are significantly low and remain within a 0.035% for 50% of the sample and within 0.055% for the 99.3%. On the other hand, Fig. 6 and Fig. 7 include the error boxplots for the power flows computed from equation (7). While the error for active power flow at the head of the feeder remains below 1.2% and 2.0% for 50% and 99.3% of the cases, it can go up to 10% and 25% for reactive power flow. Still, these values are considerably low, especially taking into account that the polynomial coefficients of the quadratic

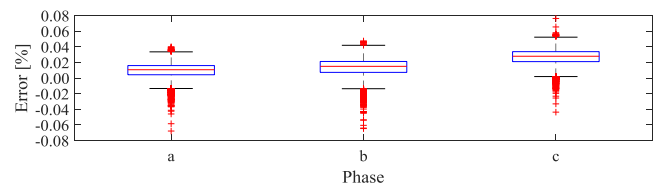


Fig. 5. Voltage closed-loop calculation error – 6pm

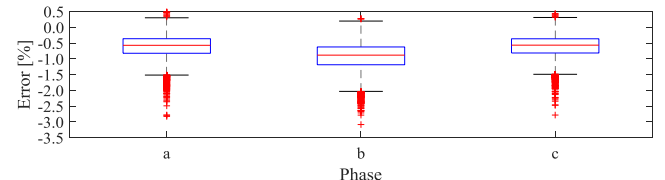


Fig. 6. Active power flow calculation error – 6pm

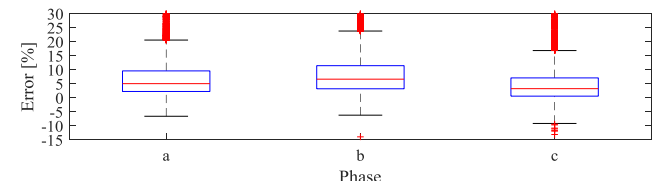


Fig. 7. Reactive power flow calculation error – 6pm

#### IV. CONCLUSIONS

This work introduces a novel methodology for remote voltages estimation in unbalanced LV feeders by only means of power flows measurements at the head of the feeder. A quadratic optimization problem that combines MLE with pseudo-measurements obtained from residential load models and simplified power flows constraints results in the most likely power consumptions for each load. These power estimations feed a closed-loop voltage calculation that provides with high accuracy three-phase voltage magnitudes for remote feeder nodes. Crucially, due to its simplicity, the MLE problem can be easily solved and applied in real-time operations.

It is worth mentioning that this methodology conceptually differs from traditional power system State Estimation. Even if similarities exist, real measurements are not included in the objective function. Actually, real monitoring represents a crucial but small component of the problem. Finally, state variables are not part of the formulation which presents real measurements as a function of the pseudo-measurements.

The results in this paper are limited to feeders with unique presence of residential loads. However, the obtained results present the methodology as a potential way to increase observability in distribution networks without the need of remote monitoring infrastructure. Further work will focus on improving the accuracy of the power flow equations in the MLE problem and considering the presence of DG units.

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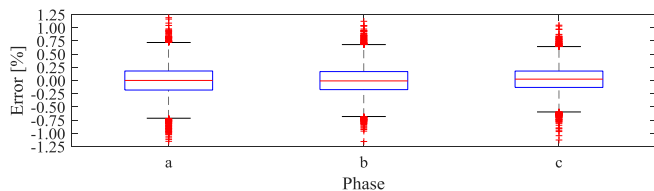


Fig. 8. Node 82 voltage estimation error – 6 pm

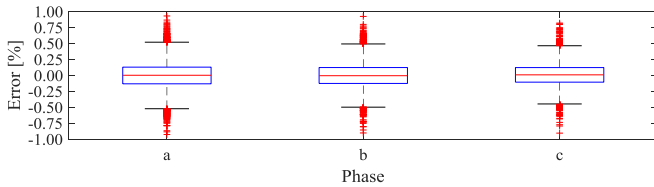


Fig. 9. Node 82 voltage estimation error – 12 pm

equation (7) are assumed constant allowing a simple formulation of the MLE problem.

In each of the 10000 scenarios, the OpenDSS values for active power and reactive power flows at the head of the feeders were captured and sent to the MALAB-GAMS interface. Once the MLE problem was solved and the most likely customers' demand obtained, the closed-loop calculation was performed and node voltages computed. Fig. 8 shows the relative error of the estimated voltages for node 82 (see Fig. 4). As this node has the longest impedance path to the head of the feeder, it is very likely to present voltage violations. From this illustration, it can be seen that 50% of the sample lays within a 0.25% error band and increases up to approximately 0.75% when englobing 99.3% of scenarios. These small values become even less significant if we put them in the context of a control strategy. A theoretical error of 0.75% like the one in Fig. 8 would allow to claim (with a 99.3% accuracy) that the lower voltage limit has not been violated as long as the estimated voltage of all nodes are higher than 0.907p.u. (lower limit of 0.9 p.u.).

Fig. 9 reveals remote estimation errors for node 82, now obtained from measurements at 12pm. For this case, divergences from real values remain within 0.15% and 0.55% for 50% and 99.3% of the sample. These smaller errors, compared to those obtained at 6pm, are due to the lower variation of residential loads parameters. With lower standard deviations of the active and reactive power pseudo-measurements, the maximum likelihood estimation results in a higher probability of having the estimated set of  $m$  measurements. It must be mentioned that this better performance is limited to the case in which only residential loads are considered. For instance, the presence of PVs included as measurements or pseudo-measurements in (8) would probably increase these errors as higher variations from the expected power injections would take place.

In terms of computational effort, the average and maximum times taken by Knitro to solve the optimization problem were 0.008 and 0.016 seconds. However, to have an idea of the real-time applicability of the methodology, the overall computational time needed to obtain the voltage of node 82 was quantified (this includes: problem compilation, "communications" and voltage calculation). This time was found to be in the order of a couple of tenths of second with average and maximum values of 1.16 and 1.21 seconds respectively. Therefore, the estimation package should not struggle to track real measurements and provide estimations in an efficient way.