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Dynamic amplification of a multi-span, continuous orthotropic bridge deck under vehicular movement

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The response of a multi-span, continuous orthotropic bridge deck during truck loading is investigated to better understand the dynamic interaction between moving vehicles and highway bridge decks. The present study is based on a recently published, semi-analytical approach for free vibration in which the modal superposition method incorporates intermodal coupling. Herein, the bridge deck is modeled as a jointless, multi-span, orthotropic plate, and the vehicle is modeled as a dynamic, multi-body system. The road surface roughness randomness is modeled as a normal, stationary, random process described by its PSD. The coupled equations of the motion vehicle/bridge deck are solved by Newmark's method. An iterative process in each time step is performed to find the equilibrium between the bridge deck and vehicle tires using an uncoupled algorithm previously developed by other authors. Two numerical application examples are presented: an isotropic and an orthotropic, three-span bridge deck both crossed by an AASHTO-based vehicle model. In example one, the intermodal coupling affects the dynamic deflection of bridge deck but only slightly. Example two demonstrates that the loading mode and the vehicle speed have a significant influence on the Dynamic Amplification Factor. However, the most important parameter to affect the

dynamic vehicle/bridge deck interaction force is the road's surface roughness, as has been shown for other bridge types under various load conditions.

Key-words: Dynamic behavior, Multi-span continuous orthotropic plate, Intermodal coupling, Interaction bridge deck/vehicle, Newmark's method

Nomenclature

A_r	Roughness coefficient ($\text{m}^3 \cdot \text{cycle}^{-1}$)
a_1, a_2	Eccentricities (m)
b	Width of the bridge deck (m)
C	Damping coefficient of the bridge deck ($\text{N} \cdot \text{s} \cdot \text{m}^{-1}$)
c_v	Damping of v th vehicle suspension ($\text{N} \cdot \text{s} \cdot \text{m}^{-1}$)
C_{ij}, c_{ij}	Modal damping, generalized damping ($\text{N} \cdot \text{s} \cdot \text{m}^{-1}$)
c_{sk}	Damping of the k th suspension of the vehicle ($\text{N} \cdot \text{s} \cdot \text{m}^{-1}$)
c_{tk}	Damping of the k th tire of the vehicle ($\text{N} \cdot \text{s} \cdot \text{m}^{-1}$)
$[C_v]$	Damping matrix of the vehicle
DAF	Dynamic Amplification Factor
D_x, D_y	Flexural rigidities for the x - and y -directions respectively (N.m)
D_{xy}	Flexural rigidity for the x - y plane (N.m)
δ	Dirac operator
Δt	Time step (s)
$\Delta \omega$	Frequency step ($\text{cycle} \cdot \text{m}^{-1}$)
E_x, E_y	Young's moduli for the x - and y -directions respectively ($\text{N} \cdot \text{m}^{-2}$)
F_{bk}^{int}	Interaction force between k th wheel of the bridge deck and vehicle (N)
$\{F_g\}$	Force vector caused by the gravitation effect of vehicle
F_{ij}	Modal forces (N)
$\{F_w^{\text{int}}\}$	Interaction force vector applied on the bridge deck
$\{F_v^{\text{int}}\}$	Interaction force vector applied on the vehicle
G_{xy}	Shear modulus in bending for the x - y plane ($\text{N} \cdot \text{m}^{-2}$)

h	Thickness of the bridge deck (m)
H	Equivalent rigidity of the bridge deck (N.m)
$I_{\theta 1}, I_{\theta 2}$	Moments of inertia of the front and rear vehicle axles respectively (kg.m^2)
$I_{\theta v}, I_{\alpha v}$	Moments of inertia of pitch and roll of the rigid block of the vehicle respectively (kg.m^2)
K_{ij}, k_{ij}	Modal rigidities, generalized rigidities (N.m^{-1})
k_{sk}	Rigidity in k th vehicle suspension (N.m^{-1})
k_{tk}	Rigidity in k th vehicle tire (N.m^{-1})
$[K_v]$	Rigidity matrix of vehicle
L	Length of the bridge deck (m)
l_r	Length of the r th span of the bridge deck (m)
m	Mode number in y -direction
\bar{m}	Mass per unit surface (kg.m^{-2})
M_{ij}, m_{ij}	Modal masses, generalized masses (kg)
m_v	Mass of the rigid block of the vehicle (kg)
$[M_v]$	Mass matrix of the vehicle
m_1, m_2	Equivalent masses of front and rear wheels with axles respectively (kg)
n	Mode number in x - direction
N	Discrete wave number in frequency domain
nf	Number of forces
q_{ij}	Generalized displacements (generalized coordinates) (m)
$r(x_k)$	Roughness of the static profile at the k th contact point (m)
S	Surface of the bridge deck (m^2)
S_r	Power Spectral Density ($\text{m}^3.\text{cycle}^{-1}$)
S_x	Spacing between the front and rear vehicle axles (m)

s_{t1}, s_{t2}	Spacing between vehicle tires of the front and the rear respectively (m)
s_1, s_2	Spacing between vehicle axles of the front and the rear respectively (m)
t	time (s)
$w(x,y,t)$	Vertical displacement of the bridge deck (m)
$w_k = w(x_k, y_k, t)$	Vertical displacement of the bridge deck in the k th contact point (m)
x, y, z	Axis of the reference system
$\{Z_v\}$	Vector of degrees of freedom of the vehicle
z_v	Vertical displacement of the rigid block of the vehicle (bounding) (m)
z_1, z_2	Vertical displacements of the front and rear vehicle axles respectively (m)
α_v	roll displacement of the rigid body of the vehicle (rad)
γ, β	Stability parameters of the Newmark's method
ε	Tolerance of convergence
ξ_{ij}	Modal damping factors
θ_i	Random phase varies between 0 and 2π
θ_v	Pitch of the rigid block of the vehicle (rad)
θ_1, θ_2	Pitch of the front and rear vehicle axles, respectively (rad)
v_x	Traveling speed (m.s^{-1})
ν_{xy}, ν_{yx}	Poisson's ratios
$\phi_{ij}(x,y)$	Mode shapes of multi-span continuous bridge deck
ω_{ij}	Natural frequencies of multi-span continuous bridge deck (rad.s^{-1})
ω_s	Spatial frequency (cycle.m^{-1})
ω_{si}	Wave number (cycle.m^{-1})
ω_{s0}	Discontinuity frequency (cycle.m^{-1})

1. Introduction

Heavy vehicles crossing highway bridges at high speeds can cause dynamic effects that amplify the effects of the static loads, because of the increase in the magnitude of the dynamic interaction forces. This problem of dynamic interaction is gaining importance bridge designers and owners, because of increased road traffic (both in intensity and passage frequency) and higher traffic speeds. For nearly a century, the dynamics of bridges under moving loads has been studied. Initial efforts were devoted to the development of analytical and semi-analytical solutions for simple cases of beams with one or more spans excited by moving forces or masses [1-8]. The dynamic interaction between a rolling slab of a bridge and a moving vehicle is very complex. In much of this work, important factors were generally identified as the rolling slab, the vehicle as the excitation source due to the speed of movement, and the road surface roughness as an interface between the vehicle and the running surface. This last factor is especially important, because it causes oscillations of both the moving vehicle and the bridge, which enables the inertial forces and damping to significantly amplify the interaction forces. Other factors have been proposed, although not universally adopted such as the axle distance to bridge span length ratio [9] and the non-dimensional speed parameter defined as the ratio of the driving frequency of the vehicle to the fundamental frequency of the beam for a simple, continuous beam [10].

The ratio of the maximum dynamic response of the structure on the maximum static response of the same structure under the action of the same load calculated at the same cross section of the bridge deck is called the dynamic amplification factor (DAF) [11]. The responses may be increased deflections, bending moments, or shear forces. Deng and Cai [12] identified the critical parameters for a coupled model by optimizing an objective function, using the residual between the measured response time history and predicted response time

history using a genetic algorithm. The factors include the dynamic properties of the vehicle and the bridge, the vehicular speed, and the pavement roughness.

Much of the research in this area has been related to quantifying the dynamic amplification factor (DAF), is an important phenomenon in the design of all bridges irrespective of application or construction material [11]. The DAF may increase deflections, bending moments, or shear forces. Some of the earliest work on DAF and road surface roughness was on done by Law and Zhu [13], in which surface roughness was modeled as a Gaussian random process with a Karhunen-Loève expansion. The bridge was modeled as a simply supported planar Euler-Bernoulli beam and the vehicle by a four degrees-of-freedom mass–spring system. This was an outgrowth of work by Green and Cebon [14], who modeled a rolling slab of a highway bridge as an isotropic single-span plate and the vehicle crossing by a mobile dynamic system with multiple DOF. They employed a modal method coupled with the convolution integral to solve the equation of motion. This interaction was similarly studied by Henchi et al. [15] but using a general finite element (FE) method combined with Newmark’s method to solve the coupled equations of motion. A strictly semi-analytical approach was introduced for this problem by Marchesiello et al. [16] where the dynamic response of the bridge slab (modeled as an isotropic, multi-span plate) was obtained by the modal method, and the frequencies and mode shapes were calculated by the Rayleigh-Ritz method. Using a very similar approach, Zhu and Law [17] modeled a multi-span, rolling slab as an orthotropic rectangular plate with several intermediate rigid supports and the pavement profile with a Power Spectral Density (PSD) to investigate the influence of the vehicle position, rolling speed, and road irregularities.

While most research has concerned itself with overall structural behavior, some more focused studies have been undertaken such as that by Huang [18] who studied the stiffness effects of the longitudinal girder in the presence of moving vehicles and road irregularities.

The bridge deck and vehicle were modeled three-dimensionally and the road irregularities by a stationary Gaussian random process. Newmark's integration method was used to solve the resulting equations of motion.

Other researchers have focused more closely on specific bridge types such as the study by Yin et al. [19] on a high-pier bridge subjected to a variable speed vehicle. In that case, the vehicle was modeled by a dynamic model with 12 DOF, and the contact between the vehicle wheels and the bridge was considered as a patch contact. The mid-span displacements were calculated and compared with measured displacements under different parameters including vehicle acceleration and vehicle deceleration. Another example is Jiang et al.'s study on Fiber Reinforced Polymer (FRP) composite materials in bridge structures [20] looking specifically at the shear stud connections under a simplified, moving truckload. This study was based on both a field test and finite element modeling. In the FE analysis, the connection between the steel girders and the FRP deck was simulated as fully and partially composite, separately. The authors demonstrated a greater response (both static and dynamic) in the partially composite model compared to the fully composite one. Additionally, the number of shear stud connections also affected the dynamic deflection, slip, and separation.

Previously, Dinga et al. [21] proposed an evolutionary spectral method to investigate road surface irregularities on DAF on a single span, concrete bridge for vehicles moving at a constant speed. The problem was modeled in two parts: the deterministic moving dynamic force induced by the vehicle weight and the random interaction for induced by the pavement roughness. Each part was calculated using the Runge-Kutta method and then summed. In an alternative approach, Asnachinda et al. [22] updated the static component (USC) technique to improve the accuracy and eliminate the difficulty of an optimal regularization selection for a three-span, continuous bridge subjected to multiple vehicles overtaking and travelling side-by-side. The topic was also considered by Zhang et al. [23] in an FE model to evaluate the

structural performance of a long-span, cable-stayed bridge, under multi-scale, dynamic loads with road surface roughness.

The work by Oliva et al. [24] considered a fully-coupled FE model using parallel road profiles pairs under the assumptions of surface isotropy and homogeneity. The cross-correlation function was solved in a semi-analytical way, and then by means of a least squares fitting technique. A simple expression was proposed for the coherence function. The approach was tested on two bridges and 10 surfaces with satisfactory result. Similar work was done by Moghimi and Ronagh [25] on a single span composite steel bridge. A 3D FE model was used that incorporated special end springs that simulated the elastomeric bearings. The approach was successful, as long as the truck weight was at least 10 percent of the total deck weight and the vehicle's placement represented only minimal eccentricity. Strong dependencies on the individual vehicle and its speed were demonstrated.

Despite this significant body of work, what remains poorly defined is the dynamic truck-bridge interaction in the presence of random irregularities for a jointless, multi-span, composite bridge, which is the topic of this paper. The bridge deck is modeled as an orthotropic, three-span plate. The truck is modeled by the H20-44 model, with seven DOF [16,17]. The road surface roughness is modeled by a random function characterized by a PSD and spectral roughness coefficient. The modal approach coupled with a numerical integration by Newmark's method is used to solve the coupled equations of the bridge-truck motion. Solving these equations is achieved in an uncoupled manner using iterative calculations.

2. Numerical modeling

2.1. Bridge deck modeling

The approach of homogenizing various bridge section types as orthotropic plates is a well-established technique verified elsewhere [e.g. 26]. The bridge deck is modeled by a thin rectangular, continuous plate of R span, orthotropic material, length l , width b , thickness h , and mass per unit surface \bar{m} . The bridge deck is simply supported on two edges and free on the other two edges (Fig. 1). The intermediate supports are simple, rigid, and perpendicular to the free edges. As per [27], the equation governing the vertical motion of the bridge deck is written as shown in Eq. (1):

$$\bar{m} \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = - \sum_{k=1}^{nf} F_{wk}^{\text{int}} \delta(x - x_k(t)) \delta(y - y_k(t)) \quad (1)$$

where $w(x, y, t)$ is the vertical displacement of the bridge deck, $D_x = E_x h^3 / 12(1 - \nu_{xy} \nu_{yx})$ and $D_y = D_x E_y / E_x$ are the bending rigidities in x - and y -directions, respectively; the term $H = \nu_{xy} D_y + 2D_{xy}$ is the equivalent flexural rigidity, and ν_{xy} and ν_{yx} are the Poisson's ratios in the x - and y -directions, respectively. Furthermore $D_{xy} = G_{xy} h^3 / 12$ is the bending stiffness in the x - y plane, G_{xy} is the shear modulus in the x - y plane, and E_x and E_y are Young's moduli in x - and y -directions, respectively. Finally, c is the viscous damping coefficient of the material of the bridge deck, F_{wk}^{int} is the interaction force between the k th wheel vehicle and the bridge deck that moves at a constant velocity v_x along the bridge length (Fig. 1), δ is the Dirac operator, and $(x_k(t), y_k(t))$ is the position of k th interaction force on the bridge deck.

The solution of Eq. (1) is achieved by the modal method by decomposing the vertical displacement of the bridge deck in the modal basis as follows:

$$w(x, y, t) = \sum_{i=1}^n \sum_{j=1}^m \phi_{ij}(x, y) q_{ij}(t) \quad (2)$$

where $q_{ij}(t)$ are the modal coordinates, $\phi_{ij}(x, y)$ are the mode shapes of a thin, orthotropic multi-span plate (taking into account intermodal coupling, which is associated with the natural frequencies ω_{ij} , as detailed in references [28] and [29]).

Substituting expression (2) into the Eq. (1), then multiplying both sides of the latter by $\phi_{kl}(x, y)$ and integrating across the surface S of the bridge deck and taking into account the orthogonality of the mode shapes, one obtains the decoupled system of modal equations as follows:

$$M_{ij} \ddot{q}_{ij} + C_{ij} \dot{q}_{ij} + K_{ij} q_{ij} = F_{ij} \quad (3)$$

with:

$$M_{ij} = \iint_S \bar{m} \phi_{ij}^2(x, y) dS \quad (4.1)$$

$$C_{ij} = \iint_S c \phi_{ij}^2(x, y) dS = 2\xi_{ij} \omega_{ij} M_{ij} \quad (4.2)$$

$$K_{ij} = \iint_S \left(D_x \frac{\partial^4 \phi_{ij}}{\partial x^4} + 2H \frac{\partial^4 \phi_{ij}}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 \phi_{ij}}{\partial y^4} \right) \phi_{ij}(x, y) dS = M_{ij} \omega_{ij}^2 \quad (4.3)$$

$$\begin{aligned} F_{ij} &= - \iint_S \sum_{k=1}^{nf} F_{wk}^{\text{int}} \delta(x - x_k(t)) \delta(y - y_k(t)) \phi_{ij}(x, y) dS \\ &= - \sum_{k=1}^{nf} F_{wk}^{\text{int}}(x_k(t), y_k(t)) \phi_{ij}(x_k(t), y_k(t)) \end{aligned} \quad (4.4)$$

Where M_{ij} , C_{ij} , K_{ij} and F_{ij} are the modal masses, modal damping, modal stiffness, and modal forces, respectively, and ξ_{ij} is the viscous modal damping factors ($\xi_{ij} = c/2\bar{m} \omega_{ij}$).

2.2. Vehicle modeling

The vehicle is modeled by a dynamic, mobile, multi-body system H20-44 with seven DOF in accordance with the American Association of State Highway and Transportation Officials (AASHTO [30]) (see Fig. 2); this is similar to that used by Marchesiello et al. [16] and Zhu and Law [17]. The rigid body of the vehicle has three DOF: the bounding z_v , the pitch θ_v , and the roll α_v . The shaking and the forward and backward roll of the vehicle axles are each represented by four DOF: z_1 and z_2 , are the vertical displacements of the front and the rear vehicle axles, respectively, and θ_1 and θ_2 are the rotations (pitches) of the front and the rear vehicle axles, respectively.

In Fig. 2, m_1 and m_2 are the masses of the wheels with their axles (front and rear, respectively). The terms m_v , I_{θ_v} , and I_{α_v} are the mass and moments of inertia (pitch and roll) of the rigid body of the vehicle, respectively. The moments of inertia of the front and the rear vehicle axles are, respectively, I_{θ_1} and I_{θ_2} , and the rigidity of the tires is k_{tk} , while c_{tk} is the tire damping. The term k_{sk} is the rigidity in the suspensions, c_{sk} is the suspension damping, s_{t1} , s_{t2} are the spacings between the contact points of tires from the front and the rear, respectively, and s_1 and s_2 are the spacing between the front and the rear vehicle axles, respectively.

Equations of motion of the vehicle model are obtained by applying the principle of dynamic equilibrium of forces and moments. The vertical displacements of the vehicle model are calculated from its static equilibrium position. Fig. 3 presents the forces and moments exerted on the system.

To determine the interaction forces of the vehicle/bridge deck, one must establish for each contact point between the bridge deck and the vehicle the displacements of the end of the springs and dashpots that are used to model the tires (Fig. 4). The interaction forces between the bridge deck and the vehicle's tires are then written as follows [27]:

$$F_1 = k_{t1} \left(z_1 - \frac{1}{2} s_{t1} \theta_1 - (w_1 + r_1) \right) + c_{t1} \left(\dot{z}_1 - \frac{1}{2} s_{t1} \dot{\theta}_1 - (\dot{w}_1 + \dot{r}_1) \right) \quad (5.1)$$

$$F_2 = k_{t2} \left(z_1 + \frac{1}{2} s_{t1} \theta_1 - (w_2 + r_2) \right) + c_{t2} \left(\dot{z}_1 + \frac{1}{2} s_{t1} \dot{\theta}_1 - (\dot{w}_2 + \dot{r}_2) \right) \quad (5.2)$$

$$F_3 = k_{t3} \left(z_2 - \frac{1}{2} s_{t2} \theta_2 - (w_3 + r_3) \right) + c_{t3} \left(\dot{z}_2 - \frac{1}{2} s_{t2} \dot{\theta}_2 - (\dot{w}_3 + \dot{r}_3) \right) \quad (5.3)$$

$$F_4 = k_{t4} \left(z_2 + \frac{1}{2} s_{t2} \theta_2 - (w_4 + r_4) \right) + c_{t4} \left(\dot{z}_2 + \frac{1}{2} s_{t2} \dot{\theta}_2 - (\dot{w}_4 + \dot{r}_4) \right) \quad (5.4)$$

With:

$$\dot{w}_k + \dot{r}_k = \left. \frac{\partial w}{\partial t} \right|_{x_k, y_k} + v_x \left(\left. \frac{\partial w}{\partial x} + \frac{\partial r}{\partial x} \right) \right|_{x_k, y_k}, \quad k = 1, \dots, 4 \quad (6)$$

In expressions (5.1-5.4), w_k and r_k are, respectively, the vertical displacements of the bridge deck and the roughness profile at the contact point k .

Determination of displacements at the ends of springs that model the vehicle's suspension permit evaluation of the suspension forces is as follows [27]:

$$f_1 = k_{s1} \left(z_v + a_1 s_x \theta_v - \frac{1}{2} s_1 \alpha_v - z_1 + \frac{1}{2} s_1 \theta_1 \right) + c_{s1} \left(\dot{z}_v + a_1 s_x \dot{\theta}_v - \frac{1}{2} s_1 \dot{\alpha}_v - \dot{z}_1 + \frac{1}{2} s_1 \dot{\theta}_1 \right) \quad (7.1)$$

$$f_2 = k_{s2} \left(z_v + a_1 s_x \theta_v + \frac{1}{2} s_1 \alpha_v - z_1 - \frac{1}{2} s_1 \theta_1 \right) + c_{s2} \left(\dot{z}_v + a_1 s_x \dot{\theta}_v + \frac{1}{2} s_1 \dot{\alpha}_v - \dot{z}_1 - \frac{1}{2} s_1 \dot{\theta}_1 \right) \quad (7.2)$$

$$f_3 = k_{s3} \left(z_v - a_2 s_x \theta_v - \frac{1}{2} s_2 \alpha_v - z_2 + \frac{1}{2} s_2 \theta_2 \right) + c_{s3} \left(\dot{z}_v - a_2 s_x \dot{\theta}_v - \frac{1}{2} s_2 \dot{\alpha}_v - \dot{z}_2 + \frac{1}{2} s_2 \dot{\theta}_2 \right) \quad (7.3)$$

$$f_4 = k_{s4} \left(z_v - a_2 s_x \theta_v + \frac{1}{2} s_2 \alpha_v - z_2 - \frac{1}{2} s_2 \theta_2 \right) + c_{s4} \left(\dot{z}_v - a_2 s_x \dot{\theta}_v + \frac{1}{2} s_2 \dot{\alpha}_v - \dot{z}_2 - \frac{1}{2} s_2 \dot{\theta}_2 \right) \quad (7.4)$$

The distribution of the vehicle weight across the four contact points with the bridge deck defines the vector of forces caused by the gravity effects [27]:

$$\{F_g\} = \{(m_v a_1 + m_1)g/2, (m_v a_1 + m_1)g/2, (m_v a_2 + m_2)g/2, (m_v a_2 + m_2)g/2\}^T \quad (8)$$

By adding the static contribution (8), the vector of interaction forces of the bridge deck acting on the vehicle then becomes:

$$\{F_{wk}^{int}\} = \{F_g\} + \{F_k\} \quad (9)$$

with:

$$\{F_k\} = \{F_1, F_2, F_3, F_4\}^T \quad (10)$$

$$\{F_{wk}^{int}\} = \{F_{w1}^{int}, F_{w2}^{int}, F_{w3}^{int}, F_{w4}^{int}\}^T \quad (11)$$

The equations of motion of the vehicle model with seven DOF are obtained by applying the dynamic equilibrium law of forces and moments for each DOF:

$$\text{for } z_v: \quad m_v \ddot{z}_v + f_1 + f_2 + f_3 + f_4 = 0 \quad (12.1)$$

$$\text{for } \theta_v: \quad I_{\theta_v} \ddot{\theta}_v + a_1 s_x (f_1 + f_2) - a_2 s_x (f_3 + f_4) = 0 \quad (12.2)$$

$$\text{for } \alpha_v: \quad I_{\alpha_v} \ddot{\alpha}_v + \frac{1}{2} s_1 (f_2 - f_1) + \frac{1}{2} s_2 (f_4 - f_3) = 0 \quad (12.3)$$

$$\text{for } z_1: \quad m_1 \ddot{z}_1 + F_1 + F_2 - f_1 - f_2 = 0 \quad (12.4)$$

$$\text{for } \theta_1: \quad I_{\theta_1} \ddot{\theta}_1 + \frac{1}{2} s_{t1} (F_2 - F_1) + \frac{1}{2} s_1 (f_1 - f_2) = 0 \quad (12.5)$$

$$\text{for } z_2: \quad m_2 \ddot{z}_2 + F_3 + F_4 - f_3 - f_4 = 0 \quad (12.6)$$

$$\text{for } \theta_2: \quad I_{\theta_2} \ddot{\theta}_2 + \frac{1}{2} s_{t1} (F_4 - F_3) + \frac{1}{2} s_2 (f_3 - f_4) = 0 \quad (12.7)$$

Substituting expressions of forces (5.1-5.4) and (7.1-7.4) into the equations of motion (12.1-12.7) enables Eq. (13) to be obtained after rearrangement and regrouping:

$$[M_v] \{\ddot{Z}_v\} + [C_v] \{\dot{Z}_v\} + [K_v] \{Z_v\} = \{F_v^{int}\} \quad (13)$$

where $\{F_v^{\text{int}}\}$ is the vector of interaction forces applied to the vehicle, $[M_v]$, $[C_v]$ and $[K_v]$ are the mass, damping, and stiffness matrices of the vehicle model, respectively (see Appendix A for further details).

2.3. Road surface modeling

The road's surface roughness is modeled by a random function characterized by a spectral roughness coefficient and a random variable. There are two approaches to define the characteristics of probabilistic random irregularities of road surfaces: auto-correlation and spectral density [27]. The static profile of the road surface can be modeled by a stationary Gaussian random process of zero mean characterized by a PSD for describing the quality of the running track. The PSD in the frequency domain S_r (depending on the spatial frequency $f_s = \omega_s/2\pi$) associated with this process is given by expression (14) as per [31,32]:

$$S_r(\omega_s) = A_r \left(\frac{\omega_s}{\omega_{s0}} \right)^{-2} \quad (14)$$

Where $A_r = A_r(\omega_{s0})$ is the spectral roughness coefficient (value of the spectral density), which characterizes the quality of the running track, and ω_{s0} is the discontinuity angular frequency ($\omega_{s0} = 1/2\pi$).

An approximate representation of a random Gaussian profile can be obtained from a PSD. This representation considers that the profile is the sum of a number of sine waves of random phases θ_i independent and uniformly distributed between 0 and 2π [31,32]:

$$r(x_k) = \sum_{i=1}^N \sqrt{4S_r(\omega_{si})\Delta\omega_s} \cos(\omega_{si}x_k + \theta_i) ; k = 1, 2, \dots, 4 \quad (15)$$

where N is the number of discretization points in the frequency domain, where ω_{si} is the wave number ($\omega_{si} = 2\pi i/L_c$), $\Delta\omega_s = 2\pi/L_c$, and L_c is typically twice the bridge deck length [15].

The expression of the discrete value of the PSD is given as (16):

$$S_r(\omega_{si}) = A_r \left(\frac{2\pi i}{L_c \omega_{s0}} \right)^{-2} \quad (16)$$

By substituting (16) into expression (15), one obtains:

$$r(x_k) = \sum_{i=1}^N \sqrt{4A_r \left(\frac{2\pi i}{L_c \omega_{s0}} \right)^{-2} \frac{2\pi}{L_c} \cos(\omega_{si} x_k + \theta_i)} ; k = 1, 2, \dots, 4 \quad (17)$$

where x_k is the longitudinal position of the k th vehicle wheel on the bridge deck.

2.4. Numerical integration of equations of motion

To solve the coupled equations of motion for the bridge deck-vehicle, Newmark's method is applied. Eq.(13), which governs the vehicle motion, is written at time $t+\Delta t$:

$$[M_v] \{\ddot{Z}_v\}_{t+\Delta t} + [C_v] \{\dot{Z}_v\}_{t+\Delta t} + [K_v] \{Z_v\}_{t+\Delta t} = \{F_v^{\text{int}}\}_{t+\Delta t} \quad (18)$$

Using Newmark's method, the displacement and velocity respectively are:

$$\{Z_v\}_{t+\Delta t} = \{Z_v\}_t + \Delta t \{\dot{Z}_v\}_t + \Delta t^2 (0,5 - \beta) \{\ddot{Z}_v\}_t + \beta \Delta t^2 \{\ddot{Z}_v\}_{t+\Delta t} \quad (19)$$

$$\{\dot{Z}_v\}_{t+\Delta t} = \{\dot{Z}_v\}_t + (1 - \gamma) \Delta t \{\ddot{Z}_v\}_t + \gamma \Delta t \{\ddot{Z}_v\}_{t+\Delta t} \quad (20)$$

where γ and β are the stability parameters of Newmark's method, and Δt is the integration time step. By replacing expressions (19) and (20) into Eq.(18) and applying factorization one obtains:

$$[S_v] \{\ddot{Z}_v\}_{t+\Delta t} + [C_v] \{\dot{Z}_v^*\}_{t+\Delta t} + [K_v] \{Z_v^*\}_{t+\Delta t} = \{F_v^{\text{int}}\}_{t+\Delta t} \quad (21)$$

with:

$$[S_v] = [M_v] + \gamma \Delta t [C_v] + \beta \Delta t^2 [K_v] \quad (22)$$

$$\{\dot{Z}_v^*\}_{t+\Delta t} = \{\dot{Z}_v\}_t + (1-\gamma)\Delta t \{\ddot{Z}_v\}_t \quad (23)$$

$$\{Z_v^*\}_{t+\Delta t} = \{Z_v\}_t + \Delta t \{\dot{Z}_v\}_t + (0,5-\beta)\Delta t^2 \{\ddot{Z}_v\}_t \quad (24)$$

By multiplying Eq.(21) by $[S_v]^{-1}$, one obtains:

$$\{\ddot{Z}_v\}_{t+\Delta t} = \{P_v\}_{t+\Delta t} - [U_v] \{\dot{Z}_v^*\}_{t+\Delta t} - [V_v] \{Z_v^*\}_{t+\Delta t} \quad (25)$$

with

$$\{P_v\}_{t+\Delta t} = [S_v]^{-1} \{F_v^{int}\}_{t+\Delta t}; [U_v] = [S_v]^{-1} [C_v]; [V_v] = [S_v]^{-1} [K_v] \quad (26)$$

Hence, the equation of motion of the bridge deck (3), at time $t+\Delta t$ can be written as:

$$\ddot{q}_{ij}^{(t+\Delta t)} + 2\xi_{ij}\omega_{ij}\dot{q}_{ij}^{(t+\Delta t)} + \omega_{ij}^2 q_{ij}^{(t+\Delta t)} = F_{ij}^{(t+\Delta t)} / M_{ij} \quad (27)$$

Using Newmark's method, the generalized displacements and velocities of the bridge deck at time $t+\Delta t$ are, respectively:

$$q_{ij}^{(t+\Delta t)} = q_{ij}^{(t)} + \Delta t \dot{q}_{ij}^{(t)} + \Delta t^2 (0,5-\beta) \ddot{q}_{ij}^{(t)} + \beta \Delta t^2 \ddot{q}_{ij}^{(t+\Delta t)} \quad (28)$$

$$\dot{q}_{ij}^{(t+\Delta t)} = \dot{q}_{ij}^{(t)} + (1-\gamma)\Delta t \ddot{q}_{ij}^{(t)} + \gamma \Delta t \ddot{q}_{ij}^{(t+\Delta t)} \quad (29)$$

By substituting expressions (28) and (29) into Eq.(27), one obtains:

$$\ddot{q}_{ij}^{(t+\Delta t)} = \left(\frac{1}{M_{ij}} F_{ij}^{(t+\Delta t)} - 2\xi_{ij}\omega_{ij}\dot{q}_{ij}^{*(t+\Delta t)} - \omega_{ij}^2 q_{ij}^{*(t+\Delta t)} \right) / \left(1 + 2\Delta t \xi_{ij}\omega_{ij} + \beta \Delta t^2 \omega_{ij}^2 \right) \quad (30)$$

with

$$q_{ij}^{*(t+\Delta t)} = q_{ij}^{(t)} + \Delta t \dot{q}_{ij}^{(t)} + (0,5-\beta)\Delta t^2 \ddot{q}_{ij}^{(t)} \quad (31.1)$$

$$\dot{q}_{ij}^{*(t+\Delta t)} = \dot{q}_{ij}^{(t)} + (1-\gamma)\Delta t \ddot{q}_{ij}^{(t)} \quad (31.2)$$

2.5. Resolution algorithm

The algorithm of resolution is the same of that used in reference [17]. Its implementation contains two loops (Fig. 5). The first corresponds to the time and the second to the iterations. Displacements, velocities, and accelerations of the bridge deck and vehicle are approximated from the previous iteration (\bar{k}), and then one calculates the interaction forces at each contact point. From there, one solves the vehicle equation of motion (18) by using Newmark's method. Then one calculates the vector of interaction forces acting on the bridge deck at each contact point. One can solve the modal Eq. (27) for the bridge deck by Newmark's method. One then makes a convergence test between the displacement $w^{(\bar{k}+1)}$ of the iteration ($\bar{k} + 1$) and the displacement $w^{(\bar{k})}$ of the previous iteration as follows:

$$\left| w^{(\bar{k}+1)}(x, y, t) - w^{(\bar{k})}(x, y, t) \right| \leq \varepsilon \quad (32)$$

If this condition is satisfied, the desired dynamic parameters can then be calculated. One then proceeds to the next time step. If, however, the condition (32) is not verified, one must apply a correction during the next iteration so that the displacement $w^{(\bar{k}+1)}$ becomes an approximation of the subsequent iteration. One then recalculates, until convergence is achieved.

3. Results

3.1. Isotropic bridge deck

In this example, the dynamic response of an isotropic three-span bridge deck is calculated (Fig. 6) [16,17]. Herein, $\nu_{xy} = \nu_{yx} = \nu$, $D_x = D_y = H = Eh^3/12(1-\nu^2) = D$, $D_{xy} =$

$D(1-\nu)/2$, and $G_{xy} = E/2(1+\nu) = G$. Data for both the bridge deck and vehicle are summarized in Table 1.

The first eight natural frequencies of the isotropic three-span bridge deck are presented in Table 2. The frequencies obtained by [27] and used herein are in general very close to those obtained by the Rayleigh-Ritz method [16,17]. The difference may be due to the effect of intermodal coupling neglected in the Rayleigh-Ritz method, which affects the frequencies of the combined bending-torsion-flexion modes. Conversely, the approach proposed by [27] takes this account into effect. Table 2 also shows the difference between the frequencies from the references [16,17], although they used the same method and number of modes ($9 \times 5 = 45$ modes).

Figs 7a and 7b represent the variation of vertical displacement in the middle of the first bridge deck span (point noted as + in Fig. 6) according to the passing time of the vehicle, which moves at speeds 32.5 m.s^{-1} and 37.5 m.s^{-1} , respectively, on a perfect road surface ($A_r = 0$). The distance between the contact points of the vehicle's right tires and the right edge of the bridge deck is 1 m (Fig. 6). The number of modes chosen (the first 45 modes) is more than sufficient for convergence of the dynamic response. The tolerance of convergence of the iterative process is $\varepsilon = 10^{-8}$, and the integration time step is $\Delta t = 3.12 \times 10^{-3} \text{ s}$. In general, two iterations are sufficient for the convergence of the iterative calculation.

The comparison between the displacements obtained in this study and those of Zhu and Law [17] (values recorded by the curve scan software) shows a slight disparity. This can be explained by the following:

- Zhu and Law [17] neglected the intermodal coupling, which affects the combined frequencies and modes and, therefore, the vertical displacement of the bridge;

- The choice of number of modes required for convergence depends on the load's spatial distribution differs. A poor choice can lead to large errors in the dynamic response [15]. In Zhu and Law [17] only 13 modes were selected, instead of the 45 proposed herein.

3.2. Orthotropic bridge deck

This symmetric, composite three span bridge deck [17] has spans of 24 m, 30 m, and 24 m. The bridge deck consists of a concrete deck slab, 5 I-steel girders, and 14 steel diaphragms (Fig. 8). All relevant characteristics are summarized in Table 3.

Zhu and Law [17] used the theory of thin orthotropic plates to model the bridge deck, whereas in the second numerical example, the structure is a composite (deck slab + diaphragms + girders). Zhu and Law [17] appear to have used a homogenization method to find orthotropic plate properties equivalent to the composite properties. Unfortunately, data of the equivalent plate does not appear explicitly in the paper, except for the equivalent rigidities D_x , D_y and D_{xy} . From this, Rezaiguia [27] made a homogenization of the composite structure based on the concept of the volume and mass fractions of a reinforced composite material to take into account all of the properties of Table 3. The characteristics of the equivalent orthotropic plate were determined and summarized as follow: $l = 78$ m, $l_1 = l_3 = 24$ m, $l_2 = 30$ m, $b = 13.715$ m, $h = 0.212$ m, $\rho = 3265.29$ kg.m⁻³, $D_x = 2.41 \times 10^9$ N.m, $D_y = 2.18 \times 10^7$ N.m, $D_{xy} = 1.14 \times 10^8$ N.m, $E_x = 3.06 \times 10^{12}$ N.m⁻², $E_y = 2.76 \times 10^{10}$ N.m⁻², $G_{xy} = 1.45 \times 10^{11}$ N.m⁻², $\nu_{xy} = 0.3$. The vehicle features used in the isotropic example were also employed herein.

Table 4 compares the values of the first 10 natural frequencies obtained based on a previously published approach [28] (using the modal method and average integration) to those of the reference based on the Rayleigh-Ritz method [17] versus those calculated by the commercial finite element program ANSYS (v.10). To obtain frequencies from ANSYS, first all characteristics of the equivalent orthotropic plate were introduced. Then the bridge deck was modeled with 28,080 shell63 type elements with 4 nodes and 6 degrees of freedom per

node. The results showed excellent agreement for all frequencies compared with those obtained by ANSYS and are described in detail in reference [28].

Table 5 presents the experimental values of the spectral roughness coefficient A_r according to the state of the track [31]. Based on the expression (17) of the road profile, Fig. 9 shows the random profile of the track for different values of the spectral coefficient roughness. The generation of the random variable θ_i is obtained by MATLAB software (v.5).

To identify the influence of the loading mode on the dynamic responses of the rolling bridge deck, a number of passages in three trajectories are simulated (Fig. 10). In the simulation, the road is assumed to be in good condition ($A_r = 15 \times 10^{-6} \text{ m}^3 \cdot \text{cycle}^{-1}$). The influences of the loading mode on the DAF in the middle of each span, and the center of each girder are presented in Table 6. The maximum static response is obtained when the truck crosses the bridge deck at a very low speed in the absence of roughness ($v_x = 10^{-3} \text{ km} \cdot \text{h}^{-1}$). The maximum dynamic response is obtained for a running speed $v_x = 108 \text{ km} \cdot \text{h}^{-1}$. Fig. 11 shows the variation of the vertical displacement in the middle of the second span (girder 1) depending on the position of the front axle on the bridge deck for three loading cases. Fig. 12 and 13 show the variation of the DAF of each girder in the middle of the second span depending upon the rolling speed of the truck (for 10 to 150 $\text{km} \cdot \text{h}^{-1}$) for loading case 1 and case 2, respectively. Findings from these results are as follows:

- DAF in the middle of the girder 1 spans 1, 2 and 3 are the highest in the third case of loading (see Table 6), because the static displacement at this point is very low.
- Near the load, the DAF is low, while the vertical static and dynamic displacements are high (Table 6, Fig. 11).

The influence of road irregularities on the interaction forces vehicle/bridge deck transmitted by the front and rear right wheels of the truck is shown in Figs 14 and 15, respectively. The truck traveled at a normal speed 80 $\text{km} \cdot \text{h}^{-1}$, along the trajectory of the load

case 1. The amplitudes of interaction forces are very sensitive to the roughness profile of the road. In Fig. 16, the interaction forces between the bridge deck and the front and rear right wheels are shown for the same coefficient spectral roughness of road profile. The interaction force exerted by a wheel (front or rear) varies with time and position around an average value that corresponds to the static force: $F_{g1} = 58.81$ kN and $F_{g3} = 32.42$ kN (when the truck is empty).

Fig. 17 shows the interactive influence of both the rolling speed and the state of the road on the DAF of girder 1 in the center of the second span for case 1 in which the DAF dramatically increases with the road deterioration, as opposed to the vehicular speed. For this example, the maximum value of DAF was observed at a speed about 60 km.h^{-1} (DAF = 1.28).

These experiments show that the most important parameter to affect the dynamic vehicle/bridge deck interaction force was the road's surface roughness. Olivia et al. came to the same conclusion [24] as did Deng and Cai [12] for a 1-axle 2-DOF vehicle model and for a 2-axle vehicle model with 4 DOF moving at a constant speed along a simply supported beam. Dinga [21] reported an increase of DAF of 25 to 39% due road surface roughness (DAF was maximum at lower speeds). Those results were compatible with the ones generated in this study.

4. Conclusions

The forced vibration of a bridge deck generated by a moving vehicle load is presented. The vehicle is modeled by a multi-body system with seven degrees of freedom, and the bridge deck is modeled by an orthotropic multi-span plate. Road surface roughness is modeled by a random function characterized by roughness coefficient and arbitrary phase. The modal

method is used to solve the equation of motion of the bridge deck taking into account the intermodal coupling. Equations of motion of the vehicle are obtained by a dynamic equilibrium law of forces and moments. The coupled equations of motion vehicle/ bridge deck are integrated numerically by Newmark's method. A computational algorithm is used to solve the integrated equations of motion with a decoupled manner and an iterative process. A computer program is elaborated using FORTRAN language.

Two numerical examples are examined: an isotropic and an orthotropic, three-span bridge deck. From the results presented, the following are concluded:

- The intermodal coupling takes into the modal superposition method and affects the dynamic response of the bridge deck, but only slightly.
- Frequencies obtained are in good agreement with those exist in literature and those calculated by ANSYS.
- The contribution of torsional modes in the mid-span deflection of the bridge deck and DAF are significant when the load is eccentric.
- Distribution of DAF on the bridge deck does not reflect a particular trend, because high values of DAF can be obtained at points where the vertical displacement is small. The DAF is significant only under the interaction force.
- The road surface roughness has a significant influence on the dynamic vehicle/bridge deck interaction forces, as has been shown for other bridge types under various load conditions.

Appendix A

Complementary to paragraph 2.2.

In matrix form, the equations of motion of the vehicle model with seven degrees of freedom are given by:

$$[M_v]\{\ddot{Z}_v\} + [C_v]\{\dot{Z}_v\} + [K_v]\{Z_v\} = \{F_v^{\text{int}}\} \quad (\text{A.1})$$

with

$$\{Z_v\} = \{z_v, \theta_v, \alpha_v, z_1, \theta_1, z_2, \theta_2\}^T \quad (\text{A.2})$$

$$\{F_v^{\text{int}}\} = \left\{ 0, 0, 0, -F_1 - F_2, \frac{1}{2}s_{t1}(F_1 - F_2), -F_3 - F_4, \frac{1}{2}s_{t2}(F_3 - F_4) \right\}^T \quad (\text{A.3})$$

$$[M_v] = \text{Diag} (m_v, I_{\theta_v}, I_{\alpha_v}, m_1, I_{\theta_1}, m_2, I_{\theta_2}) \quad (\text{A.4})$$

$$[C_v] = \begin{bmatrix} c_{v11} & c_{v12} & c_{v13} & c_{v14} & c_{v15} & c_{v16} & c_{v17} \\ & c_{v22} & c_{v23} & c_{v24} & c_{v25} & c_{v26} & c_{v27} \\ & & c_{v33} & c_{v34} & c_{v35} & c_{v36} & c_{v37} \\ & & & c_{v44} & c_{v45} & c_{v46} & c_{v47} \\ & & & & c_{v55} & c_{v56} & c_{v57} \\ & \text{Symmetric} & & & & c_{v66} & c_{v67} \\ & & & & & & c_{v77} \end{bmatrix} \quad (\text{A.5})$$

$$[K_v] = \begin{bmatrix} k_{v11} & k_{v12} & k_{v13} & k_{v14} & k_{v15} & k_{v16} & k_{v17} \\ & k_{v22} & k_{v23} & k_{v24} & k_{v25} & k_{v26} & k_{v27} \\ & & k_{v33} & k_{v34} & k_{v35} & k_{v36} & k_{v37} \\ & & & k_{v44} & k_{v45} & k_{v46} & k_{v47} \\ & & & & k_{v55} & k_{v56} & k_{v57} \\ & \text{Symmetric} & & & & k_{v66} & k_{v67} \\ & & & & & & k_{v77} \end{bmatrix} \quad (\text{A.6})$$

Elements of matrix $[C_v]$ are as follows:

$$c_{v11} = \sum_{k=1}^4 c_{sk} ; c_{v12} = (c_{s1} + c_{s2})a_1s_x - (c_{s3} + c_{s4})a_2s_x ;$$

$$c_{v13} = \frac{1}{2}s_1(-c_{s1} + c_{s2}) + \frac{1}{2}s_2(-c_{s3} + c_{s4}) ; c_{v14} = -(c_{s1} + c_{s2}) ;$$

$$c_{v15} = \frac{1}{2}s_1(c_{s1} - c_{s2}) ; c_{v16} = -(c_{s3} + c_{s4}) ; c_{v17} = \frac{1}{2}s_2(c_{s3} - c_{s4}) ;$$

$$c_{v22} = (c_{s1} + c_{s2})a_1^2s_x^2 + (c_{s3} + c_{s4})a_2^2s_x^2 ; c_{v23} = \frac{1}{2}(-c_{s1} + c_{s2})a_1s_1s_x + \frac{1}{2}(c_{s3} - c_{s4})a_2s_2s_x ;$$

$$c_{v24} = -(c_{s1} + c_{s2})a_1s_x ; c_{v25} = \frac{1}{2}(c_{s1} - c_{s2})a_1s_1s_x ; c_{v26} = (c_{s3} + c_{s4})a_2s_x ;$$

$$c_{v27} = -\frac{1}{2}(c_{s3} - c_{s4})a_2s_2s_x ; c_{v33} = \frac{1}{4}(c_{s1} + c_{s2})s_1^2 + \frac{1}{4}(c_{s3} + c_{s4})s_2^2 ; c_{v34} = \frac{1}{2}(c_{s1} - c_{s2})s_1 ;$$

$$c_{v35} = -\frac{1}{4}(c_{s1} + c_{s2})s_1^2 ; c_{v36} = \frac{1}{2}(c_{s3} - c_{s4})s_2 ; c_{v37} = -\frac{1}{4}(c_{s3} + c_{s4})s_2^2 ;$$

$$c_{v44} = c_{s1} + c_{s2} ; c_{v45} = \frac{1}{2}(-c_{s1} + c_{s2})s_1 ; c_{v46} = c_{v47} = 0 ; c_{v55} = \frac{1}{4}(c_{s1} + c_{s2})s_1^2 ;$$

$$c_{v56} = c_{v57} = 0 ; c_{v66} = c_{s3} + c_{s4} ; c_{v67} = -\frac{1}{2}(c_{s3} - c_{s4})s_2 ; c_{v77} = \frac{1}{4}(c_{s3} + c_{s4})s_2^2$$

Elements of the $[K_v]$ matrix are similar to elements of the $[C_v]$ matrix, by replacing the damping c_{sk} by rigidities k_{sk} .

5. References

- [1] S.P. Timoshenko, *On the forced vibration of bridges*, Philosophical Magazine 6 (1922) 1018.
- [2] R.S. Ayer, G. Ford, L.S. Jacobsen, *Transverse vibration of two-span beam under action of a moving constant force*, American Society of Mechanical Engineers, Journal of Applied Mechanics **17** (1950) 1-12.
- [3] H.D. Nelson, R.A. Conover, *Dynamic stability of a beam carrying moving masses*, ASME, Journal of Applied Mechanics **38** (1971) 1003-1006.
- [4] G.A. Benedetti, *Dynamic stability of a beam loaded by a sequence of moving mass particles*, ASME, Journal of Applied Mechanics **41** (1974) 1069-1071.
- [5] R. Katz, C. W. Lee, A. G. Ulsoy, R. A. Scott, *Dynamic stability and response of a beam subject to a deflection dependent moving load*, ASME, Journal of vibration, Acoustics, Stress and Reliability in Design **109** (1987) 361-365.
- [6] H.P. Lee, *Dynamic response of a beam with intermediate point constraints subjected to a moving load*, Journal of Sound and Vibration **171** (1994) 361-368.
- [7] H.P. Lee, *Dynamic response of a beam on multiple supports with a moving mass*, Structural Engineering and Mechanics **4** (1996) 303-312.
- [8] K. Henchi, G. Dhatt, M. Talbot, M. Fafard, *Dynamic behaviour of multi-span beams under moving loads*, Journal of Sound and Vibration **1** (1997) 33-45.
- [9] S.A. Hamidi, F. Danshjoo, *Determination of impact factor for steel railway bridge considering simultaneous effects of vehicle speed and axle distance to span length ratio*, Engineering Structures **32** (2010) 1369-1376.
- [10] Y.B. Yang, S.S. Liao, B.H. Lin, *Impact formulas for vehicles moving over simple and continuous beams*, Journal of Struct. Eng., ASCE, 121 (1995) 1644-1650.
- [11] C. Broquet, *Comportement des dalles de roulement des ponts en béton sollicitées par le trafic routier*, Thèse PhD, École Polytechnique Fédérale de Lausanne, (1999).

- [12] L. Deng, C. Cai, *Identification of parameters of vehicles moving on bridges*, Engineering Structures **31** (2009) 2474-2485.
- [13] S. Law, X. Zhu, *Bridge dynamic responses due to road surface roughness and braking of vehicle*, Journal of Sound and Vibration **282** (2005) 805-830.
- [14] M.F. Green, D. Cebon, *Dynamic response of highway bridges to heavy vehicle loads: theory and experimental validation*, Journal of Sound and Vibration **170** (1994) 51-78.
- [15] K. Henchi, M. Fafard, M. Talbot, *Analyse dynamique de l'interaction pont-vehicules pour les ponts routiers*, Revue Canadienne de Génie Civil **25** (1998) 161-187.
- [16] S. Marchesiello, A. Fasana, L. Garibaldi, B.A.D. Piombo, *Dynamics of multi-span continuous straight bridges subject to multi-degrees of freedom moving vehicle excitation*, Journal of Sound and Vibration **224** (1999) 541-561.
- [17] X.Q. Zhu, S.S. Law, *Dynamic load on continuous multi-lane bridge deck from moving vehicles*, Journal of Sound and Vibration **251** (2002) 697-716.
- [18] D. Huang, *Impact Behavior of Concrete Bridge Deck on Girders Due to Moving Vehicles*, ASME, Structures Congress (2013) 689-698.
- [19] X. Yin, C. Cai, Y. Liu, Z. Fang, *Experimental and Numerical Studies of Nonstationary Random Vibrations for a High-Pier Bridge under Vehicular Loads*, ASME, Journal of Bridge Engineering **18(10)** (2013) 1005–1020.
- [20] X. Jiang, Z. Ma, J. Song, *Effect of Shear Stud Connections on Dynamic Response of an FRP Deck Bridge under Moving Loads*, ASME, Journal of Bridge Engineering **18(7)** (2013) 644–652.
- [21] L. Dinga, H. Haoa, X. Zhu, *Evaluation of dynamic vehicle axle loads on bridges with different surface conditions*, Journal of Sound and Vibration **323** (2009) 826-848.
- [22] P. Asnachindaa, T. Pinkaewa, J. Laman, *Multiple vehicle axle load identification from continuous bridge bending moment response*, Engineering Structures **30** (2008) 2800-2817.
- [23] W. Zhang, C. Cai, F. Pan, *Finite element modeling of bridges with equivalent orthotropic material method for multi-scale dynamic loads*, Engineering Structures **54** (2013) 82-93.

- [24] J. Oliva, J. Goicolea, P. Antolín, M. Astiz, *Relevance of a complete road surface description in vehicle–bridge interaction dynamics*, *Engineering Structures* **56** (2013) 466-476
- [25] H. Moghimi, H. Ronagh, *Impact factors for a composite steel bridge using non-linear dynamic simulation*, *International Journal of Impact Engineering* **35** (2008) 1228-1243.
- [26] J.F. Davalos, P. Qiao, L. Shan, *Advanced fiber-reinforced polymer (FRP) composites for use in civil engineering*, *In Advanced civil infrastructure materials: Science, mechanics and applications*. ed. Wu, H.C., New York: E-Publishing Inc. (2006) 118-202.
- [27] A. Rezaiguia., *Modélisation du comportement vibratoire et acoustique d'un pont de la chaussée lors du passage des véhicules*, Thèse de Doctorat, Université d'Annaba, Algérie, 2008.
- [28] A. Rezaiguia, D.F. Laefer, *Semi-analytical determination of natural frequencies and mode shapes of multi-span bridge decks*, *Journal of Sound Vibration* **328** (2009) 291-300.
- [29] A. Rezaiguia, Y. Fisli, S. Ellagoune, D.F. Laefer, N. Ouella, *Extention of Semi-analytical approach to determine natural frequencies and mode shapes of multi-span orthotropic bridge deck*, *Journal of Structural Engineering & Mechanics* **43** (2012) 71-87.
- [30] American Association of State Highway and Transportation Officials AASHTO manual, 1998.
- [31] K. Henchi, *Analyse dynamique des ponts par éléments finis sous les sollicitations des véhicules mobiles*, Thèse de Doctorat, Université de Technologie de Compiègne, 1995.
- [32] K. Henchi, M. Fafard, M. Talbot, G. Dhatt, *An efficient algorithm for dynamic analysis of bridges under moving vehicles using a coupled modal and physical components approach*, *Journal of Sound & Vibration* **212** (1998) 633-683.

Figure captions: main text

Fig. 1. Mathematical model of the continuous, multi-span bridge deck

Fig. 2. Mathematical model of the vehicle, a): Side view, b) Front view

Fig. 3. Dynamic equilibrium of forces and moments, a): Side view, b) Front view

Fig. 4. Deformation of the contact point vehicle/bridge deck taking into account the irregularities of the track

Fig. 5. The computational algorithm for the decoupled method

Fig. 6. Geometric description of the bridge deck and vehicle

Fig. 7. Vertical displacement in the middle of the first span (point noted +) of the bridge deck, (a) : $v_x = 32.5 \text{ m.s}^{-1}$; (b) : $v_x = 37.5 \text{ m.s}^{-1}$ (values of ref. [17] are recorded using curve scan software)

Fig. 8. Continuous three-span, multi-girder bridge deck

Fig. 9. Random profile of the track for different values of roughness coefficient A_r

Fig. 10. Different trajectories used for calculating the dynamic responses of the bridge deck

Fig. 11. Influence of loading mode on the vertical displacement in the middle of the second span, girder 1, $v_x = 108 \text{ km.h}^{-1}$

Fig. 12. Variation of the Dynamic Amplification Factor for each girder in the middle of span 2 depending on the passing speed of the truck, load case 1

Fig. 13. Variation of the Dynamic Amplification Factor for each girder calculated in the middle of span 2 depending on the speed of the truck, load case 2

Fig.14. Interaction force F_{w1}^{int} under a front right wheel of the vehicle, $v_x = 80 \text{ km.h}^{-1}$, load case 1

Fig. 15. Interaction force F_{w3}^{int} under a rear right wheel of the vehicle, $v_x = 80 \text{ km .h}^{-1}$, load case 1

Fig. 16. Comparisons of the interaction forces under a front and a rear right wheels of the vehicle, $v_x = 80 \text{ km.h}^{-1}$, load case 1

Fig. 17. Dynamic Amplification Factor depending on vehicle speed and road surface roughness, center of girder 1, span two, load case 1

Table captions: main text

Table 1. Vehicle and isotropic bridge deck parameters

Table 2. Natural frequencies of the three-span bridge deck

Table 3. Parameters of the three-span, multi-girder bridge deck

Table 4. Comparison of natural frequencies of the orthotropic, three-span bridge deck

Table 5. Experimental values of A_r according to the type of the track [31]

Table 6. Distribution of the Dynamic Amplification Factor on the bridge deck