

# Re-evaluating Hedging Performance for Asymmetry: The Case of Crude Oil

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## Abstract

We examine whether the hedging effectiveness of crude oil futures is affected by *asymmetry* in the return distribution by applying tail specific metrics to compare the hedging effectiveness of both short and long hedgers. The hedging effectiveness metrics we use are based on Lower Partial Moments (LPM), Value at Risk (VaR) and Conditional Value at Risk (CVaR). Comparisons are applied to a number of hedging strategies including OLS, and both Symmetric and Asymmetric GARCH models. We find that OLS provides consistently better performance across different measures of hedging effectiveness as compared with GARCH models, irrespective of the characteristics of the underlying distribution.

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## Introduction

A large literature has examined futures based hedging strategies. The dominant hedging framework uses the variance as the risk measure despite the fact that it cannot distinguish between positive and negative returns and therefore does not provide an accurate measure of risk for asymmetric distributions. In the literature on optimal hedging, this has been addressed in a number of ways. One option is the use of hedging estimation methods that seek to minimise some measure of risk other than the variance. The second approach is the use of hedging estimation methods that allow for asymmetry in the return distribution.

In this study, we apply tail specific hedging effectiveness measures together with estimation methods that allow for asymmetry, to a dataset consisting of Crude Oil contracts. This allows us to compare the hedging effectiveness of short and long hedgers under conditions of asymmetry across one of the most important traded assets in commodity markets. Oil commodity markets were chosen as they are particularly suited for an examination into the effects of asymmetry on hedging effectiveness given, their tendency to be non-symmetrically distributed (see Kuper and Van Soest, 2006).

The hedging estimation models used are: a Naïve hedge, a rolling window OLS hedge, and two bivariate GARCH models including an asymmetric GARCH model. Four different hedging effectiveness metrics are applied: Variance, Lower Partial Moments (LPM), Value at Risk (VaR); and Conditional Value at Risk (CVaR). With the exception of the variance, these metrics can account for asymmetries as they can separately measure both left tail and right tail quantiles. This approach allows us to comprehensively examine hedging effectiveness of both short and long hedgers for

both symmetric and asymmetric distributions, and to see whether there is a dominant OHR estimation method that emerges across a broad range of hedging effectiveness metrics.

Our results show that the rolling OLS model yields the best overall performance irrespective of the distributional characteristics of the contract being hedged, or the hedging effectiveness criteria being applied. We also find that the presence of skewness in the return distribution reduces in-sample and out-of-sample hedging effectiveness. Based on these findings, it would appear that an OLS based hedging strategy is adequate for both non-skewed and skewed distributions.

## Method

The OHR is defined in the literature as the ratio that minimises the risk of the payoff of the hedged portfolio. The payoff of a hedged portfolio is given as:

$$+ r_s - \beta r_f \text{ (short hedger)} \quad (1a)$$

$$- r_s + \beta r_f \text{ (long hedger)} \quad (1b)$$

where  $r_s$  and  $r_f$  are the returns on the cash and futures respectively, and  $\beta$  is the estimated OHR. We define a short (long) hedger as being long (short) the cash asset and short (long) the futures asset. In this study we utilise five different methods for estimating OHR's. The simplest models are a Hedge Ratio (HR) of zero (no hedge) and a 1:1 or Naïve hedge ratio where each unit of the cash contract is hedged with equivalent units of the futures contract. The third method applied is an OLS HR, which is the slope coefficient of a regression of the cash on the futures returns. An OHR estimated by OLS was first used by Ederington (1979) and has been applied

extensively in the literature. Cecchetti et al., (1988) argue that the OLS method is not optimal because it assumes that the OHR is constant, whereas time-varying volatility is the rule for financial time series, and as the OHR depends on the conditional distribution of cash and futures returns, so too should the hedge ratio.

We also use two additional estimation methods that allow the OHR to be time varying. These are a symmetric and an asymmetric GARCH model. The first GARCH model that we use is the Diagonal Vech model proposed by Bollerslev, Engle and Wooldridge (1988). This model imposes a symmetric response on the variance and is useful for comparison of hedging estimation and performance as it has been extensively applied in the literature to generate OHR's. The second GARCH model that we use is an asymmetric extension of the SDVECH model. We require both a symmetric and an asymmetric GARCH model since we are examining their suitability for symmetric and asymmetric datasets. The key advantage of using an asymmetric model is that it is able to capture the asymmetries both within and between cash and futures markets. It therefore allows the volatility to respond differently to negative and positive returns. This means that dynamic hedging strategies based on an asymmetric GARCH model will differ from those models that impose symmetry, and may provide better hedging performance. The asymmetric GARCH model we use builds on the univariate asymmetric GARCH model of Glosten, Jagannathan and Runkle (GJR) (1993). The advantage of GARCH models is that they can jointly estimate the conditional variances and covariances required for optimal hedge ratios, and can also account for asymmetric effects in volatility when extended as appropriate (see, for example, Kroner and Sultan, 1993). However, the performance of these models has been mixed. Over short time horizons and in-sample they have performed well (see Conrad, Gultekin and Kaul,

1991), however, over longer hedging horizons and out-of-sample, their performance has been poor (Brooks et al, 2002). We not turn to hedging effectiveness metrics.

### **Hedging Effectiveness**

We address a gap in the literature on commodity hedging, by employing a number of hedging effectiveness metrics in addition to the variance. The hedging effectiveness metrics we use are based on the Variance, LPM, VaR, and CVaR.

The most popular performance metric applied in the literature on hedging is the percentage reduction in the variance of the cash (unhedged) position as compared to the variance of the hedged portfolio. This was proposed by Ederington (1979) and has been widely adopted. It is given as:

$$HE_1 = 1 - \left[ \frac{VARIANCE_{HedgedPortfolio}}{VARIANCE_{UnhedgedPortfolio}} \right] \quad (2)$$

The use of the variance as a measure of risk has been criticised because it fails to distinguish between the tails of the distribution, and therefore, it fails to differentiate in performance terms between short and long hedgers. Where distributions are asymmetric, the variance will over or underestimate risk. For this reason, it is not an adequate measure of risk for hedgers, except where distributions are symmetric. Also, Lien (2005) argues that the findings in the literature, namely, that the OLS hedging strategies tend to perform as well or better than more complex estimation models such as GARCH, can be attributed to the use of the Ederington effectiveness measure. This variance performance criterion is only valid for OLS based hedging strategies, and that using it to evaluate non-OLS OHR's will lead to the incorrect conclusion that OLS OHR's offer the best hedging performance. Therefore, while the variance based

measure is appropriate where the hedger seeks to minimise variance, in practice, hedgers may seek to minimise some measure of risk other than the variance. For this reason, we use three additional hedging effectiveness metrics that will allow us to make a robust comparison of different hedging models under both symmetric and asymmetric conditions.

The LPM is the first of a number of hedging effectiveness metrics that we use to address the shortcoming in the variance measure. The LPM distinguishes between the left and right tails of a distribution. Therefore, it can measure risk in the presence of asymmetries, as it is a function of the underlying distribution (Bawa, 1975). The LPM measures the probability of falling below a pre-specified target return. The LPM of order  $n$  around  $\tau$  is defined as

$$\text{LPM}_n(\tau; R) = E\left\{\left(\max[0, \tau - R]\right)^n\right\} \equiv \int_{-\infty}^{\tau} (\tau - R)^n dF(R) \quad (3)$$

where  $F(R)$  is the cumulative distribution function of the investment return  $R$ , and  $\tau$  is the target return parameter. The value of  $\tau$  will depend on the level of return or loss that is acceptable to the investor. Some values of  $\tau$  that may be considered are, zero or the risk free rate of interest. For a hedger a small negative return may be acceptable to reflect the cost of hedging. The parameter  $n$  is the weighting applied to shortfalls from the target return. The more risk averse an investor the higher the weight ( $n$ ) that would be attached. Fishburn (1977) shows that  $0 < n < 1$  is suitable for a risk seeking investor,  $n = 1$  for risk neutral, and  $n > 1$  for a risk adverse investor. We can form a complete set of downside risk measures by changing the  $\tau$  and  $n$  parameters to reflect the position and risk preferences of different types of hedger. The more risk averse investors may set  $\tau$  as the disaster level of return and have utilities that would reflect an LPM with  $n=2$ ,

3... For example, an investor who views the consequences of a disaster level return as being unacceptable may use an order of LPM with  $n = 5$ .

Interest in one sided risk measures has increased in recent years as investors do not attach the same importance to positive and negative outcomes, but focus on downside or tail specific risk. The LPM therefore serves as an intuitive measure of risk that is in line with the risk preferences of many investors (Lee and Rao, 1988). A number of studies have compared the hedging effectiveness of short and long hedgers using the LPM methodology (see, for example, Demirer and Lien, 2003). The results indicate that hedging effectiveness for long hedgers differs from that of short hedgers, with long hedgers deriving more benefit from hedging in terms of risk reduction as measured by reduced LPM's.

Since the majority of hedgers seek to limit losses, it may be appropriate for hedgers to seek to minimise a risk measure that is tail specific. Two additional metrics that are closely related to the LPM and which also have similar advantages as measures of risk for hedging are VaR and CVaR. Both of these measures are also tail specific and may be used to measure downside risk. These are considered in due course.

We calculate the lower partial moment using a target return of zero since the general aim of hedging is to avoid negative outcomes. We also use  $n=3$  as the order of LPM, as this corresponds to a strongly risk averse investor. The metric we apply to evaluate

hedging effectiveness based on the LPM is the percentage reduction in the LPM of each hedging estimation method as compared with a no hedge position<sup>1</sup>.

This is calculated as:

$$HE_2 = 1 - \left[ \frac{LPM_{3HedgedPortfolio}}{LPM_{3UnhedgedPortfolio}} \right] \quad (4)$$

The third hedging effectiveness metric is VaR<sup>2</sup>. This is the loss level of a portfolio over a certain period that will not be exceeded with a specified probability. VaR has two parameters, the time horizon (N) and the confidence level (x). Generally VaR is the (100-x)<sup>th</sup> percentile of the portfolio over the next N days. We calculate VaR using  $x = 99$  and  $N = 1$ . This corresponds to the first percentile of the return distribution of each portfolio over a one-day period which is consistent with the OHR estimation period used in this study. Similar to the LPM, the performance metric employed is the percentage reduction in the VaR.

$$HE_3 = 1 - \left[ \frac{VaR_{1\% HedgedPortfolio}}{VaR_{1\% UnhedgedPortfolio}} \right] \quad (5)$$

A shortcoming in VaR is that it is not a coherent measure of risk, as it is not subadditive<sup>3</sup>. Also in practice two portfolios may have the same VaR but different potential losses. This is because VaR does not account for losses beyond the (100-x)<sup>th</sup> percentile. We address this by estimating an additional performance metric; CVaR, that

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<sup>1</sup> The percentage reduction in the relevant performance measure is generally compared with a no-hedge position however there may be some cases where a no-hedge position does not yield the worst hedging performance, whereby the comparison is then made against the hedging model that yields the worst hedging performance.

<sup>2</sup> As stated we choose a weight of  $n = 3$  to describe a strongly risk averse investor. However, our measures of VaR and CVaR can also encompass attitudes to risk. VaR can be viewed as a special case of the LPM in equation (11) setting  $n = 0$  and CVaR sets  $n = 1$ . By fixing the probability  $LPM_0$ , the corresponding VaR/CVaR can be calculated.

<sup>3</sup> Coherent measures of risk are subadditive. What this means is that the risk of two positions when added together is never greater than the sum of the risks of the two individual positions (see, for example, Artzner et al, 1999).

addresses the shortcoming in VaR as it is a coherent measure of risk. CVaR is the expected loss conditional that we have exceeded the VaR. It is given as:

$$CVaR = E[L|L > VaR] \quad (6)$$

This measures the expected value of our losses,  $L$ , if we get a loss in excess of the VaR. CVaR is preferable to the VaR because it estimates not only of the probability of a loss, but also the magnitude of a possible loss. In calculating CVaR we use the 1% confidence level which gives us the expected shortfall beyond the 1% VaR. The performance metric we use to evaluate hedging effectiveness is the percentage reduction in CVaR as compared with a no hedge position.

$$HE_4 = 1 - \left[ \frac{CVaR_{1\% \text{ HedgedPortfolio}}}{CVaR_{1\% \text{ UnhedgedPortfolio}}} \right] \quad (7)$$

Few studies in the hedging literature have applied either VaR or CVaR as measures of hedging effectiveness, however, Giot and Laurent (2003) use both the VAR and CVaR measures to examine the risk of short and long trading positions over a one day time horizon. They estimate volatility using both symmetric and asymmetric GARCH type approaches. Their findings show that symmetric models underperform models that account for asymmetry; however, their analysis is only applied to unhedged positions. Next we turn to our application of both the hedging models and performance measures outlined to examine hedging effectiveness under conditions of asymmetry.

## **Data**

Our data consist of daily cash and futures closing prices of Crude Oil. Crude Oil was chosen as it potentially exhibits significant asymmetric effects, because it is the largest traded commodity in the world in terms of monetary value, because of its economic importance, and because it is widely hedged. The oil contract is the NYMEX West

Texas Light Sweet Crude contract, which was chosen as it is used as an international oil pricing benchmark given its liquidity and price transparency.<sup>4</sup> All data was obtained from Commodity Systems and daily returns were calculated as the differenced logarithmic prices. Continuous series were formed using the nearby contract with rollover occurring about one week before maturity. Trading volume was used as the criterion in deciding the rollover date meaning that the price of the largest traded contract by volume was used, and that the price switched from one contract to the next when that contract's volume fell below the volume of the next traded contract.

Because we examine the influence of asymmetry and its effects on hedging effectiveness, we require data that exhibits both skewed and non-skewed characteristics in order to facilitate a comparison. These considerations motivated our choice of sample periods. Our initial sample is daily logarithmic returns from January 1, 2000 through December 31, 2010. For each contract our initial search criterion is to have one year each of symmetric and asymmetric returns. Our second search criterion is choosing both in-sample and out-of-sample periods that are consistently symmetrically or non-symmetrically distributed to ensure consistency between in-sample and out-of-sample performance results. From this sample period we were able to extract two separate datasets, one skewed and one non-skewed. The first dataset is asymmetric and runs from January 2001 – December 2001. The second dataset is symmetric and runs from January 2002 – December 2002. For each period, the first 160 observations are used for the in-sample estimation of the hedging models. The

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<sup>4</sup> The contract trades in units of 1,000 barrels on the NYMEX through open outcry. Further details of the contract and its trading characteristics are available at [http://www.nymex.com/lsc0\\_fut\\_descri.aspx](http://www.nymex.com/lsc0_fut_descri.aspx).

remaining 100 observations were used to facilitate out-of-sample comparisons for model evaluation. The characteristics of the data are now examined.

INSERT TABLE I HERE

Summary statistics for the data are displayed in Table I. We can see that the characteristics of the return distributions of the two series are markedly different. The first dataset for Crude Oil (2001) is non-symmetric and is characterised by significant skewness. In contrast, the second dataset of Crude Oil (2002) can be characterised as symmetric given the insignificant skewness figure. We can also see that the volatility differs considerably between the skewed and non-skewed periods. For example the standard deviation for Crude Oil Cash is 2.7% for the skewed data as compared to 2.1% for the non-skewed data. This is further demonstrated if we examine Figure I which plots the cash and futures return series.

INSERT FIGURE I HERE

The data were checked for stationarity using Dickey Fuller unit root tests. As expected, the log returns for cash and futures are stationary. The Engle (1982) Lagrange Multiplier (LM) test with 4 lags was used to check for ARCH effects. The findings indicate significant ARCH effects present for the symmetric period (2002) only. This may limit the advantages of the GARCH models over hedging strategies that assume a constant variance in estimating hedge strategies for the Crude Oil contract.

#### **IV. Empirical Results**

We compare the short and long hedging effectiveness of five different OHR estimation methods for distributions with symmetric and asymmetric characteristics. We construct hedged portfolios using (1a) and (1b) and then evaluate hedging effectiveness

using the performance metrics outlined in section II. The out-of-sample hedge ratios are graphically represented in Figures IIa and IIb together with the associated summary statistics in Table II.

INSERT FIGURES IIa and IIb HERE

INSERT TABLE II HERE

A quick glance at Figures IIa and IIb indicates that the different hedging models produce different hedge ratios. In particular we observe from Figure IIa that the ASDVECH model differs from the SDVECH model. From Table II, we can see that the range of the hedge ratios is much narrower for the SDVECH model as compared to the ASDVECH model. For example, the OHR's for the asymmetric period range from 0.79 – 0.99 for the SDVECH model as compared with a range of 0.54 – 1.07 for the ASDVECH model. Again, from Table II, the OHR's range from about 0.55 to 1.07 for the symmetric Crude Oil series, using the ASDVECH model. We also observe that the OHR's for both series appear to be stationary. Therefore, while we would expect to see different hedging performance for each of the different hedging models, the performance gap may be narrow in many cases. We now turn to an examination of the hedging performance of the difference models for both short and long hedgers for both the Symmetric and Asymmetric data.

Tables III and IV present both in-sample and out-of-sample results for hedging effectiveness. Examining first the in-sample results from Table III, these show that hedging is effective at reducing risk as measured by the variance reduction criterion. For example, if we examine Crude Oil for the asymmetric period, using a benchmark no-hedge we find a 65.90% reduction in the variance using an OLS hedge. In terms of the overall hedging performance, the average in-sample hedging performance across

both asymmetric and symmetric periods and for all hedging models and performance metrics is just 54%.

INSERT TABLE III HERE

We also compare the in-sample hedging performance of Crude Oil based on distributional characteristics. We find better hedging performance across all models and risk measures for the symmetric data. For example, variance reductions for the symmetric period average 85% as compared with 64% for the asymmetric period when all hedging models are used. This finding persists if we use the tail specific measures with average differences in hedging performance between the symmetric and asymmetric periods of 27% and 31% for short and long hedgers respectively.

We derive a number of conclusions from these results. First, the in-sample hedging performance of the Crude Oil contract is better for symmetric distributions. This suggests that hedging the Crude Oil commodity using futures is of relatively limited use when returns may be skewed. Secondly, hedging effectiveness is not as good as could be expected when compared with other assets. For example equity hedges typically show effectiveness in excess of 85% (Cotter and Hanly, 2006).

Examining next the tail specific performance metrics, hedging is not as effective in reducing VaR and CVaR as it is at reducing the LPM. For example, there are differences in hedging effectiveness typically ranging from 15%-50%. Also, the different hedging models yield poor performance in terms of reducing the VaR and especially the CVaR. Using the best performing models for example, the results for the asymmetric data series indicate only an 11% reduction in the CVaR for short hedgers and a 29% reduction for long hedgers. These results may be related to the ability of VaR and CVaR

metrics to model extreme tail events whereas the LPM is a more general metric that doesn't pick up the most extreme outliers in the same way.<sup>5</sup> Therefore, hedging Crude Oil may be quite limited in reducing extreme losses as measured by the VaR or CVaR. Thus, tail specific hedging effectiveness metrics such as VaR and CVaR may be appropriate for use by strongly risk averse investors, as a measure of risk for distributions that may be affected by a small number of observations. A key point is that hedges may not be as effective in volatile markets that are skewed. In hedging terms, this means that investors may face the risk that their hedges will not fulfil their function of risk reduction during stressful markets conditions when they are most needed.

We also make a statistical comparison of the hedging effectiveness of short and long hedgers using Efrons (1979) bootstrap methodology, by employing t-tests of the differences between short and long hedgers based on the point estimates of our results<sup>6</sup>. This approach is also adopted in tests of model hedging effectiveness and allows us to make statistical as well as economic inferences from our results<sup>7</sup>. Taking the Crude Oil asymmetric sample first, the hedging effectiveness differences between short and long hedgers are significant at the 1% level in every single case whereas for the symmetric sample they are significant in only 50% of cases in-sample. The differences between short and long hedgers even for the symmetric period

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<sup>5</sup> While each of these measures is one sided, the LPM with a target rate set  $t=0$  will include all observations less than 0, whereas both the VaR and CVaR calculated at the 1% interval will include only extreme observations located in the left or right tails of the distribution. Also, modelling the tail of the distribution is more statistically reliable as compared with the exceeding method that relates to the LPM.

<sup>6</sup> The returns of the hedged portfolios as compiled using equations 1a and 1b were bootstrapped by resampling with replacement from the returns. 100 simulations were used allowing for the construction of confidence intervals around each point estimate.

<sup>7</sup> For each hedging model we compare the performance metric for short hedgers with the performance metric for long hedgers again employing bootstrap confidence intervals. To illustrate, examining Crude Oil hedges and taking the asymmetric data for example, the difference between the Naïve model VaR figures of 13.19 and 26.38 respectively is statistically significant at the 1% level.

demonstrates the importance of using tail specific hedging effectiveness metrics irrespective of the characteristics of the return distribution. This finding is supported across each of the different tail specific metrics of hedging performance.

We now turn to the out-of-sample results which are of importance in determining the hedging performance under real world conditions. From Table IV, the first thing to note is that hedging is still effective in terms of reducing risk across each of the risk measures. For example, Variance reductions are of the order of 80% for the asymmetric sample and 72% for the symmetric sample. Also, we can again see the limited use of hedging in reducing risk as measured by VaR and CVaR metrics. For example using the symmetric period, we can see that effectiveness as measured by CVaR is as low as 2.84% using the ASDVECH model.

INSERT TABLE IV HERE

Using the tail specific performance metrics, we also examine the out-of-sample hedging performance of short and long hedgers for asymmetric as compared to symmetric periods. For the asymmetric data, the differences in hedging effectiveness between short and long hedgers are significant at the 1% level in 92% of cases out-of-sample whereas for the symmetric data they are significant in only 69% of cases out-of-sample. These results indicate the importance of using tail specific performance measures even for data with symmetric distributional characteristics.

We now turn our attention towards model performance. Tables V and VI present absolute figures for the performance effectiveness measures of each of the hedging models, for in-sample and out-of-sample hedges respectively. The lower values represent better hedging performance. For example, the variance figure of 0.481 in

Table V for the asymmetric crude oil hedge represents the risk associated with an unhedged position, whereas the OLS model is the best hedging model as it yields the lowest risk with a variance of 0.164. Looking first at the in-sample results, we can see that the best hedging model depends on the hedging performance metric. For example, we can see from Table V, using the asymmetric data for short hedgers, that the OLS model yields the best performance in terms of both the Variance (0.164) and the VaR (4.19), whereas using the LPM as the performance criterion, the Naïve hedge (0.328) is the best performer.

INSERT TABLES V AND VI HERE

In terms of the best overall model, Table VII presents a summary of the best performing model for both short and long hedgers; in-sample and out-of sample.. The best overall in-sample hedging model for the asymmetric data is the OLS model which performs best in 50% of cases across both short and long hedgers. For the symmetric data the best overall model is the SDVECH model which performs best for five out of eight performance metrics spread across both short and long hedgers. The out-of sample results show that the OLS and ASDVECH models are the best overall performers for the asymmetric and symmetric data respectively

INSERT TABLE VII HERE

While these findings highlight the best hedging models for a given scenario, a key issue is whether there are significant differences in the performance of the different hedging models. To test this we compared the performance of the different hedging models, again employing Efrons (1979) bootstrap methodology. The results of these tests are presented in Tables V and VI. We find significant differences between model hedging performances in 54% of cases in-sample and 56% of cases out-of-sample. Indeed if we look at the actual performance metrics for the different hedging models,

while there may be statistical differences between them, the absolute differences between models are small and not economically significant. What this demonstrates is that the OLS and GARCH models tend to provide better hedging performance than a Naïve hedge. These findings show that when hedging commodities such as Crude Oil, the choice of hedging model is important but it also indicates that the OLS model provides consistently good performance across different assets.

The failure of the asymmetric GARCH model to provide better performance for asymmetric distributions contrasts with that of Giot and Laurent (2003). This may relate to the earlier point relating to the ability of the ASDVECH model to model positive and negative return innovations separately, however, this is not the same as being able to model skewness in the distribution. Based on these findings, we would have to conclude that there is little to be gained from the more complex GARCH models in performance terms over the simpler OLS model irrespective of the characteristics of the return distribution. This finding supports the broad literature on optimal hedging, that the OLS model provides an efficient outcome across a selection of risk measures.

## **V. Conclusions**

This paper compares the hedging effectiveness of Crude Oil for both symmetric and asymmetric distributions. We also compare the hedging effectiveness of short and long hedgers using a variety of hedging estimation methods that are tail specific. We find that both in-sample and out-of-sample hedging effectiveness is significantly reduced by the presence of skewness in the return distribution. This is an important finding as it means that hedging may not be as effective during asymmetric return periods, and therefore investors may not be effectively hedging during the periods when they most require it.

We also find larger differences in hedging performance between the short and long hedgers for the asymmetric distribution when compared with a symmetric distribution. Therefore the use of one-sided hedging performance measures that are consistent with modern risk management techniques such as VaR and CVaR is to be recommended, as the traditional variance reduction criterion is not adequate and will provide inaccurate measures of risk for different types of hedgers both for symmetric, and especially asymmetric distributions.

Also, the best hedging estimation model to emerge is the OLS model. This provides the best overall hedging performance across all measures of hedging effectiveness for both short and long hedgers. This finding suggests that there is little economic benefit to be gained by the use of more complex hedging estimation models over the simpler OLS model irrespective of the characteristics of the return distribution.

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**Table I: Summary Statistics**

This table presents summary statistics for the log returns of each cash and futures series for Crude Oil. The excess skewness statistic measures asymmetry where zero would indicate a symmetric distribution. The excess kurtosis statistic measures the shape of a distribution where a value of zero would indicate a normal distribution. The Bera-Jarque B-J statistic combines skewness and kurtosis in comparison to normality. LM with 4 lags is the Lagrange Multiplier test for ARCH effects proposed by Engle 1982 and is distributed  $\chi^2$ . Stationarity is tested using the Dickey-Fuller unit root test. The correlation coefficient between each set of cash and futures is also given. \*Denotes Significance at the 1% level. \*\*Denotes Significance at the 5% level.

	Crude Oil-Asymmetric 2001		Crude Oil-Symmetric 2002	
	Cash	Futures	Cash	Futures
<b>Mean</b>	-0.0012	-0.0012	0.0018	0.0017
<b>Min</b>	-0.1653	-0.1654	-0.0586	-0.0627
<b>Max</b>	0.1039	0.0807	0.0613	0.0595
<b>Std Dev</b>	0.0277	0.0261	0.0212	0.0208
<b>Skewness</b>	-0.633*	-1.003*	-0.066	-0.041
<b>Kurtosis</b>	6.746*	6.762*	0.421	0.384
<b>B-J</b>	512.36*	541.14*	2.112	1.678
<b>LM</b>	7.17	1.53	9.48	11.41**
<b>Stationarity</b>	-16.25*	-15.43*	-16.83*	-17.51*
<b>Correlation</b>	0.92		0.92	

**Table II: Summary Statistics of Optimal Hedge Ratios**

This table presents summary statistics for the estimated optimal hedge ratios for each hedging model for Crude Oil.

Hedging Model	2001			2002		
	OLS	SDVECH	ASDVECH	OLS	SDVECH	ASDVECH
<b>Mean</b>	0.8722	0.8877	0.8993	0.8994	0.9274	0.9150
<b>Min</b>	0.7857	0.7972	0.5446	0.8554	0.5781	0.5504
<b>Max</b>	0.9409	0.9928	1.0781	0.9407	1.0197	1.0724
<b>Std Dev</b>	0.0336	0.0312	0.0660	0.0248	0.0507	0.0680

**Table III: In –Sample Hedging Effectiveness – Short Vs Long Comparison**

This table presents the In-Sample hedging performance for Crude Oil for both the asymmetric and symmetric datasets. Figures are in percentages. HE<sub>1</sub> – HE<sub>4</sub> give the percentage reduction in the performance metric from the hedged model as compared with the worst hedged position. Statistical comparisons are made between the performance of short and long hedgers on a metric by metric basis using Efron's 1979 bootstrap technique. \* Indicates that the percentage reduction in the risk metric is significantly better comparing long to short hedges at the 1% confidence level. For example, using the Crude Oil asymmetric dataset, and the VaR HE<sub>3</sub> as our performance metric, we can see that the in-sample performance of long hedgers, is significantly better than that of the short hedgers for each of the four hedging models.

	HE <sub>1</sub> Variance x10 <sup>-2</sup>	HE <sub>2</sub> LPM x10 <sup>-2</sup>	HE <sub>3</sub> VaR x10 <sup>-2</sup>	HE <sub>4</sub> CVaR x10 <sup>-2</sup>	HE <sub>1</sub> Variance x10 <sup>-2</sup>	HE <sub>2</sub> LPM x10 <sup>-2</sup>	HE <sub>3</sub> VaR x10 <sup>-2</sup>	HE <sub>4</sub> CVaR x10 <sup>-2</sup>
<b>Panel A: SHORT HEDGERS</b>					<b>Panel B: LONG HEDGERS</b>			
<b>CRUDE OIL-Asymmetric</b>								
None	0.00	0.00	0.00	11.61*	0.00	0.00	0.00	0.00
Naïve	64.45	76.18*	13.19	0.00	64.45	36.55	26.38*	29.07*
OLS	65.90	69.97*	20.61	9.91	65.90	55.27	26.55*	22.57*
SDVECH	64.24	72.13*	18.25	3.97	64.24	44.93	30.05*	24.53*
ASDVECH	65.07	74.17*	17.77	3.05	65.07	43.30	29.25*	26.09*
<b>CRUDE OIL-Symmetric</b>								
None	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Naïve	82.95	92.02	42.01	33.18	82.95	87.05	47.17	51.43*
OLS	83.33	90.08	45.59	48.94*	83.33	90.82	47.77	40.47
SDVECH	84.48	93.29	45.45	36.26	84.48	89.26	55.38*	56.17*
ASDVECH	83.52	92.76	43.97	33.86	83.52	88.00	52.74*	54.35*

**Table IV: Out-of-Sample Hedging Effectiveness – Short Vs Long Comparison**

This table presents the Out-of-Sample hedging performance for both Crude Oil and S&P500 returns and for both the asymmetric and symmetric datasets. Figures are in percentages. HE<sub>1</sub> – HE<sub>4</sub> give the percentage reduction in the performance metric from the hedged model as compared with the worst hedged position. For example, short hedging the Crude Oil contract for the asymmetric dataset with the Naïve model yields an 80.54% reduction in the variance as compared with a No-Hedge strategy. Statistical comparisons are made between the performance of short and long hedgers on a metric by metric basis using Efron's 1979 technique. \* Indicates that the percentage reduction in the risk metric is significantly better comparing long to short hedges at the 1% confidence level. For example, using the Crude Oil asymmetric dataset, and the LPM HE<sub>2</sub> as our performance metric, we can see that the out-of-sample performance of long hedgers, is significantly better than that of the short hedgers for each of the four hedging models.

	HE <sub>1</sub> Variance x10 <sup>-2</sup>	HE <sub>2</sub> LPM x10 <sup>-2</sup>	HE <sub>3</sub> VaR x10 <sup>-2</sup>	HE <sub>4</sub> CVaR x10 <sup>-2</sup>	HE <sub>1</sub> Variance x10 <sup>-2</sup>	HE <sub>2</sub> LPM x10 <sup>-2</sup>	HE <sub>3</sub> VaR x10 <sup>-2</sup>	HE <sub>4</sub> CVaR x10 <sup>-2</sup>
<b>Panel A: SHORT HEDGERS</b>					<b>Panel B: LONG HEDGERS</b>			
<b>CRUDE OIL-Asymmetric</b>								
None	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Naïve	80.54	79.97	64.02*	59.77*	80.54	93.56*	40.77	36.06
OLS	81.11	85.37	51.69	62.99*	81.11	93.53*	52.08	47.85
SDVECH	80.22	80.87	62.88*	64.23*	80.22	94.12*	37.44	40.00
ASDVECH	80.37	81.62	62.86*	64.90*	80.37	94.14*	37.58	40.55
<b>CRUDE OIL-Symmetric</b>								
None	0.00	0.00	0.00	2.90*	0.00	0.00	0.00	0.00
Naïve	70.38	76.82*	15.40	0.00	70.38	62.83	33.43*	26.25*
OLS	71.55	75.63	24.40	16.75	71.55	71.64	29.94	19.71
SDVECH	71.55	78.25*	17.58	3.06	71.55	66.62	35.28*	28.25*
ASDVECH	72.43	81.02*	19.05	2.84	72.43	66.92	35.50*	36.48*

**Table V: Statistical Comparison of Hedging Model Performance - In-Sample**

Table V presents the in-sample hedged portfolio statistics upon which we base our performance measures. The best performing model is the model that yields the lowest value for each risk measure and is denoted by <sup>a</sup>. For example, the OLS model yields the lowest variance of 0.164 when hedging the Crude Oil Asymmetric data. Statistical comparisons are made for each hedging model against the best performing model using Efron's 1979 technique. For example, taking the asymmetric dataset for a short hedger, we can see that there is a significant difference between the in-sample variance of the No Hedge 0.481 as compared with the best performing OLS model 0.164. \* Denotes a better performance being recorded for the best performing benchmark relative to that measure at the 1% significance level.

	Variance x10 <sup>-3</sup>	LPM x10 <sup>-5</sup>	VaR x10 <sup>-2</sup>	CVaR x10 <sup>-2</sup>	Variance x10 <sup>-3</sup>	LPM x10 <sup>-5</sup>	VaR x10 <sup>-2</sup>	CVaR x10 <sup>-2</sup>
<b>Panel A: SHORT HEDGERS</b>					<b>Panel B: LONG HEDGERS</b>			
<b>CRUDE OIL-Asymmetric</b>								
NONE	0.481*	1.375*	5.278*	5.700 <sup>a</sup>	0.481*	0.722*	5.917*	8.107*
NAIVE	0.171	0.328 <sup>a</sup>	4.582*	6.449*	0.171	0.458*	4.356	5.750 <sup>a</sup>
OLS	0.164 <sup>a</sup>	0.413*	4.190 <sup>a</sup>	5.810	0.164 <sup>a</sup>	0.323 <sup>a</sup>	4.346	6.277*
SDVECH	0.172	0.383	4.315	6.193	0.172	0.398*	4.139 <sup>a</sup>	6.118*
ASDVECH	0.168	0.355	4.340	6.252	0.168	0.410*	4.186	5.992*
<b>CRUDE OIL-Symmetric</b>								
NONE	0.522*	1.075*	5.115*	5.587*	0.522*	0.895*	5.518*	6.131*
NAIVE	0.089*	0.086*	2.966*	3.733*	0.089*	0.116*	2.915*	2.978*
OLS	0.087*	0.107*	2.783 <sup>a</sup>	2.853 <sup>a</sup>	0.087*	0.082 <sup>a</sup>	2.882*	3.650*
SDVECH	0.081 <sup>a</sup>	0.072 <sup>a</sup>	2.790	3.561*	0.081 <sup>a</sup>	0.096*	2.462 <sup>a</sup>	2.687 <sup>a</sup>
ASDVECH	0.086	0.078	2.866	3.695*	0.086	0.107*	2.608*	2.799*

**Table VI: Statistical Comparison of Hedging Model Performance - Out-of-Sample**

Table VI presents the out-of-sample hedged portfolio statistics upon which we base our performance measures. The best performing model is the model that yields the lowest value for each risk measure and is denoted by <sup>a</sup>. Statistical comparisons are made for each hedging model against the best performing model using Efron's 1979 technique. For example, taking the Crude Oil asymmetric dataset for a short hedger, we can see that there is a significant difference between the in-sample variance of both the No Hedge model 1.240 as compared with the best performing OLS model 0.234. \* Denotes a better performance being recorded for the best performing benchmark relative to that measure at the 1% significance level.

	Variance x10 <sup>-3</sup>	LPM x10 <sup>-5</sup>	VaR x10 <sup>-2</sup>	CVaR x10 <sup>-2</sup>	Variance x10 <sup>-3</sup>	LPM x10 <sup>-5</sup>	VaR x10 <sup>-2</sup>	CVaR x10 <sup>-2</sup>
<b>Panel A: SHORT HEDGERS</b>					<b>Panel B: LONG HEDGERS</b>			
<b>CRUDE OIL-Asymmetric</b>								
NONE	1.240*	2.862*	12.183*	16.531*	1.240*	8.110*	9.399*	10.395*
NAIVE	0.241	0.573*	4.383 <sup>a</sup>	6.650*	0.241	0.522	5.567*	6.647*
OLS	0.234 <sup>a</sup>	0.419 <sup>a</sup>	5.885*	6.118	0.234 <sup>a</sup>	0.524	4.504 <sup>a</sup>	5.421 <sup>a</sup>
SDVECH	0.245	0.548*	4.522	5.913	0.245	0.476	5.880*	6.237*
ASDVECH	0.243	0.526*	4.525	5.803 <sup>a</sup>	0.243	0.475 <sup>a</sup>	5.867*	6.180*
<b>CRUDE OIL-Symmetric</b>								
NONE	0.341*	0.634*	4.278*	4.312*	0.341*	0.469*	4.974*	5.353*
Naïve	0.101*	0.147*	3.619*	4.441*	0.101*	0.174*	3.311	3.948*
OLS	0.097	0.154*	3.234 <sup>a</sup>	3.697 <sup>a</sup>	0.097	0.133 <sup>a</sup>	3.485*	4.298*
SDVECH	0.097	0.138*	3.526*	4.305*	0.097	0.157*	3.219	3.841*
ASDVECH	0.094 <sup>a</sup>	0.120 <sup>a</sup>	3.463*	4.315*	0.094 <sup>a</sup>	0.155*	3.208 <sup>a</sup>	3.400 <sup>a</sup>

## Table VII: Summary of Best Performing Hedging Model

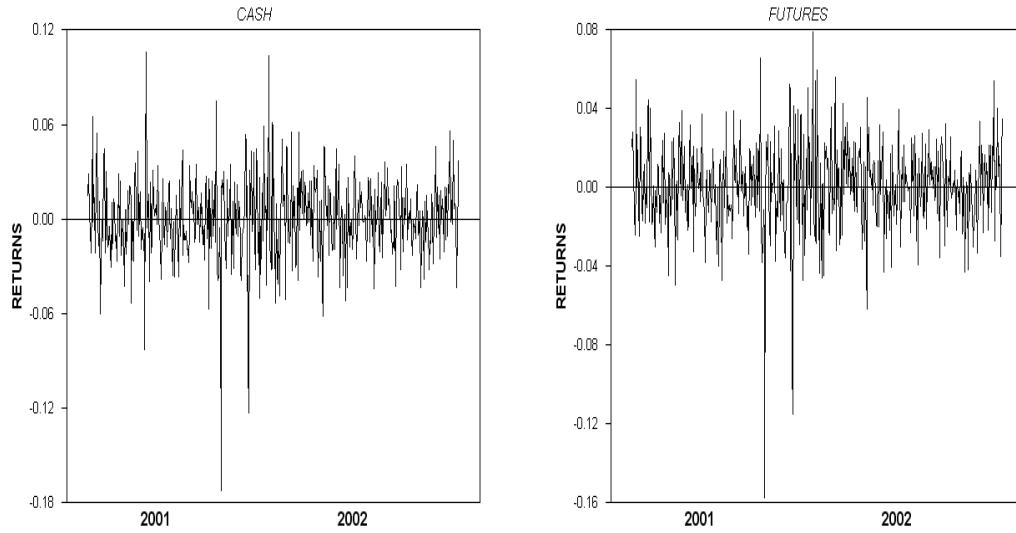
Table VII summarises the best hedging model for both short and long hedgers for each measure of hedging effectiveness. For example the best in-sample hedging model in terms of risk reduction for the asymmetric Crude Oil dataset is the OLS model in four out of eight cases across both sets of hedgers.

	Variance	LPM	VaR	CVaR	Variance	LPM	VaR	CVaR
<b>Panel A: SHORT HEDGERS</b>								
<b>IN-SAMPLE CRUDE OIL</b>								
<b>Asymmetric</b>	OLS	NAÏVE	OLS	NONE	OLS	OLS	SDVECH	NAÏVE
<b>Symmetric</b>	SDVECH	SDVECH	OLS	OLS	SDVECH	OLS	SDVECH	SDVECH
<b>OUT-OF-SAMPLE CRUDE OIL</b>								
<b>Asymmetric</b>	OLS	OLS	NAÏVE	ASDVECH	OLS	ASDVECH	OLS	OLS
<b>Symmetric</b>	ASDVECH	ASDVECH	OLS	OLS	ASDVECH	OLS	ASDVECH	ASDVECH

## Figure I: Returns of Crude Oil

Figure I displays both the cash and futures return series for Crude Oil. Returns vary over time. For example, the Crude Oil series displays greater volatility for the year 2001 than for 2002.

### CRUDE OIL



## Figure IIa: Out-of-Sample Optimal Hedge Ratios Crude Oil

Figure IIa displays the time varying OHR's for the Crude Oil series. Three OHR's are presented for both the asymmetric and symmetric distributions which are 2001 and 2002 respectively. The OHR's are the rolling window OLS, Symmetric GARCH SDVECH and Asymmetric GARCH ASDVECH models.

