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# Optimal Allocation of Wind Generation Subject to Voltage Stability Constraints

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**Abstract**—In power systems the occurrence probability of operating points close to network limits may be increased as a result of high wind penetration. Consequences of such scenarios include inefficient exploitation of both wind and economic resources. A well chosen allocation of wind capacity not only is in line with the trend of renewables integration in power systems but also allows for limiting the occurrence probability of unsafe operating points that may require costly remedies. In this work, a voltage stability constrained optimal power flow (VSC-OPF) framework is presented for transmission system planning and applied to wind capacity allocation. This framework captures multiple wind and demand scenarios within the OPF. The pattern of wind capacity allocation is studied in order to assess its impact on voltage stability and the total wind capacity allocation. The results emphasize the effect of the capacity allocation pattern on improvement of voltage stability

**Index Terms**—Voltage stability; Optimal power flow; Wind; Capacity allocation; Capability diagram

## I. NOMENCLATURE

$b$ : set of buses  
 $no$ : normal operating point  
 $st$ : set of stressed operation conditions  
 $s$ : set of scenarios  
 $V_b$ : voltage at bus,  $b$   
 $V^{max}$ : highest voltage allowed  
 $V^{min}$ : lowest voltage allowed  
 $I_{Br}$ : current flow through branch,  $Br$   
 $I_{Br}^{rated}$ : current limit of branch,  $Br$   
 $P_G$ : active dispatch of generator,  $G$   
 $Q_G$ : reactive dispatch of generator,  $G$   
 $P_G^{min}$ : minimum active output of generator,  $G$   
 $P_G^{max}$ : maximum active output of generator,  $G$   
 $\omega_G$ : Status of generator,  $G$  (on: 1, off: 0)  
 $C_{WF}$ : capacity of wind farm,  $WF$   
 $C_{WF}^{min}$ : least capacity required for wind farm  $WF$   
 $C_{WF}^{max}$ : maximum capacity allowed for wind farm  $WF$   
 $CF_{WF}$ : Capacity factor of wind farm,  $WF$   
 $Q_{WF}$ : reactive dispatch of wind farm,  $WF$

$\mu_{1,WF}$ : maximum ratio between reactive power output capacity of wind farm,  $WF$

$\mu_{2,WF}$ : minimum ratio between reactive power output capacity of wind farm,  $WF$

$\lambda$ : Maximum allowed instantaneous wind penetration

$P_D$ : active consumption of demand,  $D$

$Q_D$ : reactive consumption of demand,  $D$

$\alpha_D$ : percentage of constant power in demand,  $D$

$\beta_D$ : percentage of constant current in demand,  $D$

$\gamma_D$ : percentage of constant impedance in demand  $D$

$DF_D$ : load factor of demand,  $D$

$LI_D$ : Load increment factor of demand,  $D$

$\kappa$ : required load increment percentage

## II. INTRODUCTION

Power generated from wind turbines is displacing conventional generators in power systems. Increasing wind penetration may occasionally force the power system towards operating points close to the physical and technical limits. Stability is one of the concerns in this regard. A loading reduction in the network due to the stability limits may lead to costly remedies such as load shedding and network reinforcement [1]. Voltage instability may occur following small disturbances such as incremental load increase or large disturbances such as loss of a generator, faults or circuit contingencies. Voltage stability assessment entails study time frames ranging from a few seconds to several minutes. Dynamics of fast-acting network components such as loads require short-term studies. Conversely, slow operating equipment e.g. tap-changers involve long term studies [2]. Long term small disturbance steady state voltage stability is the focus of this paper.

AC optimal power flow (ACOPF) is a potent framework for power systems planning and operation studies. Inclusion of steady state voltage stability constraints in ACOPF has been addressed in several works. Reference [3] uses an orthonormal decomposition of the power flow Jacobian matrix in order to monitor its minimum singular value. This value tends to zero at operating points close to instability. The voltage stability constrained OPF proposed in [4] considers the total transfer capability limit for cost minimization in reactive power planning. In [5] a series of preventive-corrective tools are introduced to the OPF in order to consider both grid code steady-state voltage limits and stability margin. Bender's decomposition

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technique is used to decompose the base and contingent cases in order to supply signals for the preventive-corrective tools. In [6] the L index is employed as an indicator of voltage stability margin. The OPF problem is solved with the aid of an improved particle swarm optimization algorithm in such a way that it provides a minimum stability margin. Reference [7] proposes reactive reserve based contingency constrained OPF. The method studies post contingency operation in order to find the optimum reactive reserve required for power system voltage stability.

A multi-scenario voltage stability constrained AC OPF (VSCOPF) framework for application in power systems planning studies is implemented here using the enhanced two sets of variables (TSV) approach introduced in [4]. The voltage stability criteria employed is the loadability margin (LM) subject to constraints such as generator capability, line rating and secure voltage range. This captures both saddle node and limit imposed bifurcation in the power flow equations. The multi-scenario aspect allows for consideration of multiple wind and demand levels. The difference of the proposed methodology with the previous works is the multi-scenario aspect of the VSCOPF and its application to wind capacity allocation. Variability of both wind and demand in the system may result in worst case scenarios other than those taken into account traditionally for transmission system planning. Furthermore, the connection bus of the wind generation in the network may also increase the level of uncertainty of the worst case scenarios. Thus it is essential to consider a reasonable number of scenarios covering all wind and demand levels in the wind generation planning studies.

Voltage stability may improve with an optimal siting and sizing of wind farms in the network. The framework implemented here is used to study the wind capacity allocation pattern to assess its impact on voltage stability and total wind capacity allocation in a 35 bus test system based on the topology of a subset of the Irish high voltage transmission system.

In Section III formulation of the methodology and its implementation are explained in details. The 35 bus test network and results of the application of the methodology is shown in Section IV. Section V concludes this paper.

### III. METHODOLOGY

#### A. Methodology Overview

The multi scenario VSCOPF non-linear problem is formulated as:

$$\min f(x_{no}^s, x_{st}^s) \quad (1)$$

$$s.t. \quad g(x_{no}^s, x_{st}^s) = 0 \quad (2)$$

$$h^{min} \leq h(x_{no}^s, x_{st}^s) \leq h^{max} \quad (3)$$

where  $x$ ,  $f(\cdot)$ ,  $g(\cdot)$ ,  $h(\cdot)$ ,  $h^{min}$ , and  $h^{max}$  stand for the vector of controllable variables, objective function, equality constraints function, inequality constraints function, upper bound and lower bound for inequality constraints, respectively. In (1)-(3), each wind-demand scenario is defined by a set

of variables representing the normal operating point of the system. Based on the the normal operating point, a further group of variables is introduced denoting the state of the system for a number of stressed conditions. Each stressed condition describes the system operating point after being subject to a load increase at a load serving bus mimicking incremental load increase in continuation power flow. Thus the number of operating points being considered in a single run is  $[(number\ of\ loads + 1) \times number\ of\ scenarios]$ . Here a load increment factor links the normal operating point to stressed operating points; this is basically a multiplier reflecting the percentage of increase in power transfer to a load serving bus in the system at stressed operating points with respect to the corresponding normal operating point. Treating the load increment factor as a decision variable in the optimization environment, allows for optimization of the network loadability margin. Fig.1 illustrates the structure of the formulation.

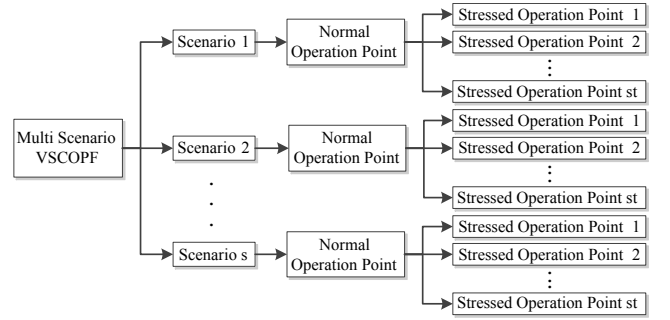


Fig. 1. Multi scenario VSCOPF problem

#### B. Multi Scenario VSCOPF Formulation

For stressed condition,  $st$ , defined as a subset of demands,  $D$ , the constraints and objective functions are formulated as follows.

##### 1) Equality constraints:

$$\sum_D^G (P_G^{s,no} \omega_G^s)_b + \sum^{WF} (C_{WF} C_{F_{WF}}^s)_b + \sum^{Br} (P_{Br}^{s,no})_b = \sum_D \{(\alpha_D + \beta_D V_b^{s,no} + \gamma_D V_b^{s,no^2}) DF_D^s P_D\}_b \quad (4)$$

$$\sum_D^G (Q_G^{s,no} \omega_G^s)_b + \sum^{WF} (Q_{WF}^{s,no})_b + \sum^{Br} (Q_{Br}^{s,no})_b = \sum_D \{(\alpha_D + \beta_D V_b^{s,no} + \gamma_D V_b^{s,no^2}) DF_D^s Q_D\}_b \quad (5)$$

$$\sum_D^G (P_G^{s,st} \omega_G^s)_b + \sum^{WF} (C_{WF} C_{F_{WF}}^s)_b + \sum^{Br} (P_{Br}^{s,st})_b = \sum_D \{(\alpha_D + \beta_D V_b^{s,st} + \gamma_D V_b^{s,st^2}) DF_D^s P_D\}_b + \sum_{D=st} \{(\alpha_D + \beta_D V_b^{s,st} + \gamma_D V_b^{s,st^2}) LI_D^s DF_D^s P_D\}_b \quad (6)$$

$$\begin{aligned} \sum_D^G (Q_G^{s,st} \omega_G^s)_b + \sum_{WF} (Q_{WF}^{s,st})_b + \sum_{Br} (Q_{Br}^{s,st})_b = \\ \sum_D \{(\alpha_D + \beta_D V_b^{s,st} + \gamma_D V_b^{s,st^2}) DF_D^s Q_D\}_b + \\ \sum_{D=st} \{(\alpha_D + \beta_D V_b^{s,st} + \gamma_D V_b^{s,st^2}) LI_D^s DF_D^s Q_D\}_b \end{aligned} \quad (7)$$

2) *Inequality constraints:*

$$P_G^{min} \leq P_G^{s,no}, P_G^{s,st} \leq P_G^{max} \quad (8)$$

$$Q_G^{min} \leq Q_G^{s,no}, Q_G^{s,st} \leq Q_G^{max} \quad (9)$$

$$\mu_2^s C_{WF} \leq Q_{WF}^{s,no}, Q_{WF}^{s,st} \leq \mu_1^s C_{WF} \quad (10)$$

$$\sum_D^G P_G^{s,no} \omega_G^s + \sum_{WF} C_{WF} C F_{WF}^s \geq \lambda \sum_{WF} C_{WF} C F_{WF}^s \quad (11)$$

$$I_{Br}^{s,no}, I_{Br}^{s,st} \leq I_{Br}^{rated} \quad (12)$$

$$V^{min} \leq V_b^{s,no}, V_b^{s,st} \leq V^{max} \quad (13)$$

$$LI_D^s \geq \kappa \quad (14)$$

Equations (4)-(7) are standard power flow equations and require active and reactive power injected at every bus to be equal to the power drawn from it in all operating points. The extra term in (6) and (7) applies the load increase factor in the stressed condition. Active and reactive power limits of conventional generators, are imposed through (8) and (9), respectively. Equation (10) defines the bounds on reactive power of wind farms;  $\mu_1$  and  $\mu_2$  may be extracted from the reactive capability digram of any type of wind farm at each scenario. Equation (11) ensures that the instantaneous wind penetration does not exceed a certain percentage in any of the study scenarios. In (12) and (13) the power flow through each branch and the voltage at each bus are required to be within the physical and grid code specified limits, respectively. Equation (14) requires the load increment factor to be higher than a minimum value,  $\kappa$ , at all load serving buses in all of the stressed operating points. This ensures that a minimum loadability margin is available throughout the network in all of the study scenarios.

3) *Objective functions:*

For the purpose of comparison two separate objective functions were considered, wind capacity allocation maximization and voltage stability maximization. These are denoted as (15) and (16), respectively. The former allocates wind capacity regardless of its effect on voltage stability while the latter considers this effect and employs it to enhance voltage stability in the system. Each objective function is run individually to find the optimal wind capacity allocation. Here the decision variables are  $C_{WF}$  and  $P_G$  for wind capacity allocation maximization and  $C_{WF}$ ,  $P_G$  and  $\kappa$  for voltage stability maximization.

$$a) \max \sum_{WF} C_{WF} \quad (15)$$

$$b) \max \kappa \quad (16)$$

C. *Implementation*

The VSCOPF formulation presented in this paper is implemented using the COmmon Optimization Python Repository (COOPR) [8]. KNITRO [9] non-linear solver is employed to solve the problem.

## IV. CASE STUDY

A. *Test System*

The 3-area 35 bus network illustrated in Fig.2 is employed as a test system. This network is based on the topology of a subset of the Irish transmission system originally introduced in [10]. The line parameters were altered to represent the 110 kV transmission system. Demand power factor at each bus was derived from [11]. 15 buses in the network are specified as candidate for wind capacity allocation. Table 1 lists these buses. Further data on the test system may be found in the appendix. Combination of 8 demand (30-100% load factors) and 10 wind (10-100% wind capacity factors) levels form the 80 scenarios to be considered by the VSCOPF. Generator status matrix,  $\omega_G^s$ , was built based on unit commitment carried for a range of loading conditions representing the study scenarios using the FAST unit commitment and dispatch tool [12]. Conventional generators were set to regulate voltage at their connection bus to 1 pu [13]. For the purpose of this study, wind farms allocated are required to inject power at unity power factor in all operating points. This is accomplished by fixing  $\mu_1$  and  $\mu_2$  to zero. Maximum allowed instantaneous wind penetration,  $\lambda$ , was set to 60%. VSCOPF allocates wind farm capacity and decides on conventional generator active dispatch at all operating points.

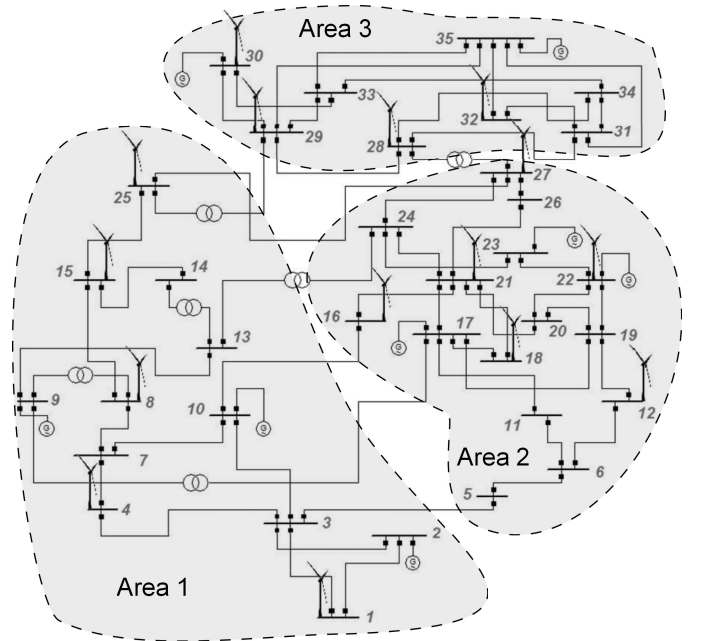


Fig. 2. Irish "All-Island" 35 bus test network [10]

TABLE I  
WIND CAPACITY ALLOCATION CANDIDATE BUSES

Candidate Buses for Wind Capacity Allocation	1, 4, 8, 12, 15, 16, 18, 21, 22, 25, 27, 28, 29, 30, 32
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B. Results

1) Total Capacity:

Table II presents the total wind capacity allocated in the network by VSCOPF for each objective function case. It is noted that the total wind capacity allocated in order to maximize voltage stability is approximately 8% lower than the maximum wind capacity that can be allocated; this shows that further wind capacity allocation in this case leads to either network limits breaching or reduction in the loadability margin in the system.

2) Voltage Stability:

Assessment of the loadability margin as a metric of network voltage stability shows that despite the higher wind capacity allocated with the wind maximization objective function, the network loadability margin is limited to 18.69% of the actual load due to weakness in area 3 of the network. Conversely, with voltage stability maximization used as the objective function significant improvement in the loadability margin is achieved. The minimum loadability margin ( $\kappa$ ) in the studied cases is listed Table II. Fig.3 illustrates the loadability margin for every load serving bus within the test system at each of the 80 study scenarios. Saturated points at 100% loadability margin level represent scenarios where the loadability margin is greater or equal to 100%. It is noted that in general voltage stability maximization leads to better loadability margins compared to the wind maximization case; this is evident in area 3 of the network where the binding loadability margin has increased from 18.69%, occurring at bus 28, to 47%, occurring at bus 30. The PV curve at bus 30 in the 100% wind capacity and 60% demand factor scenario is shown in Fig.4. The loadability margin for the voltage stability maximization case is approximately 17 MW higher than wind capacity maximization case. With both objective functions, voltage limit breaching at bus 30 is the binding factor for the loadability margin.

TABLE II  
TOTAL ALLOCATED WIND CAPACITY AND LOADABILITY MARGIN

Objective Function	Allocated Wind Capacity (MW)	Minimum Loadability Margin, $\kappa$ (%)
Max Wind Capacity	300	18.69
Max Voltage Stability	277.8	47

3) Allocation Pattern:

Table III presents the allocated wind capacity to candidate buses (buses not listed have zero allocation). It is seen that wind is mostly allocated at buses 1 and 4 with both wind

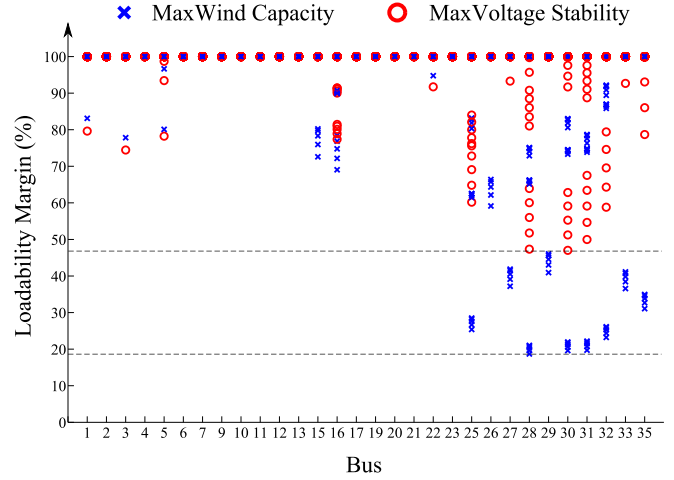


Fig. 3. Test system loadability margin

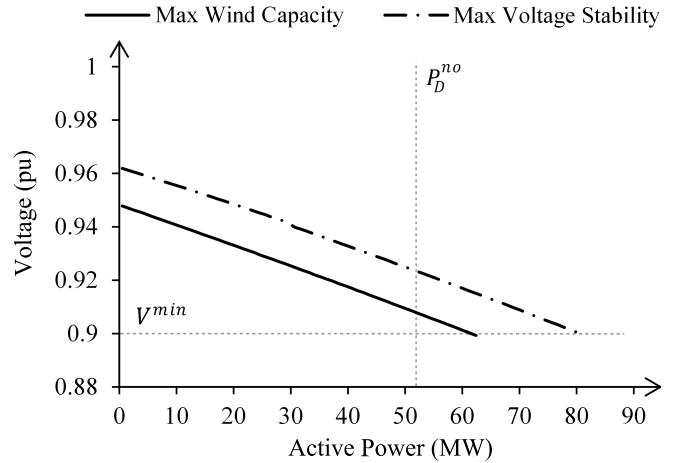


Fig. 4. PV curve at bus 30 for 100% wind capacity factor and 60% demand factor scenario

TABLE III  
PATTERN OF WIND CAPACITY ALLOCATION (MW)

Bus	Max Wind Capacity	Max Voltage Stability
1	150	150
4	150	91.56
30	0	20.68
32	0	15.58

maximization and voltage stability maximization objective functions. Buses 1 and 4 are electrically distant from major load centers and connected through high capacity lines to the rest of the network. A centralized allocation to these buses results in potentially higher MW losses and consequently a larger wind capacity allocation without breaching the network limits. It is also noted that in the voltage stability maximization case approximately 36 MW is allocated to buses 30 and 32. Allocation of capacity to these buses results in voltage rise in area 3 of the network explaining the higher loadability

margin realized in this case compared to the wind capacity maximization case. This denotes the difference of the buses, in terms of voltage stability improvement, where wind generation planning is considered on transmission systems.

## V. CONCLUSION

A multi scenario VSCOPF was implemented for power system planning by employing the loadability margin criteria subject to network constraint; This considers both saddle node and limit imposed bifurcation in the power flow equations. The multi scenario aspect of the methodology allowed for capturing of multiple wind and demand levels. The pattern of wind capacity allocation affects both the total wind capacity allocation and loadability margin in the network. Maximization of wind capacity is an objective function commonly used for wind capacity integration however, it does not address voltage stability issues arisen as an aftereffect of conventional generator displacement. On the other hand while there is a potential for improving voltage stability by allocating wind capacity to certain buses in the network, a trade off of such a scheme is reduction in the total wind capacity allocation realized. This reduction is dependent on the network topology and equipment limits. Further studies will consider various reactive power control schemes for wind farms and assess their impact on voltage stability as well as the pattern of wind capacity allocation.

## APPENDIX A 35 BUS TEST SYSTEM DATA

TABLE IV  
35 BUS TEST SYSTEM LINE PARAMETERS (P.U. ON 100 MVA BASE)

fr	to	Z	B	MVA	fr	to	Z	B	MVA
1	2	.007+.04j	.03	4.3	18	21	.014+.088j	.184	1.61
1	3	.007+.04j	.03	4.3	19	20	.009+.02j	.016	1.72
2	3	.004+.022j	.019	4.3	19	22	.009+.02j	.016	3.44
3	4	.012+.078j	.059	2.69	20	21	.009+.02j	.016	3.23
3	5	.022+.15j	.113	2.42	20	22	.009+.02j	.016	3.44
3	10	.029+.146j	.105	3.23	21	24	.007+.04j	.029	3.01
4	7	.026+.168j	.127	2.15	21	26	.012+.076j	.058	4.41
5	6	.007+.04j	.03	3.01	22	23	.002+.006j	.722	3.98
6	11	.019+.12j	.091	1.83	23	24	.003+.016j	.02	3.98
6	12	.024+.152j	.115	1.08	24	27	.016+.106j	.08	4.73
7	8	.002+.014j	.173	4.3	25	27	.031+.19j	.144	2.58
7	10	.019+.122j	.092	2.37	25	29	.05j	0	4.84
8	9	.084j	0	4.09	26	27	.009+.06j	.046	3.55
8	15	.024+.154j	.116	2.9	27	28	.05j	0	4.73
9	13	.005+.046j	.53	4.62	28	29	.003+.022j	.049	1.94
9	17	.158j	0	3.76	28	31	.005+.037j	.092	1.61
10	16	.076+.16j	.007	1.88	28	34	.01+.072j	.195	1.08
11	17	.016+.102j	.078	1.88	29	30	.003+.022j	.049	2.47
12	19	.014+.092j	.07	1.88	29	33	.005+.027j	.057	1.4
13	14	.08j	0	2.69	29	35	.008+.056j	.152	1.08
13	24	.092j	0	4.09	30	33	.005+.04j	.107	3.55
14	15	.009+.058j	.044	2.58	31	32	.002+.01j	.014	1.18
15	25	.024+.152j	.115	3.06	31	34	.008+.059j	.098	1.18
16	21	.038+.188j	.135	2.37	31	35	.005+.04j	.067	1.4
17	18	.007+.044j	.034	1.72	32	35	.005+.039j	.104	1.4
17	19	.012+.072j	.054	2.26	33	34	.002+.013j	.021	.97
17	21	.005+.032j	.023	3.55	33	35	.005+.04j	.092	1.08

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## REFERENCES

- [1] J. McCalley, V. Vittal, H. Wan, Y. Dai, and N. Abi-Samra, "Voltage risk assessment," in *Power Engineering Society Summer Meeting, 1999. IEEE*, vol. 1, 1999, pp. 179–184 vol.1.
- [2] P. Kundur, N. Balu, and M. Lauby, *Power system stability and control*, ser. EPRI power system engineering series. McGraw-Hill, 1994.
- [3] R. Avalos, C. Canizares, and M. Anjos, "A practical voltage-stability-constrained optimal power flow," in *Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE*, 2008, pp. 1–6.
- [4] W. Zhang, F. Li, and L. Tolbert, "Voltage stability constrained optimal power flow (vscopf) with two sets of variables (tsv) for reactive power planning," in *Transmission and Distribution Conference and Exposition, 2008. IEEE*, 2008, pp. 1–6.
- [5] E. Vaahedi, Y. Mansour, C. Fuchs, S. Granville, M. Latore, and H. Hamadanizadeh, "Dynamic security constrained optimal power flow/var planning," *Power Systems, IEEE Transactions on*, vol. 16, no. 1, pp. 38–43, 2001.
- [6] T. Niknam, M. R. Narimani, J. Aghaei, and R. Azizpanah-Abarghoee, "Improved particle swarm optimisation for multi-objective optimal power flow considering the cost, loss, emission and voltage stability index," *Generation, Transmission Distribution, IET*, vol. 6, no. 6, pp. 515–527, 2012.
- [7] H. Song, B. Lee, S. hyuk Kwon, and V. Ajarapu, "Reactive reserve-based contingency constrained optimal power flow (rccopf) for enhancement of voltage stability margins," *Power Systems, IEEE Transactions on*, vol. 18, no. 4, pp. 1538–1546, 2003.
- [8] A common optimization python repository. [Online]. Available: <https://software.sandia.gov/coopr>
- [9] Knitro optimization package. [Online]. Available: <http://www.ziena.com/>
- [10] D. Burke and M. O'Malley, "Maximizing firm wind connection to security constrained transmission networks," *Power Systems, IEEE Transactions on*, vol. 25, no. 2, pp. 749–759, 2010.
- [11] Eirgrid transmission forecast statement 2008-2014. [Online]. Available: <http://www.eirgrid.com/>
- [12] A. Shortt and M. O'Malley. The FAST Unit-Commitment Algorithm. Working Paper. [Online]. Available: <http://erc.ucd.ie/files/perma/FAST.pdf>
- [13] P. Cuffe, P. Smith, and A. Keane, "Capability chart for distributed reactive power resources," *Power Systems, IEEE Transactions on*, In Press, 2013.