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Dual wavelength digital holographic Laplacian reconstruction

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Access to the spatial derivatives of an optical wave field can be used to enhance edge detection, focusing, and holographic imaging. It was recently shown that, by using digital holographic techniques, the Laplacian of an object field can be extracted. Here it is demonstrated that equivalent results can be found using two holograms captured at either two distances or with two appropriately related wavelengths. Experimental and numerical results confirming the theoretical analyses are presented. The proposed two-wavelength-based system requires no mechanical repositioning of the object and is shown to provide superior performance. © 2010 Optical Society of America
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The practical realization of megapixel digital holography (DH) systems in the mid 1990s [1], and the continuing advancements of various enabling technologies (e.g., CCDs and PCs), has resulted in the semi-ubiquitous field of DH. DH systems by which whole-field information is retrieved involve either (i) off-axis or (ii) in-line recording geometries. In (i), the twin and object image are angularly separated, but the available spatial resolution is reduced as compared to (ii).

In 2004, Zhang *et al.* [2] described an in-line reconstruction method using two axially displaced intensity measurements. An extension of this method [3] involves altering the phase of the reference beam in order to extract the holographic data in a method akin to that used in phase-shifting DH [4]. Under specific conditions, the transport of intensity equation (TIE) [5,6] can be applied to in-line DH to minimize the effects due to the twin image. Recently it has been shown that, by Fresnel propagating the difference of two such holograms, recorded at two displaced planes, the resultant reconstruction is the second-order spatial differentiation or Laplacian of the reconstructed object wave field [7,8]. In this Letter we extend this concept, showing that, by recording two holograms using different wavelengths, the reconstructed image also approximates the Laplacian of the wave field. Significantly, using the proposed two-wavelength method, no object or camera movement is required.

The intensity hologram, $H(x, y, z, \lambda)$, captured at the camera plane, a distance z from the object, is the coherent superposition of the object wave field, $o(x, y, z, \lambda)$, and the reference plane wave, $r(x, y, z, \lambda)$:

$$H(x, y, z, \lambda) - \text{DC} = \underbrace{o(x, y, z, \lambda)r^*(x, y, z, \lambda)}_{\text{object}} + \underbrace{o^*(x, y, z, \lambda)r(x, y, z, \lambda)}_{\text{twin}}. \quad (1)$$

It consists of a zero-order or DC = $|r(x, y, z, \lambda)|^2 + |o(x, y, z, \lambda)|^2$ term, the unwanted twin image, and the desired object image. We apply scalar diffraction theory [7,8] and use $O(\xi, \eta, 0, \lambda)$ to denote the Fourier transform

(FT) of $o(x, y, 0, \lambda)$ at the object plane, $z = 0$. $\mathfrak{S}^{-1}()$ represents the inverse FT operation. We then apply the Fresnel paraxial approximation to describe the resulting complex object wave field:

$$o(x, y, z, \lambda) = \exp\left(+i\frac{2\pi}{\lambda}z\right)\mathfrak{S}^{-1}\{O(\xi, \eta, 0, \lambda) \times \exp[-i\pi z\lambda(\xi^2 + \eta^2)]\}. \quad (2)$$

To begin, we briefly review the two-plane method [7,8]. Two digital holograms of the same object are captured at distances z and $z + \Delta z$. The resulting difference hologram, $\Delta H_z = H(x, y, z + \Delta z, \lambda) - H(x, y, z, \lambda)$ is

$$\Delta H_z = r(x, y, z, \lambda)[o^*(x, y, z + \Delta z, \lambda) - o^*(x, y, z, \lambda)] + r^*(x, y, z, \lambda)[o(x, y, z + \Delta z, \lambda) - o(x, y, z, \lambda)]. \quad (3)$$

In the far field, when $\Delta z \ll z$, the differences in the DC terms are assumed negligible and are thus omitted from Eq. (3). Substituting from Eq. (2) into Eq. (3) and neglecting the phase,

$$\Delta H_z = \Delta z r(x, y, z, \lambda)\mathfrak{S}^{-1}\{+i\pi\lambda(\xi^2 + \eta^2) \times O^*(-\xi, -\eta, 0, \lambda) \exp[+i\pi\lambda z(\xi^2 + \eta^2)]\} + \Delta z r^*(x, y, z, \lambda)\mathfrak{S}^{-1}\{-i\pi\lambda(\xi^2 + \eta^2)O(\xi, \eta, 0, \lambda) \times \exp[-i\pi\lambda z(\xi^2 + \eta^2)]\}. \quad (4)$$

The Fresnel propagation operator $P_{-z}(-)$ produces the effect of propagating from the camera plane a distance $-z$ back to the object plane. Applied to the difference hologram,

$$\begin{aligned}
P_{-z}(\Delta H_z) &= \Delta z r(x, y, z, \lambda) \mathfrak{F}^{-1} \{ +i\pi\lambda(\xi^2 + \eta^2) \\
&\quad \times O^*(-\xi, -\eta, 0, \lambda) \exp[+i2\pi\lambda z(\xi^2 + \eta^2)] \} \\
&\quad + \Delta z r^*(x, y, z, \lambda) \mathfrak{F}^{-1} \{ -i\pi\lambda(\xi^2 + \eta^2) \\
&\quad \times O(\xi, \eta, 0, \lambda) \}. \tag{5}
\end{aligned}$$

Recalling the differential properties of the FT,

$$\begin{aligned}
P_{-z}(\Delta H_z) &= \frac{i\lambda\Delta z}{4\pi} r^*(x, y, z, \lambda) \\
&\quad \times \nabla_{xy}^2 o(x, y, 0, \lambda) - \text{twin}. \tag{6}
\end{aligned}$$

The first term in Eq. (6) is related to the Laplacian of the object, while the second is related to the Laplacian of the twin image [Eq. (9) in [7]] reconstructed a distance $2z$ from the object image. Thus, based on the difference of two holograms captured a distance Δz apart, the reconstructed object image is an approximation to the Laplacian of the object wave.

Alternatively, two holograms of the same object can be recorded using two different illuminating wavelengths, λ and $\lambda + \Delta\lambda$ a distance z from the camera. Under these condition, the objects' resulting complex amplitudes, $o(x, y, z, \lambda)$ and $o(x, y, z, \lambda + \Delta\lambda)$, are given by Eq. (2) and

$$\begin{aligned}
&\exp\left(+i\frac{2\pi}{\lambda + \Delta\lambda}z\right) \mathfrak{F}^{-1} \{ O(\xi, \eta, 0, \lambda \\
&\quad + \Delta\lambda) \exp[-i\pi z(\lambda + \Delta\lambda)(\xi^2 + \eta^2)] \}, \tag{7}
\end{aligned}$$

respectively. In this case,

$$\begin{aligned}
H(x, y, z, \lambda) - \text{DC} &= r^*(x, y, z, \lambda) \mathfrak{F}^{-1} \{ O(\xi, \eta, 0, \lambda) \\
&\quad \times \exp[-i\pi z\lambda(\xi^2 + \eta^2)] \} \\
&\quad + r(x, y, z, \lambda) \mathfrak{F}^{-1} \{ O^*(-\xi, -\eta, 0, \lambda) \\
&\quad \times \exp[+i\pi z\lambda(\xi^2 + \eta^2)] \}. \tag{8}
\end{aligned}$$

Two holograms are captured. If $\Delta\lambda \ll \lambda$, and assuming $r(x, y, z, \lambda) \approx r(x, y, z, \lambda + \Delta\lambda)$, the resulting difference, $\Delta H_\lambda = H(x, y, z, \lambda + \Delta\lambda) - H(x, y, z, \lambda)$, is

$$\begin{aligned}
\Delta H_\lambda &= r^*(x, y, z, \lambda) \Delta\lambda \mathfrak{F}^{-1} \{ -i\pi z(\xi^2 + \eta^2) O(\xi, \eta, 0, \lambda) \\
&\quad \times \exp[-i\pi z\lambda(\xi^2 + \eta^2)] \} + r(x, y, z, \lambda) \\
&\quad \times \Delta\lambda \mathfrak{F}^{-1} \{ +i\pi z(\xi^2 + \eta^2) O^*(-\xi, -\eta, 0, \lambda) \\
&\quad \times \exp[+i\pi z\lambda(\xi^2 + \eta^2)] \}. \tag{9}
\end{aligned}$$

Backpropagating and recalling the differential properties of the FT results in the Laplacian of the object field:

$$P_{-z}(\Delta H_\lambda) = \frac{iz\Delta\lambda}{4\pi} r^*(x, y, z, \lambda) \nabla_{xy}^2 o(x, y, 0, \lambda) - \text{twin}. \tag{10}$$

Equation (10) is directly analogous to Eq. (6), and, therefore, reconstructing the difference of two holograms captured at two wavelengths (separated by $\Delta\lambda$), at a single distance z from the CCD, provides an approximation to

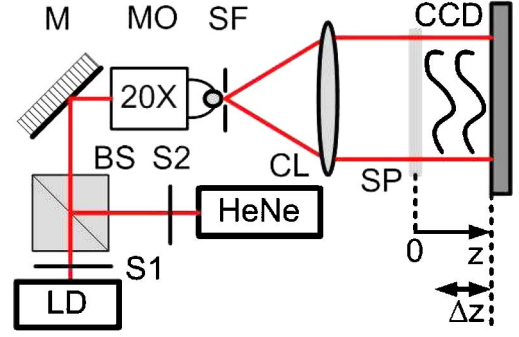


Fig. 1. (Color online) Schematic of the experimental setup.

the Laplacian of the original object field. Comparing the analyses culminating in Eqs. (6) and (10), it can be shown that they produce the expected Fresnel equivalency [9] when $\Delta z/z = \Delta\lambda/\lambda$.

The experimental setup is presented in Fig. 1. It contains an He-Ne laser of wavelength $\lambda = 632.8$ nm and a laser diode (LD) (DealExtreme 05900 Red 5 mW laser module) with a measured wavelength of $\lambda + \Delta\lambda = 652.5$ nm, i.e., $\Delta\lambda = 19.7$ nm. In the system, the illuminating source is controlled by two independent shutters (S1 and S2). Both beams are combined using a beam splitter (BS), redirected using a mirror (M), and focused by a 20× microscopic objective (MO) onto a 15 μm diameter pinhole (SF). Collimation is provided by a $f = 20$ cm lens (CL). The object is located in the sample plane (SP), and the hologram is recorded by an Imperx-4M15-G digital camera (15.15 mm × 15.15 mm, 2048 × 2048 pixels). The camera is mounted on a UTS 50PP translation stage driven by a Newport SMC100 controller. The first object is a 150-μm-thick transparent glass slide with the black letters “UCD” transferred onto it from a Letraset transfer sheet L92. The second object imaged was a commonly available small jewelers’ flathead screwdriver (Radionics part number RS537-861 with narrowest side width 300 μm). The data processing was performed using MATLAB (R2007a).

All eight images presented in Fig. 2 are for fields of view of 9.6 mm × 4.8 mm (1300 × 650 pixels). All five images presented in Fig. 3 are for fields of view of 3.78 mm × 1.89 mm (512 × 256 pixels). In each case, four holograms were captured. Two were illuminated using $\lambda = 632.8$ nm and captured with the object positioned at $z = 160$ mm and $z + \Delta z = 165$ mm. The other two were illuminated with $\lambda + \Delta\lambda = 652.5$ nm. These parameter values were chosen so to approximately satisfy the Fresnel equivalency, i.e., 0.03125–0.03113.

In Fig. 2(a), the magnitude of the object wave field, $|o(x, y, z = 0, \lambda)|$, is estimated by backpropagating $H(x, y, z, \lambda)$. The magnitude of the Laplacian is calculated directly by applying the numerical operation “del2(-)” [MATLAB (R2007a)] to this object wave field; this is provided in Fig. 2(b). In Fig. 2(c), a two-plane difference hologram, ΔH_z , for $\lambda = 632.8$ nm is given, with the corresponding magnitude of the Laplacian reconstruction, $|P_{-z}(\Delta H_z)|$, in Fig. 2(d). Similarly, a difference hologram, ΔH_λ , for two wavelengths, is presented in Fig. 2(e). The corresponding magnitude of the Laplacian reconstruction at z , $|P_{-z}(\Delta H_\lambda)|$, is given in Fig. 2(f). By using Eq. (10), we also construct a difference hologram

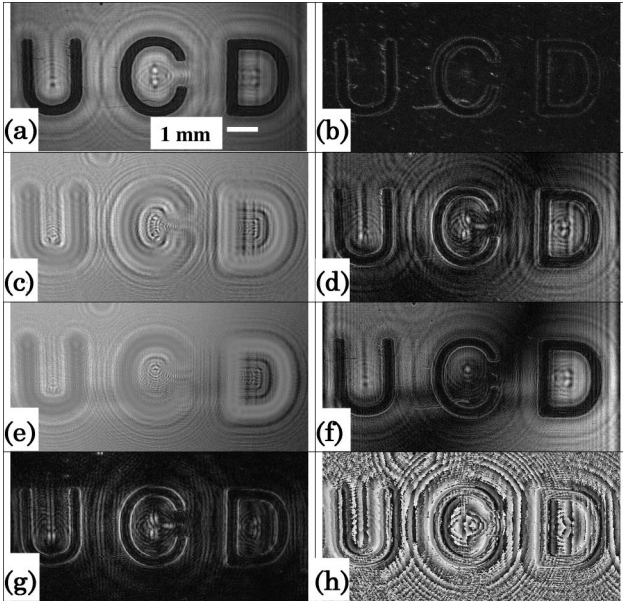


Fig. 2. In (a), a horizontal 1 mm bar is present in the reconstructed image $|o(x, y, z = 0, \lambda)|$ and, in (b), $|\nabla_{xy}^2 o(x, y, z = 0, \lambda)|$, calculated directly from a single hologram using MATLAB. (c) ΔH_z and (d) the corresponding magnitude of the resulting Laplacian differential reconstruction at $z = 160$ mm [see Eq. (6)] with $z + \Delta z = 165$ mm and $\lambda = 632.8$ nm. (e) ΔH_λ with $z = 160$ nm and $\lambda = 632.8$ nm and $\lambda + \Delta\lambda = 652.5$ nm. (f) Magnitudes of Laplacian differential reconstructions: (d) $z = 160$; see Eq. (10). (g) Magnitude and (h) phase, calculated from a difference hologram $\Delta H_{z\lambda}$ by combining both methods.

$\Delta H_{z\lambda} = H(x, y, z + \Delta, \lambda + \Delta\lambda) - H(x, y, z, \lambda)$. We then propagate $\Delta H_{z\lambda}$ to the object plane. In Fig. 2(g), we present the magnitude of this variation, while Fig. 2(h) shows the resulting phase map. This illustrates the λ/z degeneracy [9].

In Fig. 3, we provide a quantitative comparison between the direct numerical calculation using the complex object field, the two-plane method, and the two-wavelength method. Figure 3(a) shows the magnitude of the object wave field estimated by backpropagating $H(x, y, z, \lambda)$. The Laplacian is again calculated directly, giving Fig. 3(b). A difference hologram, ΔH_λ , for two wavelengths, is presented in Fig. 3(c), while $|P_{-z}(\Delta H_\lambda)|$ is given in Fig. 3(d). Under ideal conditions, Figs. 3(b), 3(d), and 3(e) should be identical, since they all represent the magnitude of the Laplacian of the same object field reconstructed with the same wavelength λ at the same plane z . To compare these three results, lines of 200 pixel values [indicated by thin white lines in Figs. 3(b), 3(d), and 3(e)], were extracted and normalized with respect to their average values. The results are plotted in Fig. 3(f). It is clear that the two-wavelength result is the best, as it is (i) the most symmetric, (ii) consistently has the lowest values in the dark object field center, and (iii) the locations of the two edges (object extent) are more easily identifiable.

Much of the variation among the experimental results can be explained by (a) difficulties in maintaining

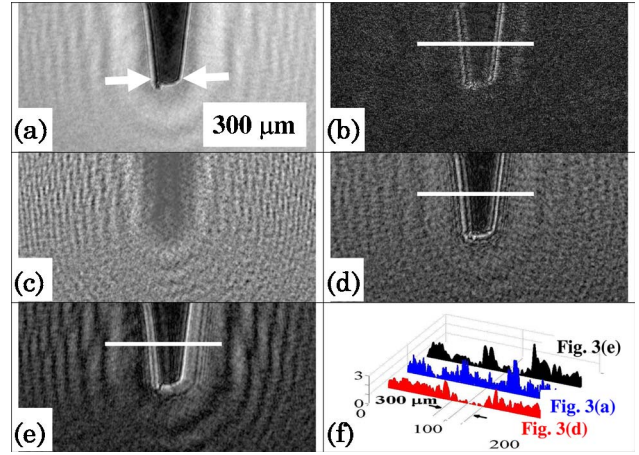


Fig. 3. (Color online) In (a), $|o(x, y, z = 0, \lambda)|$ and, in (b), $|\nabla_{xy}^2 o(x, y, z = 0, \lambda)|$, calculated directly from a single hologram using MATLAB. (c) Two-wavelength difference hologram, ΔH_λ , with $z = 160$ mm, and $\lambda = 632.8$ nm and $\lambda + \Delta\lambda = 652.5$ nm, while (d) shows the resulting reconstructed Laplacian magnitude from (c). (e) Magnitude of the two-plane Laplacian differential reconstruction at $z = 160$ mm [see Eq. (6)], with $z + \Delta z = 165$ mm and $\lambda = 632.8$ nm. Mean normalized pixel values from Figs. 2(c)–2(e) are replotted and shown in (f). Note the symmetry and narrow peaks indicating the edges, as well as the low values in the center, in (d).

accurate mechanical alignment follow displacement in the two-plane case and (b) issues related to the uniformity of illumination alignment. Further work to more accurately quantify these effects is being undertaken. However, the results are consistent with those in the literature [2,7–9].

The practical advantage of the two-wavelength-based method is that it eliminates the need to accurately change the position of the object relative to the camera; this removes mechanical alignment issues that can arise when using the two-position method. Improved performance, over that of either the two-position or direct numerical methods, has been demonstrated.

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