DC Constrained Fuzzy Power Flow for Transmission Expansion Planning Studies

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Abstract

In restructured power systems, the adequacy of the transmission network may be defined as the ability to meet reasonable demands by transmission of electricity (as stated by the Directive 2009/72/EC). The symmetric/constrained fuzzy power flow (SFPF/CFPF) was recently proposed as a suitable tool to quantify that adequacy. In this paper, the use of the SFPF/CFPF is extended in order to support the decision process of investment in network components in order to accomplish a specific adequacy criteria. A technique based on dual variables, obtained from the linear formulation of the CFPF, is used. The importance of the duality information concerning the adequacy indices is explained. The proposed methodology is applied on IEEE 14 bus reliability test system to demonstrate its applicability.

Keywords: Transmission Expansion Planning, Fuzzy Power Flow, Adequacy, Repression, Constrained, Dual Variable.

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1. Introduction

The transmission network is a critical component for the operation of the electrical power systems. This justifies the large amount of research activities related to the transmission system network planning that has been developed in the last decades. A significant part of that research was done assuming the concept of vertically integrated power systems and, often, handling the adequacy of both the generation and transmission subsystems.

The restructuring of the power systems, namely concerning the separation of the traditional generation, transmission and distribution activities, increased the complexity of the transmission network planning process [1]. In fact, restructuring implies that the transmission network should be able to accommodate a wider range of generation dispatches due, for instance, to power exchanges between regions or countries [2]. Moreover, the transmission systems tend to be operated much closer to their limits [2]. What is more, the classic paradigm of composite system adequacy evaluation is no longer appropriate, the transmission system adequacy assessment should be done independently from the generation system [3][4].

Concerning the transmission adequacy at European Union level, the Directive 2009/72/EC states that each TSO must ensure "long term ability of the system to satisfy reasonable demands for the transmission of electricity". Therefore, the regulatory authority of each UE state should define the rules concerning the adequacy of transmission network. In this context, evaluating the adequacy is more than just performing a load forecast exercise. Actually, the adequacy of the network depends on the "requests" made by both loads and generators. Furthermore, these requests are often linked with relevant uncertainties, which should be accounted for assessing the adequacy of the transmission system [5].The existence of such uncertainties led to the development of power flow models that are able to handle the associated uncertainties, while supporting the evaluation of the transmission system adequacy. The probabilistic power flow (PPF) models [6][7] were the first attempt to achieve this goal. In those models, the uncertainties are treated through using probability distributions to model the generation and load values, as well as the potential uncertainties related to the system components (e.g. reliability of system lines). A different approach, named fuzzy power flow (FPF), was proposed by Miranda and Matos [8]. This model is based on the fuzzy description of loads and generation and has the advantage of not requiring knowledge about probability density functions. This is particularly useful for modeling events whose probability distributions are difficult or even impossible to be built. This is due to the lack of data or even because of the non-probabilistic nature of the events. In such circumstances, using past experiences to evaluate events in the future can be an incomplete approach [7].

In these cases, the use of judgments based on human accumulated knowledge and experience is a helpful alternative. Those judgments are translated into linguistic declarations such as "load certainly between 1 MW and 2.4 MW, however, should not exceed 4.7 MW or be less than 0" or "generation around 50 MW". These kinds of sentences are typical examples of vague information that result, for instance, from the experience of the System Operator [5]. Some variants of the FPF have been also proposed in the literature, namely the interval power flow [9], the boundary power flow [10], as well as recent contributions [11][12][13][14], that falls under the same philosophy of classical FPF.

More recently, an extension of the FPF, known as Symmetric Fuzzy Power Flow (SFPF), was presented in order to treat the slack bus as any other bus of the system [15][16][17]. In SFPF, the slack bus does not assume all the uncertainties regarding the remaining buses of the system (as it happens in the classic FPF), since fuzzy values of generation/load information are also defined for the slack bus. This allows avoiding unjustifiable asymmetrical situations that distort the results and "create" artificial uncertainty).The addition of power flows constraints to the SFPF formulation leads to the Constrained Fuzzy Power Flow (CFPF). Some results comparing classical FPF and SFPF can be found at [16][17].

The PPF [18] and FPF [19] models have been used in different approaches devoted to support the definition of transmission system expansion plans. However, the PPF seems not completely adequate in cases of lack of information and the FPF presents the problems related with the mentioned asymmetrical situations. Despite their advantages, the CFPF model was not yet used in order to support the definition of transmission system expansion plans. CFPF is a linear optimization problem. As it is intended to optimize the power injected/absorbed in the buses, the knowledge of the dual variables, as will be seen, becomes very important since it allows identifying the best options of branch reinforcements. Therefore, the main purpose of this paper is to provide evidence on how the CFPF model can be successfully used in order to support the evaluation of the most promising reinforcements to be performed, by just considering the adequacy of the transmission network. Note that the proposed adequacy evaluation may be integrated in the multicriteria/multiobjective problem of defining expansion plans for the transmission network (which should account for other criteria besides the network adequacy, including the cost of the expansion plans).

The remainder of the paper is organized as follows: the CFPF formulation is briefly reviewed in Section 2; Section 3 shows how the dual variables of the CFPF problem may be used to: i) identify the branches of a transmission system that are the best options to be reinforced, in order to accomplish adequacy requirements; and ii) find the adequate level of reinforcement for those branches; Section 4 provides an application of the proposed approach and discusses decision making aspects; Section 5 concludes the paper.

2. DC constrained fuzzy power flow (DC CFPF)

2.1 Fuzzy numbers

As previously mentioned, fuzzy numbers are a suitable tool to translated linguistic declarations such as "load certainly between 1 MW and 2.4 MW, however, should not exceed 4.7 MW or be less than 0". This declaration can be expressed as a trapezoidal fuzzy number defined by the values $(a_1 = 0\ MW,\ a_2 = 1\ MW,\ a_3 = 2.4\ MW,\ a_4 = 4.7\ MW)$, as shown in Figure 1. A membership function is also needed in order to completely characterize the trapezoidal fuzzy number. That function, represented on expression 1, denotes the possibility of a specific value of x occur [20]. For instance, for the above trapezoidal number, the occurrence of a load value x < 0 MW or x > 4.7 MW is not possible and a value 1 ≤ x ≤ 2.4 MW has the maximum possibility of occur.

$$p_A(x)=\begin{cases}0 & ,x\le a_1\\ \dfrac{x-a_1}{a_2-a_1} & ,a_1<x\le a_2\\ 1 & ,a_2<x\le a_3\\ \dfrac{a_4-x}{a_4-a_3} & ,a_3<x\le a_4\\ 0 & ,x>a_4\end{cases} \qquad (1)$$

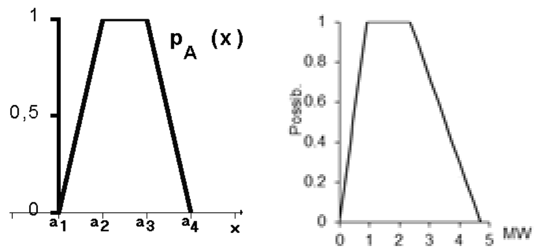

Figure 1 – Fuzzy load representation

Considering a fuzzy generation $\tilde{G}$ and a fuzzy load $\tilde{L}$ for the same bus, the fuzzy injected power for that bus may be obtained through the application of the rules of fuzzy arithmetic:

$$\tilde{G}-\tilde{L}=[a_1,a_2,a_3,a_4]-[b_1,b_2,b_3,b_4]$$
$$\tilde{G}-\tilde{L}=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1) \qquad (2)$$

Note that expression (2) remains applicable when the fuzzy numbers present an alternative form as, for instance, the triangular or the rectangular forms. In fact, when the triangular form is used, it is only need to assume $a_2=a_3$. For the rectangular form it should be assumed that $a_1=a_2$ and $a_3=a_4$.

2.2 CFPF

The DC formulation of the CFPF assumes a linearized model of the network, which only considers the active power flows. The active power injected at each bus of the system is a fuzzy number. Therefore, the CFPF consists of calculating, the maximum and minimum values that each state variable may get for all the possible values (with degree of membership greater than or equal to α) of the external variables [16]:

$$
\begin{array}{lll}
\max/\min & \tilde{Z}(\alpha) = f(P_1, P_2, \ldots) & \\
st: & P_i \in \tilde{P}_i(\alpha) & all\ buses\ i \\
 & \sum_i P_i = 0 & \\
 & |A.P| \leq P_{LIM} &
\end{array}
\quad (3)
$$

where $\tilde{P}_i(\alpha)$ is the α-level cut interval of the nodal injected power $\tilde{P}_i$, A is the sensitivity matrix of the DC model, P the vector of P_i , and P_{LIM} is the vector containing the branch limits. The α-level cut ($\tilde{P}_i(\alpha)$) means that the membership level of all uncertain parameters ($p_A(x)$) have a value grater than α [21]. The objective function defined in (3) is the power injected into the buses subject to the line constraints (considering the fuzzy values of generation and load in the system buses). To construct the fuzzy number P_i, a maximization and minimization problem is solved for each alpha level.

In contrast to the classical formulations of the FPF, no slack bus is defined in CFPF formulation. This feature avoids the concentration of all uncertainties arising from the system on a specific bus (the slack bus) [16][22].

The formulation presented on (3) allows to recognize the situations where the power flow of one or more branches surpass the relevant limit (defined on P_{LIM} vector). In such circumstances (power flow of a branch surpassing the relevant limit) a situation of demand repression exists. These repressions compromise the adequacy of the transmission system to “satisfy reasonable demands by electricity transferring”. The level of the “inadequacy” may be quantified by the following indices [15][17]:

Degree of Repression (DOR) - maximum value of α ensuring that no repressed generation/demand exists.

Individual Severity of Repression (ISR) – local index representing the repressed fuzzy power injection at each bus, expressed in MW. This index corresponds to the active power that cannot be satisfied.

Global Severity of Repression (GSR) – sum of the individual active fuzzy injections that cannot be satisfied. This index, expressed in MW, reflects the total repression due to the network limitations.

Figure 2 provides an example of adequacy indices for a generic bus. In this case, a degree of repression of 0.8 exists. The shadow area in the figure corresponds to the severity of repression. A more exhaustive explanation of those indices can be found in [16-17].

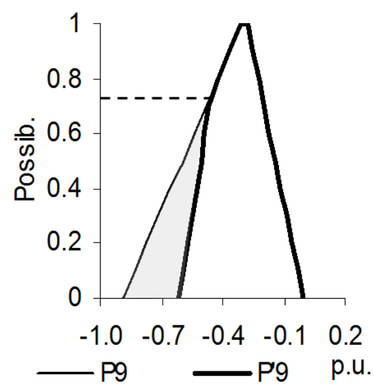

Figure 2 - Repressed demand in a generic bus

3. Dual variables

In constrained optimization, the shadow price is the instantaneous change (per unit) of the constraint, in the objective value of the optimal solution of an optimization problem. This change is obtained by relaxing the constraint. In other words, it is the marginal cost of strengthening the constraint. Each constraint in an optimization problem has a shadow price or dual variable [23]. Therefore, the dual variable is a natural result of a linear problem (LP), which may be used, in the CFPF framework, to capture the effects on the objective function (expression (3)) that result from changes in the branch limits (PLIM). In other words, the dual variable, associated with a specific constraint (in this case associated to branch limits), "measures" the variation on the value of the objective function produced by the change in the Right Hand Side (RHS) of the constraints [23].

In a general way, for each α-level interval, the decrease of the repression (DR) on active power can be obtained by (4):

$$DR(\alpha) = P_i^R(\alpha) - P_i^b(\alpha) = \lambda(\alpha) \, . \, \Delta P_{ik\max} \qquad (4)$$

where:

$$\Delta P_{ik\max} = P_{ik}^R - P_{ik\max} = \frac{P_i^R(\alpha) - P_i^b(\alpha)}{\lambda(\alpha)} \qquad (5)$$

where: P_{ikmax} is the initial value of branch limit; ΔP_{ikmax} is the increased quantity at the transmission capacity of the branch; P_i^R is the new value of the active power that may be injected at bus i (after the reinforcement of the branch i-k); P_i^b is the initial value of active power injected at bus i (before the reinforcement of branch i-k); λ is the dual variable at a specific RHS interval (Figure 3); and P_{ik}^R is the new proposed branch limit.

Note that, dual value (λ) is a constant for a specific interval of RHS values. Assuming the example of Figure 3, applicable for a generic branch, the dual value λ_1 only is valid for P_{ik} values belonging to the range [A, B]. This range defines the maximum value of ΔP_{ik} (ΔP_{ikmax1}) that ensures the validity of the dual value λ_1. Thus, for values ΔP_{ik} that ensure a P_{ik} value inside the range [A, B] (that is $\Delta P_{ik} < \Delta P_{ikmax1}$) the variation on the limit of the branch changes the magnitude of the basic variables of the linear optimization problem but not the optimal point of the solution. Now, if we consider a reinforcement action such that $\Delta P_{ik} > \Delta P_{ikmax1}$, two distinct dual values (and distinct optimal points) will emerge: λ_1 for $\Delta P_{ik} \leq \Delta P_{ikmax1}$; and λ_2 for $\Delta P_{ikmax1} \leq \Delta P_{ik} < \Delta P_{ikmax3}$. In this case, the expression (4) must be rewritten as follows:

$$DR = P_i^R(\alpha) - P_i^b(\alpha) = \lambda_1(\alpha) \,.\, \Delta P_{ik\max 1} + \lambda_2(\alpha) \,.\Delta P_{ik\max 2} \qquad (6)$$

Generically, for several dual values, the expression (2) becomes:

$$DR = P_i^R(\alpha) - P_i^b(\alpha) = \sum_{g=1}^{n-1} \lambda_g(\alpha) \,.\, \Delta P_{ik\max\, g} + \lambda_n(\alpha) \,.\Delta P_{ik\max\, n} \cdot \frac{\delta}{100} \qquad (7)$$

where: δ is the percentage of branch reinforcement in the dual variable interval (see Figure 3).

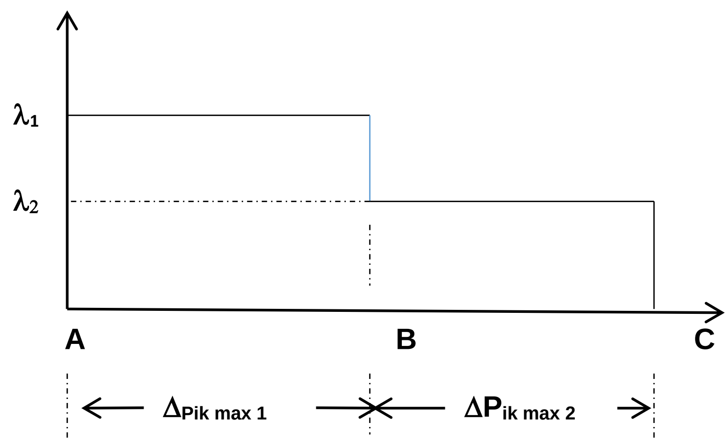

Figure 3. Dual variable variation for a generic reinforcement action

The limits of the RHS interval can be directly obtained from the LP problem. Note taht for each “dual” interval, a post-optimization must occur to find the limits of the next interval if they exist.

The branches causing repression situations at each node may be identified by solving the CFPF problem described by expression (3) and assessing the resulting dual variables (dual variable greater than zero means a repression situation). Note that the dual values are generally available in linear problems solvers such as CPLEX [24], LINDO [25], or

MATLAB optimization toolbox [26]. Assuming that the information provided on Table I was obtained for a generic network with 4 buses and 4 branches, the following conclusions may be extracted: i) the repression situations at bus 4 are caused by branch i3-k4; ii) this branch also contributes to the repression situations occurring at nodes 1 and 3 ($\lambda_1 \ldots \lambda_5$ are the values of the active dual variables).

TABLE I

DUAL INFORMATION OBTAINED FROM CFPF

Supported by this information, the decision maker can identify and choose the network branches to reinforce, in order to reduce or eliminate the situations of repression. Thus, this evaluation allows to identify the branches to be reinforced (for repression reduction at the nodes) as well as to find the most adequate reinforcement values to a specific branch ($\Delta P_{ik\ max}$). The amount of reinforcement to be chosen will be conducted naturally fulfilling the technical, economic and environmental criteria.

4. Case Study

4.1. Test system

In order to show the applicability of the purposed methodology the 14 bus, 20 branches, IEEE network (Figure 4) is used as test system. The system data can be found in [27], wherein the active power values will be assumed as the central values of the trapezoidal fuzzy numbers. The remaining characteristic points of such numbers are assumed to be 0.0, 0.95, 1.05 and 3.0 pu of the central values. The large α=0 intervals are used to enhance the repression situations. Once the IEEE 14 bus test system does not provide data regarding the branch limits, the values presented on Table II are assumed for this purpose. A power base equal to 100MVA is considered for simulation.

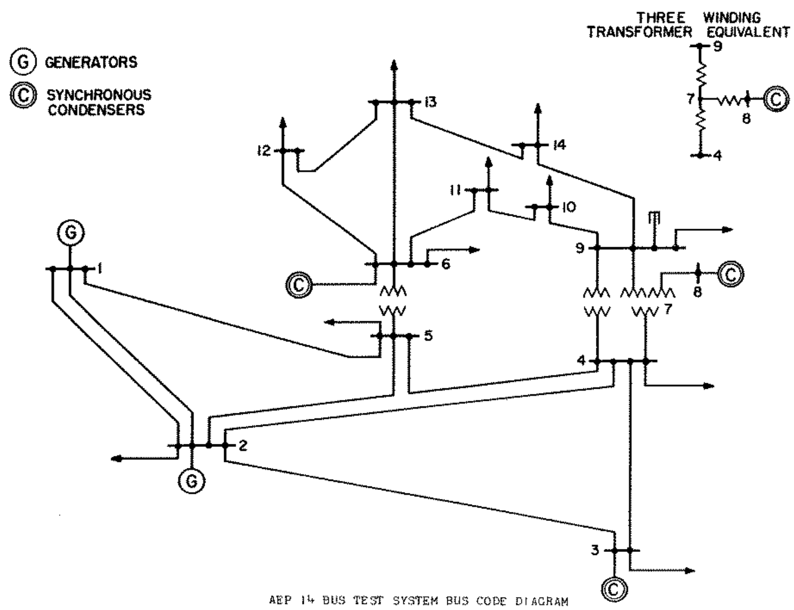

Figure 4 – IEEE 14 bus reliability test system

TABLE II BRANCH LIMITS

4.2. System Adequacy Assessment – Base Case

In order to assess the system adequacy for the base case the CFPF optimization problem described by expression (3) was run for all system nodes. The execution of the CFPF, on the above circumstances, revealed situations of active power repression at seven system nodes, as shown in Table III. Figures 5 and 6 show some fuzzy results for the cases of nodes 1, 3, 9 and 10. Note that the repression occurring at bus 1 is a generation repression while load repressions occur at buses 3, 9 and 10. In such figures

(Figure 5 and Figure 6), Pi denotes the specified possibility distributions (i.e. the intended injections) for each bus i (including the reference bus) and Pi’ denotes the possibility distributions resulting from the CFPF.

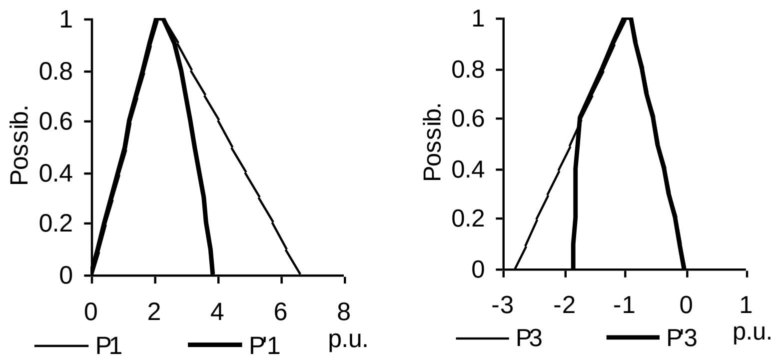

Figure 5 - Repression of generation and load, respectively at nodes 1 and 3

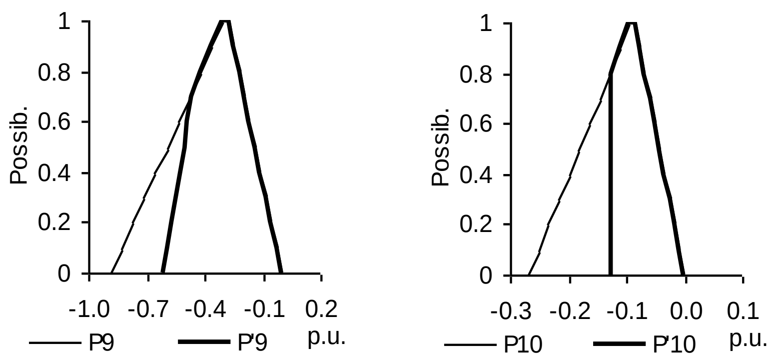

Figure 6 – Load repression at nodes 9 and 10

From Table III it is possible to conclude that bus 1 presents the larger value of individual severity of repression, accounting for about 67% of the global severity of repression of the system. Thus, the bus 1 represents a serious situation of local inadequacy due to the network limitations. The maximum DOR value equals 1 (at bus 1) and the GSR index is around 172 MW.

TABLE III

DOR/SEVERITY OF REPRESSION

4.3. Dual information

Table IV shows the active dual variables regarding branches constraints for each bus and for α cut equal to zero. Those variables contain important information about the impact of the ampacity of the branches on the node repressions. For instance, the ampacity limit of branch 4-5 is most likely the principal reason for the repression occurring at bus 1. Moreover, the information of Table IV shows that: i) bus repressions are not always caused by neighborhood branches (see the case of nodes 1, 9 and 13); ii) the same branch can contribute for situations of repression on several nodes (see the case of branches 3-4, 6-13, 7-9, 9-10, 10-11 and 12-13).

In order to find the RHS intervals where dual variables are applicable, it is necessary to perform several post-optimizations problems over expression (5), incrementing the P_{LIM} value (the maximum bound of RHS interval). This kind of information is helpful to support decisions regarding the choice of reinforcement strategies. Figure 7 shows the result of performing this exercise for the case of bus 1 (worst inadequacy situation). Note that three post optimizations were done on such case. The result reveals that, for the cut level α=0, the maximum gain in terms of injected power at bus 1 occurs when P_{45}max is 1.46 pu (value to which a null dual variable exist). Based on that information, the decision maker may find solutions to reinforce the branch 4-5 (taking into consideration relevant aspects as, for instance, economic ones) in order to reduce the existing repression at node 1.

TABLE IV

DUAL INFORMATION FOR $\alpha = 0$

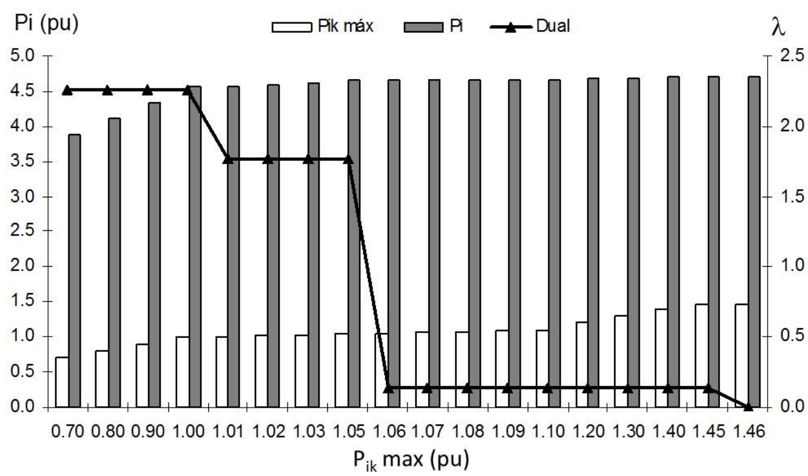

Figure 7 - Dual variable and Pi evolution due to P4-5 reinforcement, bus 1

4.4. Transmission system reinforcement strategy

The reinforcement of a specific transmission system may be implemented following different strategies, namely: Strategy A - Decrease the worst DOR value; Strategy B - Decrease the worst ISR value; Strategy C - Decrease the GSR

4.4.1. Strategy A

To reduce the worst DOR index, (the one corresponding to node 1, DOR = 1.0) it is necessary to obtain the dual information of active constraints for all α cuts. Figure 8 presents such information, obtained as a result of the optimization problem described by expression (3).

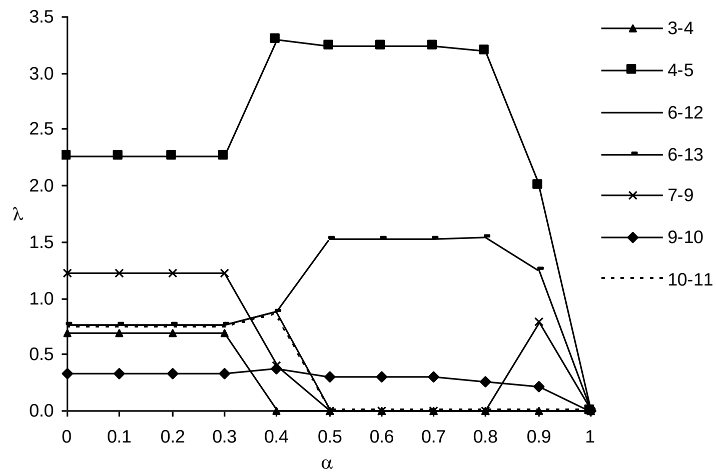

Figure 8 - Dual variables evolution for each α cut at node 1

As previously referred, the branch 4-5 presents the bigger dual value at the neighborhood of the cut α=1. Thus, a suitable decision of the decision maker (the TSO) is to reinforce this branch in order to achieve a P_{LIM}=1.46 pu. After performing this exercise, the new DOR value would be equal to 0.7 and the new dual variables corresponds to the ones are presented on Figure 9. The constraints of branches 3-4, 4-5 and 10-11 are no longer active for the bus 1 repression. However new constraints were activated, namely at branches 1-5 and 2-3. If the decision maker considers that the new DOR remains higher than the desirable threshold, the most promising branch to be reinforced is the branch 7-9, as shown in Figure 9.

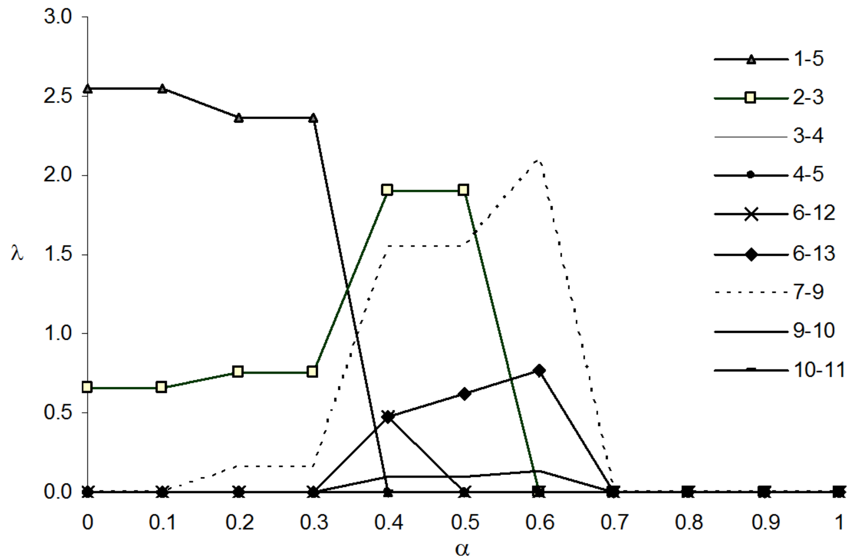

Figure 9 - Dual variables evolution for each α cut at node 1 after branch 4-5 reinforcement

4.4.1. Strategy B

In this strategy, the objective is to reduce the value of the worst ISR. Resuming the base case, it can be concluded from Table III that bus 1 presents the worst ISR value. Considering the base case for bus 1 (see Figure 8), the decision maker should act at firstly branch 4-5, the same used to reduce the DOR. The reinforcement of branch 4-5 will provide a new ISR of 0.4896 pu (Table V).

Note that the use of same branch to handle the two indices (ISR and DOR) is a coincidence and not a rule. See the information of Figure 9: here if is desired reduce the DOR, the natural choice will be the branch 7-9 to be reinforced (this branch is the one with highest dual value in the upper alpha cuts). But if the reduction of ISR is the decision maker scope, so he must reinforce branch 1-5. That means the reinforcement of branch 7-9 will enable decrease DOR (and also the ISR), but reinforce of node 1-5 certainly must not cause changes at this index since this constraint is not active for the actual DOR (0.7). To check this and after provide the reinforcement of branch 4-5 (most promising option to reduce ISR at bus 1) we proceed to the reinforcement of branches 7-9 and 1-5 separately (both branch reinforcements coincident with values of α=0 for the constraint – major decrease of P_1). We conclude, as expected, that the reinforcement of branch 7-9 will allow a lower DOR and the reinforcement of node 1-5 provides only a lower ISR. All results are available at Table V and at Figures 10 to 12. At this Figures, P_1 and P_1' and P_1'' represents, respectively the possibility distributions for the intended injections (without branch limits), repression for the base case (with the branch limits of Table II) and repression for the reinforcement actions: branch 4-5, branch 4-5 plus branch 7-9, and branch 4-5 plus branch 1-5.

TABLE V

INDIVIDUAL DEGREE/SEVERITY OF REPRESSION AFTER BRANCH REINFORCEMENT (BUS 1)

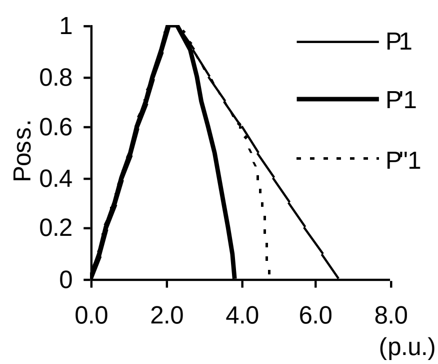

Figure 10 - Reinforce of branch 4-5

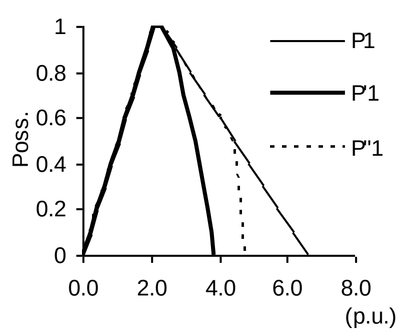

Figure 11 - Reinforce of branches 4-5 and 7-9

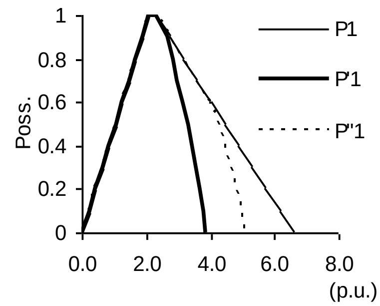

Figure 12 - Reinforce of branches 4-5 and 1-5

4.4.1. Strategy C

Now, the concern is related to the minimization of the GSR value (that is, the minimization of the total MW repression existing in the system). The first step in order to ensure that objective consists in determining the maximum gain in injected active power at each bus (that means reach null dual variables for the selected active constraints). This is achieved following, for each system bus, a procedure similar to the one used in section 4.3 for the case of bus 1 (where a value P_{LIM}=1.46 pu was found). Table VI contains the result of such a procedure (obtained for α=0). Note that, in Table VI, P_{icb} and P_{ikcb} are, respectively, the limits of the active injected power at each bus and of the active power flow at each branch for the base case. P_{iR} and P_{ikR} have the meaning described in section 3. The final results are coincident with the major dual variables in each bus (Table IV).

Table VII contains the values of the ISR obtained after performing the reinforcement options (1 to 7) referred on Table VI. The column "BC" corresponds to the base case, which is the initial situation of the system. The last line of Table VII contains de GSR values for all the assumed situations. Those values allow concluding that all reinforcement actions allows to reduce the GSR values, wherein reinforcement corresponding to option 1 leads to the smallest value of GLR. Figure 13 shows the ISR for buses 1, 3 and 9 and the GSR.

TABLE VI

BEST REINFORCMENT OPTIONS FOR EACH BUS (pu)

TABLE VII

SEVERITY OF REPRESSION BEFORE (BC) AND AFTER BRANCH REINFORCMENT (pu)

Other important conclusion is the fact of bus 3 suffers changes when branches 4-5, 7-9, 9-10 or 6-13 (non-adjacent branches) are reinforced (Table VII). To understand why this situation occurs the TSO only have to consult the dual information for node 3 at the base case (Figure 14).

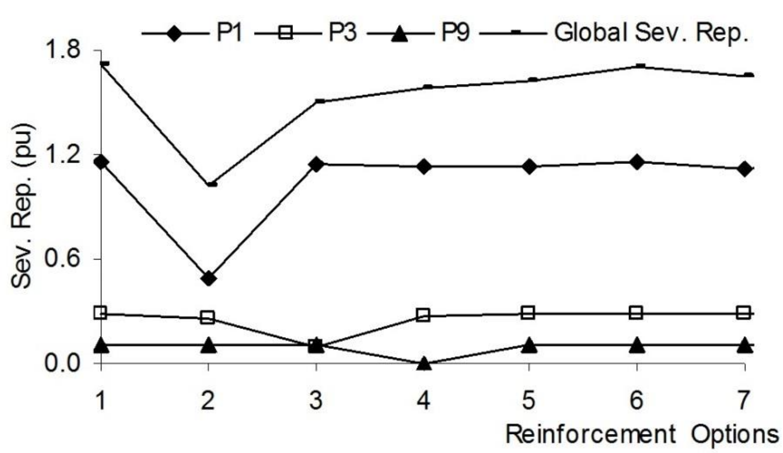

Figure 13 - Individual severity of repression for nodes 1, 3 and 9 and global severity of repression

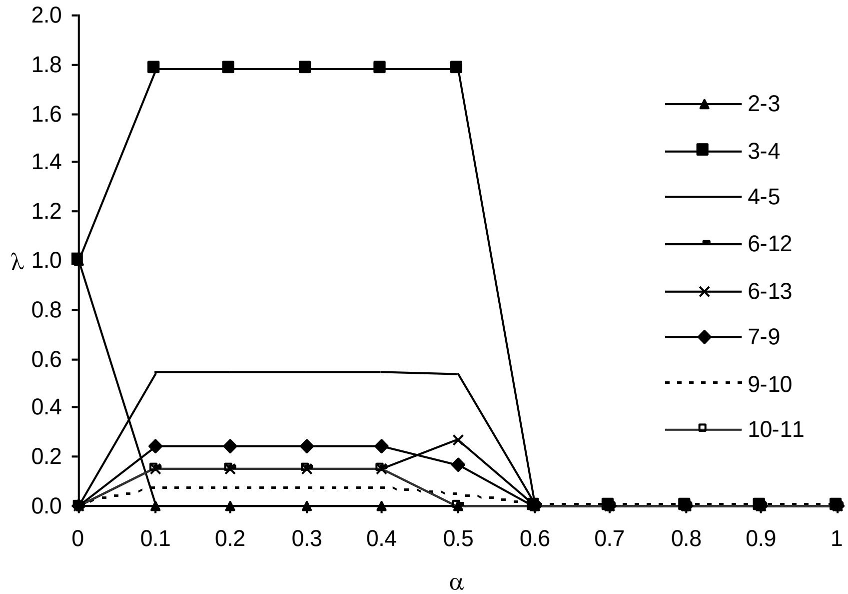

Figure 14 – Dual variables evolution for each α cut at node 3

4.5. DECISION MAKING ASPECTS

Often, the decision making about transmission network reinforcement actions has to consider other criteria beyond the impacts on the system adequacy. Different issues such as the environmental impacts and the cost of the actions are frequently considered in the decision process. Some costs are associated with investments and the other costs are related to the network operation. The environmental impacts are generally lower during branch operation but may exist as a result of the reinforcement action (e.g. the enlargement of the high voltage corridors). Assuming the adequacy A(x), the cost C(x) and the environmental impact EI(x) as the criteria to be considered in the process of choosing between alternative reinforcement actions, an objective function to be minimized could be:

$$G(x) = -A(x) + \beta.C(x) + \sigma.EI(x) \qquad (8)$$

Note that, in expression (6), β and σ are weights that should be defined by the decision maker in order to account for the relative importance of each criteria.

The decision problem to be solved simultaneously considers the minimization of attributes C(x) and EI(x) and the maximization of attribute A(x) which, often, are in conflict. The value A(x) for each reinforcement action may be obtained by using the methodology proposed in the previous sections of this paper.

5. Conclusions

In this paper, the DC SFPF/CFPF formulation is used to support the definition of network investments on transmission networks (investment plans), in order to accomplish a specific adequacy criteria. This criteria may be based on a local index (defined for each bus of the system) or on a global index (defined at system level). Naturally, a more complex criteria based on an association of individual and global objectives may be defined. The SFPF/CFPF model allows a fuzzy description of loads and generation, thus having the advantage of not require knowledge about probability density functions. This is particularly useful for modeling events whose probability distributions are difficult or even impossible to be built, due to lack of data or even because of the non-probabilistic nature of the events.

A technique based on dual variables, obtained from the linear formulation of the SFPF/CFPF, is used to study and quantify the most promising network reinforcement alternatives in order to achieve the desired adequacy level (repressions reduction). Thus, the proposed approach permits to evaluate the impact of reinforcement options on the system adequacy. Moreover, this evaluation is done in a suitable way to be integrated into optimization and decision-aid procedures.

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TABLE I

DUAL INFORMATION OBTAINED FROM CFPF

branch\bus	1	2	3	4
i1-k3	λ1			
i1-k2	λ2			
i3-k4	λ3		λ4	λ5
i2-k3				

TABLE II

BRANCH LIMITS

End Buses	Limit (p.u.)	End buses	Limit (p.u.)
1-2	3.5400	6-11	0.1000
1-5	1.3400	6-12	0.1100
2-3	1.2400	6-13	0.1900
2-4	1.0700	7-8	10.000
2-5	0.9500	7-9	0.3300
3-4	0.6100	9-10	0.0830
4-5	0.7000	9-14	0.1200
4-7	0.6100	10-11	0.0453
4-9	0.4100	12-13	0.0300
5-6	0.7800	13-14	0.1200

TABLE III

DOR/SEVERITY OF REPRESSION

Bus	ISR (pu)	DOR
1	**1.1617**	**1.0**
2	0.0000	0.0
3	**0.2867**	**0.6**
4	0.0000	0.0
5	0.0000	0.0
6	0.0000	0.0
7	0.0000	0.0
8	0.0000	0.0
9	**0.1032**	**0.8**
10	**0.0573**	**0.9**
11	0.0000	0.0
12	**0.0079**	**0.4**
13	**0.0295**	**0.6**
14	**0.0760**	**0.8**
GSR (p.u.)	1.7223	

TABLE IV

DUAL INFORMATION FOR $\alpha = 0$

branch/bus	1	3	9	10	12	13	14
2-3		1.0000	---	---	---	---	---
3-4	0.6957	1.0000	---	---	---	---	---
4-5	2.2597	---	---	---	---	---	---
6-12	0.7649	---	---	---	---	---	---
6-11	---	---	0.2764	---	---	---	---
6-12	---	---	---	---	1.0000	---	---
6-13	0.7649	---	---	---	---	1.2107	---
7-9	1.2273	---	1.5738	---	---	---	---
9-10	0.3250	---	---	1.0000	---	0.1366	---
9-14	---	---	---	---	---	---	1.0000
10-11	0.7387	---	1.3843	1.0000	---	0.6322	---
12-13	---	---	---	---	---	---	---
12-13	---	---	---	---	1.0000	1.0000	---
13-14	---	---	---	---	---	---	1.0000

TABLE V

INDIVIDUAL DEGREE/SEVERITY OF REPRESSION AFTER BRANCH REINFORCEMENT (BUS 1)

Reinforcement	ISR (pu)	DOR
4-5	0.4896	0.7
4-5 plus 7-9	0.4675	0.6
4-5 plus 1-5	0.4183	0.7

TABLE VI

BEST REINFORCMENT OPTIONS FOR EACH BUS (pu)

Opt.	Bus	Reinf.	P_i^{cb}	P_i^R	P_{ik}^{cb}	P_{ik}^R	DR
1	1	4-5	3.8777	4.7123	0.700	1.4600	0.8346
2	3	3-4	-1.8500	-2.3222	0.610	1.0822	0.4722
3	9	7-9	-0.6097	-0.8850	0.330	0.5100	0.2753
4	10	9-10	-0.1283	-0.2700	0.083	0.2300	0.1417
5	12	12-13	-0.1400	-0.1502	0.030	0.0730	0.0102
6	13	6-13	-0.3000	-0.3872	0.190	0.2630	0.0872
7	14	9-14	-0.2400	-0.4470	0.120	0.3280	0.8346

TABLE VII

SEVERITY OF REPRESSION BEFORE (BC) AND AFTER BRANCH REINFORCMENT (pu)

Bus	BC	ISR (pu) for reinforcement option n.º						
		1	2	3	4	5	6	7

P1	1.162	0.490	1.139	1.127	1.133	1.162	1.125	1.162
P3	0.287	0.261	0.094	0.277	0.283	0.287	0.282	0.287
P9	0.103	0.103	0.103	0.000	0.103	0.103	0.103	0.103
P10	0.057	0.057	0.057	0.057	0.000	0.057	0.057	0.057
P12	0.008	0.008	0.008	0.008	0.008	0.000	0.008	0.008
P13	0.030	0.029	0.029	0.029	0.020	0.021	0.001	0.029
P14	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.000
GSR	1.722	1.025	1.507	1.575	1.623	1.706	1.652	1.646