



Title	The influence of speed on the dynamic amplification of two loads crossing a simply supported bridge
Authors(s)	O'Brien, Eugene J., Li, Yingyan, González, Arturo, Brady, Sean P.
Publication date	2005-04-13
Publication information	O'Brien, Eugene J., Yingyan Li, Arturo González, and Sean P. Brady. "The Influence of Speed on the Dynamic Amplification of Two Loads Crossing a Simply Supported Bridge." Thomas Telford, April 13, 2005. https://doi.org/10.1680/bmf.33542 .
Conference details	Bridge Managment 5, Proceedings of the 5th International Conference on Bridge Management, London, UK, 11-13 April 2005
Publisher	Thomas Telford
Item record/more information	http://hdl.handle.net/10197/9227
Publisher's version (DOI)	10.1680/bmf.33542

Downloaded 2026-05-02 00:30:14

The UCD community has made this article openly available. Please share how this access benefits you. Your story matters! (@ucd_oa)



© Some rights reserved. For more information

The influence of speed on the Dynamic Amplification of two loads crossing a simply supported bridge

E.J. O'BRIEN

Department of Civil Engineering, University College Dublin, Ireland

Y.Y. LI

Department of Civil Engineering, University College Dublin, Ireland

A. GONZALEZ

Department of Civil Engineering, University College Dublin, Ireland

S.P. BRADY

Texcel Pty Ltd, Brisbane, Queensland, Australia

ABSTRACT

It is possible to use statistical data for the determination of traffic load models for the design of bridges. For existing bridges, traffic simulations based on measured traffic data, can provide a more accurate prediction of the characteristic load effect for assessment purposes. However, this procedure only provides the characteristic *static* loading or load effect. The total static-plus-dynamic load is often estimated based on conservative factors due to the high degree of uncertainty involved in the dynamic interaction between traffic, road profile and bridge. This paper aims to reduce this uncertainty by using simple dynamic models to obtain an understanding of the speeds and axle spacings that cause the highest dynamic amplifications.

KEYWORDS Bridge Dynamic Amplification, Speed, WIM.

INTRODUCTION

The Eurocode for bridge traffic loading was developed using Weigh-in-Motion (WIM) data and extensive simulation of bridge loading events to calculate characteristic values for load effects in a wide range of bridges. An allowance for dynamic interaction between the bridge and the traffic load was made through the application of a Dynamic Amplification Factor (DAF). The DAF's used were based on tests of vehicles crossing bridges where "frequency matching" (between bridge natural frequency and the frequency associated with the vehicle speed) was identified as a key source of dynamic amplification [1].

Bridge vehicle dynamic interaction is influenced by many factors. For example, road roughness can play a most significant role in exciting the dynamics of the vehicle and the level of excitation can vary greatly for profiles within a given roughness category. Of all the factors influencing bridge vehicle interaction, vehicle speed and its relationship with bridge natural frequency, is among the most important [2-3]. In this paper, the influence of vehicle speed on bridge dynamic amplification is examined.

Previously authors have considered multiple vehicle crossings by constructing elaborate finite element models or undertaking field tests [4]. Although both of these methods give valuable information regarding the magnitude of dynamic amplification, the results tend to be site-specific and give limited insight into the reasons why large amplifications sometimes occur. This paper gives dynamic amplification results for a simply supported bridge being crossed by two loads. The numerical point load models are too simple to provide accurate estimates of the magnitudes of the DAF. However, they can be used to determine the critical load velocities and relative load positions on the bridge. While greatly simplified, this model gives a valuable insight into the nature of bridge dynamic amplification and the critical speeds at which it can occur.

NUMERICAL MODEL

The numerical model used in this work is represented in figure 1. The equations of motion of this system are based on the work of Frýba [5] and they are numerically solved using the Runge-Kutta-Nyström method [6].

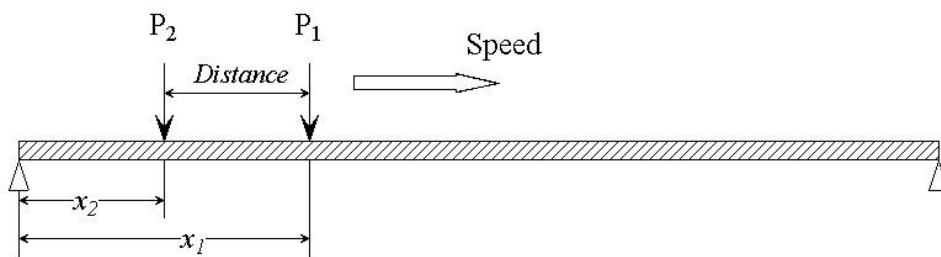


Fig 1. Model of constant forces crossing a bridge

A dimensionless bending moment is defined as the total moment divided by the maximum static moment due to a mid-span point load of $(P_1 + P_2)/2$. This is evaluated using equation 1:

$$M(\xi, \tau) = \sum_{i=1}^2 M_{Ri}(\xi, \tau) + M_{\mu}(\xi, \tau) \quad (1)$$

where:

$$M_{Ri}(\xi, \tau) = \left\{ \begin{array}{l} \frac{8}{\bar{P}l} \frac{1}{l} \varepsilon_i P_i (l - x_i) x = 4 \varepsilon_i \bar{P}_i (1 - \xi_i) \xi \text{ for } \xi_i \geq \xi \\ \frac{8}{\bar{P}l} \frac{1}{l} \varepsilon_i P_i (l - x) x_i = 4 \varepsilon_i \bar{P}_i (1 - \xi) \xi_i \text{ for } \xi_i < \xi \end{array} \right\} \quad (2)$$

and

$$M_{\mu}(\xi, \tau) = -\frac{1}{12} \alpha^2 \sum_{j=1}^{\infty} \frac{1}{j^2} \ddot{q}_{(j)}(\tau) \sin j\pi\xi \quad (3)$$

and where ξ is the dimensionless position of each load and τ is the dimensionless time coordinate; the value of ε_i determines if a load is on the beam. \bar{P} is the total weight of the two following loads; P_i is the value of each load and \bar{P}_i is the dimensionless load magnitude; α is the speed parameter and $\ddot{q}_{(j)}(\tau)$ can be obtained for each mode of vibration j and each moment in dimensionless time, τ . It can be shown that the moment due to the two forces is the sum of the moment due to each of the individual forces, i.e., superposition applies.

ONE AND TWO-LOAD SYSTEMS

Equation (1) has been used by the authors to show the relationship between DAF and vehicle speed for one and two point loads. Loads of 10 tonnes were used on a 25-m bridge with a mass per unit length of 18.36 tonne/m, a second moment of area of 1.39 m⁴ and a modulus of elasticity of 3.5×10¹⁰ N/m².

Figure 2 shows the relationship between DAF and speed for the case of zero damping. When a single load is present, there are certain critical speeds at which the dynamic amplification reaches a local peak. As speed increases, these peaks recur and are larger each time. Figure 2 represents a typical 25m span simply supported bridge with a natural frequency of 25.71 rad/s (4.09 Hz). It is worth noting that a speed of 100 km/hr, an upper limit for a typical loaded truck, corresponds to a load circular frequency, $\pi c/l$, of 3.49 rad/s (0.56 Hz). Similar graphs can be plotted for bridges of different span (and hence frequency), the only difference being in the critical speeds at which the peaks occur.

When a second force is present on the bridge, the nature of the relationship with speed is significantly different as can be seen in the figure 2. It is assumed here that both loads are travelling at the same speed. The nature of the difference is influenced strongly by the distance between the loads which in this example is 10 m or 0.4 of the span. Clearly in the two-load case shown, the peaks are not only greater in magnitude but also occur at different speeds. This can be explained by considering figure 3. It can be seen in figure 3(a) that the spacing between loads is in phase with the bridge vibration. Hence, a peak in the oscillating moment resulting from the

crossing of the first load coincides with the first peak due to the second load. The result is a greatly increased peak corresponding to the position of the second load. This "constructive interference" does not always occur. Figure 3(b) illustrates a case where the load spacing is out of phase with the bridge vibration and "destructive interference" has the effect of reducing the second peak.

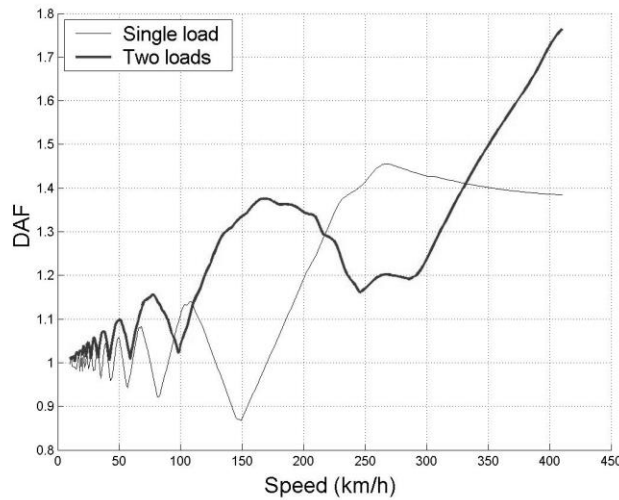
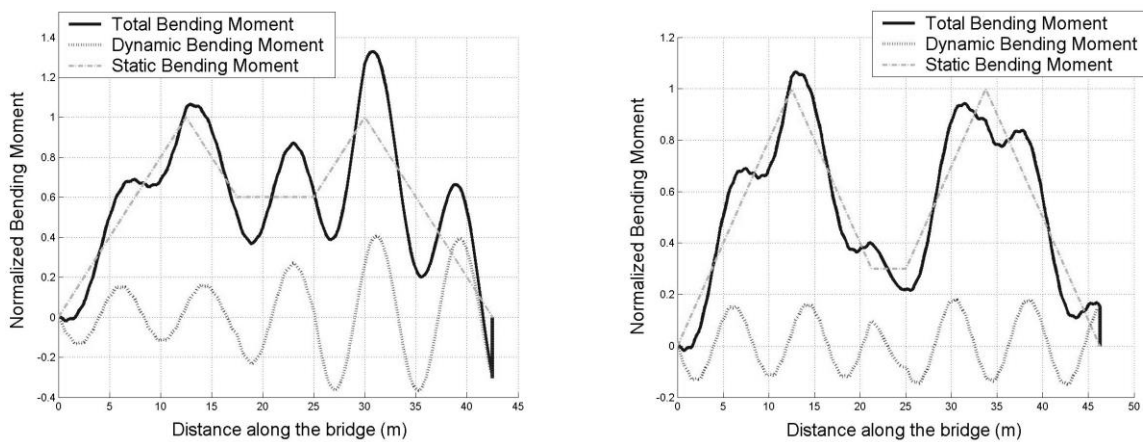


Fig. 2. Dynamic amplification factor versus speed for a 25-m beam being traversed by one load, and by two loads spaced at 10 m



(a) Constructive interference due to two loads at spacing of 17.5m (b) Destructive interference due to two loads at spacing of 21.25m

Fig 3. Normalised Bending moment for load circular frequency of 4.19 rad/s

Dynamic amplifications for the full range of possible speeds and Inter-Load Spacings (*ILS*) are presented in figure 4 for a 25-m span bridge. *ILS* is defined herein as load spacing divided by span length. Figure 2 which involves an inter-load spacing of 0.4 of the span can be found by taking a cross-section through figure 4 at *ILS* = 0.4. The same pattern of dynamic amplification applies for other bridges but the speed axis is affected by the first natural frequency so the critical speeds are different. This is represented in figure 4 through the Frequency Ratio (*FR*),

defined as the ratio of load circular frequency to bridge first natural frequency. FR is shown on the top of the figure

It is clear that there are two regions in figure 4 with distinctly different patterns. For ILS less than 0.5, i.e., loads less than half a span apart, the second load arrives on the bridge before the first reaches the critical mid-span point. The pattern in this region is often of the form of figure 2, i.e., one of a series of peaks which get higher as speed gets higher. While there are critical low speeds, there is a general trend in examples such as figure 2 towards higher dynamic amplification as speed increases. In the region of figure 4 for which $ILS < 0.5$, the contours give an indication of the critical speeds that might apply to a two-axle vehicle travelling alone over a short or medium-span bridge. Clearly the highest dynamic amplification will often occur at the highest speeds recorded and at axle spacings that fall into the dark zones in the figure.

A different pattern is evident in figure 4 for ILS in excess of 0.5. In this region, the contours give an indication of critical speed and spacing combinations for two following vehicles on a long-span bridge. It is significant that, for a given bridge, the local peaks occur at a finite number of key speeds. Thus, it is possible to identify the critical speeds first and then identify the corresponding ranges of inter-load spacing that will result in high DAF.

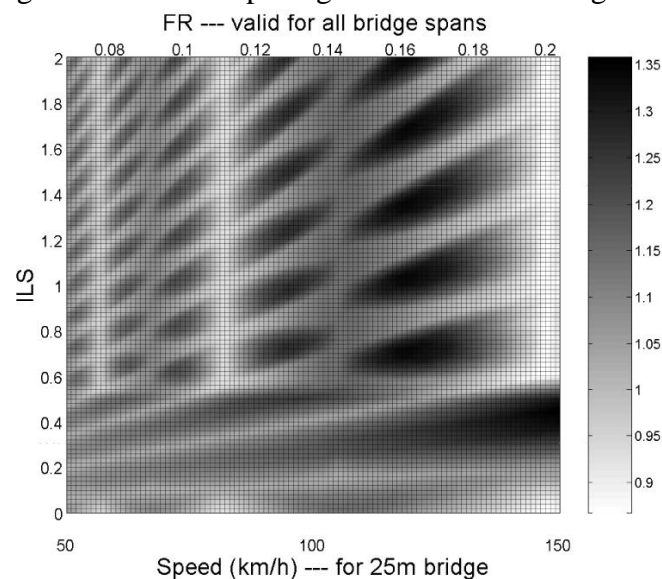


Fig. 4. DAF's for 25 m bridge for range of speeds and Inter-Load Spacing and for bridges of any span where FR represents the ratio of the load circular frequency to the bridge first natural frequency (zero damping assumed)

For $ILS > 0.5$, the first load has passed the critical mid-span point when the second load arrives on the bridge. It is important to note that amplifications far in excess of the single load case (corresponding to the limit as ILS approaches zero) can occur. Even when the spacing between loads exceeds the span ($ILS > 1$), significant amplifications result from the interaction between

the two loads. Figure 5 illustrates an example of constructive interference when $ILS = 1.3$. It can be seen that the dynamic amplification is significantly greater for the second load. The bridge vibration is clearly initiated by the first load and increased further by the second. This phenomenon will be much less pronounced when damping is present.

Figure 6 presents the dynamic amplifications for the full range of frequency ratios and inter-load spacings for 3% damping. It can be seen that, for higher ILS , the influence of damping is significant. However, the general pattern of local peaks of critical speed and ILS is still present.

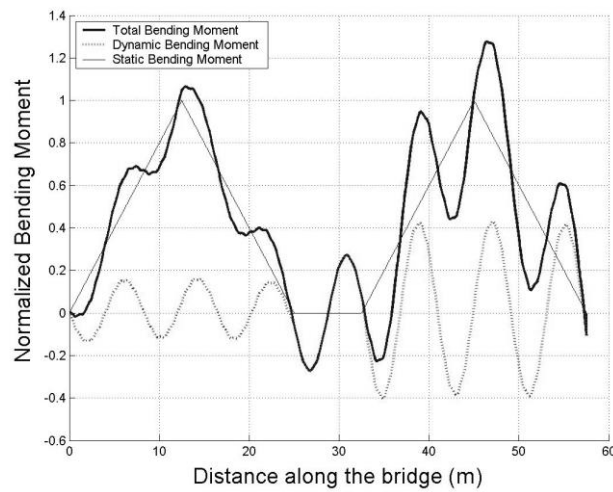


Fig. 5. Normalised bending moment for load circular frequency of 4.19 rad/s and load spacing of 32.5m on a 25m bridge.

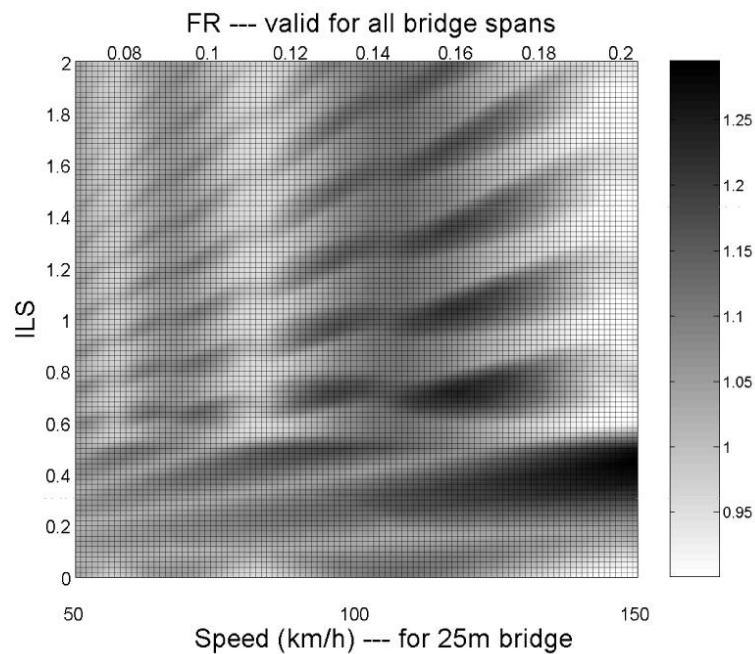


Fig. 6. DAF's for range of speeds and Inter-Load Spacing (3% bridge damping)

CONCLUSIONS

The theoretical bridge response due to a series of moving constant loads have been analysed. This simple model only takes into account bridge dynamics, but it is able to identify the existence of critical speeds and load-spacings that subjects the bridge to high dynamic amplification. Contour plots of maximum dynamic amplification factor versus frequency ratio (or speed) and inter-load spacing show a clear pattern of dynamic excitation consisting of a series of 'hills', the peak of each being associated with a different speed. As speed increases, these 'hills' tend to be wider and the corresponding peak higher, resulting in more significant dynamics. This trend is limited by the maximum vehicle speeds to be expected on a motorway. In the near future, it is expected to relate these critical parameters identified by simple models to more complex situations, so the uncertainty on static and dynamic traffic components can be reduced and a more realistic traffic dynamic load model facilitated.

REFERENCES

1. DIVINE Programme, OECD, Dynamic Interaction of Heavy Vehicles with Roads and Bridges, DIVINE Concluding Conference, Ottawa, Canada, 1997.
2. OLSSON M, On the Fundamental Moving Load Problem. *Journal of Sound and Vibration*, 1991, Vol. 154, No.2, pp 299-307.
3. LIU C, HUANG D & WANG T L, Analytical dynamic impact study Based on Correlated Road Roughness, *Computers and Structures*, 2002, Vol. 80, pp 1639-1650.
4. GONZALEZ A, Development of Accurate Methods of Weighting Trucks in Motion Ph.D. Thesis, Department of Civil Engineering, Trinity College, Dublin, Ireland, 2001.
5. FRYBA L, *Vibration of Solids and Structures under Moving Loads*, Noordhoff International Publishing, Groingen, The Netherlands, 1971.
6. KREYSZIG E, *Advanced Engineering Mathematics*, John Wiley and Sons, 1973.