Title	A method for modeling voltage regulators in probabilistic load flow for radial systems
Authors(s)	Melhorn, Alexander C., Dimitrovski, A.
Publication date	2014-07-10
Publication information	Melhorn, Alexander C., and A. Dimitrovski. "A Method for Modeling Voltage Regulators in Probabilistic Load Flow for Radial Systems." IEEE, July 10, 2014. https://doi.org/10.1109/PMAPS.2014.6960598.
Publisher	IEEE
Item record/more information	http://hdl.handle.net/10197/8631
Publisher's version (DOI)	10.1109/PMAPS.2014.6960598

A Method for Modeling Voltage Regulators in Probabilistic Load Flow for Radial Systems

A.C. Melhorn
School of Electrical, Electronic
and Communications Engineering
University College Dublin
Dublin, Ireland
Email: acmelhorn@gmail.com

A. Dimitrovski
Electrical and Electronics
Systems Research Division
Oak Ridge National Laboratory
Oak Ridge, TN USA
Email: dimitrovskia@ornl.gov

***Abstract*—Probabilistic load flow is becoming a more useful and needed power system analysis technique with the increase of stochastic generation and demand. This paper improves the usefulness of probabilistic load flow by demonstrating an algorithm for modeling voltage regulators within the probabilistic load flow solution. Previously, the voltage regulators have been removed from the system before analysis. The proposed technique is verified by comparing the solutions to those obtained by Monte Carlo simulation.**

***Index Terms*—Power distribution systems, probabilistic load flow, three phase, voltage regulators**

I. Introduction

Load flow, also referred to as power flow, is one of the basic and most used tools in power system analysis [1]. There are a variety of load flow techniques, most of which are deterministic, accurate, and allow for a high level of detail to be modeled within a system. However, traditional deterministic load flow (DLF) algorithms rely on precise input variables, which are generally unknown. As systems become more uncertain, determining suitable input variables will become difficult, if not unrealistic [2]. For example, to statistically represent the output of wind generation in [3], [4] various probability density functions (PDF) are proposed.

Probabilistic load flow (PLF) is a tool for modeling the statistical uncertainties in generation and demand. Such uncertainties in the power system are becoming of particular interest today with push for renewable generation and dynamic loads. [5], [6] are some of the first papers to propose a PLF solution, however they relied upon a DC model. [7], [8] introduced PLF with AC models. Research has continued to progress with [9]–[11], however they use the **Y**-bus system matrix and assume the shapes of the random variable's (RV) PDFs can be broken into a finite number of moments. [2] introduces a direct PLF technique which takes advantage of the topology of radial systems. This allows the solution to be found without explicitly using the **Y**-bus matrix. [12] expands the PLF solution to three-phase, unbalanced radial systems.

None of the cited references account for voltage regulators (VR). The ability to model VRs is becoming more important as the addition of more dynamic loads and generation on the distribution system will have adverse effects on the voltage in the system [13]. This paper presents a method for modeling VRs for use with PLF. The proposed method, with modifications, could be applied to other controllable variables: capacitor banks, reactors, etc.

This manuscript has been authored by UT-Battelle, LLC, under Contract No. DE-AC05-00OR22725 with the U.S. Department of Energy. The United States Government retains and the publisher, by accepting the article for publication, acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

II. Voltage Regulators

Distribution system voltage levels can vary considerably due to the high variation of system loads. The American National Standards Institute (ANSI) requires that the customers' voltage stays within an acceptable level [14]. There are several ways to control voltages in a system, one of which is the step-voltage regulator.

A. Voltage Regulator Model

VRs can be divided into two main types: off-load and on-load. Off-load VRs cannot change their voltage-ratio with an active load connected. On-load VRs on the other hand can actively change the voltage-ratio while a load is connected. Here, it is not important which type of VR is in the system as they both work in the same manner.

VRs consist of two main components: an autotransformer and a tap changing mechanism. The voltage is regulated by the autotransformer's tap settings. The taps in turn are controlled by the line drop compensator.

The generalized equations for the tap changing mechanism and autotransformer, derived from [15], are:

$$TC = \begin{cases} ceil\left(\frac{V_{LL}-V_{meas}}{SS}\right) & \text{for } V_{meas} < V_{LL} \\ floor\left(\frac{V_{UL}-V_{meas}}{SS}\right) & \text{for } V_{meas} > V_{UL} \\ 0 & \text{otherwise} \end{cases}, \quad (1)$$

$$tap = tap + TC, \quad (2)$$

$$d = \frac{1}{1 - SS \cdot tap}, \quad (3)$$

$$V_{out} = d \cdot V_s, \quad (4)$$

$$I_{out} = \frac{1}{d} \cdot I_s, \quad (5)$$

where:

TC	tap change;
V_{meas}	measured line drop compensator voltage;
V_{LL}	voltage lower-limit;
V_{UL}	voltage upper-limit;
SS	step-size;
tap	tap setting;
d	voltage ratio;
V_s, I_s	source voltage and current;
V_{out}, I_{out}	output voltage and current.

V_{meas} can be directly measured from a given node or it can be approximated with a line drop compensator. In this paper it is assumed that the regulated voltage is measured directly from the voltage at the respective node.

III. Forward Backward Sweep

In a radial system, the load flow can be solved using a forward backward sweep (FBS) technique. FBS allows the solution to be found without explicitly using the **Y**-bus matrix, as with the Newton-Raphson method which is a common load flow algorithm. FBS is also robust and can typically converge in a minimal number of iterations, even with a bad initial estimate. A description of the modified FBS that includes VRs follows.

A. Backward Sweep

The first step in every iteration is to define the loads:

$$S_k = P_{nom_k} \left(\frac{|V_k|}{V_{nom}} \right)^{np_k} + jQ_{nom_k} \left(\frac{|V_k|}{V_{nom}} \right)^{nq_k}, \quad (6)$$

where V_{nom} is the system's rated voltage and np_k and nq_k depend on the corresponding load models (typically Z, I, PQ, or a combination thereof). V_k is typically initialized with the slack voltage for all nodes.

The current at each node is then:

$$I_k = \left(\frac{S_k}{V_k} \right)^*, \quad (7)$$

and the current flowing through the branches is:

$$I_{i-j} = \sum_{k \in \Lambda_j} d_k \cdot I_k, \quad (8)$$

where Λ_j denotes the set of all of the nodes supplied via node j including node j and d_k is voltage ratio at the node k.

B. Forward Sweep

The next step is to update nodal voltages starting with the slack bus and working forward (or down) through the radial system. The change in voltage is:

$$\Delta V_{i-j} = Z_{i-j} \cdot I_{i-j}, \quad (9)$$

and the voltage for each node is:

$$V_k = d_k \cdot V_{Uk} - \Delta V_{i-j}, \quad (10)$$

where V_{Uk} is the upstream voltage of node k.

C. Voltage Regulator Ratio

After the voltages are updated, the status of the VRs are checked (1) and the voltage ratios are updated if necessary (3).

D. Convergence

The forward and backward sweeps are repeated with updated voltage values until the convergence criterion has been met:

$$max|V_{k_{old}} - V_{k_{new}}| < V_{thres}, \quad (11)$$

or after the maximum number of iterations have been executed.

E. System Losses

After the load flow has converged fully, the system's losses can be calculated as follows:

$$\Delta S_{i-j} = Z_{i-j} \cdot |I_{i-j}|^2. \quad (12)$$

The total losses are just the sum of all of the line losses:

$$\Delta S_{tot} = \sum_{i-j \in \beta} \Delta S_{i-j}, \quad (13)$$

where β denotes the set of all the elements in the network.

IV. Probabilistic Load Flow

PLF is basically an extension of the load flow calculation. PLF is used in cases when there is statistical uncertainty regarding the load flow's input variables. The input variables are modeled with random variables and probability theory methods are used to find the solution of the PLF.

The nonlinearity of the load flow equations is typically addressed by linearizing around the expected operating points. The solution of the PLF then becomes a sum of independent RVs weighted by their sensitivity coefficients, (14), simplifying the complex relations within the load flow equations.

$$f(x) = \sum_{i}^{n} \frac{1}{c_i} f_i \left(\frac{y_i}{c_i} \right), \quad (14)$$

where:

f_i	PDF of random variable y_i (the i-th element) ;
c_i	sensitivity coefficient for y_i.

The solution can then be obtained by convolution, (15), [16] using one of the number of numerical techniques.

$$f(x) = \frac{1}{|c_1|} f_1 \left(\frac{y_1}{c_1} \right) * \frac{1}{|c_2|} f_2 \left(\frac{y_2}{c_2} \right) * \cdots * \frac{1}{|c_n|} f_n \left(\frac{y_n}{c_n} \right) \quad (15)$$

The most common and efficient techniques are based on the Fast Fourier Transform algorithm.

For this paper the PLF is modeled from [12]. It follows a similar FBS algorithm as the deterministic load flow shown in Section III, however it is not iterative. A description of the PLF used for this paper follows.

A. Backward Sweep

The power flows for the branches are found using (6); however, they are RVs and not deterministic values. P_k and Q_k are the real and reactive parts of S_k, respectively:

$$P_k = \mathbb{R}\left(S_k\right), \tag{16}$$

$$Q_k = \mathbb{I}\left(S_k\right). \tag{17}$$

If the power losses are neglected at the moment, the real and reactive power flows in the element $i-j$ for a radial system become:

$$P_{i-j} = \sum_{k\in\Lambda_j} P_k, \tag{18}$$

$$Q_{i-j} = \sum_{k\in\Lambda_j} Q_k, \tag{19}$$

where Λ_j denotes the set of all the nodes supplied via node j including node j.

If (18) is rearranged around the expected values, $\bar{P}_k$, of the input RVs, it becomes:

$$P_{i-j} = \sum_{k\in\Lambda_j} \bar{P}_k + \sum_{k\in\Lambda_j} \left(P_k - \bar{P}_k\right). \tag{20}$$

The first term on the right hand side of (20) is an approximation of P_{i-j}. When all of the inputs are at their expected values, the approximation can be replaced with P_{i-j_0}. This is the value obtained from the DLF, where the inputs are the respective expected values which implicitly takes into account power losses. Equation (20) becomes:

$$P_{i-j} = P_{i-j_0} + \sum_{k\in\Lambda_j} \left(P_k - \bar{P}_k\right). \tag{21}$$

The same can be applied for the reactive power:

$$Q_{i-j} = Q_{i-j_0} + \sum_{k\in\Lambda_j} \left(Q_k - \bar{Q}_k\right). \tag{22}$$

B. Forward Sweep

Assuming the voltages in the distribution network operate at a medium or low voltage level, the following two approximations can be made:

- Since the voltages in every node of the network do not differentiate much from the rated voltage, the nominal voltage can be used instead of the actual voltage in calculating the voltage drop.
- Since the imaginary part of the voltage drop in any element of the network comparing to the real one is much smaller, it can be neglected. This is equivalent to saying that the phase angle differences among the voltages at different nodes are small.

Then, (9) becomes:

$$\Delta V_{i-j} \approx \mathbb{R}\left[Z_{i-j} \cdot \left(\frac{P_{i-j} + jQ_{i-j}}{V_{nom}}\right)^*\right]. \tag{23}$$

The total voltage drop from the substation (slack) to node k, is the sum of all the voltage drops in the elements in the supply path, Π_k, starting from the slack up to node k. The total voltage drop for node k is:

$$\Delta V_k = \sum_{(i-j)\in\Pi_k} \Delta V_{i-j}. \tag{24}$$

Along with the approximations above, the voltage drop is linearized around its expected values in much the same way as (21) and (22):

$$\Delta V_k \approx \Delta V_{k_0} + \sum_{i=1}^{n} \mathbb{R}\left[Z_{ik} \cdot \left(\frac{\left(P_i - \bar{P}_i\right) + j\left(Q_i - \bar{Q}_i\right)}{V_{nom}}\right)^*\right]. \tag{25}$$

The voltage for node k is then:

$$V_k = V_0 - \Delta V_k. \tag{26}$$

C. System Losses

As in Sections IV-A and IV-B the system losses, (27), are split into their real and reactive components:

$$\Delta S_{i-j} = \left(R_{i-j} + jX_{i-j}\right) \frac{{P_{i-j}}^2 + {Q_{i-j}}^2}{|V_{i-j}|^2}, \tag{27}$$

and linearized around their expected values:

$$\Delta P_{i-j} \approx \Delta P_{i-j_0} + \mathbb{R}\Bigg[\frac{2Z_{i-j}}{V_{nom}} \cdot \Bigg(\sum_{k\in\Lambda_j} P_k \cdot \sum_{k\in\Lambda_j} \bar{P}_k + \sum_{k\in\Lambda_j} Q_k \cdot \sum_{k\in\Lambda_j} \bar{Q}_k - \left(\sum_{k\in\Lambda_j} \bar{P}_k\right)^2 - \left(\sum_{k\in\Lambda_j} \bar{Q}_k\right)^2\Bigg)\Bigg], \tag{28}$$

$$\Delta Q_{i-j} \approx \Delta Q_{i-j_0} + \mathbb{I}\Bigg[\frac{2Z_{i-j}}{V_{nom}} \cdot \Bigg(\sum_{k\in\Lambda_j} P_k \cdot \sum_{k\in\Lambda_j} \bar{P}_k + \sum_{k\in\Lambda_j} Q_k \cdot \sum_{k\in\Lambda_j} \bar{Q}_k - \left(\sum_{k\in\Lambda_j} \bar{P}_k\right)^2 - \left(\sum_{k\in\Lambda_j} \bar{Q}_k\right)^2\Bigg)\Bigg]. \tag{29}$$

The total real and reactive power loss in the network become:

$$\Delta P_{tot} = \sum_{i-j\in\beta} \Delta P_{i-j}, \tag{30}$$

$$\Delta Q_{tot} = \sum_{i-j\in\beta} \Delta Q_{i-j}, \tag{31}$$

where β denotes the set of all the elements in the network.

V. METHODOLOGY

A VR can be accounted for in PLF with a simple deterministic methodology. Fig. 1 shows a three bus part of a system that includes a VR. This is just a portion with the VR of a larger system. The rest of the system is solved by using the previously explained and referenced methodology. The VR is assumed to be lossless and it is monitoring the voltage at node three, V_3. For simplicity the VR is able to read the voltage directly from node three (or the VR's line drop compensator models $line_{2-3}$'s impedance perfectly).

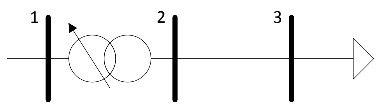

Fig. 1. Example three node voltage regulator part of a system.

To account for a VR in PLF a simple recursive shifting method is applied:

1) Solve the PLF for the system without the voltage regulator.
2) Determine the section of the monitored voltage PDF that is within the bandwidth of the VR (as in Fig. 1).
3) If the complete voltage PDF is within the bandwidth of the VR or the upper and lower tap limits have been reached, go to step 7.
4) Shift the sections of the original voltage PDF outside of the VR's bandwidth to at tap change of ± 1 accordingly. Increased tap for lower part and decreased tap for upper part of PDF.
5) Determine the equivalent section of the PDF within the VR's bandwidth: sum the shifted sections with the section of PDF in the bandwidth of the VR.
6) Repeat steps 3-5 until the entire PDF of the monitored voltage has been accounted for or until the tap is at its max/min.
7) If the tap limit has been reached, add the corresponding voltage PDF sections shifted accordingly by the tap settings to those within the VR's bandwidth.
8) Repeat steps 2-7 for all phases in a polyphase VR.

A flowchart of the corresponding code demonstrating the methodology above is shown in Fig. 2. The flowchart gives a more detailed look at how the methodology works and should be repeated for all phases of the VR.

A visual illustration of the solution methodology is shown in Fig. 3. It shows how the regulated voltage PDF is chopped and shifted into the acceptable voltage bandwidth and the corresponding probabilities are summed together. The first plot is the PDF of the monitored voltage from the PLF solution. The next plot shows the section of the PDF that is within bandwidth of the VR, while in its neutral setting. The third plot shows the section of the PDF within the VR bandwidth and that of the original voltage PDF which is in the VR bandwidth after the tap is moved up by one. The last plot shows the two PDFs summed together giving the PDF of the voltage with

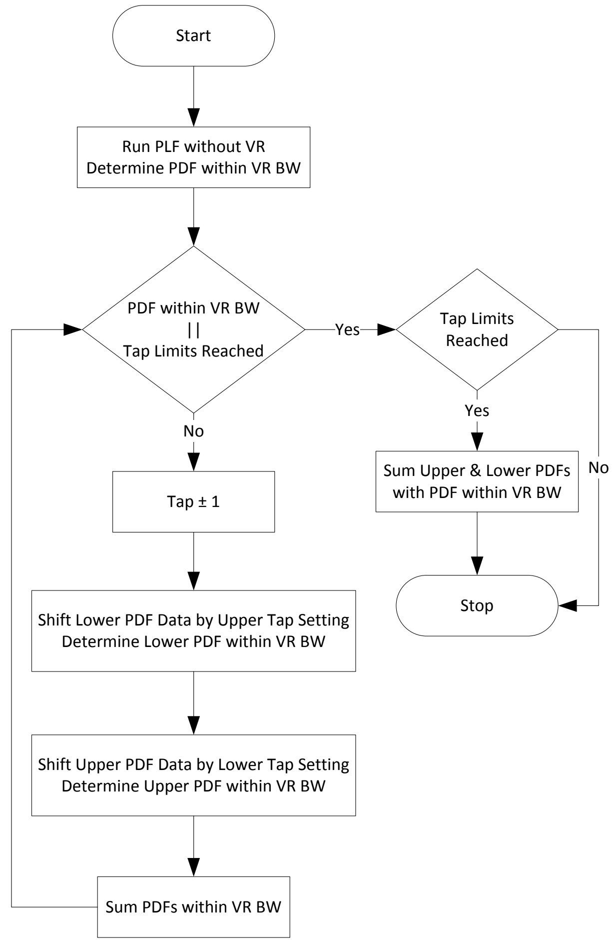

Fig. 2. Flowchart for modeling voltage regulator output for a single phase or balanced three phase system.

the VR in the system. Fig. 4 compares the final plot in Fig. 3 to that found using a Monte Carlo simulation as explained in the following section.

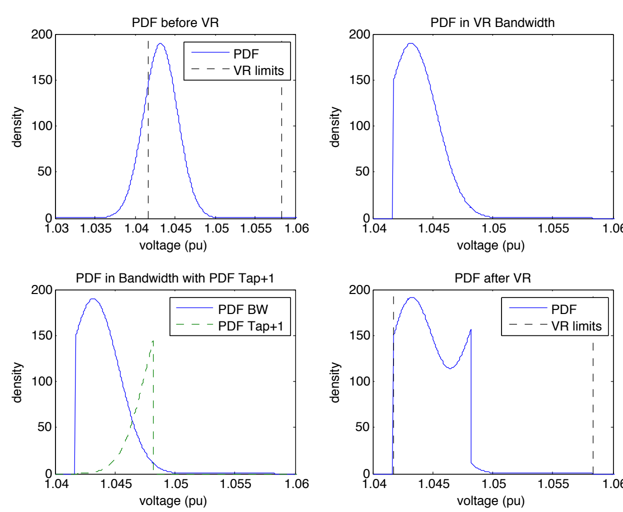

Fig. 3. Step-by-step process of modeling voltage output of a voltage regulator.

VI. Example

In order to illustrate the proposed method in a general unbalanced case, the subsystem shown in Fig. 1 is used. The focus on the three bus subsystem allows for easy display of the response of the VR and the effectiveness of the proposed methodology in accounting for the VR within the PLF solution of the whole system. The data for the subsystem are given in Table I. For simplicity, the loads in all three phases are represented as constant PQ models in a wye configuration. For the PLF, the loads are considered to be correlated with one another and the load PDFs are assumed to be Gaussian. Once the solution of the PLF is found, the proposed method is applied to each phase to account for the VR between nodes one and two. As before, the voltage at node three is the controlling voltage.

TABLE I
Data for example subsystem.

	Branch	Voltage Level	Bandwidth	Ref. Node
VR	1-2	126 volts	2 volts	3

Branch	Impedance (pu)		
2-3	0.045+0.079j	0.009+0.029j	0.009+0.029j
	0.009+0.029j	0.045+0.079j	0.009+0.029j
	0.009+0.029j	0.009+0.029j	0.045+0.079j

Node	Load (pu)		
3	0.209+0.002j	0.157+0.001j	0.194+0.003j

The voltage level of the VR is set to 126 volts with the standard bandwidth of 2 volts on a 120 volt base. This gives the VR a mean voltage of 1.05 pu, a minimum voltage of 1.0417 pu and a maximum voltage of 1.0583 pu.

To verify the solution, a Monte Carlo simulation is performed using one million experiments of the FBS load flow described in Section III. Figures 4 - 6 show a comparison of the voltages at node three produced by the proposed method and those obtained by the Monte Carlo simulation. The overlap of simulation with analytical results prove the validity of the proposed approach. The proposed method also has a faster solution time than the Monte Carlo simulation, seconds compared to minutes, respectively.

For all three phases, the voltage at node three falls outside of the VR's bandwidth. The proposed method accounts for any degree of the voltage outside of the VR's bandwidth. Fig. 4 shows the phase A voltage at node three. As shown in the previous section, about one third of the voltage from the PLF originally falls below the bandwidth of the VR. The methodology implementing the VR is more than capable of accounting for the times the voltage drops to low. In the case of the phase B voltage, shown in fig. 5, only the tail of PDF is outside of the VR's bandwidth before the VR is accounted for in the methodology. The phase C voltage PDF from the PLF is below the VR bandwidth approximately fifty percent of the time. Again, the proposed method compensates for the low voltage occurrences as shown by the Monte Carlo simulations in Fig. 6.

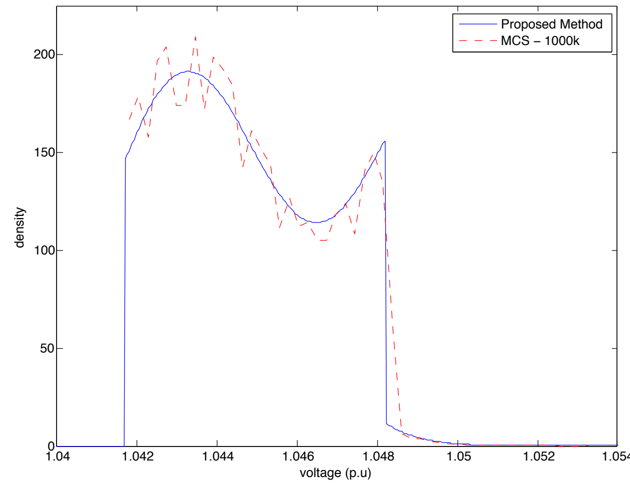

Fig. 4. PDF of phase A voltage at node 3.

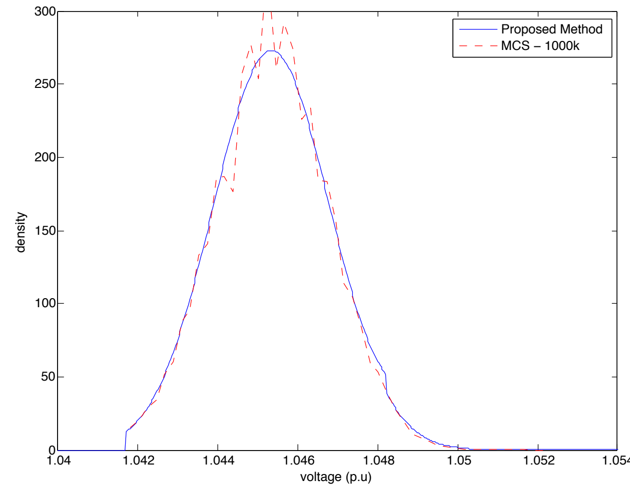

Fig. 5. PDF of phase B voltage at node 3.

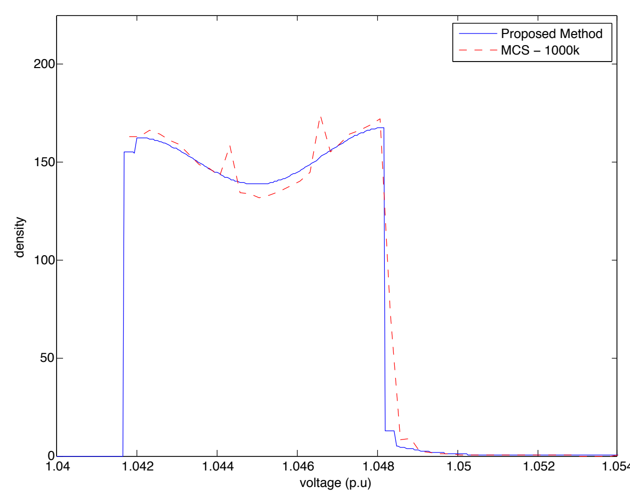

Fig. 6. PDF of phase C voltage at node 3.

VII. Conclusion

PLF solutions provide more qualitative information about a power system compared to the standard deterministic approach. However, it is difficult to model nonlinear components, like VRs, in a PLF. VRs play an important role in distribution systems, but previous research in the area has ignored them due to their non-linearity. This paper proposes a simple iterative method which successfully models voltage regulators within the PLF algorithm with improved solutions times over Monte Carlo simulation.

References

[1] A. Bergen and V. Vittal, *Power Systems Analysis*. Prentice Hall, 1999.

[2] A. Dimitrovski and R. Ačkovski, "Probabilistic load flow in radial distribution networks," in *Transmission and Distribution Conference, 1996. Proceedings., 1996 IEEE*, sep 1996, pp. 102 –107.

[3] M. Mohanpurkar and R. Ramakumar, "Probability density functions for power output of wind electric conversion systems," in *Power and Energy Society General Meeting, 2010 IEEE*, july 2010, pp. 1 –7.

[4] X. Li, J. Pei, and S. Zhang, "A probabilistic wind farm model for probabilistic load flow calculation," in *Power and Energy Engineering Conference (APPEEC), 2010 Asia-Pacific*, march 2010, pp. 1 –4.

[5] B. Borkowska, "Probabilistic load flow," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-93, no. 3, pp. 752–759, 1974.

[6] R. Allan, B. Borkowska, and C. Grigg, "Probabilistic analysis of power flows," *Electrical Engineers, Proceedings of the Institution of*, vol. 121, no. 12, pp. 1551–1556, 1974.

[7] R. Allan and M. R. G. Al-Shakarchi, "Probabilistic a.c. load flow," *Electrical Engineers, Proceedings of the Institution of*, vol. 123, no. 6, pp. 531–536, 1976.

[8] ——, "Probabilistic techniques in a.c. load-flow analysis," *Electrical Engineers, Proceedings of the Institution of*, vol. 124, no. 2, pp. 154–160, 1977.

[9] P. Caramia and P. Varilone, "Probabilistic ac/dc 3-phase load flow," in *Harmonics And Quality of Power, 1998. Proceedings. 8th International Conference on*, vol. 2, oct 1998, pp. 1018 –1028 vol.2.

[10] P. Caramia, G. Carpinelli, P. Varilone, and P. Verde, "Probabilistic three-phase load flow," *International Journal of Electrical Power and Energy Systems*, vol. 21, no. 1, pp. 55 – 69, 1999.

[11] P. Caramia, G. Carpinelli, M. Pagano, and P. Varilone, "Probabilistic three-phase load flow for unbalanced electrical distribution systems with wind farms," *Renewable Power Generation, IET*, vol. 1, no. 2, pp. 115 –122, june 2007.

[12] A.C. Melhorn, A. Dimitrovski, and K. Tomsovic, "Three phase probabilistic load flow in radial distribution networks," in *Proceedings of the 12th International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, June 2012.

[13] P. Richardson, D. Flynn, and A. Keane, "Impact assessment of varying penetrations of electric vehicles on low voltage distribution systems," in *Power and Energy Society General Meeting, 2010 IEEE*, 2010, pp. 1–6.

[14] "American national standard for electric power systems and equipment voltage ratings (60 hertz)," *ANSI C84.1-2011*, 2011.

[15] W. Kersting, "The modeling and application of step voltage regulators," in *Power Systems Conference and Exposition, 2009. PSCE '09. IEEE/PES*, 2009, pp. 1–8.

[16] A. Papoulis, *Probability, Random Variables and Stochastic Processes*. McGraw-Hill Inc.,New York, 1965.