



Title	Computers, obsolescence, and productivity
Authors(s)	Whelan, Karl
Publication date	2000-01-05
Publication information	Whelan, Karl. "Computers, Obsolescence, and Productivity." Federal Reserve, January 5, 2000. https://doi.org/10.2139/ssrn.220289 .
Series	Finance and Economics Discussion Series, No. 2000-06
Publisher	Federal Reserve
Item record/more information	http://hdl.handle.net/10197/244
Publisher's version (DOI)	10.2139/ssrn.220289

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Computers, Obsolescence, and Productivity

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February, 2000

Abstract

What effect have computers had on U.S. productivity growth? This paper shows that increased productivity in the computer-producing sector and the effect of investment in computers on the productivity of those who use them together account for the recent acceleration in U.S. labor productivity. In calculating the computer-usage effect, standard NIPA measures of the computer capital stock are inappropriate because they do not account for technological obsolescence; this occurs when machines that are still productive are retired because they are no longer near the technological frontier. Using a framework that accounts for technological obsolescence, alternative stocks are developed that imply larger computer-usage effects.

*Mail Stop 80, 20th and C Streets NW, Washington DC 20551. Email: kwhelan@frb.gov. I wish to thank Eric Bartelsman, Darrel Cohen, Steve Oliner, Dan Sichel, Larry Slifman, Stacey Tevlin, and participants in seminars at the University of Maryland, the Federal Reserve Bank of St. Louis, and the 2000 AEA meetings for comments. I am particularly grateful to Steve Oliner for providing me with access to results from his computer depreciation studies. The views expressed in this paper are my own and do not necessarily reflect the views of the Board of Governors or the staff of the Federal Reserve System.

1 Introduction

Recent years have seen an explosion in the application of computing technologies by U.S. businesses. Real business expenditures on computing equipment grew an average of 44 percent per year over 1992-98 as plunging computer prices allowed firms to take advantage of ever more powerful hardware and, consequently, the ability to use increasingly sophisticated software. These developments have helped improve the efficiency of many core business functions such as quality control, communications, and inventory management, and, in the case of the Internet, have facilitated new ways of doing business. They have also coincided with an improved productivity performance for the U.S. economy: Private business output per hour grew 2.2 percent per year over the period 1996-98, a rate of advance not seen late into an expansion since the 1960s.¹ This confluence of events raises some fascinating questions. Are we finally seeing a resolution to the now-famous Solow Paradox that the influence of computers is seen everywhere except in the productivity statistics? And if so, is the recent pace of productivity growth likely to continue?

This paper addresses these questions by focusing on two separate computer-related effects on aggregate productivity. First, there has been an enormous productivity increase in the computer-producing sector, a development that on its own contributes to increased aggregate productivity. Second, the resulting declines in computer prices have induced a huge increase in the stock of computing capital. I show that this deepening of the computer capital stock - the *computer-using* effect - combined with the direct effect of increased productivity in the *computer-producing* sector together account for the improvement in productivity growth over the period 1996-98 relative to the previous 20 years.

Most of the paper is devoted to analyzing and estimating the computer-using effect, because it is here that the paper uses a new methodology. This effect has been the subject of a number of previous studies, most notably the work of Steve Oliner and Dan Sichel (1994), updated in Sichel (1997).² Using a growth accounting framework, these studies concluded that computer capital accumulation had only a small effect on aggregate productivity because computers were a relatively small part of aggregate capital input: In this

¹All figures in this paper refer to 1992-based National Income statistics, and not the 1996-based figures published in October 1999. The paper relies extensively on NIPA capital stock data, and capital stocks consistent with the revised NIPA figures will not be published until at least Spring 2000.

²Other studies include Kevin Stiroh (1998) and Dale Jorgenson and Kevin Stiroh (1999).

sense, computers were not “everywhere”. This paper comes to a different conclusion, in part because computer capital stocks, however measured, have become a more important part of capital input in recent years, and in part because I use new estimates of the computer capital stock that are larger than the conventionally used measures.

The new computer capital stocks used in this paper are motivated by the following observation. The National Income and Product Accounts (NIPA) capital stock data used in most growth accounting exercises are measures of the replacement value of the capital stock and are thus measures of wealth: They weight past real investments according to a schedule for economic depreciation, which describes how a unit of capital loses value as it ages. However, in general, these wealth stocks will not equal the “productive” stock that features in the production function: Productive stocks need to weight up past real investment according to a schedule for physical decay, which describes a unit’s loss in productive capability as it ages. In this paper, I document the NIPA procedures for constructing computer capital stocks, use the vintage capital model of Solow (1959) to derive the conditions under which these wealth stocks equal their productive counterparts, and then show that the evidence on computer depreciation is inconsistent with these conditions. An alternative vintage capital model is presented that explains the evidence on computer depreciation by allowing for *technological obsolescence*: This occurs when computers are retired while they still retain productive capacity. Alternative productive stocks are presented that are consistent with this model and that are significantly larger than their NIPA counterparts.

The paper relates to a number of existing areas of research. First, in calculating both computer-producing and computer-using effects, it updates the approach of Stiroh (1998). Second, the focus on the retirement of capital goods as an endogenous decision and the argument that explicit modelling of this decision can improve our understanding of the evolution of productivity, echoes the conclusions of Feldstein and Rothschild (1974), and also the contribution of Goolsbee (1998). Finally, the paper sheds new light on the recent productivity performance of the U.S. economy, a topic also explored by Gordon (1999).

The contents are as follows. Sections 2 to 6 develop the new estimates of the contribution of computer capital accumulation to output growth, defining wealth and productive capital stocks, documenting the NIPA procedures for constructing computer stocks, and using a new theoretical approach to develop alternative estimates. Section 7 calculates the direct effect of increased productivity in the computer-producing sector and discusses the recent productivity performance of the U.S. economy. Section 8 concludes.

2 Wealth and Productive Capital Stocks

We will start with some definitions.

Definition (Wealth Stock): The *Nominal Wealth Stock* for a type of capital is the total current dollar cost of replacing all existing units of this type. The *Real Wealth Stock* is the replacement value of all existing units expressed in terms of some base-year's prices. *Economic Depreciation* is the decline in the replacement value of a unit of capital (relative to the price of new capital) that occurs as the unit ages.

Definition (Productive Stock): Assume there is a production function

$$Q(t) = F(K_1^p(t), K_2^p(t), \dots, K_n^p(t), X_1(t), \dots, X_m(t))$$

describing real output as a function of capital and other inputs, such that

$$K_j^p(t) = \sum_{\tau=0}^{\infty} I_j(t-\tau) \lambda(\tau)$$

where $I_j(t)$ is the number of units of capital of type j . Then $K_j^p(t)$ is defined as the *Real Productive Stock*. The *Nominal Productive Stock* equals $P_j(t) K_j^p(t)$ where $P_j(t)$ is the current value of the price index for capital of type j . *Physical Decay* refers to the pattern by which a unit of capital becomes less capable of producing output as it ages, as determined by the sequence $\lambda(\tau)$.

In theory, nominal wealth stocks could be estimated by obtaining the current replacement values for all units of capital, new and old, and adding them up. In practice, of course, it is impossible to obtain all this information. Instead, these stocks have been constructed from cross-sectional studies of economic depreciation based on used-asset prices, the most important being those of Charles Hulten and Frank Wykoff (1981). These studies provide a schedule for economic depreciation, $\delta_e^j(\tau)$, which describes the value of a piece of capital of type j and of age τ relative to a piece of type- j capital of age zero ($\delta_e^j(0) = 1$, $(\delta_e^j)'(\tau) \leq 0$). Using this schedule, the real wealth stock is defined as:

$$K_j^w(t) = \sum_{\tau=0}^{\infty} I_j(t-\tau) (1 - \delta_e(\tau))$$

The nominal wealth stock is then constructed as $P_j(t) K_j^w(t)$.

Examples of wealth stocks include the capital stock series of the U.S. National Income and Product Accounts (NIPA), which are formally known as the "Fixed Reproducible

Tangible Wealth” data. These series, largely based on geometric depreciation rates from the Hulten-Wyckoff studies, are used to provide estimates of the current-dollar loss in the value of the capital stock associated with production, the NIPA variable “Consumption of Fixed Capital” that is subtracted from GDP to arrive at Net Domestic Product.³

Consider now the relationship between wealth and productive capital stocks. For the moment, we will restrict discussion to the case where there is no embodied technological change. Suppose that capital of type i physically decays at a geometric rate δ_i . Let $p_v^i(t)$ be the price at time t of a unit of type- i capital produced in period v and assume there is an efficient rental market for new and used capital goods, so that a new unit of type- i capital is available for rent at rate $r_i(t)$ where this equals its marginal productivity. No-arbitrage in the capital rental market requires that the present value of the stream of rental payments for a capital good should equal the purchase price of the good. Given a discount rate r , this implies

$$p_{t-v}^i(t) = \int_t^\infty r_i(s) e^{-\delta_i(t-v)} e^{-(r+\delta_i)(s-t)} ds = e^{-\delta_i(t-v)} p_t^i(t)$$

Under these circumstances, then, the rate of economic depreciation equals the rate of physical decay and thus the real wealth stock equals the real productive stock.

It is well known, however, that this identity rests upon the assumption of geometric decay. For example, consider a one-time investment in an asset with a one-hoss-shay pattern of physical decay, whereby the asset produces a fixed amount for n periods and then expires (think of a light-bulb). In this case, the productive stock follows a one-zero path while the wealth stock gradually declines as the asset approaches expiration.⁴ Nevertheless, despite such counter-examples, the underlying pattern of economic depreciation has usually been found to be close enough to geometric for real wealth stocks to be considered good proxies for productive stocks; moreover, even those productivity studies that have constructed productive stocks from non-geometric patterns of physical decay have based these stocks on estimates of economic depreciation.⁵

³See Katz and Herman (1997) for a description of the NIPA capital stocks.

⁴See Jorgenson (1973) for the general theory on the relationship between wealth and productive concepts of the capital stock. Hulten and Wyckoff (1996) and Triplett (1996) are two recent papers that articulately explain the distinctions between physical decay and economic depreciation.

⁵For example, the Bureau of Labor Statistics (BLS) publishes an annual Multifactor Productivity calculation using productive stocks constructed according to a non-geometric “beta-decay” schedule that falls off to zero according to a specified service life. However, BLS uses the economic depreciation rates underlying the NIPA wealth stocks to set their service life assumptions, and in practice the BLS and NIPA stocks are

3 Capital Stocks with Embodied Technological Change

Embodied technological change occurs when new machines of type i are more productive than new type- i machines used to be. The focus of this paper, computing equipment, provides the most obvious example of this phenomenon: Today’s new PCs can process information considerably more efficiently than new PCs could five years ago. In this section, we consider some issues concerning the measurement of wealth and productive stocks with embodied technological change. We discuss the NIPA procedures for constructing wealth stocks for computing equipment, and use Solow’s (1959) model of vintage capital to outline the conditions under which these NIPA stocks can be interpreted as productive stocks.

3.1 The NIPA Real Wealth Stocks for Computing Equipment

In principle, the measurement of nominal wealth stocks is the same with embodied technological change as without. Even if capital of type i is improving every period, the only thing we need to calculate a wealth stock is a schedule for economic depreciation for this type of capital. We can use this schedule to weight up past type- i investment quantities and then use the current price to arrive at a nominal wealth stock. Quality improvement does not have to be taken into account.

For the purposes of calculating wealth, this procedure is fine. However, while one does not have to take quality improvement explicitly into account in the measurement of wealth stocks, this does not mean this issue is unimportant for the National Accounts. Measurement of the real output of the PC industry based only on the number of PCs produced would completely miss the increased ability of this industry to produce *computing power*. Given that computing power is an economically valuable product (people are willing to pay extra for more powerful computers), it seems more sensible to define the real output of the computer industry on a “quality-adjusted” basis. Since 1985 the U.S. NIPAs have followed this approach, and thus the real investment series for computing equipment are based on quality-adjusted prices, constructed from so-called hedonic regressions that control for the effects on price of observed characteristics such as memory and processor speed.

The fact that real investment in computing equipment is measured in quality-adjusted units has important implications for the calculation of wealth stocks. As Steve Oliner

very similar. See BLS (1983) for a description of their methodology.

(1989) has demonstrated, once one is using quality-adjusted real investment data, then the construction of the real wealth stock cannot use an economic depreciation rate estimated for non-quality-adjusted units. The availability of superior machines at lower prices is one of the principal reasons that computers lose value as they age. However, once we have converted our real investment series to a constant-quality basis, to use a depreciation rate for non-quality-adjusted units would be to double-count the effect of quality improvements. Instead Oliner (1989, 1994) proposed using the coefficient on age $(t - v)$ from hedonic vintage price regressions of the form

$$\log(p_v(t)) = \beta_t + \theta \log(X_v) - \tilde{\delta}_e(t - v) \quad (1)$$

where $p_v(t)$ is the price at time t of a machine introduced at time v , and X_v describes the features embodied in the machine. We will call $\tilde{\delta}_e$ the *quality-adjusted economic depreciation rate*.⁶ Since 1997, Oliner’s depreciation schedules have formed the basis for the NIPA wealth stocks for computing equipment. We will take a closer look at these schedules in Section 4.

3.2 The Solow Vintage Model

The relationship between wealth and productive capital stocks is more complicated when there is embodied technological change. To illustrate, we will use a slightly embellished version of Solow’s (1959) vintage capital model.⁷

There are two types of capital, one of which, computers, features embodied technological change and another (“ordinary capital”) which does not. Computers physically decay at rate δ : This is best thought of as a process by which a fraction of the remaining stock of machines from each vintage “explodes” each period. The technology embodied in new computers improves each period at rate γ , meaning that associated with each vintage of computers is a production function of the form

$$Q_v(t) = A(t) L_v(t)^{\alpha(t)} K_v(t)^{\beta(t)} \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right)^{1-\alpha(t)-\beta(t)} \quad (2)$$

where $I(v)$ is the number of computers purchased at time v , $L_v(t)$ and $K_v(t)$ are the quantities of labor and other capital that work with computers of vintage v at time t , and

⁶Oliner (1989) labeled this a “partial depreciation rate”.

⁷I have added a couple of features, such as disembodied technological change and multiple types of capital to help shed light on some issues in empirical growth accounting, but the logic of the model is from Solow.

$A(t)$ is disembodied technological change. Technology is of the putty-putty form, implying flexible factor proportions. The price of output and ordinary capital are assumed to be constant and equal to one. The price of computers (without adjusting for the value of embodied features) changes at rate g ($< \gamma$). Finally, labor and capital are obtained from spot markets with the wage rate being $w(t)$, a unit of ordinary capital renting at a price of $r^o(t)$, and a unit of computer capital of vintage v renting at rate $r_v(t)$.

The flow of profits obtained from operating computers of vintage v is

$$\begin{aligned} \pi_v(t) = & A(t) L_v(t)^{\alpha(t)} K_v(t)^{\beta(t)} \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right)^{1-\alpha(t)-\beta(t)} \\ & - r_v(t) I(v) e^{-\delta(t-v)} - r^o(t) K_v(t) - w(t) L_v(t) \end{aligned} \quad (3)$$

Firms choose how much labor and ordinary capital should work with vintage v so as to maximize the profits generated by the vintage. Re-arranging first-order conditions, the allocation of labor and ordinary capital to vintage v at time t is

$$L_v(t) = \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right) A(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha(t)}{w(t)} \right)^{\frac{1-\beta(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta(t)}{r^o(t)} \right)^{\frac{\beta(t)}{1-\alpha(t)-\beta(t)}} \quad (4)$$

$$K_v(t) = \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right) A(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha(t)}{w(t)} \right)^{\frac{\alpha(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta(t)}{r^o(t)} \right)^{\frac{1-\alpha(t)}{1-\alpha(t)-\beta(t)}} \quad (5)$$

So output from vintage v is

$$Q_v(t) = \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right) A(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha(t)}{w(t)} \right)^{\frac{\alpha(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta(t)}{r^o(t)} \right)^{\frac{\beta(t)}{1-\alpha(t)-\beta(t)}} \quad (6)$$

Given this allocation of factors to each vintage, consider the determination of rental rates and prices for computer capital. No-arbitrage in the rental market implies

$$r_v(t) = \frac{\partial Q_v(t)}{\partial (I(v) e^{-\delta(t-v)})}$$

and that the price of a new unit of computer capital is

$$p_v(v) = \int_v^\infty r_v(s) e^{-(r+\delta)(s-v)} ds$$

Differentiating this expression with respect to v we get

$$\dot{p}_v(v) = (r + \delta)p_v(v) + \int_v^\infty e^{-(r+\delta)(s-v)} \frac{dr_v(s)}{dv} ds - r_v(v)$$

From equation (6) we know that $\frac{dr_v(s)}{dv} = \gamma r_v(s)$: Rental rates decline cross-sectionally with age at rate γ . Using this property and re-arranging we have

$$r_v(v) = p_v(v) \left(r + \delta + \gamma - \frac{\dot{p}_v(v)}{p_v(v)} \right) \quad (7)$$

Note that when there is no embodied technological change ($\gamma = 0$) this formula reduces to the standard Jorgensonian rental rate. Because computer prices change at rate g , the expression simplifies to:

$$r_t(t) = (r + \delta + \gamma - g) e^{gt} \quad (8)$$

Combining the information that the rental rate for new vintages changes at rate g each period, and that, at any point in time, rental rates decline at rate γ with age tells us that

$$\frac{\dot{r}_v(t)}{r_v(t)} = g - \gamma < 0 \quad (9)$$

Note what this equation implies for the allocation of factors across vintages. From equation (6) we know that, at each point in time, the equilibrium level of output from each vintage is a multiplicative function of the initial quantity of investment $I(v)$. The declining rental rate implies that, despite the existence of disembodied technological change, the output produced by each remaining unit of a vintage of computers falls over time. A full solution to the model reveals that this occurs because $L_v(t)$ and $K_v(t)$ fall over time: Firms optimize profits by re-allocating other factors to work with newer vintages of computers. A simple way to view this process is that the utilization rate for a computer falls as it ages. In reality, this process of decreasing utilization may take many different forms. As improved PCs emerge, old PCs may be passed from high-human-capital workers to low-human-capital workers, then used as backups, and so on. Similarly, new investments in complementary factors such as the latest software will be allocated only to the newest vintages.

Finally, note that the price of a unit of vintage v computer capital is

$$p_v(t) = \int_t^\infty r_v(s) e^{-(r+\delta)(s-t)} ds = e^{gt} e^{-(\gamma+\delta)(t-v)} \quad (10)$$

Thus, the rate of economic depreciation is $\gamma + \delta$: Computers decline in price as they age not only because of physical decay but also because the introduction of new and improved computing technologies implies falling rates of utilization.

3.3 Wealth and Productive Stocks in the Solow Vintage Model

Consider now the calculation of aggregate capital stocks for computing equipment under the conditions of the Solow vintage model.

Wealth Stocks

As described earlier, there are two ways to calculate wealth stocks when there is quality-change. The first calculates a real wealth stock in terms of non-quality-adjusted units. These units decline in value at rate $\gamma + \delta$, implying an aggregate real wealth stock of

$$K^w(t) = \int_{-\infty}^t I(v) e^{-(\gamma+\delta)(t-v)} dv$$

This can be converted to a nominal wealth stock by multiplying by the non-quality-adjusted price of computing equipment, $p_t(t)$ (which equals e^{gt}).

In contrast, the second method, implemented in the construction of the NIPA computer stocks, uses quality-adjusted prices, quantities, and depreciation rates. By inserting the quality variable X_v (such that $\theta \log(X_v) = \gamma v$) into the vintage asset price equation, we change the equation from

$$\log(p_v(t)) = gt - (\gamma + \delta)(t - v)$$

to

$$\log(p_v(t)) = (g - \gamma)t - \theta \log(X_v) - \delta(t - v)$$

Thus, adjusting for embodied features, the price index for new computers changes at rate $g - \gamma$ and, importantly, *the quality-adjusted depreciation rate equals the rate of physical decay*. The quality-adjusted real wealth stock is

$$\tilde{K}^w(t) = \int_{-\infty}^t I(v) e^{\gamma v} e^{-\delta(t-v)} dv = e^{\gamma t} K^w(t)$$

As might have been expected, this real wealth stock grows γ percent faster than the non-quality-adjusted version. Note though, that the *nominal* wealth stock obtained from multiplying $\tilde{K}^w(t)$ by the quality-adjusted price index is the same as that obtained from the using the non-quality-adjusted data.

Productive Stocks

An elegant feature of the Solow vintage model is the fact that it can be neatly aggregated. Defining the aggregate stock of computing equipment as

$$C(t) = \int_{-\infty}^t I(v) e^{\gamma v} e^{-\delta(t-v)} dv \quad (11)$$

and aggregating equations (4), (5), and (6) across vintages, we get

$$L(t) = \int_{-\infty}^t L_v(t) dv = A(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha(t)}{w(t)} \right)^{\frac{1-\beta(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta(t)}{r^o(t)} \right)^{\frac{\beta(t)}{1-\alpha(t)-\beta(t)}} C(t) \quad (12)$$

$$K(t) = \int_{-\infty}^t K_v(t) dv = A(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha(t)}{w(t)} \right)^{\frac{\alpha(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta(t)}{r^o(t)} \right)^{\frac{1-\alpha(t)}{1-\alpha(t)-\beta(t)}} C(t) \quad (13)$$

$$Q(t) = A(t)^{\frac{1}{1-\alpha(t)-\beta(t)}} \left(\frac{\alpha(t)}{w(t)} \right)^{\frac{\alpha(t)}{1-\alpha(t)-\beta(t)}} \left(\frac{\beta(t)}{r^o(t)} \right)^{\frac{\beta(t)}{1-\alpha(t)-\beta(t)}} C(t) \quad (14)$$

Re-arranging this expression for aggregate output gives

$$Q(t) = A(t) L(t)^{\alpha(t)} K(t)^{\beta(t)} C(t)^{1-\alpha(t)-\beta(t)} \quad (15)$$

Thus, we have two important results:

- Aggregate output can be modelled using a Cobb-Douglas production function similar to that associated with each vintage, replacing the vintage-specific computer capital with an aggregate productive stock of computer capital, $C(t)$.
- $C(t) = \tilde{K}^w(t)$; In other words, the productive stock of computing equipment is identical to the quality-adjusted real wealth stock. This result comes from the fact that the quality-adjusted economic depreciation rate equals the rate of physical decay.

This second result is crucial: It implies that the NIPA real stocks for computing equipment, although intended as measures of wealth, can be used in aggregate productivity calculations. Unfortunately, though, it turns out that the evidence on quality-adjusted economic depreciation for computers is not consistent with the Solow vintage model. To understand why, we need to look more closely at the depreciation schedules derived by Oliner and used in the construction of the NIPA stocks.

4 Evidence on Computer Depreciation

Oliner (1989, 1994) studied depreciation patterns for four categories of computing equipment: Mainframes, storage devices, printers, and terminals. Figure 1 shows the quality-adjusted depreciation schedules from these studies that the Commerce Department's Bureau of Economic Analysis (BEA) has used to construct the NIPA wealth stocks. Figure 2 shows the (negative of) the corresponding depreciation rates. Oliner found evidence that quality-adjusted economic depreciation rates had increased over time and so BEA applies different schedules to investment data from different vintages.⁸

If the Solow vintage capital model is correct, then these quality-adjusted economic depreciation schedules should correspond to the schedules for physical decay. However, these estimates do not seem to be measuring physical decay for computers. I will note three facts that seem inconsistent with a physical decay interpretation, in ascending order of importance. First, with the exception of printers, the schedules show a marked non-geometric pattern, with depreciation rates increasing as the machines age. This contrasts with the results for other assets, for which geometric depreciation has proved a useful approximation. Second, the downward shifts over time in these schedules seem inconsistent with a physical decay interpretation since one would expect that, if anything, computing equipment has probably become more reliable over time, not less.

Third, and most serious, these numbers simply appear to be too high to be physical decay rates. Table 1 shows the 1997 NIPA depreciation rates for all categories of equipment. Remarkably, the quality-adjusted depreciation rates based on Oliner's studies are higher than the depreciation rates for all other categories of equipment except cars. This is all the more notable when one considers that for all other types of equipment, the NIPA depreciation rates are not based on a quality-adjustment approach and hence they combine the effects of both physical decay and embodied technological change. Casual observation suggests it is very unlikely that physical decay rates for computers are so much higher than for other types of equipment. Adding to the puzzle is the fact that Oliner's studies focused on IBM equipment, which, at the time, was automatically sold with pre-packaged service maintenance contracts: IBM guaranteed to repair or replace any damage due to equipment due to wear and tear. Thus, for the equipment in these studies, the estimated physical decay rates *should have been zero* since IBM absorbed the cost of physical decay. Oliner's

⁸These depreciation schedules can be found in Department of Commerce (1999).

(1994) study of computer peripherals also argues that these figures do not measure physical decay and in addition to the wealth stocks, presents productive stocks based on an assumed physical decay rate of zero.

Together, these arguments strongly suggest that the data have not been generated by the Solow vintage model. Next, we will present a simple extension to this model that will explain all three of the patterns noted here about the quality-adjusted depreciation schedules: The non-geometric shape, the downward shifts over time, and the fact that quality-adjusted economic depreciation rates that are larger than the rates of physical decay.

First, though, we need to point out an anomaly on Table 1, which is the NIPA depreciation rate for Personal Computers (PCs). Oliner's studies did not include PCs. In the absence of evidence for this category, and thus evidence that PCs are depreciating faster over time, BEA chose to use a schedule for mainframes estimated by Oliner (1989) that did not allow the pace of depreciation to vary over time. Since the schedules for each of the other categories of computing equipment have shifted down over time, this left PCs as the slowest depreciating category. BEA has acknowledged that this is anomalous and intends to introduce new capital stock estimates for PCs reflecting depreciation rates closer to those used for the other computing categories.⁹

5 Computing Support Costs and Endogenous Retirement

The following extension of the Solow vintage model is motivated by two observations. The first is that the basic model is inconsistent with technological obsolescence as defined in the introduction. It predicts that firms will never choose to retire a machine that retains productive capacity. Rather, it suggests the optimal strategy is simply to let the flow of income from a computer gradually erode over time. The second observation is that computer systems are usually complex in nature and can only be used successfully in conjunction with technical support and maintenance. The explosion in demand in recent years for Information Technology (IT) positions such as PC network managers is a clear indication of the need to back up computer hardware investments with outlays on maintenance and support. Indeed, research by the Gartner Group (1999), a private consulting firm, shows

⁹See Moulton and Seskin (1999), page 12.

that, as of 1998, for every dollar firms spent on computer hardware there was another 2.3 dollars spent on wages for IT employees and consultants. The model presented here uses the existence of support costs to motivate the phenomenon of technological obsolescence: Once the marginal productivity of a machine falls below the support cost, the firm will choose to retire it.

5.1 Theory

I will use a very simple formulation of support costs: For each remaining computer from vintage v , firms need to incur a support cost each period equal to a fraction s of the original purchase price, $p_v(v)$. Thus, if the firm purchased the machine for \$1000 and $s = 0.15$, then the firm has to pay \$150 per year to support it. The firm's profit function can now be expressed as

$$\begin{aligned} \pi_v(t) = & A(t) L_v(t)^{\alpha(t)} K_v(t)^{\beta(t)} \left(I(v) e^{\gamma v} e^{-\delta(t-v)} \right)^{1-\alpha(t)-\beta(t)} \\ & - r_v(t) I(v) e^{-\delta(t-v)} - r_o(t) K_v(t) - w(t) L_v(t) - s p_v(v) I(v) e^{-\delta(t-v)} \end{aligned} \quad (16)$$

How does the introduction of the support cost affect the model? First, note what has not changed. The additive support cost has no direct effect on the marginal productivity of the other factors that work with a vintage of computer capital. Thus, the first-order conditions for the allocation of labor and ordinary capital across vintages are unchanged, apart from one important new wrinkle. As before, declining utilization implies that the marginal productivity of a unit of computer capital falls over time at rate $\gamma - g$. Now, though, instead of allowing the marginal productivity to gradually erode towards zero, once a computer reaches the age, T , where it cannot cover its support cost, it is considered *obsolete* and is scrapped. The expression for the aggregate computer capital stock is changed to

$$C(t) = \int_{t-T}^t I(v) e^{\gamma v} e^{-\delta(t-v)} dv \quad (17)$$

and, given this new expression, aggregate output can still be described by the aggregate Cobb-Douglas production function in equation (15).

Figure 3 helps to tease out the implications of this pattern for economic depreciation. It shows the paths over time for the marginal productivity of a vintage of capital for a fixed set of values of r, δ , and $\gamma - g$ and for two values of the support cost parameter: $s = 0$,

in which case the model reduces to the Solow vintage model, and $s = .07$, shown as the horizontal line on the chart.¹⁰ Because firms now have to pay a support cost to operate the computer, the usual equality between the rental rate and the marginal productivity of capital needs to be amended to

$$r_v(t) = \frac{\partial Q_v(t)}{\partial (I(v)e^{-\delta(t-v)})} - sp_v(v) \quad (18)$$

For the purchase of a computer to be worthwhile, the present discounted value of these rents must still equal the purchase price.

$$p_v(t) = \int_t^{v+T} \left(\frac{\partial Q_v(n)}{\partial (I(v)e^{-\delta(n-v)})} - sp_v(v) \right) e^{-r(n-t)} e^{-\delta(n-v)} dn \quad (19)$$

Thus, for a given purchase price, the marginal productivity of a unit of computer capital must be higher when there is a support cost.

Consider now the path of the price of a computer as it ages. In terms of Figure 3, this price is determined by the area above the support cost and below the marginal productivity curve. Importantly, as the machine ages, this area declines at a faster rate than does the marginal productivity of the computer, reaching zero at retirement age. Since this marginal productivity declines at rate $g - \gamma$ over time, this implies that the price of the computer falls over time at a faster rate than $g - \gamma - \delta$ and so the economic depreciation rate for computers is greater than $\delta + \gamma$.

The model is solved formally in an appendix. The retirement age T is derived as the solution to the nonlinear equation

$$e^{(r+\delta+\gamma-g)T} = (r + \delta + \gamma - g) \left(\frac{1}{s} + \frac{1}{r + \delta} \right) e^{(r+\delta)T} - \frac{\gamma - g}{r + \delta} \quad (20)$$

While the solution to the equation will in general require numerical methods, one can show it has the intuitive property that the faster is the rate of quality-adjusted price decline for new computers, $\gamma - g$, and the higher is the support cost, the shorter is the time to retirement. Defining $\tau = t - v$, it can also be shown that the quality-adjusted economic depreciation schedule calculated from an Oliner-style study will be

$$\begin{aligned} d_v(t) &= e^{-\delta\tau} \left[1 + \frac{s}{r + \delta} - \left(\frac{se^{-(r+\delta)T}}{r + \delta + \gamma - g} \right) \left(\frac{\gamma - g}{r + \delta} + e^{(r+\delta+\gamma-g)\tau} \right) \right] \\ &\quad - e^{-(\delta+\gamma-g)\tau} \left(\frac{s}{r + \delta} \right) \left(1 - e^{-(r+\delta)(T-\tau)} \right) \end{aligned} \quad (21)$$

¹⁰The parameter values for the figure are $\gamma - g = .2$, $r = .03$, $\delta = .09$.

This extension of the Solow vintage model (which we will call the “obsolescence model”) can explain all three of the anomalies noted in our discussion of the evidence on computer depreciation. Non-geometric quality-adjusted depreciation, shown formally in equation (21), is an intuitive feature of the model, as explained by Figure 3. The downward shifts over time in the quality-adjusted economic depreciation schedules are consistent with an increase in the pace of embodied technological progress, a pattern that seems to fit with the apparent acceleration in technological change in the computer industry since the early 1980s. Finally, and most importantly, this model explains why the quality-adjusted economic depreciation rates, used to construct the NIPA real wealth stocks, are so high. Even if the rate of physical decay were zero, the expectation of early retirement would imply that computers still lose value as they age at a faster rate than the decline in quality-adjusted prices. Combined with significant anecdotal evidence for the importance of technological support and early retirements of computing equipment, these patterns point towards the need to explicitly account for technological obsolescence.

5.2 Alternative Estimates of Productive Stocks

The obsolescence model suggests that the quality-adjusted depreciation rates used to construct the NIPA real wealth stocks for computing equipment will be higher than the corresponding rates of physical decay. Thus, these real wealth stocks will be smaller than the appropriate productive stocks. The model also suggests an alternative strategy for estimating these productive stocks. Given values for s , δ , r , and $\gamma - g$, we can jointly simulate equations (20) and (21) to obtain both the retirement age and the schedule for quality-adjusted economic depreciation. Using the observed rate of quality-adjusted relative price decline to estimate $\gamma - g$, and assuming a value for r , we can obtain the values of s and δ that are most consistent with the observed depreciation schedules. The estimated δ 's can then be used to construct productive stocks.

Table 2 shows the estimated values of s and δ obtained from this procedure for the four classes of computing equipment in Oliner’s studies.¹¹ These values were based on the most recent depreciation schedules for each type of equipment and were obtained from

¹¹A value of $r = .0675$ was used. As explained in Appendix B, this value was also used in the calibrations of the marginal productivity of capital in our growth accounting exercises. The estimates of s and δ were not sensitive to this choice.

a grid-search procedure to find the values giving the depreciation profiles that best fitted Oliner’s schedules. The table shows that for mainframes, storage devices, and terminals, the obsolescence model’s depreciation schedules fit far better than any geometric alternative: Root-Mean-Squared-Errors of the predicted depreciation schedules relative to the observed schedules are far lower for the obsolescence model. Also, for mainframes and terminals, the parameter combinations that fit best are those that have a physical decay rate of zero. An exception to these patterns is printers, which as seen on Figure 2, show an approximately geometric pattern of decay. I have interpreted this as a rejection of the obsolescence model for this category. The estimated values for the support cost parameters for mainframes and terminals of 0.17 and 0.15 suggest a substantial additional expenditure, beyond the purchase price, over the lifetime of the computing equipment, but are low relative to what has been suggested by some studies, such as the Gartner Group research cited above.

The estimated values of s and δ imply a unique value of T , which was used to fit the economic depreciation schedules. This value of T could also be used to calculate the productive stock for each type of equipment according to equation (17). We can do a little better, however. While the model predicts that all machines of a specific vintage are retired on the same date, reality is never quite so simple: In practice, there is a distribution of retirement dates. Given a survival probability distribution, $d(\tau)$ that declines with age, the appropriate expression for the productive stock needs to be changed from equation (17) to

$$C(t) = \int_{-\infty}^t d(t-v) I(v) e^{\gamma v} e^{-\delta(t-v)} dv \quad (22)$$

This problem also needs to be confronted in the construction of economic depreciation schedules. If these schedules are constructed using only information on prices of assets of age τ , they will underestimate the average pace of depreciation: There is a “censoring” bias because we do not observe the price (equal to zero) for those assets that have already been retired. Hulten and Wykoff’s (1981) methodology corrects for this censoring problem by multiplying the value of machines of age τ by the proportion of machines that remain in use up to this age. Oliner’s depreciation studies followed the same approach and I have used his retirement distributions to construct estimates of productive stocks for computing equipment that are consistent with equation (22).¹²

Finally, we do not have a schedule to fit for PCs. As described in Section 4, the NIPA

¹²An implicit assumption here is that all retirements are voluntary, rather than being due to physical decay “explosions”. However, given our very low estimates of physical decay, this is a reasonable simplification.

depreciation rate for PCs is far lower than for the other categories of computing equipment. However, there is no evidence to support this assumption and BEA intends to revise the NIPA stock for PCs to bring this category into line with the other types of computing equipment. As a result, I have chosen to treat PCs symmetrically to mainframes, using the depreciation schedule applied by BEA for mainframes to construct a “NIPA-style” stock for PCs, and using identical schedules to derive the obsolescence model’s productive stocks for both PCs and mainframes.

Figure 4 displays the productive stocks implied by the obsolescence model and compares them with the NIPA real wealth stocks. Printers are not shown since we could not find sufficient evidence that the obsolescence model applied to this category. The low estimated rates of physical decay for the obsolescence model imply productive stocks that, in 1997 (the last year for which we have published NIPA stocks), ranged from 24 percent (for storage devices) to 72 percent (for mainframes) higher than their NIPA real wealth counterparts. The wide range in these ratios comes in part from the variation in the average age of these stocks: The NIPA stocks place far lower weights on old machines than the alternative stocks, and the stock of mainframes contains more old investment than the stock of storage devices. For PCs, by far the largest category in 1997, the obsolescence model implies a stock that is 44 percent larger than that implied by the NIPA-style stock.

6 Calculating the Computer-Usage Effect

We now consider the implications of these alternative estimates of productive stocks for the contribution of computer capital accumulation to aggregate output growth.

6.1 Methodology

Starting from a general production function:

$$Q(t) = F(X_1, X_2, \dots, X_n, t)$$

Solow (1957) defined the contribution of technological progress to output growth as that proportion of the change in output that cannot be attributed to increased inputs:

$$\frac{A'(t)}{A(t)} = \frac{1}{Q(t)} \frac{\partial F(X_1, X_2, \dots, X_n, t)}{\partial t}$$

Taking derivatives with respect to time we get

$$\frac{Q\dot{(t)}}{Q(t)} = \frac{A\dot{(t)}}{A(t)} + \sum_{i=1}^n \frac{X_i(t)}{Q(t)} \frac{\partial F(X_1, X_2, \dots, X_n, t)}{\partial X_i} \frac{X_i\dot{(t)}}{X_i(t)} \quad (23)$$

The contribution to growth of technological progress, known also as Total Factor Productivity (TFP), is calculated by subtracting a weighted average of growth in inputs from output growth, where each input's weight is determined by the quantity of the input used times its marginal productivity. As is well known, if the production function displays constant returns to scale with respect to inputs and factors are being paid their marginal products then these growth accounting weights will sum to one and the weight for a factor will equal its share of aggregate income.

In the case where output is a function of labor input, $L(t)$, and n capital inputs, $K_i(t)$, we have

$$\frac{Q\dot{(t)}}{Q(t)} = \frac{A\dot{(t)}}{A(t)} - \alpha(t) \frac{L\dot{(t)}}{L(t)} - \sum_{i=1}^n \beta_i(t) \frac{K_i\dot{(t)}}{K_i(t)} \quad (24)$$

Since labor's share of income is an observable parameter, we can use this as a time series for $\alpha(t)$. While we cannot observe the actual payments of factor income to different types of capital, the standard implementation of empirical growth accounting follows Jorgenson and Grilliches (1967) and uses theoretically-based measures of the marginal productivity of capital

$$r_i(t) = \frac{\partial F(X_1, X_2, \dots, X_n, t)}{\partial X_i}$$

to calculate growth accounting weights for each type of capital

$$\beta_i(t) = \frac{r_i(t) K_i(t)}{Q(t)} \quad (25)$$

The *contribution to growth* of accumulation of capital of type i is defined as $\beta_i(t) \frac{K_i\dot{(t)}}{K_i(t)}$.¹³ There are three areas where the calculation of the contribution of computer capital to growth differs depending on whether we model the data as being generated by the Solow vintage model or by the obsolescence model.

¹³Using theoretically-specified measures of the marginal productivity of each type of capital does not ensure that our growth accounting weights sum to one. In practice, then, we force this to be the case by restricting the weights for each type of capital to sum to capital's share of income, with the relative size of the weight for capital of type i being proportional to our estimate of $r_i(t) K_i(t)$. This procedure is discussed in more detail in the appendix.

Computer Capital Stock Growth Rates ($\frac{\dot{K}_i(t)}{K_i(t)}$): Perhaps surprisingly, these are almost identical under both the Solow vintage model (in which case we use the NIPA stocks) and the obsolescence model (in which case we use the alternative stocks). While the levels of the alternative stocks are higher than the levels of the NIPA stocks, the growth rates in the 1990s are very similar.

The Marginal Productivity of Computer Capital: The productive stock of computer capital is measured in quality-adjusted units. Thus, we need an estimate of the marginal productivity of adding another quality-adjusted unit. For both models, we know that declining utilization implies that the marginal productivity of non-quality-adjusted computer units declines cross-sectionally with age at rate γ

$$r_v(t) = r_t(t) e^{-\gamma(t-v)}$$

However, dividing by $e^{\gamma v}$, this means that, in terms of quality-adjusted units, the marginal productivity of capital equals $\tilde{r}(t) = r_t(t) e^{-\gamma t}$ for all units.

The formula for $r_t(t)$ differs in our two models. In the Solow vintage model we have

$$r_t(t) = (r + \delta + \gamma - g) e^{gt}$$

Letting $q(t) = e^{(g-\gamma)t}$ be the quality-adjusted computer price index, $\tilde{r}(t)$ is given by the traditional Jorgensonian rental rate:

$$\tilde{r}(t) = q_t(t) \left(r + \delta - \frac{\dot{q}_t(t)}{q_t(t)} \right) \quad (26)$$

In Appendix B, I show that the corresponding formula for the obsolescence model is

$$\tilde{r}(t) = q_t(t) \left[\left(r + \delta - \frac{\dot{q}_t(t)}{q_t(t)} \right) + s \left(1 - \frac{1 - e^{-(r+\delta)T}}{r + \delta} \frac{\dot{q}_t(t)}{q_t(t)} \right) \right] \quad (27)$$

These equations are the algebraic expression of the pattern shown on Figure 3: Introducing a support cost implies that the marginal productivity of capital must be higher to compensate for both the payment of the support cost and early retirement. Perhaps surprisingly, then, Table 3 shows that the estimates of $\tilde{r}(t)$ under the assumption that the Solow vintage model is correct are fairly similar to the estimates for the obsolescence

model.¹⁴ The reason for this is that the models give very different estimates of δ , with the obsolescence model being consistent with low values and the Solow model consistent with very high values. So, because of the high rates of economic depreciation, both models agree that the marginal productivity of computer investments should be high. However, they arrive at this conclusion via different reasoning: The obsolescence model sees that firms need to be compensated for support costs and early retirement, the Solow model that firms need to be compensated for high rates of physical decay.

The calculated values for $\tilde{r}(t)$ from the two models differ principally because of the effect of quality-adjusted price declines, $\frac{q_t(t)}{q_t(t)}$. This variable has a stronger effect on $\tilde{r}(t)$ in the obsolescence model because of the influence that faster embodied technological change has in shortening service lives. Thus, the obsolescence model's value of $\tilde{r}(t)$ is notably higher for PCs, because this is the category with the fastest rate of price decline.

The Level of Computer Capital Stocks: The final difference between these two models in the calculation of the contribution to growth of computer capital accumulation is what we have already shown - that the levels of the stocks consistent with the obsolescence model are higher than the NIPA stocks consistent with the Solow vintage model. This results in a higher contribution to growth for the obsolescence model for a simple reason: While both models agree that the stock of computer capital is growing fast and has high marginal productivity, this cannot have much effect on aggregate output if this stock is too small.

6.2 Results

Our empirical implementation is for the U.S. private business sector, the output of which equals GDP minus output from government and nonprofit institutions and the imputed income from owner-occupied housing.¹⁵ Table 4 gives a summary for both models of the combined contributions to output growth of the five types of computer capital. Figure 5 gives a graphical illustration.

The results for both models show similar patterns over time with the contributions from the obsolescence model consistently about 50 percent higher than those from the Solow vintage model. The results from the Solow model for the 1980s are very similar

¹⁴The values in Table 3 use 1997-based prices. In other words, $q(t) = 1$ for each category.

¹⁵Appendix B contains a detailed description of the empirical growth accounting calculations.

to those of Oliner and Sichel (1994), who, using essentially the same methodology, found that during this period computer capital accumulation contributed about two-tenths of a percentage point per year to aggregate output growth. However, both approaches agree that the contribution of computer capital accumulation has picked up substantially over the past few years, with average contributions over 1996-98 (in percentage points) of 0.57 for the Solow model and 0.82 for the obsolescence model.¹⁶ Both approaches also see this contribution accelerating as computers become a more important part of capital input. By 1998, the obsolescence model indicates that this contribution was worth almost a full percentage point for economic growth, 0.32 percentage points higher than for the Solow model. One interpretation of these results is that they provide a partial reversal of Oliner and Sichel's original resolution of the Solow paradox: Computers may not be everywhere but they are more prevalent than the NIPA capital stocks suggest, and even the NIPA series are growing very rapidly.

7 Computers and The Acceleration in Productivity

7.1 The Computer Sector and Aggregate TFP

Our results so far have suggested that the substantial investments in computing technologies made by U.S. businesses in recent years have had a very important influence on output growth. Note, though, that the models have been silent on the cause of this massive accumulation of computing power: Why has the price of computing power fallen so rapidly? Our models have assumed that output can be expressed in terms of an aggregate production function, which allows for two possibilities. The first is that computers are produced using the same technology as all other goods. However, this raises the question of why their relative price would decline. The second is that all computers have been imported, which clearly does not fit the reality of the U.S. economy. Thus, an alternative approach, which recognizes that computers may be produced using a different technology to other goods, seems appropriate.

Suppose that Sector 1 produces consumption goods and ordinary capital according to

¹⁶Oliner and Sichel (2000, forthcoming) also present figures for recent years that are very similar to those presented here for the Solow vintage model.

an aggregated production function, derived from a vintage structure as in previous sections:

$$Q_1(t) = A_1(t) L_1(t)^{\alpha(t)} K_1(t)^{\beta(t)} C_1(t)^{1-\alpha(t)-\beta(t)} \quad (28)$$

and that Sector 2 produces quality-adjusted computers according to a production function that is identical up to the multiplicative disembodied technology term:

$$Q_2(t) = A_2(t) L_2(t)^{\alpha(t)} K_2(t)^{\beta(t)} C_2(t)^{1-\alpha(t)-\beta(t)} \quad (29)$$

We are interested in estimating the behavior of $A_2(t)$ relative to $A_1(t)$. For the United States, we do not have sufficient information on capital stocks by industry to allow for direct estimation of this series for the computer industry using a growth accounting method. Instead, I will estimate this series under the simplifying assumption that both sectors are perfectly competitive, so prices are set equal to marginal cost. It is easily shown that, given the wage rate, $w(t)$, and the rental rates for ordinary capital, $r^o(t)$, and quality-adjusted computer capital $\tilde{r}(t)$, the cost function is

$$C_i(w(t), r^o(t), \tilde{r}(t), Q_i(t)) = \frac{Q_i(t)}{A_i(t)} \left(\frac{w(t)}{\alpha(t)} \right)^{\alpha(t)} \left(\frac{r^o(t)}{\beta(t)} \right)^{\beta(t)} \left(\frac{\tilde{r}(t)}{1-\alpha(t)-\beta(t)} \right)^{1-\alpha(t)-\beta(t)} \quad (30)$$

Thus, the ratio of Sector 1's price to Sector 2's is

$$\frac{p_1(t)}{p_2(t)} = \frac{\partial C_1(t)}{\partial Q_1(t)} / \frac{\partial C_2(t)}{\partial Q_2(t)} = \frac{A_2(t)}{A_1(t)} \quad (31)$$

Under these assumptions, we can use the relative decline in quality-adjusted computer prices to measure the relative rates of TFP growth in the computer and non-computer sectors:

$$\frac{\dot{A}_2(t)}{A_2(t)} - \frac{\dot{A}_1(t)}{A_1(t)} = \gamma - g \quad (32)$$

Consider now the behavior of a Tornqvist aggregate of $Q_1(t)$ and $Q_2(t)$. This aggregation procedure, which is a close theoretical approximation to the Fisher chain-aggregation method that has been used to construct real GDP since 1996, weights the real growth rates for each category according to its share in nominal output. The growth rate of this aggregate will be:

$$\frac{\dot{Q}(t)}{Q(t)} = (1 - \mu_t) \frac{\dot{Q}_1(t)}{Q_1(t)} + \mu_t \frac{\dot{Q}_2(t)}{Q_2(t)} \quad (33)$$

where μ_t is the share of the computer sector in nominal output. Applying the standard growth accounting equation to each sector, we get

$$\frac{Q_i \dot{(t)}}{Q_i(t)} = \frac{A_i \dot{(t)}}{A_i(t)} + \alpha(t) \frac{L_i \dot{(t)}}{L_i(t)} + \beta(t) \frac{K_i \dot{(t)}}{K_i(t)} + (1 - \alpha(t) + \beta(t)) \frac{C_i \dot{(t)}}{C_i(t)} \quad (34)$$

Performing an aggregate TFP calculation with the Tornqvist measure of aggregate output, we get

$$\begin{aligned} & \frac{Q \dot{(t)}}{Q(t)} - \alpha(t) \frac{L \dot{(t)}}{L(t)} - \beta(t) \frac{K \dot{(t)}}{K(t)} - (1 - \alpha(t) - \beta(t)) \frac{C \dot{(t)}}{C(t)} \\ &= (1 - \mu_t) \frac{A_1 \dot{(t)}}{A_1(t)} + \mu_t \frac{A_2 \dot{(t)}}{A_2(t)} \\ &= \frac{A_1 \dot{(t)}}{A_1(t)} + \mu_t (\gamma - g) \end{aligned} \quad (35)$$

The effect of faster TFP growth in the computer sector in boosting aggregate TFP growth can be measured as the product of the share of the computer industry in nominal output (μ_t) times the rate of relative price decline for computers, $(\gamma - g)$. Figure 6 describes this calculation.¹⁷ The upper panel shows that despite enormous declines in quality-adjusted prices, the nominal output of the computer industry has fluctuated around 1.5 percent of business output since 1983, ticking up a bit since the mid-1990s. The middle panel shows that the pace of quality-adjusted price declines accelerated rapidly after the mid-1990s. As a result, the boost to aggregate TFP growth from the computer sector, which had fluctuated around 0.25 percentage points a year between 1978 and 1995 has picked up considerably in recent years, averaging almost 0.5 percentage points a year in 1997 and 1998.

These figures are likely to be a lower bound on the contribution of the computer sector to aggregate TFP growth because of the assumption of perfect competition. Equating the TFP growth differential between computer and non-computer sectors with the relative decline in computer prices implies that a given set of factors' ability to generate *nominal* output should be the same in the computer and non-computer industry. However, even looking only at the computer industry's ability to produce nominal output, there still appears to

¹⁷There is no official measure of the output of the computer industry. The measure of nominal computer output used here is the sum of consumption, investment, and government expenditures on computers plus exports of computers and peripherals and parts minus imports for the same category. The measure of real output is the Fisher chain-aggregate of these 5 components.

have been large productivity improvements. Perhaps surprisingly, despite maintaining its share in aggregate nominal output, employment in SIC Industry 357 (computer and office machinery) has declined almost continuously from a high of 522,000 in 1985 to about 380,000 in 1998.¹⁸ Moreover, while we do not have estimates of the capital stocks of computing and non-computing equipment being used in the computer industry, the NIPA capital stocks show that the proportions of both stocks in use in the two-digit industry that contains the computer industry (SIC 35) have been declining since the mid-1980s. Thus, if anything, TFP growth in the computer industry has been stronger than we have assumed and that part of the improved productivity of the computer sector may have shown up as higher markups over marginal cost.

7.2 Interpreting Recent Productivity Developments

Our results have shown that the effects on aggregate output growth of both computer usage and improved productivity in the computer sector increased substantially over the period 1996-98. This period also saw a notable step-up in the growth rate of labor productivity, an unusual development late into an expansion: Business sector labor productivity averaged 2.15 percent during this period, a full 1 percentage point more than the average rate over the previous 22 years.

We can calculate the role of computer-related factors in the acceleration in labor productivity by subtracting hours growth from both sides of our growth accounting equation:

$$\begin{aligned} \frac{Q\dot{(t)}}{Q(t)} - \frac{H\dot{(t)}}{H(t)} &= \frac{\dot{A}_C(t)}{A_C(t)} + \frac{\dot{A}_{NC}(t)}{A_{NC}(t)} + \alpha(t) \left(\frac{L\dot{(t)}}{L(t)} - \frac{H\dot{(t)}}{H(t)} \right) \\ &\quad + \beta(t) \left(\frac{K\dot{(t)}}{K(t)} - \frac{H\dot{(t)}}{H(t)} \right) + (1 - \alpha(t) - \beta(t)) \left(\frac{C\dot{(t)}}{C(t)} - \frac{H\dot{(t)}}{H(t)} \right) \end{aligned} \quad (36)$$

Productivity growth is a function of TFP growth (here divided into the contributions of the computer and non-computer sectors, labeled C and NC), of computer and non-computer capital accumulation, and of improvements in the quality of labor input (represented as an increase in labor input relative to hours). I will focus on the two computer-related elements of productivity growth, $\frac{\dot{A}_C(t)}{A_C(t)}$ and $(1 - \alpha(t) - \beta(t)) \left(\frac{C\dot{(t)}}{C(t)} - \frac{H\dot{(t)}}{H(t)} \right)$, and represent the productivity growth due to all other factors as a residual.

¹⁸Source: Bureau of Labor Statistics, *Employment and Earnings*.

Table 5 shows the results of this decomposition using computer capital accumulation effects from our preferred obsolescence model. Computer capital accumulation and computer sector TFP growth together account for 1.23 percentage points of the 2.15 percent a year growth in business sector productivity over 1996-98. Moreover, a remarkable 0.73 percentage points of the 1 percentage point increase in labor productivity growth over 1996-98 can be explained by computer-related factors.¹⁹ In fact, the calculated 0.26 percentage point acceleration due to other factors probably overstates the true effect of these factors since, as Gordon (1999) has discussed, methodological changes in price measurement introduced into the GDP statistics that were not fully “backcasted” to earlier periods probably contributed around three-tenths a year to the acceleration in measured productivity in our data.²⁰

Our results indicate that computers have played a crucial role in the recent pickup in aggregate productivity growth.²¹ These calculations should be interpreted carefully, however. While the results appear to endorse the popular belief that there is a connection between high-tech investments and improved productivity, they also contradict the position of some of the more enthusiastic believers in the benefits of technology investments. In particular, we have assumed that all capital investments earn the same net rate of return. Thus, the common belief that high-tech investments earn supernormal returns and are thus more profitable than other investments, would if correct, show up here as an improvement in productivity growth due to “All Other Factors”, which (accounting for measurement factors) we do not see.

What of the outlook for future productivity growth? Will current rates of productivity growth persist or evaporate? Both upside and downside risks are apparent. The downside risks center around the dependence of the recent positive performance on one sector of the economy. The recent period of spectacular rates of productivity improvement in the computer sector, and the associated acceleration in quality-adjusted price declines, may

¹⁹Stiroh (1998) is another paper that examines the combined effects of computer capital accumulation and computer-sector TFP growth. For his sample, which ends in 1991, Stiroh reports total computer-related effects on growth that are notably lower than those in this paper, largely because of the differences in the treatment of computer capital accumulation.

²⁰This problem has been rectified with the October 1999 benchmark revision to the NIPAs.

²¹These calculations are similar to those of Gordon (1999) in stressing the important role that increased productivity in the computer sector has played in directly boosting aggregate productivity growth in recent years. However, they differ starkly from Gordon’s calculations in attributing an even more important role to the effect of computer capital accumulation on productivity throughout the economy. Gordon does not assign a role to this factor. His analysis instead emphasizes the effects of cyclical utilization.

turn out to be a flash in the pan. Indeed, given historical patterns, it seems unlikely that the recent pace of computer-related technological advance can be sustained. Given that we did not find any evidence that TFP growth has picked up outside this sector, a slowdown in aggregate productivity growth would be the most likely outcome.

The upside potential has two elements. First, the recent burst of productivity growth does not appear to be particularly cyclical in nature: Increased utilization would show up as an increase in productivity growth due to “All Other Factors”. Thus, there is little reason to believe that we will see a period of sluggish productivity growth as “payback” for the current period. Second, thus far, it does not appear as though the computer industry is close to exhausting the potential for producing faster and cheaper computers. Moreover, one lesson from the expansion of the Internet is that businesses are still taking advantage of declines in the price of computing power by finding new and (hopefully) productive uses for computing technologies.

8 Conclusions

The purpose of this paper has been part methodological, part substantive. The methodological contribution has been to outline the issues surrounding capital stock measurement in the presence of embodied technological change and technological obsolescence. In particular, the paper provides a number of arguments against the use of the NIPA computer capital stocks for growth accounting and suggests an alternative approach. The substantive contribution has been to document the role that computers have played in the recent productivity performance of the U.S. economy: A marked pickup in the rate of computer capital deepening combined with improved productivity in the computer-producing industry have accounted for almost all of the recent acceleration in aggregate productivity.

I will conclude by pointing to the need for further empirical research in this area. Most of the calculations in this paper have relied on estimates of things that are difficult to measure (quality-adjusted prices for computing equipment) or studies that may themselves have become obsolete (Oliner’s depreciation schedules). Given the increasing importance of computing technologies, further empirical work on the measurement of prices and depreciation for computing equipment would be extremely useful for refining and extending the analysis in this paper.

References

- [1] Bureau of Labor Statistics (1983). *Trends in Multifactor Productivity, 1948-1981*. BLS Bulletin No. 2178.
- [2] Feldstein, Martin and Michael Rothschild (1974). "Towards an Economic Theory of Replacement Investment", *Econometrica*, 42, 393-423.
- [3] Gartner Group (1999). 1998 IT Spending and Staffing Survey Results, report available at www.gartnergroup.com.
- [4] Goolsbee, Austan (1998). "The Business Cycle, Financial Performance, and the Retirement of Capital Goods", *Review of Economic Dynamics*.
- [5] Gordon, Robert J. (1999). Has the New Economy Rendered the Productivity Slowdown Obsolete?, mimeo, Northwestern University.
- [6] Gravelle, Jane (1994). *The Economic Effects of Taxing Capital Income*, Cambridge: MIT Press.
- [7] Hulten, Charles and Frank Wykoff (1981a). "The Estimation of Economic Depreciation Using Vintage Asset Prices: An Application of the Box-Cox Power Transformation", *Journal of Econometrics*, 15, 367-396.
- [8] Hulten, Charles and Frank Wykoff (1981b). "The Measurement of Economic Depreciation" in Charles Hulten, ed., *Depreciation, Inflation, and the Taxation of Income from Capital*, Washington DC: The Urban Institute Press. 367-396.
- [9] Hulten, Charles and Frank Wykoff (1996). "Issues in the Measurement of Economic Depreciation: Introductory Remarks", *Economic Inquiry*, 33, 10-23.
- [10] Jorgenson, Dale (1973). "The Economic Theory of Replacement and Depreciation" in W. Sellekaerts, ed., *Econometrics and Economic Theory*, New York: Macmillan.
- [11] Jorgenson, Dale and Zvi Griliches (1967). "The Explanation of Productivity Change", *Review of Economic Studies*, 34, 249-280.
- [12] Jorgenson, Dale and Kevin Stiroh (1999). "Information Technology and Growth", *American Economic Review*, May, 109-115.

- [13] Katz, Arnold and Shelby Herman (1997). “Improved Estimates of Fixed Reproducible Tangible Wealth, 1929-95”, *Survey of Current Business*, May, 69-92.
- [14] Moulton, Brent and Eugene Seskin (1999). “A Preview of the 1999 Comprehensive Revision of the National Income and Product Accounts”, *Survey of Current Business*, October, 6-17.
- [15] Oliner, Stephen (1989). “Constant-Quality Price Change, Depreciation, and Retirement of Mainframe Computers”, in *Price Measurements and Their Uses*, ed. Allan Young, Murray Foss, and Marilyn Manser, Chicago: University of Chicago Press.
- [16] Oliner, Stephen (1994). Measuring Stocks of Computer Peripheral Equipment: Theory and Application, mimeo, Federal Reserve Board.
- [17] Oliner, Stephen and Daniel Sichel (1994). “Computers and Output Growth Revisited: How Big is the Puzzle?”, *Brookings Papers on Economic Activity*, 2, 273-317.
- [18] Oliner, Stephen and Daniel Sichel (2000). “The Resurgence of Growth in the Late 1990s: Are Computers the Story?”, forthcoming, *Journal of Economic Perspectives*.
- [19] Sichel, Daniel (1997). *The Computer Revolution: An Economic Perspective*, Washington DC: Brookings Institution Press.
- [20] Solow, Robert (1957). “Technical Change and the Aggregate Production Function”, *Review of Economics and Statistics*, 39, 312-320.
- [21] Solow, Robert (1959). “Investment and Technical Progress” in *Mathematical Methods in the Social Sciences*, 1959, eds. Kenneth Arrow, Samuel Karlin, and Patrick Suppes, 89-104. Stanford, CA: Stanford University Press.
- [22] Stiroh, Kevin J. (1998). “Computers, Productivity, and Input Substitution”, *Economic Inquiry*, 36, 175-191.
- [23] Triplett, Jack (1996). “Depreciation in Production Analysis and in Income and Wealth Accounts”, *Economic Inquiry*, 33, 93-115.
- [24] U.S. Department of Commerce, Bureau of Economic Analysis (1999). *Fixed Reproducible Tangible Wealth, 1925-94*, Washington DC: U.S. Government Printing Office.

A Solution to the Obsolescence Model

This appendix derives the formal solution for the depreciation schedule under the assumptions of the obsolescence model.

The Marginal Productivity of Capital: As described in the text, the computer price arbitrage formula is

$$p_v(t) = \int_t^{v+T} \left(\frac{\partial Q_v(n)}{\partial (I(v)e^{-\delta(n-v)})} - sp_v(v) \right) e^{-r(n-t)} e^{-\delta(n-v)} dn$$

Denoting the marginal productivity of a unit of capital as

$$r_v^*(t) = \frac{\partial Q_v(t)}{\partial (I(v)e^{-\delta(t-v)})}$$

We get the following formula for the purchase price

$$p_v(v) = \frac{1}{1 + \frac{s}{r+\delta} (1 - e^{-(r+\delta)T})} \int_v^{v+T} r_v^*(n) e^{-(r+\delta)(n-v)} dn$$

Now, differentiating the price of new computers with respect to v we get

$$\dot{p}_v(v) = (r + \delta + \gamma) p_v(v) - \frac{r_v^*(v)}{1 + \frac{s}{r+\delta} (1 - e^{-(r+\delta)T})} + \frac{r_v^*(v+T) e^{-(r+\delta)T}}{1 + \frac{s}{r+\delta} (1 - e^{-(r+\delta)T})}$$

At the time of scrappage, the computer must be just covering the support cost, implying $r_v^*(v+T) = sp_v(v)$. Making this substitution, using $p_t(t) = e^{gt}$ and re-arranging gives us the marginal productivity of new computer capital:

$$r_v^*(v) = \left[(r + \delta + \gamma - g) + s \left(1 + \frac{\gamma - g}{r + \delta} (1 - e^{-(r+\delta)T}) \right) \right] e^{gv}$$

This equation shows that the introduction of computing support costs implies a higher marginal productivity of computer capital than in our previous model.

The Retirement Age: Since all the conditions for the allocation of other factors to each vintage are as before, the marginal productivity of a unit of computer capital still declines over time at rate $g - \gamma$, so we can now define the retirement age T from

$$\left[(r + \delta + \gamma - g) + s \left(1 + \frac{\gamma - g}{r + \delta} (1 - e^{-(r+\delta)T}) \right) \right] e^{(g-\gamma)T} e^{gv} = se^{gv}$$

Re-arranging, we get

$$e^{(r+\delta+\gamma-g)T} = (r + \delta + \gamma - g) \left(\frac{1}{s} + \frac{1}{r + \delta} \right) e^{(r+\delta)T} - \frac{\gamma - g}{r + \delta}$$

Given values for r, δ, s , and $\gamma - g$, this nonlinear equation can be solved numerically to give us the retirement age, T .

Economic Depreciation: Given a path for the marginal productivity of a unit of computer capital, we can now explain the pattern of economic depreciation implied by this path.

$$p_v(t) = \int_t^{v+T} r_v^*(n) e^{-r(n-t)} e^{-\delta(n-v)} dn - sp_v(v) \int_t^{v+T} e^{-r(n-t)} e^{-\delta(n-v)} dn$$

To keep this calculation simple, we will break it into two, defining

$$\begin{aligned} p^*(t) &= \int_t^{v+T} r_v^*(n) e^{-r(n-t)} e^{-\delta(n-v)} dn \\ &= r_v^*(v) e^{rt} e^{(\delta+\gamma-g)v} \int_t^{v+T} e^{-(r+\delta+\gamma-g)n} dn \\ &= r_v^*(v) e^{-(\delta+\gamma-g)(t-v)} \frac{\left(1 - e^{-(r+\delta+\gamma-g)(T-t+v)}\right)}{r + \delta + \gamma - g} \end{aligned}$$

Now, we use the fact that $r_v^*(v) = se^{gv} e^{(\gamma-g)T}$. Inserting this, re-arranging, and defining the age of the vintage as $\tau = t - v$, we get

$$p_v^*(t) = e^{gt} e^{-(\delta+\gamma)\tau} \left(\frac{se^{-(r+\delta)T}}{r + \delta + \gamma - g} \right) \left(e^{(r+\delta+\gamma-g)T} - e^{(r+\delta+\gamma-g)\tau} \right)$$

Finally, inserting the expression for $e^{(r+\delta+\gamma-g)T}$ into equation 20 we get

$$p_v^*(t) = e^{gt} e^{-(\delta+\gamma)\tau} \left[1 + \frac{s}{r + \delta} - \left(\frac{se^{-(r+\delta)T}}{r + \delta + \gamma - g} \right) \left(\frac{\gamma - g}{r + \delta} + e^{(r+\delta+\gamma-g)\tau} \right) \right]$$

Thus

$$\begin{aligned} p_v(t) &= e^{gt} e^{-(\delta+\gamma)\tau} \left[1 + \frac{s}{r + \delta} - \left(\frac{se^{-(r+\delta)T}}{r + \delta + \gamma - g} \right) \left(\frac{\gamma - g}{r + \delta} + e^{(r+\delta+\gamma-g)\tau} \right) \right] \\ &\quad - e^{gv} e^{-\delta\tau} \left(\frac{s}{r + \delta} \right) \left(1 - e^{-(r+\delta)(T-\tau)} \right) \end{aligned}$$

And the quality-adjusted economic depreciation schedule calculated from an Oliner-style study by comparing the price of an old vintage with the price of new computers and then subtracting off the quality-improvement in the new computers gives

$$d_v(t) = e^{-\delta\tau} \left[1 + \frac{s}{r+\delta} - \left(\frac{se^{-(r+\delta)T}}{r+\delta+\gamma-g} \right) \left(\frac{\gamma-g}{r+\delta} + e^{(r+\delta+\gamma-g)\tau} \right) \right] - e^{-(\delta+\gamma-g)\tau} \left(\frac{s}{r+\delta} \right) \left(1 - e^{-(r+\delta)(T-\tau)} \right)$$

B Empirical Growth Accounting Details

Capital Stocks: The calculations used detailed disaggregated capital stock data: In addition to the 5 types of computing equipment, we use the 26 types of non-computing equipment shown on Table 1, 11 types of non-residential structures, and tenant-occupied housing (rental income from such housing is part of business output). For all non-computer stocks, we use the NIPA real wealth capital stocks, altered in two ways. First, these data are published through 1997. However, real investment data for 1998 are available. Thus, I extended each of the published capital stock series by growing them out using the 1998 investment data and the depreciation rates published in Katz and Herman (1997). Second, these stocks refer to year-end values. Since the growth accounting analysis seeks to explain year-average growth rates, year-average stocks were constructed by averaging adjacent year-end stocks. The same transformation was applied to the computer stocks for the obsolescence model.

Rental Rates: For all capital except computers in the obsolescence model, our empirical analysis proxied the marginal productivity of capital using the Hall-Jorgenson tax-adjusted rental rate

$$r_i(t) = p_i(t) \left(r + \delta_i - \frac{\dot{p}_i(t)}{p_i(t)} \right) \left(\frac{1 - c_i - \tau z_i}{1 - \tau} \right)$$

where $p_i(t)$ is the price of capital of type i relative to the price of output, r is the real interest rate, δ_i is the NIPA depreciation rate for capital of type i , τ is the marginal corporate income tax rate, z_i is the present discounted value of depreciation allowances per dollar invested, and c_i is the investment tax credit.

The real rate of return on capital, r , was set equal to 6.75 percent: This produces a series for the “required” income flow from capital that, on average, tracks with the observed

series for business sector capital income over our sample. The $\frac{\dot{q}_t(t)}{q_t(t)}$ term is calculated for each type of capital as a three-year moving average of the rate of change of the price of capital relative to the price of output. The tax terms were calculated for each type of capital using the information on tax credit rates and depreciation service lives presented in Gravelle (1994).

Growth Accounting Weights: We start by imputing factor shares for each type of capital. For labor, we use $\alpha(t)$, the labor share of income in the business sector. For capital, things are a bit more complicated. In theory, if we had perfect measures of the marginal productivity of each type of capital, then we could just use equation (25) to estimate the factor shares. In practice, theoretical estimates can produce a set of factor shares that do not sum to one. There are two standard methods for dealing with this problem. One is to vary the value of r each period so that $\sum_{j=1}^n r_j(t) K_j(t)$ equals total capital income. The second method, implemented in this paper, is to define the growth accounting weights for capital so that they sum to capital's share of income, letting the share for capital of type i be proportional to $r_i(t) K_i(t)$, by using the formula

$$\beta_i(t) = (1 - \alpha(t)) \left(\frac{r_i(t) K_i(t)}{\sum_{j=1}^n r_j(t) K_j(t)} \right)$$

The final growth accounting weights were constructed by averaging factor shares from adjacent years.

Computing Equipment:		Electrical Transmission	0.05
Mainframes	0.30	Cars	0.28
Terminals	0.27	Trucks, Buses, and Trailers	0.18
Storage Devices	0.28	Aircraft	0.08
Printers	0.35	Ships and Boats	0.06
Personal Computers	0.11	Railroad Equipment	0.06
Other Office Equipment	0.31	Household Furniture	0.14
Communications Equipment	0.11	Other Furniture	0.12
Instruments	0.14	Farm Tractors	0.15
Photocopying Equipment	0.18	Construction Tractors	0.16
Fabricated Metals	0.09	Agricultural Machinery	0.12
Steam Engines and Turbines	0.05	Construction Machinery	0.16
Internal Combustion Engines	0.21	Mining and Oilfield Machinery	0.15
Metalworking Machinery	0.12	Service Industry Machinery	0.15
Special Industrial Machinery	0.10	Other Electrical Equipment	0.18
General Industrial Machinery	0.11	Miscellaneous Equipment	0.15

Table 1: 1997 NIPA Depreciation Rates for Equipment

	Mainframes	Storage	Printers	Terminals
s	0.17	0.05	0	0.15
δ	0	0.06	0.35	0
RMSE	0.01	0.04	0.02	0.01
RMSE - Geometric	0.06	0.10	0.02	0.06

Table 2: Calibrating The Obsolescence Model

	Mainframes	PCs	Storage	Printers	Terminals
Solow Vintage Model	0.61	0.64	0.31	0.62	0.39
Obsolescence Model	0.67	0.76	0.20	0.62	0.33

Table 3: The Marginal Productivity of a Unit of Quality-Adjusted Computer Capital

	Solow Vintage Model	Obsolescence Model
1970-79	0.14	0.20
1980-89	0.24	0.39
1990-95	0.21	0.33
1996-98	0.57	0.82
1996	0.48	0.67
1997	0.55	0.80
1998	0.66	0.98

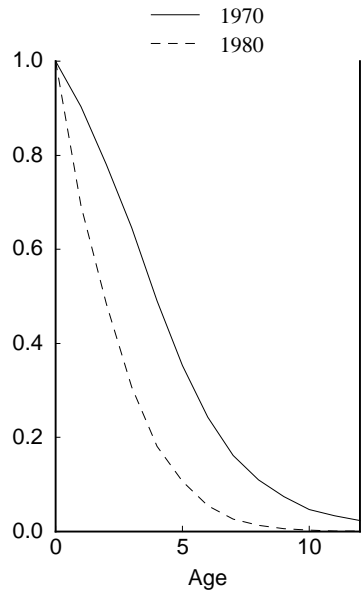
Table 4: The Contribution of Computer Capital Accumulation to Output Growth

	1974-95	1996-98
Growth in Labor Productivity	1.16	2.15
Effect of Computer Capital Accumulation	0.30	0.76
Effect of Computer TFP Growth	0.20	0.47
Total Computer-Related Effect	0.50	1.23
All Other Factors	0.66	0.92

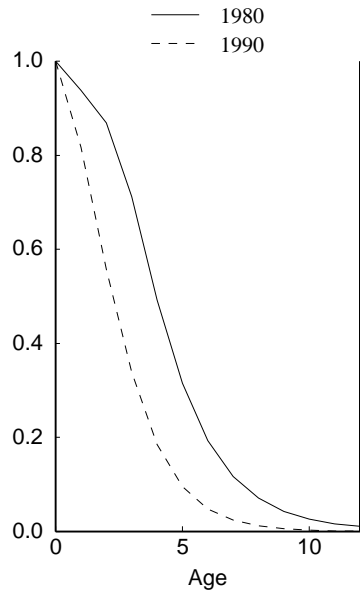
Table 5: Computers and Business Sector Productivity

Figure 1
Depreciation Schedules for Computing Equipment

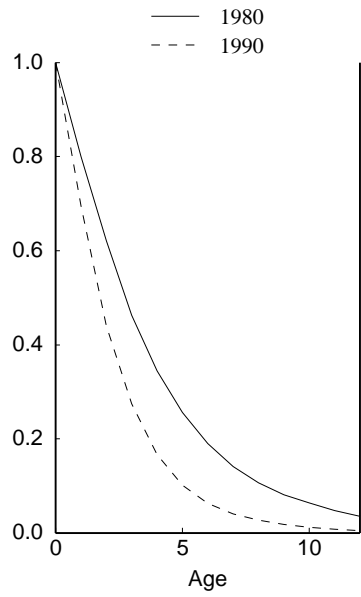
Mainframes



Storage Devices



Printers



Terminals

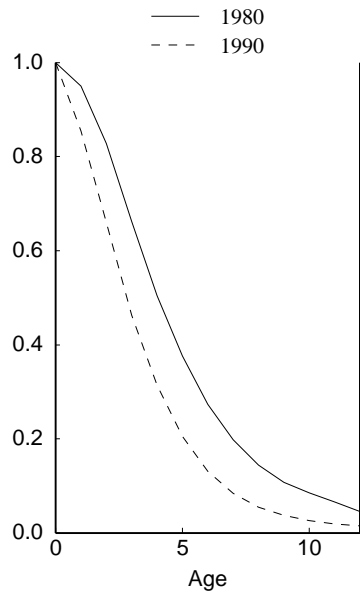
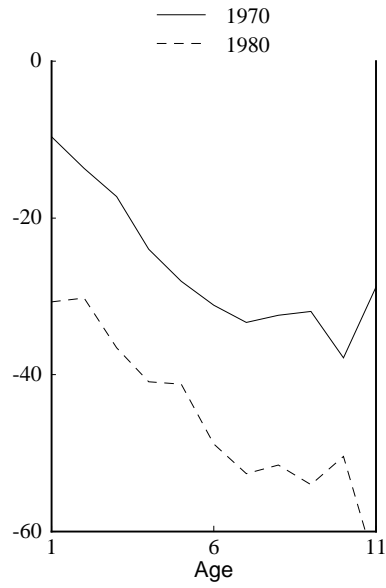
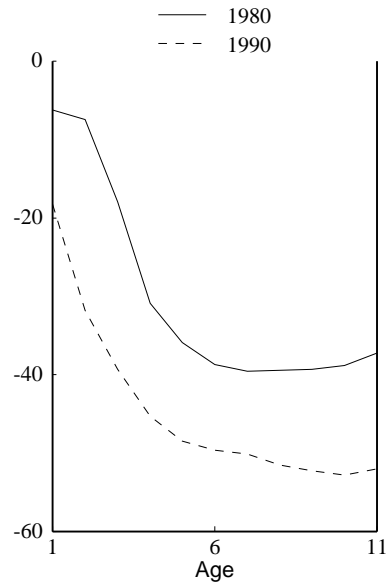


Figure 2
 Depreciation Rates for Computing Equipment

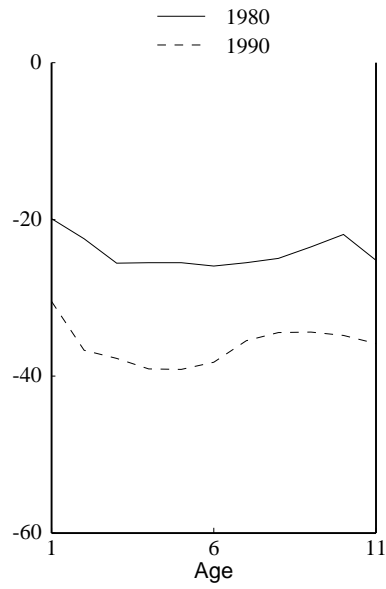
Mainframes



Storage Devices



Printers



Terminals

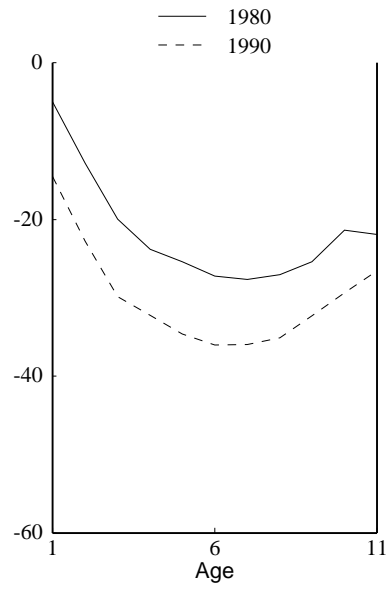


Figure 3
Endogenous Retirement and the Marginal Productivity of Capital

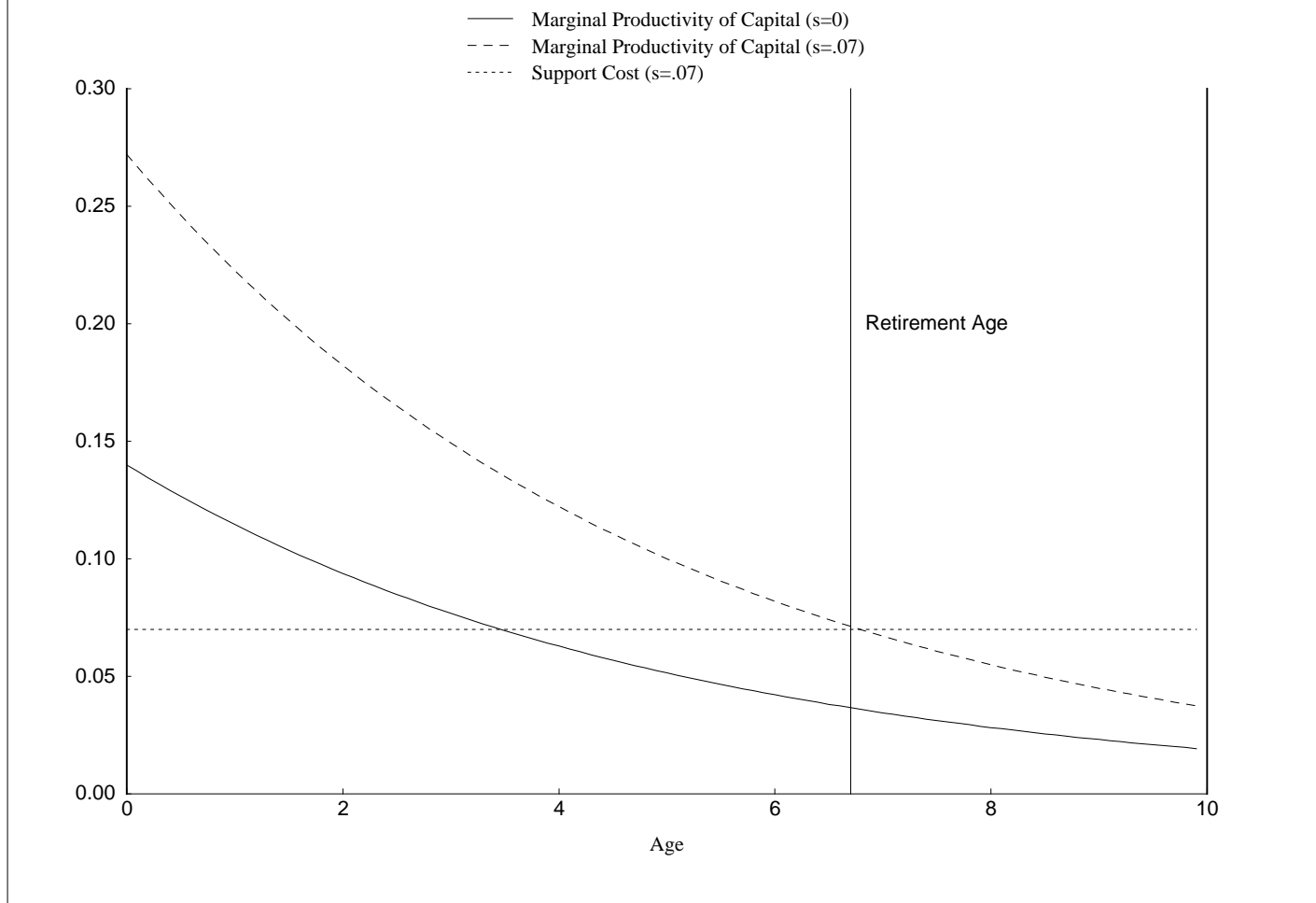


Figure 4
Alternative Measures of the Productive Capital Stock
 Billions of 1992 Dollars

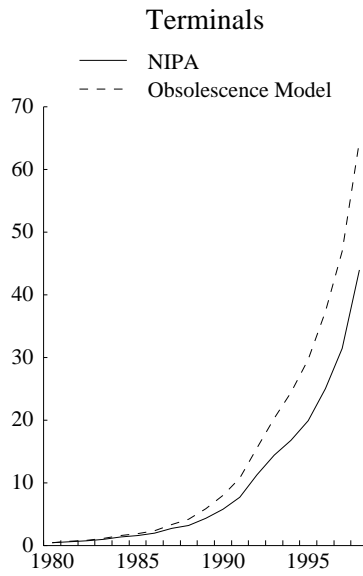
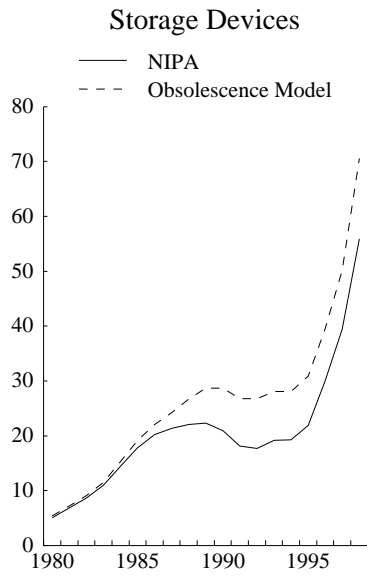
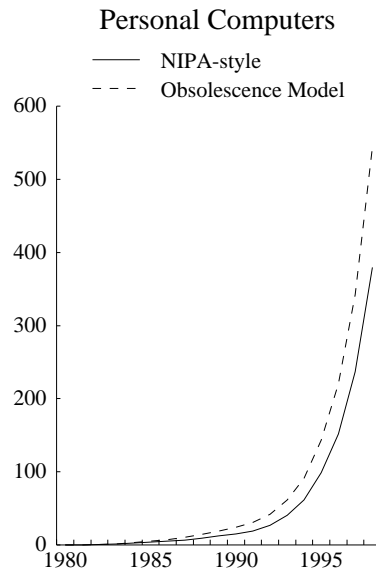
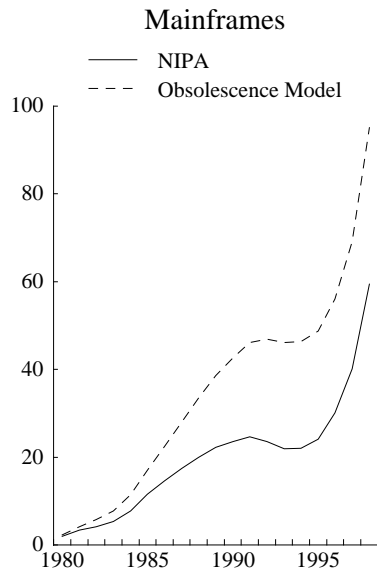


Figure 5
The Contribution of Computer Capital Accumulation to Aggregate Growth

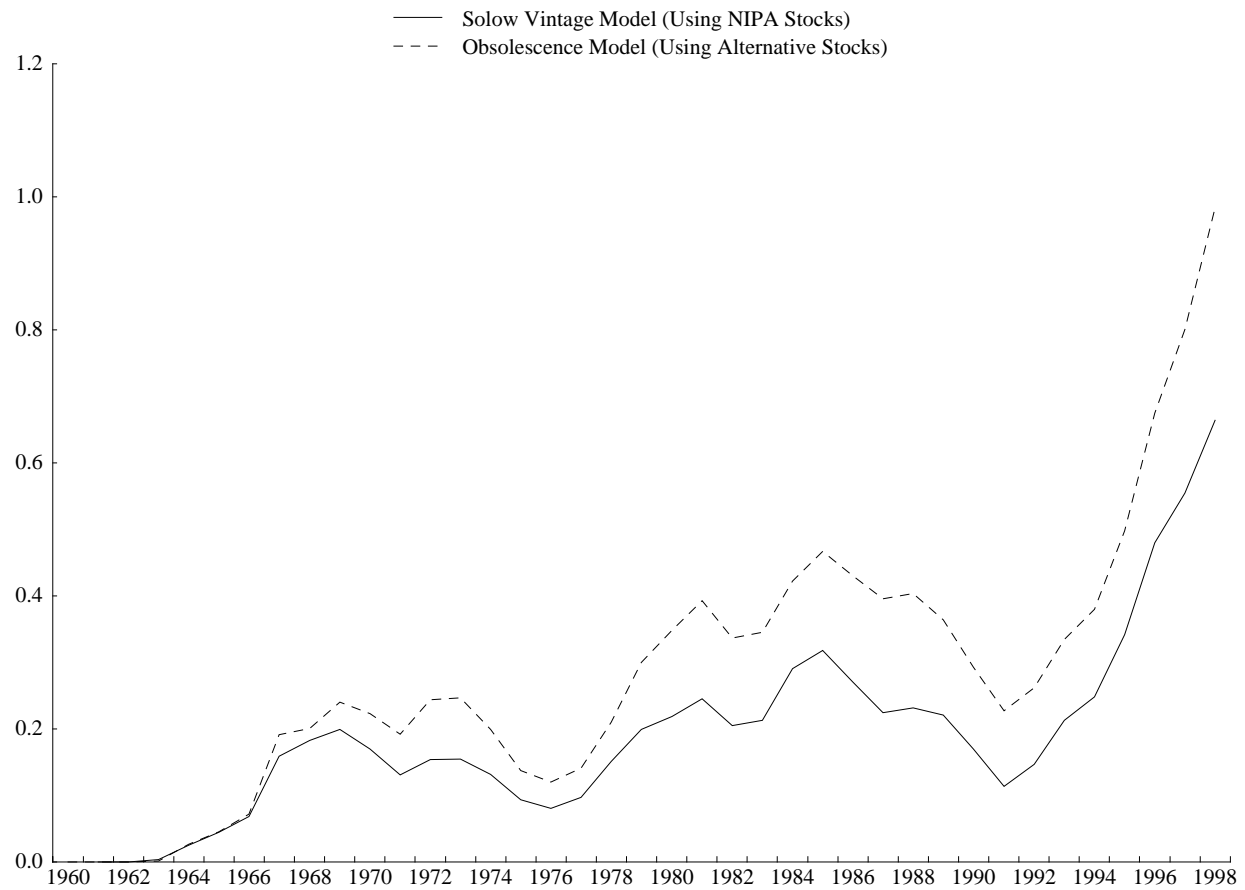


Figure 6
Computers and Aggregate TFP

