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Stochastic Network Constrained Payment Minimization in Electricity Markets

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Abstract: This paper presents a novel framework to incorporate the uncertainties associated with load fluctuations and components' availability in a day-ahead joint energy-reserve Payment Cost Minimization (PCM). The payments are calculated based on Locational Marginal Prices (LMPs). Considering these uncertainties, appropriate definitions are not available for energy and reserve LMPs that suit the probabilistic PCM formulation and reflect the market characteristics. A tri-level optimization framework is proposed. The optimization variables in the first level optimization include commitment status. In the second level optimization, the production schedule and allocated reserves are the optimization variables. The third level problem includes different sub-problems, each of which related to optimization of units' production in a single scenario. This tri-level optimization is managed to be converted to a linear single-level optimization and is solved using an off-the-shelf branch-and-cut solver. The 10-unit system is first used to show the impacts of varying the degree of uncertainties. The IEEE Reliability Test System is next analyzed to validate the proposed formulation for reserve LMPs.

Nomenclature

Constants

a_{ljts}	Generation shift factor of line *l* with respect to change in injected power at bus *j* in scenario *s*.
$f_{lts}^{\max}$	Maximum power flow limit of line *l*.
GA_{its}	Availability indicator of unit *i* (hour *t*, scenario *s*).
MSR_i	Maximum sustainable ramp rate of unit *i*.
NL_{it}	No load cost of unit *i* in hour *t*.
Pd_{jts}	Demand at bus *j* in hour *t* in scenario *s*.
Q_{it}^{gdn}	Rate offered by unit *i* for down-going reserve.
Q_{it}^{gup}	Rate offered by unit *i* for up-going reserve.
SD_{it}	Shut-down cost of unit *i* in hour *t*.
rec_l	Receiving bus of line *l*.
$send_l$	Sending bus of line *l*.
SL_{itm}	Marginal cost of the *m*th block of the *i*th unit.
SU_{it}	Start-up cost of unit *i* in hour *t*.
X_{jk}	Element *jk* in inverse DC load flow matrix.
x_l	Reactance of line *l*.
π_s	Weight of scenario *s*.

Variables

$ELMP_{its}$	Energy LMP of unit *i* in hour *t* in scenario *s*.
p_{itsm}	Generation of unit *i* in scenario *s* in block *m*.
Pg_{jt}	Output power scheduled for unit *i* in hour *t*.
Pg_{jts}	Output power of unit *i* in hour *t* in scenario *s*.
R_{it}^{gdn}	Committed down-going reserve of unit *i* in hour *t*.
R_{it}^{gup}	Committed up-going reserve of unit *i* in hour *t*.
$RLMP_{it}^{gdn}$	Down-going reserve LMP of unit *i* in hour *t*.
$RLMP_{it}^{gup}$	Up-going reserve LMP of unit *i* in hour *t*.
u_{it}	Selection status of unit *i* in hour *t*.
y_{it}	Start-up indicator of unit *i* in hour *t*.
z_{it}	Shut-down indicator of unit *i* in hour *t*.

Indices and Sets

B_i	Index of the bus at which unit *i* is located.
G_j	Set of indices of the generation units at bus *j*.
N_b	Number of buses indexed by *j*.
N_l	Number of lines indexed by *l*.
N_g	Number of generation units indexed by *i*.
N_s	Number of scenarios indexed by *s*.
NS_{it}	Number of blocks (indexed by *m*) in the offer submitted by unit *i* for hour *t*.
T	Number of hours of the UC period indexed by *t*.

1. Introduction

Electricity is mostly traded in wholesale day-ahead markets. These markets should be designed to securely provide electricity at least cost to costumers. In electricity market design, the design of clearing mechanism, i.e., the main objective of the market operator, and also the pricing scheme are of the premium importance. Most Independent System Operators (ISOs) use an Offer Cost Minimization (OCM) mechanism to select supply bids in single side auction wholesale electricity markets minimizing the total supply cost offered by the suppliers. A pay at Market Clearing Price (MCP) or Locational Marginal Prices (LMPs) scheme is then usually used to calculate the payments. These mechanisms are supported by the well-known theory of maximization of social welfare if the supply bids represent the *real production cost* of generating units. Such an assumption does not usually hold [1, 2], since the producers take strategic approaches to submit their bids which maximize their profits. This leads to a high consumers' payment. The minimized system cost is also different from the actual payment and therefore, auction and settlement mechanisms are inconsistent [3-5]. With widely accepted marginal pricing schemes [6], the alternative is Payment Cost Minimization (PCM) mechanism which directly minimizes the total payment.

In a day-ahead electricity market, ISO solves an optimization problem, i.e., Unit Commitment (UC) problem to select the winning offers and generation of each supplier for the next day. The input data are not known precisely and can drift, in some way, around the expected values. A slight perturbation can profoundly affect the *feasibility* and or *optimality* of the solution. Commitment status, generation schedule and marginal prices depend on the system loads, availability of the system components and production of the renewable resources, which all are uncertain variables. These uncertainties should be included in market clearing. The resulting problem is referred to as stochastic Security Constrained Unit Commitment (SCUC) problem.

The focus of this paper is on proposing a proper framework to consider the system uncertainties in PCM mechanism which allows inclusion of the reserve payment to select the offers that lead to the minimum total payment with probabilistic reserve criterion. The place of reserves and interaction between energy and reserve markets are taken into account. The main challenges include implementing the proper definition of Reserve LMPs (RLMPs) and Energy LMPs (ELMPs) in a multi-level payment minimization and converting the problem to a single-level Mixed Integer Linear Programming (MILP) problem (which can effectively be solved using an off-the-shelf branch-and-cut solver). The definitions of LMPs should suit the market characteristics and can be used along with stochastic programming framework.

PCM mechanisms can be categorized into Consumers Payment Minimization (CPM) mechanism that minimizes the consumers' payment and Payment to Suppliers Minimization (PSM) which minimizes the total payment to suppliers. PSM is selected here for the reasons discussed in Section 2.

1.1. Review of the Related Literature

Most previous studies compared the PCM and OCM mechanisms considering a simple market model. Network constraints [5, 7] and simplified reserve models [8, 9] were included in PCM formulation. Many substantial studies and different optimization techniques applied to solve PCM problem were reviewed in [10].

Different simple PCM frameworks were proposed in [8, 9] for a joint energy-reserve market with *deterministic* reserve criteria. The effects of reserve location and the lost opportunity of selling energy for the units which contributes in reserve provision were not included. In [8, 9], the energy payments were calculated using the demand side LMPs, while the reserve payment was calculated based on a simple supply side reserve MCP. A locational reserve marginal pricing scheme was modelled in [11]. However, the reserve requirement was not set probabilistically.

Different uncertainty handling approaches for stochastic OCM SCUC were compared in [12]. Scenario-based approaches are more suitable to be applied here according to the type of uncertainties considered [12]. A PCM UC was proposed in [13] to incorporate the uncertainty associated with wind generation. The optimization problem was solved using Genetic Algorithm (GA). The reserve payment was excluded from the objective function. The solution optimality cannot be guaranteed using the method used in [13]. A custom-designed GA was applied in [14] to solve a simplified joint energy-reserve PCM UC.

A simplified stochastic PCM was proposed in [15]. The problem was a MILP problem and was solved using branch and cut technique. Since the network constraints were neglected in [15], a simplified formulation was proposed to model the marginal prices in a single-level optimization. For instance, the reserve marginal price was defined as the maximum accepted reserve offer. Such definition does not hold considering the network constraints. In other words, the effects of the location of the reserve providers invalidate the definition of LMPs and formulation developed in [15] when the system network is modelled. Network constraints cannot be neglected in most power systems. In comparison to reference [15], the main contribution of this paper is summarized here. 1) The network constraints and energy and reserve *locational* marginal prices are modeled in stochastic payment minimization problem and the uncertainty associated with components' availability is also considered here. 2) A tri-level formulation is developed for this problem, which as will be shown is inevitable. 3) The tri-level optimization is converted to a single level linear optimization problem without compromising the solution accuracy.

As shown in [16], individual marginal prices for each type of reserve cannot be defined at *demand side*. Having this fact in mind, a tri-level framework is proposed to formulate ELMPs and RLMPs at *supply side*. ELMPs depend on the uncertain variables. A set of ELMPs are assigned to each scenario. Thus, there should be different Optimal Power Flows (OPFs) each regarding a certain scenario. LMPs are indeterminable prior to optimization. Existence of LMPs in PCM objective function causes a self-referring issue. Bi-level programming was applied to cope with such issue [5].

1.2. Contributions of the Present Paper

To implement the proper definition of RLMPs and ELMPs (Section 4) and to avoid the self-referring issue, a tri-level optimization framework is proposed. In the first level, decision variables include commitment status which should be determined in a way that the solution satisfies the optimality and feasibility conditions in the lower level optimizations. Generation schedule, allocated reserves and RLMPs are the decision variables of the second level optimization. The optimality and feasibility of the third level sub-problems should be guaranteed in the second level optimization. The third level problem includes different sub-problems, in which every single sub-problem is related to optimization of unit productions in a single scenario. The ELMPs are found in this level for each scenario.

To solve the tri-level optimization problem, the third level problems are replaced with the equivalent Karush–Kuhn–Tucker (KKT) conditions in the second level. The unified second level optimization is also linearized and replaced with its equivalent KKT conditions in the first level optimization. Then, the resulting single-level optimization is linearized. To linearize the problem formulation in each level, the objective function and constraints are rearranged and expressed as the linear functions in terms of decision variables (using additional variables) without compromising the model accuracy.

The main contribution of this study resides in developing an appropriate model and formulation for stochastic PCM problem but not proposing a novel solution technique. Instead, the problem is solved using an available effective off-the-shelf branch-and-cut solver. The contributions of this paper are fourfold:

1) To model the stochastic PSM in a tri-level framework. Sub-section 3.4 explains why this framework is inevitable.
2) To model the stochastic reserve problem in PCM problem.
3) To propose a framework for proper energy and reserve pricing that suits the stochastic formulation.
4) To linearize and unify the tri-level formulation so that it can be solved using available branch-and-cut solvers.

2. Why payment to suppliers minimization

In this section, different options for the objective function of PCM mechanism are analyzed. The first option is to consider the sum of consumers' payments based on LMPs. This may lead to uncovered out of market cost (sum of the units' start-up, shut-down, and minimum costs). On the other hand, the demand side LMPs cannot be used for reserve market settlement. In other words, it should be noted that the RLMP at a certain bus cannot be defined as the rate of increase in the reserve cost with respect to the increase in demand at this bus. The reason is that based on [16] individual RLMPs cannot be defined at demand side. On the other hand, even if such definition holds, it may lead to uncovered reserve cost. Settlement approach for reserve market is also vague.

The second option is minimization of the consumers' payment with the addition of full compensation for out of market costs. The objective function is then neither the consumers' payment nor the payment to the suppliers. The reserve payment is also an issue with this objective function. In [8, 9], the reserve payment was added to the consumers' payment and out of market cost based on the supply side reserve MCP to form the objective function. In this case, the question is how to find the share of each consumer from the out of market costs and the reserve payment and the objective function is again not calculated fully at demand- or supply-side. If the objective function is calculated fully at demand or supply side, at least one problem is fully solved and the individual payments are determined at demand or supply side.

The third option for objective function that is also used here is to consider the sum of payment to suppliers as the objective function. The sum of the total energy payment based on ELMPs, reserve payment based on RLMPs, and out of market costs is considered as the objective function of this paper. With this objective function, the payments to individual suppliers are readily determined.

3. Stochastic PSM auction formulation

A two-stage stochastic programming is proposed. The first stage variables include the units' status, generation schedule and proper level of reserves. This decisions are the same for all scenarios. At the second stage, the decisions include the units' generation and LMPs in each scenario.

To model the network constraints, DC power flow is used. Stochastic SCUC problem is a large-scale mixed integer nonlinear optimization problem with huge number of variables and constraints, which takes a long time to be solved with exact network models. Therefore, application of DC power flow for market clearing has become the defacto industry standard. The solution feasibility can then be validated using a full AC power flow and the solution is modified in case of significant constraint violations.

The reasons why a tri-level programming framework is inevitable are discussed in subsection 3.4. A "1", "2" or "3" is added to the notations used for decision variables of level 1, 2 or 3, respectively. The notations with no number are used for constants. In each level, the variables with superscript * are the decision variables. All the decision variables and multipliers are positive.

3.1. First Level Problem

The objective function of this level is stochastic payment to suppliers and is denoted by Obj^{1st} which includes payments for energy, reserve, and units' start-up/shut-down costs (1). Only spinning reserves are modelled.

$$Obj^{1st} = \sum_{s=1}^{N_s} \pi_s . \sum_{t=1}^{T}\sum_{i=1}^{N_g}\{ELMP3_{its} .Pg3_{its} + GA_{its}.[u1^*_{it} \times NL_{it} + y1^*_{it} .SU_{it} + z1^*_{it} .SD_{it}]\} + \sum_{t=1}^{T}\{\sum_{i=1}^{N_g}[RLMP2^{gup}_{it} .R2^{gup}_{it} + RLMP2^{gdn}_{it} .R2^{gdn}_{it}]\} \quad (1)$$

The constraints on the problem binary variables include (2) and minimum up-/down-time constraints [17]. Optimality and feasibility of the second level optimization should also be included in the constraints of this level.

$$\begin{aligned} & y1^*_{i(t+1)} - z1^*_{i(t+1)} = u1^*_{i(t+1)} - u1^*_{it} \\ & 0 \le y1^*_{it} + z1^*_{it} \le 1 \\ & u1^*_{it},\ y1^*_{it},\ z1^*_{it} \in \{0,1\} \qquad \forall i,t \end{aligned} \quad (2)$$

3.2. Second Level Problem

The objective of the second level optimization is OCM. The decision variables in this level are the allocated reserves and output schedule of the units. The RLMPs are calculated using the Lagrange multipliers of the constraints (Section 4). The objective function of this level is given in (3). In this paper, generation-side up-/down-going reserve is the generation capacity that is ready to meet the demand increase/decrease or generation decrease/increase in 10 minutes [16]. Supply bid level constraints are given in (4)-(5).

$$Obj^{2nd} = \sum_{t=1}^{T}\sum_{i=1}^{N_g}\left\{Q^{gup}_{it} .R2^{*gup}_{it} + Q^{gdn}_{it} .R2^{*gdn}_{it}\right\} \quad (3)$$

Though the objective of second level seems to be reserves cost minimization based on (3), this formulation along with the optimizations of the third level co-optimizes the energy and reserves costs. In other words, the second level optimization should guarantee the optimality and feasibility of the third level optimizations. In order to guarantee the optimality and feasibility of the third level optimizations, these optimizations are replaced with their equivalent KKT conditions (see Section 5) which are sufficient for optimality and feasibility of the third level optimizations, having this fact in mind that the objective functions and constraints of the third level optimizations are convex [11] (even linear in this case). Such KKT conditions are included in the second level constraints along with other constraints of this level. Constraints (4) show the effect of decision variables of the upper level optimization on the solution of this level.

$$\begin{aligned} & -u1_{it}.Pg^{\max}_{i} + Pg2^*_{it} + R2^{*gup}_{it} \le 0 && (\mu^{\max}_{it}) \\ & u1_{it}.Pg^{\min}_{i} - Pg2^*_{it} + R2^{*gdn}_{it} \le 0 && (\mu^{\min}_{it}) \\ & -Pg2^*_{it} \le 0 && (\mu^{pg}_{it}) \\ & -R2^{*gup}_{it} \le 0 && (\mu^{up}_{it}) \\ & -R2^{*gdn}_{it} \le 0 && (\mu^{dn}_{it}) \end{aligned} \quad (4)$$

$$\begin{aligned} & R2^{*gup}_{it} - \min\left\{10 \times MSR_i , R^{gup,\max}_{it}\right\} \le 0 && (\lambda^{gup}_{it}) \\ & R2^{*gdn}_{it} - \min\left\{10 \times MSR_i , R^{gdn,\max}_{it}\right\} \le 0 && (\lambda^{gdn}_{it}) \end{aligned} \quad (5)$$

3.3. Third Level Problem

Here, the optimization variables include power output of each generator in each scenario. ELMPs are calculated at this level using the appropriate Lagrange multipliers (Section 4). The objective function for scenario s is given in (6). The power balance constraint is given in (9) for each hour.

$$Obj_s^{3rd} = \sum_{t=1}^{T}\sum_{i=1}^{N_g}\sum_{m=1}^{NS_{it}} SL_{itm} \times p3^*_{itsm} \quad (6)$$

$$-GA_{its}.Pg_i^{\min}.u1_{it} + Pg3^*_{its} - \sum_{m=1}^{NS_{it}} p3^*_{itsm} = 0 \;\forall i,t \;\; (\eta_{its}) \quad (7)$$

$$\begin{aligned} &-p3^*_{itsm} \le 0 && \forall i,t,m && (\sigma^{\min}_{itsm}) \\ &-p^{\max}_{itm} + p3^*_{itsm} \le 0 && \forall i,t,m && (\sigma^{\max}_{itsm}) \end{aligned} \quad (8)$$

$$\sum_{j=1}^{N_b} Pd_{jts} - \sum_{i=1}^{N_g} Pg3^*_{its} = 0 \quad \forall t \quad (\gamma_{ts}) \quad (9)$$

Only the generators which are available in scenario *s* and their offers have been accepted to contribute in reserve provision can change their outputs to satisfy the system load balance constraint in scenario *s*. The values of these changes are limited by (10). This constraint links the individual optimizations for different scenarios and shows the effect of the second level optimization on the decision variables of the last level optimizations and vice versa. The power flow in the network lines cannot exceed their capacity (11). The ramping-up and -down constraints are presented in (13).

$$\begin{aligned} &-GA_{its}.Pg2_{it} - GA_{its}.R2^{gup}_{it} + Pg3^*_{its} \le 0 \;\forall i,t \;(\xi^{\max}_{its}) \\ &GA_{its}.Pg2_{it} - GA_{its}.R2^{gdn}_{it} - Pg3^*_{its} \le 0 \quad \forall i,t \;(\xi^{\min}_{its}) \end{aligned} \quad (10)$$

$$\begin{aligned} &-f^{\max}_{lts} + \sum_{j=1}^{N_b} a_{ljts} \cdot \left(-Pd_{jts} + \sum_{i\in G_j} Pg3^*_{its} \right) \le 0 \;\; \forall l \;\; (\varphi^+_{lts}) \\ &-f^{\max}_{lts} - \sum_{j=1}^{N_b} a_{ljts} \cdot \left(-Pd_{jts} + \sum_{i\in G_j} Pg3^*_{its} \right) \le 0 \;\; \forall l \;\; (\varphi^-_{lts}) \end{aligned} \quad (11)$$

$$a_{ljs} = \frac{X_{send_l,j} - X_{rec_l,j}}{x_l} \quad (12)$$

$$\begin{aligned} &-RU_i - y1_{i(t+1)}Pg_i^{\min} + Pg3^*_{i(t+1)s} - Pg3^*_{its} \le 0 \quad \forall i,t \\ &-RD_i - z1_{i(t+1)}Pg_i^{\max} + Pg3^*_{its} - Pg3^*_{i(t+1)s} \le 0 \quad \forall i,t \end{aligned} \quad (13)$$

3.4. Rationale behind the Proposed Tri-level Framework

The reasons behind necessity of the proposed tri-level optimization framework are listed below:

1) To avoid the self-referring issue due to the presence of LMPs, PSM problem should be formulated as a multi-level optimization [7, 18].
2) LMPs are included in the lower level decision variables. These LMPs cannot be defined as the rate of change of *payment* to the change in demand at the system buses [18].
3) With the same units' on/off status, there is no difference between OCM and PSM OPF results [11]. Units' on/off states are the decision variables of the *first level*.
4) Reason 2 above declares it is necessary to formulate the lower level(s) as OCM problems and reason 3 declares that replacement of the lower level problem(s) with OCM problem does not change the results. Therefore, the lower level problems are OCM problems in this paper.
5) There are two approaches that can be taken at this point for the lower level(s). This approaches are compared here to determine why application of the proposed tri-level framework in *inevitable*.

The first approach is to solve the problem via a bi-level framework with the total stochastic energy and reserve cost as the objective function of the lower level. The Lagrangian of such second level optimization is presented in (14) for a single-hour market clearing without network constraints, where EC_s and RC_s are respectively the energy and reserve costs in scenario *s*. Lagrange multiplier γ_s is the rate of decrease in the total objective function (*EC*+*RC*) to the perturbed relaxation of this constraint. Therefore, it is not possible to find the ELMP in scenario *s* independent from the ELMPs in other scenarios and RLMPs. From the point of view of electricity markets, if we relax the regarding constraint, by decreasing Pd_s, the reserve cost may also decrease. Under such setup, γ_s cannot be used to determine the ELMP in scenario *s* and therefore, there is no way to formulate the first level optimization problem based on the decision variables of the second level optimization.

$$\ell = \sum_{s=1}^{N_s} EC_s + \sum_{s=1}^{N_s} RC_s + ... + \gamma_s \left(Pd_s - \sum_{i=1}^{N_g} Pg_{is} \right) + ... \quad (14)$$

In market clearing step, the system future state is unknown. Therefore, it is necessary to assign a proper set of ELMPs to each scenario to formulate the stochastic payment minimization. Marginal price of energy at bus *j* in each scenario can be defined as the rate of change of tomorrow's system cost (generation cost) with respect to the increase in consumption at bus *j* if that scenario happens. With the proposed formulation (as the second approach), this change would not affect the reserve cost, since the reserve cost is fixed, no matter which scenario would happen (3). Therefore, to formulate ELMPs in each scenario, different optimizations are necessary each of which related to a certain scenario.

It should be noted that the proposed framework is not a sequential optimization. The optimality and feasibility of the third level optimizations is considered among the constraints of the second level optimization. The optimality and feasibility of this unified second level optimization are also included in the constraints of the first level optimization. Under such setup, all the decision variables should be found simultaneously. The tri-level optimization is first converted to an equivalent single-level optimization and then is solved.

4. ELMPs and RLMPs at generation side

The ELMPs in each scenario can be found using Lagrange multipliers of the third level optimization for this scenario. Among the constraints of the third level optimizations, Pd_{jts} can be found in constraints (9) and (11). The rate of change of objective function with respect to the change in the demand at each bus in scenario *s* can be found using the Lagrange multipliers of constraints (9) and (11) considering the coefficients of Pd_{jts} in these constraints (15).

$$ELMP_{jts} = \gamma_{ts} - \sum_{l=1}^{N_l} a_{ljs} \cdot \left(\varphi^+_{lst} - \varphi^-_{lst} \right) \quad (15)$$

Here, the RLMP of a certain type of reserve provided by a reserve supplier is defined as the value of decrease in the system cost when 1 MW free additional reserve of this type is offered by the regarding provider. Marginal prices of the committed reserves can be found using (16). The first equation of (16) is explained further. The constraints that impose a maximum limit on the accepted generation-side up-going reserve are the first parts of (4) and (5). If each of these constraints is relaxed by an incremental value, the rate of decrease in the value of the objective function of the second level optimization is equal to the Lagrange multiplier of this constraint. Therefore, in order to find the marginal price of this type of reserve for unit *i*, the Lagrange multipliers of these constraints should be aggregated and added to the offered rate of this reserve. It is interesting to note that based on (16), the unit maximum power constraint affects the marginal price of this type of reserve when the first constraint of (4) is activated. This accounts for the lost opportunity of this unit for selling the convertible products.

$$\begin{aligned} RLMP_{it}^{gup} &= \lambda_{it}^{gup} + \mu_{it}^{\max} + Q_{it}^{gup} \\ RLMP_{it}^{gdn} &= \lambda_{it}^{gdn} + \mu_{it}^{\min} + Q_{it}^{gdn} \end{aligned} \qquad (16)$$

5. Solution Methodology

The KKT conditions of the third level optimizations are linearized and passed to the second level. The resulting second level problem is replaced in the first level optimization with KKT conditions. The final single-level problem is then linearized. Hereinafter, the terms Higher Level (HL) and Lower Level (LL) optimizations are used for the first level optimization and linearized second level optimization in which the KKT conditions of the third level optimizations are also included, respectively.

There is no need to apply any reserve to satisfy the load balance constraint in the base scenario [19]. Therefore, the power scheduled for unit *i* ($Pg2_{it}$) is equal to the output power of this unit in the base case scenario which is referred to as scenario 1 afterwards. It is necessary to modify the formulation developed so far for the third level optimization regarding scenario 1. The variables of this optimization include merely the power scheduled for the units. On the other hand, the only variables needed to be determined from the previous levels are the units' on/off status. Therefore, this optimization can be transferred into the second level problem.

5.1. Replacing the Third Level problems with KKT Conditions in the Second Level Problem

The linearized formulation of the lower level problem is given in (17)-(34). *lo*, *pf*, *df*, and *cs* stand for the Lagrange optimality, primal feasibility, dual feasibility, and complementary slackness conditions, respectively. Lagrange optimality, primal feasibility and dual feasibility conditions of the third level optimizations are specified in (23)-(24), (25)-(26) and (27), respectively (see Appendix 1). Complementary slackness conditions are linearized in (28)-(33). Such linearization introduces new binary variables (34).

$$\begin{aligned} Obj^{\mathrm{LL}} = \pi_1 \sum_{t=1}^{T}\sum_{i=1}^{N_g}\sum_{m=1}^{NS_{it}} SL_{itm} \times p2_{itm} \\ + \sum_{t=1}^{T}\sum_{i=1}^{N_g}\left\{Q_{it}^{gup}.R2_{it}^{gup} + Q_{it}^{gdn}.R2_{it}^{gdn}\right\} \end{aligned} \qquad (17)$$

$$\begin{aligned} &-u1_{it}.Pg_i^{\max} + Pg2_{it} + R2_{it}^{gup} \le 0 && (\mu_{it}^{\max}) \\ &u1_{it}.Pg_i^{\min} - Pg2_{it} + R2_{it}^{gdn} \le 0 && (\mu_{it}^{\min}) \\ &-Pg2_{it} \le 0 && (\mu_{it}^{pg}) \\ &-R2_{it}^{gup} \le 0 \quad (\mu_{it}^{up}), \qquad -R2_{it}^{gdn} \le 0 && (\mu_{it}^{dn}) \end{aligned} \qquad (18)$$

$$\begin{aligned} &R2_{it}^{gup} - \min\left\{10 \times MSR_i, R_{it}^{gup,\max}\right\} \le 0 && (\lambda_{it}^{gup}) \\ &R2_{it}^{gdn} - \min\left\{10 \times MSR_i, R_{it}^{gdn,\max}\right\} \le 0 && (\lambda_{it}^{gdn}) \end{aligned} \qquad (19)$$

$$-Pg_i^{\min}.u1_{it} + Pg2_{it} - \sum_{m=1}^{NS_{it}} p2_{itm} = 0 \quad \forall i,t \quad (\eta_{it}) \qquad (20)$$

$$-p2_{itm} \le 0 \ \ (\sigma_{itm}^{\min}), \qquad -p_{itm}^{\max} + p2_{itm} \le 0 \ \ (\sigma_{itm}^{\max}) \qquad (21)$$

$$\sum_{j=1}^{N_b} Pd_{jt} - \sum_{i=1}^{N_g} Pg2_{it} = 0 \qquad \forall t \qquad (\gamma_t) \qquad (22)$$

$$\frac{\partial \mathcal{L}^{3rd}}{\partial p3_{itsm}} = SL_{itm} - \eta_{its} - \sigma_{itsm}^{\min} + \sigma_{itsm}^{\max} = 0 \qquad (\varsigma_{itsm}^{lo,p}) \qquad (23)$$

$$\begin{aligned} \frac{\partial \mathcal{L}^{3rd}}{\partial Pg3_{its}} = \eta_{its} + \gamma_{ts} - \xi_{its}^{\min} + \xi_{its}^{\max} \\ + \sum_{l=1}^{N_l} a_{lB_i s}.\varphi_{lts}^{+} - \sum_{l=1}^{N_l} a_{lB_i s}.\varphi_{lts}^{-} = 0 \qquad (\varsigma_{its}^{lo,Pg}) \end{aligned} \qquad (24)$$

$$-GA_{its}.Pg_i^{\min}.u1_{it} + Pg3_{its} - \sum_{m=1}^{NS_{it}} p3_{itsm} = 0 \ \forall i,t \ \ (\varsigma_{its}^{pf}) \qquad (25)$$

$$\sum_{j=1}^{N_b} Pd_{jts} - \sum_{i=1}^{N_g} Pg3_{its} = 0 \qquad \forall t \qquad (\varsigma_{ts}^{pf}) \qquad (26)$$

$$\begin{aligned} &-\eta_{its} \le 0 \ \ \left(\varsigma_{its}^{df}\right), && -\gamma_{ts} \le 0 \ \ \left(\varsigma_{ts}^{df}\right) \\ &-\sigma_{itsm}^{\min} \le 0 \ \ \left(\varsigma_{itsm}^{df,\min}\right), && -\sigma_{itsm}^{\max} \le 0 \ \ \left(\varsigma_{itsm}^{df,\max}\right) \\ &-\xi_{its}^{\min} \le 0 \ \ \left(\varsigma_{its}^{df,\min}\right), && -\xi_{its}^{\max} \le 0 \ \ \left(\varsigma_{its}^{df,\max}\right) \\ &-\varphi_{lts}^{+} \le 0 \ \ \left(\varsigma_{lts}^{df,+}\right), && -\varphi_{lts}^{-} \le 0 \ \ \left(\varsigma_{lts}^{df,-}\right) \end{aligned} \qquad (27)$$

$$\begin{aligned} &-p3_{itsm} \le 0 && \left(\varsigma_{itsm}^{cs1,\min}\right) \\ &p3_{itsm} + H_{itsm}^{\min} e_{itsm}^{\min} - H_{itsm}^{\min} \le 0 && \left(\varsigma_{itsm}^{cs2,\min}\right) \\ &\sigma_{itsm}^{\min} - M_{itsm}^{\min} e_{itsm}^{\min} \le 0 && \left(\varsigma_{itsm}^{cs3,\min}\right) \\ &H_{itsm}^{\min} = p_{itm}^{\max}, \qquad M_{itsm}^{\min} = SL_{itm} - \min_{k,n}\left\{SL_{ktn}\right\} \end{aligned} \qquad (28)$$

$$\begin{aligned} &-p_{itm}^{\max} + p3_{itsm} \le 0 && \left(\varsigma_{itsm}^{cs1,\max}\right) \\ &p_{itm}^{\max} - p3_{itsm} + H_{itsm}^{\max} e_{itsm}^{\max} - H_{itsm}^{\max} \le 0 && \left(\varsigma_{itsm}^{cs2,\max}\right) \\ &\sigma_{itsm}^{\max} - M_{itsm}^{\max} e_{itsm}^{\max} \le 0 && \left(\varsigma_{itsm}^{cs3,\max}\right) \\ &H_{itsm}^{\max} = p_{itm}^{\max}, \qquad M_{itsm}^{\max} = \max_{k,n}\left\{SL_{ktn}\right\} - SL_{itm} \end{aligned} \qquad (29)$$

$$\begin{aligned} &GA_{its}.Pg2_{it} - GA_{its}.R2_{it}^{gdn} - Pg3_{its} \le 0 && \left(\varsigma_{its}^{cs1,\min}\right) \\ &-GA_{its}.Pg2_{it} + GA_{its}.R2_{it}^{gdn} + Pg3_{its} \\ &\qquad + H_{its}^{\min} e_{its}^{\min} - H_{its}^{\min} \le 0 && \left(\varsigma_{its}^{cs2,\min}\right) \\ &\xi_{its}^{\min} - M_{its}^{\min} e_{its}^{\min} \le 0 && \left(\varsigma_{its}^{cs3,\min}\right) \\ &H_{its}^{\min} = Pg_{its}^{\max} - Pg_{its}^{\min} \\ &M_{its}^{\min} = \max_{n}\left\{SL_{itn}\right\} - \min_{k,n}\left\{SL_{ktn}\right\} \end{aligned} \qquad (30)$$

$$\begin{aligned} &-GA_{its}Pg2_{it} - GA_{its}R2_{it}^{gup} + Pg3_{its} \le 0 && \left(\varsigma_{its}^{cs1,\max}\right) \\ &GA_{its}Pg2_{it} + GA_{its}R2_{it}^{gup} - Pg3_{its} \\ &\qquad + H_{its}^{\max} e_{its}^{\max} - H_{its}^{\max} \le 0 && \left(\varsigma_{its}^{cs2,\max}\right) \\ &\xi_{its}^{\max} - M_{its}^{\max} e_{its}^{\max} \le 0 && \left(\varsigma_{its}^{cs3,\max}\right) \\ &H_{its}^{\max} = Pg_{its}^{\max} - Pg_{its}^{\min} \\ &M_{its}^{\max} = \max_{k,n}\left\{SL_{ktn}\right\} - \min_{n}\left\{SL_{itn}\right\} \end{aligned} \qquad (31)$$

$$
\begin{aligned}
&-f_{lts}^{\max} - \sum_{j=1}^{N_b} a_{ljs} \cdot \left(-Pd_{jts} + \sum_{i \in G_j} Pg3_{its} \right) \le 0 && \left(\varsigma_{lts}^{cs1,-}\right) \\
&f_{lts}^{\max} + \sum_{j=1}^{N_b} a_{ljs} \cdot \left(-Pd_{jts} + \sum_{i \in G_j} Pg3_{its} \right) \\
&\qquad\qquad + H_{lts}^{-}.e_{lst}^{-} - H_{lts}^{-} \le 0 && \left(\varsigma_{lts}^{cs2,-}\right) \\
&\varphi_{lts}^{-} - M_{lts}^{-}.e_{lts}^{-} \le 0 && \left(\varsigma_{lts}^{cs3,-}\right) \\
&H_{lts}^{-} = 2f_{lts}^{\max},\ M_{lts}^{-} = \max_{i,m}\{SL_{itm}\} - \min_{i,m}\{SL_{itm}\}
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
&-f_{lts}^{\max} + \sum_{j=1}^{N_b} a_{ljs} \cdot \left(-Pd_{jts} + \sum_{i \in G_j} Pg3_{its} \right) \le 0 && \left(\varsigma_{lts}^{cs1,+}\right) \\
&f_{lts}^{\max} - \sum_{j=1}^{N_b} a_{ljs} \cdot \left(-Pd_{jts} + \sum_{i \in G_j} Pg3_{its} \right) \\
&\qquad\qquad + H_{lst}^{+}.e_{lst}^{+} - H_{lst}^{+} \le 0 && \left(\varsigma_{lts}^{cs2,+}\right) \\
&\varphi_{lst}^{+} - M_{lst}^{+}.e_{lst}^{+} \le 0 && \left(\varsigma_{lts}^{cs3,+}\right) \\
&H_{lst}^{+} = 2f_{lts}^{\max},\ M_{lst}^{+} = \max_{i,m}\{SL_{itm}\} - \min_{i,m}\{SL_{itm}\}
\end{aligned} \tag{33}
$$

$$
e_{itsm}^{\min}, e_{itsm}^{\max}, e_{its}^{\min}, e_{its}^{\max}, e_{lst}^{-}, e_{lst}^{+} \in \{0,1\} \tag{34}
$$

5.2. Replacing LL Optimization with Equivalent KKT Conditions in the HL Problem

The LL problem is also replaced with the KKT conditions. The equivalent single-level optimization is linearized and solved using the branch-and-cut method. This method assumes that an algorithm ("sub-algorithm") does exist to find solutions of the continuous problem with the additional bounds on integer variables.

Since the third level optimizations are convex, they can be replaced with KKT conditions. However, linearization of the complementary slackness conditions of the third level introduces some integer variables to the second level problem (34), so replacing the unified second level optimization with KKT conditions is not quite right due to the non-convexity introduces by the integrality constraints.

At the starting node, branch and cut algorithm ignores the integer restrictions as though all the variables are real-valued. At this step a LP algorithm can be used as "sub-algorithm", since all the variables are real-valued and the lower level optimization is convex and KKT conditions can be applied to convert the bi-level optimization into a single-level LP problem. In the continuous problem associated with the other nodes in the search tree, some *additional bounding constraints* are introduced by branch-and-cut algorithm. Fortunately, these constraints are linear. However, if the branching process is related to an integer variable of lower level problem, these constraints should be considered in the lower level problem. It is possible to consider all of bounding constraints regarding the lower level optimization introduced by branch-and-cut algorithm at each node and form the KKT condition of the linear and convex LP relaxed lower level problem including the additional bound introduced by branch-and-cut algorithm. Now the continuous problem of each node can be solved using LP, and since the "sub-algorithm" is finite and the integer variables are bounded, an optimal integer solution will be reached after finite number of branching operations. The only issues that still stand are complementary slackness conditions of the additional bounds and variable number of optimization variables due to Lagrange multiplier of these additional bounds. These constraints are not included in the standard MILP solvers and these solvers cannot solve the problems with variable number of optimization variables. Luckily, the integer variables of the lower level optimization are all binary. In a branching operation on such binary variables (e) there are only two options, i.e., e=0 or e=1. Therefore, the additional bounding constraints are equality constraints, so there is no complementary slackness condition regarding such additional bounding constraints. The primal feasibility constraints regarding such additional bounds are automatically handled using a standard solver. In the formulation of the relaxed LP, (34) is replaced with (35). The subscript and superscript of the binary variables are omitted.

$$
-e \le 0 \quad \left(\upsilon^{-}\right), \qquad e - 1 \le 0 \quad \left(\upsilon^{+}\right) \tag{35}
$$

In the relaxed LL LP problem, the primal and dual feasibility, Lagrange optimality conditions and inequality constraint of the complementary conditions are linear. The complementary slackness conditions are linearized using the Fortuny-Amat technique [11]. The product terms of the LMPs and lower level decision variables in the objective function of the higher level optimization cause nonlinearity in the formulation that has not been addressed yet.

The product terms of ELMPs and power output of each generator in each scenario (except scenario 1) is linearized here. Using (23), the first term in the second line of (36) can be replaced by the first term of (37). The second term of (36) can be replaced by the second line of (37) using (38) to (39). Equation (38) is one of the complementary slackness conditions associated with the inequality constraints of (11). In a same way, the third term of the second line in (36) can be replaced by the third line of (37). The last two terms of (37) give the linearized formulation of the product terms of ELMPs and power output of generators.

$$
\begin{aligned}
\sum_{i=1}^{N_g} ELMP3_{its}.Pg3_{its} &= \sum_{i=1}^{N_g} \left[\left(\gamma_{ts} - \sum_{\substack{l=1 \\ j=B_i}}^{N_l} a_{ljs} \cdot \left(\varphi_{lst}^{+} - \varphi_{lst}^{-} \right) \right) .Pg3_{its} \right] \\
&= \gamma_{ts} \sum_{i=1}^{N_g} Pg3_{its} - \sum_{i=1}^{N_g} \sum_{\substack{l=1 \\ j=B_i}}^{N_l} a_{ljs}.\varphi_{lst}^{+}.Pg3_{its} + \sum_{i=1}^{N_g} \sum_{\substack{l=1 \\ j=B_i}}^{N_l} a_{ljs}.\varphi_{lst}^{-}.Pg3_{its}
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
\sum_{i=1}^{N_g} ELMP3_{its}.Pg3_{its} &= \gamma_{ts} \sum_{j=1}^{N_b} Pd_{jts} \\
&- \sum_{l=1}^{N_l} \varphi_{lst}^{+} \cdot f_{lts}^{\max} - \sum_{l=1}^{N_l} \sum_{j=1}^{N_b} \varphi_{lst}^{+}.a_{ljs}.Pd_{jts} \\
&- \sum_{l=1}^{N_l} \varphi_{lst}^{-} \cdot f_{lts}^{\max} + \sum_{l=1}^{N_l} \sum_{j=1}^{N_b} \varphi_{lst}^{-}.a_{ljs}.Pd_{jts} \\
&= \sum_{j=1}^{N_b} Pd_{jts} \cdot \left(\gamma_{ts} - \sum_{l=1}^{N_l} a_{ljs}.(\varphi_{lst}^{+} - \varphi_{lst}^{-}) \right) - \sum_{l=1}^{N_l} \left(\varphi_{lst}^{+} + \varphi_{lst}^{-} \right) \cdot f_{lts}^{\max} \\
&= \sum_{j=1}^{N_b} ELMP3_{jts}.Pd_{jts} - \sum_{l=1}^{N_l} \left(\varphi_{lst}^{+} + \varphi_{lst}^{-} \right) \cdot f_{lts}^{\max}
\end{aligned} \tag{37}
$$

$$
\left[-f_{lts}^{\max} + \sum_{j=1}^{N_b} a_{ljs} \cdot \left(-Pd_{jts} + \sum_{i \in G_j} Pg3^{*}_{its} \right) \right] .\varphi_{lst}^{+} = 0 \tag{38}
$$

$$\varphi^{+}_{lst}.\sum_{j=1}^{N_b} a_{ljs}.\sum_{i\in G_j} Pg3^{*}_{its} = \varphi^{+}_{lst}.f^{\max}_{lts} + \varphi^{+}_{lst}.\sum_{j=1}^{N_b} a_{ljs}.Pd_{jts} \quad (39)$$

$$\sum_{l=1}^{N_l}\sum_{\substack{i=1,\\ j=B_i}}^{N_g} \varphi^{+}_{lst} a_{ljs} Pg3_{its} = \sum_{l=1}^{N_l}\varphi^{+}_{lst} f^{\max}_{lts} + \sum_{l=1}^{N_l}\sum_{j=1}^{N_b}\varphi^{+}_{lst} a_{ljs} Pd_{jts} \quad (40)$$

5.3. Linearizing the Objective Function

In order to linearize the remaining nonlinear parts of the objective function of the HL problem, (17) is rewritten as (41). This objective function is divided into two parts. The first part has been linearized so far. The second part includes the payment of scenario 1 and the reserve payments. None of these terms can be linearized separately. Here, both terms are linearized simultaneously. Considering the objective function and the constraints of the LL optimization (17)-(34), only the first/second constraint of (18) imposes a maximum/minimum limit on the producible power in the base case scenario.

$$Obj^{HL} = \left\{\begin{array}{l} \sum_{s=2}^{N_s}\pi_s.\sum_{t=1}^{T}\sum_{i=1}^{N_g}\{ELMP3_{its}.Pg3_{its} \\ +\sum_{s=1}^{N_s}\pi_s.\sum_{t=1}^{T}\sum_{i=1}^{N_g}\{GA_{its}.[u1_{it}NL_{it}+y1_{it}SU_{it}+z1_{it}SD_{it}]\} \end{array}\right\} \quad (41)$$
$$+\left\{\begin{array}{l} \pi_1.\sum_{t=1}^{T}\sum_{i=1}^{N_g}\{ELMP_{it}.Pg2_{it}\} \\ +\sum_{t=1}^{T}\{\sum_{i=1}^{N_g}[RLMP2^{gup}_{it}.R2^{gup}_{it}+RLMP2^{gdn}_{it}.R2^{gdn}_{it}]\} \end{array}\right\}$$

$$\begin{array}{l} \pi_1.ELMP_{it}Pg2_{it}+RLMP2^{gup}_{it}R2^{gup}_{it}+RLMP2^{gdn}_{it}R2^{gdn}_{it} \\ =\pi_1\sum_{m=1}^{NS_{it}}SL_{itm}v_{itm}.p2_{itm}+\mu^{\max}_{it}Pg2_{it}-\mu^{\min}_{it}Pg2_{it} \\ +Q^{gup}_{it}R2^{gup}_{it}+\mu^{\max}_{it}R2^{gup}_{it}+\lambda^{gup}_{it}R2^{gup}_{it} \\ +Q^{gdn}_{it}R2^{gdn}_{it}+\mu^{\min}_{it}R2^{gdn}_{it}+\lambda^{gdn}_{it}R2^{gdn}_{it} = \\ Q^{gup}_{it}R2^{gup}_{it}+Q^{gdn}_{it}R2^{gdn}_{it}+\pi_1\sum_{m=1}^{NS_{it}}SL_{itm}v_{itm}.p2_{itm} \\ +\lambda^{gup}_{it}R2^{gup}_{it}+\lambda^{gdn}_{it}R2^{gdn}_{it} \\ +\mu^{\max}_{it}(Pg2_{it}+R2^{gup}_{it})+\mu^{\min}_{it}(-Pg2_{it}+R2^{gdn}_{it}) \end{array} \quad (42)$$

The constraints that impose a maximum limit on the accepted level of the up-/down-going reserves are the first/second parts of (18) and (19). Equation (42) shows how to calculate the sum of the probabilistic energy payment for scenario 1 and the reserve payment to unit *i* in hour *t*. In (42) the binary variable v_{itm} is 1 if *m* is the index of the last accepted block of the generation offer of unit *i* in hour *t*.

Constraints of type (43) should be included in the formulation presented for the LL problem. The binary variable w_{itm} is 1 if the *m*th block of the generation offer of unit *i* is accepted. The nonlinear terms of (42) can be converted to the linear ones using complementary slackness conditions associated with constraints (18) and (19), which are presented in (44) and (25), respectively. Finally, the objective function of the HL problem is presented in (46) in terms of some linear and binary-linear product terms which are underlined. These terms should be converted into the equivalent linear terms to achieve an absolutely linear formulation as follows. The term $\kappa.\delta$, where κ and δ are binary and continuous variables, respectively, is replaced by the linear and always positive term ζ considering the constraints introduced in (47). The value of the large enough constant *M* is also specified in (47).

$$\begin{array}{ll} p^{\max}_{itm}w_{itm} \geq p2_{itm} & \\ w_{it1}=u_{it} & \\ w_{it\bar{m}} \leq v_{it\bar{m}} & \bar{m}=NS_{it} \\ w_{itm}+v_{it(m-1)} \leq 1 & \forall m \geq 2 \\ v_{itm}-w_{itm} \leq 0 & \forall m \geq 1 \\ w_{it(m-1)}-w_{it(m-1)}-v_{it(m-1)} \leq 0 & \forall m \geq 2 \end{array} \quad (43)$$

$$\begin{array}{l} \mu^{\max}_{it}(Pg2_{it}+R2^{gup}_{it}) = Pg^{\max}_{i}u1_{it}.\mu^{\max}_{it} \\ \mu^{\min}_{it}(-Pg2_{it}+R2^{gdn}_{it}) = -Pg^{\min}_{i}u1_{it}.\mu^{\min}_{it} \end{array} \quad (44)$$

$$\begin{array}{l} \lambda^{gup}_{it}.R2^{gup}_{it} = R2^{up,\max}_{it}.\lambda^{gup}_{it} \\ \lambda^{gdn}_{it}.R2^{gdn}_{it} = R2^{dn,\max}_{it}.\lambda^{gdn}_{it} \\ R2^{up,\max}_{it} = \min\left\{10\times MSR_i, R^{gup,\max}_{it}\right\} \\ R2^{dn,\max}_{it} = \min\left\{10\times MSR_i, R^{gdn,\max}_{it}\right\} \end{array} \quad (45)$$

$$\begin{array}{l} Obj^{HL} = \sum_{s=2}^{N_s}\pi_s\sum_{t=1}^{T}\sum_{j=1}^{N_b}\left\{\begin{array}{l} Pd_{jts}.\gamma_{ts}-\sum_{l=1}^{N_b}[a_{ljs}Pd_{jts}+f^{\max}_{lts}].\varphi^{+}_{lst} \\ +\sum_{l=1}^{N_b}[a_{ljs}Pd_{jts}-f^{\max}_{lts}].\varphi^{-}_{lst} \end{array}\right\} \\ +\sum_{s=1}^{N_s}\pi_s\sum_{t=1}^{T}\sum_{i=1}^{N_g}\left\{GA_{its}.[NL_{it}u1_{it}+SU_{it}y1_{it}+SD_{it}z1_{it}]\right\} \\ +\sum_{t=1}^{T}\sum_{i=1}^{N_g}\left\{\begin{array}{l} Q^{gup}_{it}R2^{gup}_{it}+Q^{gdn}_{it}R2^{gdn}_{it} \\ +Pg^{\max}_{i}\underline{u1_{it}.\mu^{\max}_{it}}-Pg^{\min}_{i}\underline{u1_{it}.\mu^{\min}_{it}} \\ +R2^{up,\max}_{it}.\lambda^{gup}_{it}+R2^{dn,\max}_{it}.\lambda^{gdn}_{it} \end{array}\right\} \\ +\pi_1\sum_{t=1}^{T}\sum_{i=1}^{N_g}\sum_{m=1}^{NS_{it}}SL_{itm}.\underline{v_{itm}.p2_{itm}} \end{array} \quad (46)$$

$$\delta-M.(1-\kappa)\leq\zeta,\quad 0\leq\zeta\leq\delta,\quad \zeta\leq M.\kappa,\quad M=\max\{\delta\} \quad (47)$$

6. Computational Complexity

The proposed model has been implemented on a personal computer with 8 processors at 2.4 GHz and 16 GB of RAM using CPLEX under GAMS. It should be mentioned that even with the linearized formulation, the solution time is still very high. Some reasons come from the high number of binary and continuous variables and constraints of the proposed framework, but the main reason is the distinct features of PCM mechanisms.

The efficiency of solving linearized PCM model is lower than OCM which is not an issue with the proposed model but rather a general difficulty of applying PCM mechanisms. The reason is that after LP relaxation, the feasible region which is restricted by the constraints defining the LMPs and units' status variables is enlarged. Such feature of PCM problem, comparing to OCM, causes an optimum with fractional elements in each dimension which is difficult to be cut off to obtain a convex hull. This leads to a large number of branching operations and a high solution time [20].

7. Case Studies

7.1. Basic 10-unit System

The main target of this case study is to test the proposed framework on a simple system without network constraints, where it is possible to present and analyze the full results of the proposed stochastic PSM framework and to compare them with those obtained for OCM problem. The second target is to show the effects of the degree of uncertainty on the results to conduct a sensitivity analysis.

In this section the proposed algorithm is tested on the 10-unit system in a 24-hour time horizon. The UC data for this system can be found in [21]. The reserve offers are given in Table 1. The standard deviation of hourly load is 2% of the average load. For the sake of tractability, all the units except unit 3 are assumed to be fully reliable and the forced outage rate of unit 3 is 5%. Load scenarios are generated using the adopted moment matching technique [22, 23]. The correlation between the hourly loads is neglected. 25 different scenarios have been also generated to model the availability of unit 3 each of which related to outage of unit 3 in each hour of the UC time horizon. These scenarios have been combined and ten scenarios have been selected after backward scenario reduction [24]. The repair times of the units are higher than 24 hours. Therefore, after the outage of unit 3, it is out for the rest of scheduling horizon. Each scenario represents the load profile in 24 hours and the outage time of unit 3. These ten scenarios are almost the permutations of two outage scenarios for unit 3 and five load scenarios (average, low, very low, high and very high load). Finding the number of scenarios that are required to model the system future is beyond the scope of this paper. In this subsection, the main target is to compare the results of OCM and PSM mechanisms and for both mechanisms, the same 10 scenarios are applied. In the next section, the same problems are solved with higher number of scenarios to show the effects. Generally, in order to find the required number of scenarios, a thorough analysis should be conducted to make a compromise between accuracy and computing time. One approach is to gradually increase the number of scenarios in offline simulations, until the UC results are not widely changed.

The relative optimality stopping criterion has been set to 2%. Via this option, a relative tolerance has been set on the gap between value of the objective function of the best integer feasible solution and the objective function value of the best node remaining (best bound). It should be noted that in most cases, the value of objective function never hits or even approaches the best bound during the future branching process. Therefore, selecting the value of 0.02 for relative optimality criterion does not mean that the final solution is suboptimal by 2%. With the selected relative optimality criterion, the solution performance of CPLEX for PCM problem is demonstrated in the case studies. However, in order to make sure that the value set for this parameter does not impose a high risk on solution optimality, all the optimizations in the case studies were also solved with this parameter set to 0.01 and the results didn't change.

The integrality and feasibility tolerances have been set to 1e-004 and 1e-005, respectively. The other parameters of CPLEX have been set to the default values. Total number of decision variables for the illustrative study is 143713 where about half of these variables are binary.

Fig. 1 shows the units' status under both mechanisms neglecting the uncertainties. Table 2 gives the system cost and payment. The units' status considering the uncertainties is also given in Fig. 1. More units are committed to satisfy the load increase in high loads scenarios. Unit 3 is avoided in most hours in OCM, since it is unavailable in some scenarios and its minimum cost is relatively high. However, unit 3 is on in hour 12 due to high system load. Since the start-up of unit 3 is inevitable in hour 12, it is also on in adjacent high load hours. On the other hand, though the no load cost of unit 3 is relatively high, it is on in most hours under PSM mechanism, to reduce the MCP in most of high-probability scenarios.

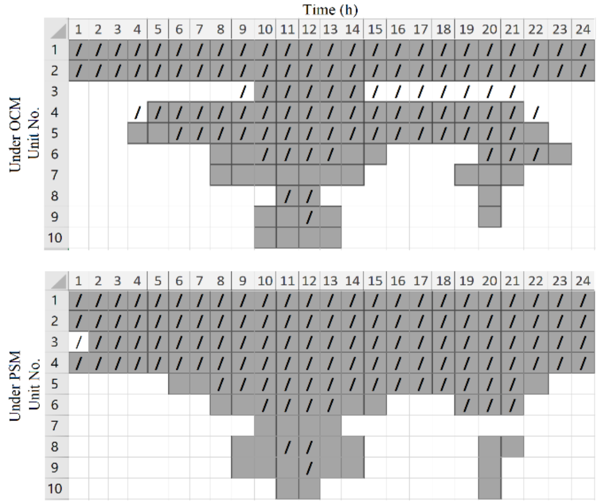

Fig. 1. *UC status under both mechanisms, cells with sign "/": units are "on" neglecting uncertainties, dark cells: units are "on" considering system uncertainties and dark cells with "/": units are "on" with and without considering uncertainties.*

Table 1 Reserve Offered Rates ($/MWh), 10-Unit System

Unit	1	2	3	4	5	6	7	8	9	10
Q^{gup}	5.5	4.5	5	5	3.5	3	2.5	3	2	2
Q^{gdn}	4.5	3.5	4	4	2.5	2	1.5	2	1	1

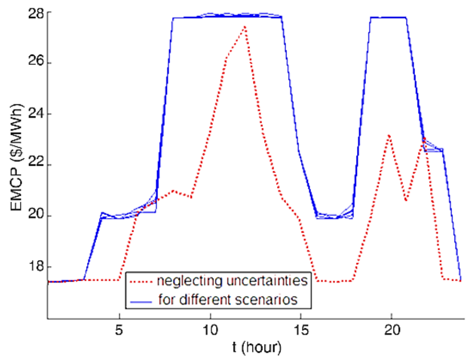

Fig. 2. *MCPs under OCM mechanism, 10-unit system.*

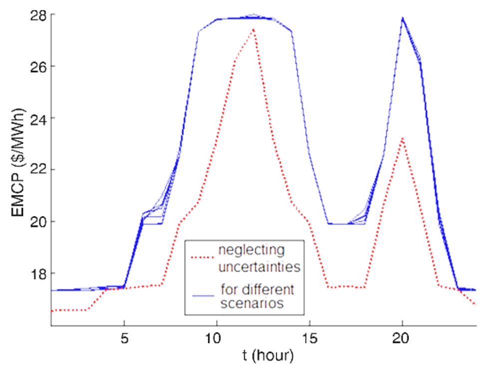

Fig. 3. *Market clearing prices under PSM, 10-unit system.*

The MCP in different hours for different scenarios and the system MCP neglecting the uncertainties are depicted in Fig. 2 under OCM mechanism. The MCPs are lower for PSM mechanism (Fig. 3). Without uncertainty, the MCP follows the load profile (given in [21]) under PSM mechanism.

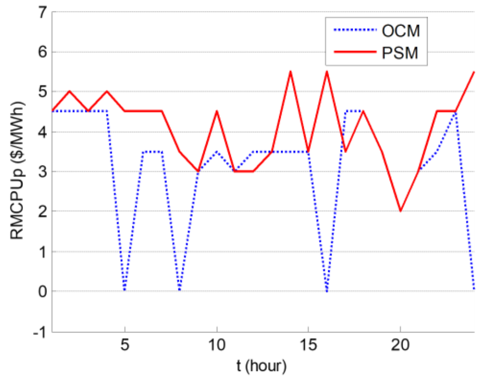

Fig. 4. *Up-going reserve MCP under OCM/PSM mechanism.*

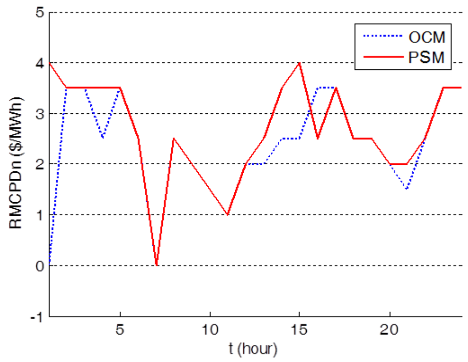

Fig. 5. *Down-going reserve MCP under both mechanisms.*

Table 2 Payment/Cost (1000 $), Neglecting Uncertainties

Mechanism	System Cost	Total Payment
OCM	552.24	575.79
PSM	560.85	564.41

Table 3 Payments/Costs (1000 $), Including Uncertainties

	System cost	Energy cost	Reserve cost	Total payment	Energy payment	Reserve payment
OCM	572.65	566.59	6.06	658.63	653.80	4.83
PSM	598.32	581.21	17.11	627.80	622.66	5.14

The MCPs are higher for the case including the system uncertainties under both mechanisms. The reason is that some units have left a share of their capacity for reserve. This leads to commitment of more expensive units. Table 3 shows the components of the system cost and total payment under both mechanisms. Comparing to Table 2, the energy cost has increased for both mechanisms as well as energy payment due to aforementioned phenomenon. The PSM has raised the cost by 4.5% and the increase in payment under OCM mechanism is 5%. It is also interesting that the reserve payment is higher under PSM mechanism. It seems that the PSM mechanism tries to decrease the energy payment as much as possible since the energy payment has the higher share in final payment, so the reserve payment is higher.

Figs. 4 and 5 show the up-going Reserve MCPs (RMCPUp) and down-going reserve MCPs (RMCPDn), respectively. The RMCPs are higher under PSM mechanism. The total up-going capacity in 24 hours is 1267.7 for OCM and 3240.3 MWh for PSM. Total down-going capacity is 996 and 1616.5 MWh for OCM and PSM, respectively.

In a separate study, with no unit outage the standard deviation of the load in different hours varied from 0 to 3%. The results are depicted in Fig 6. Under PSM mechanism, the payment is more stable as the degree of uncertainty changes. This leads to lower financial risk for the costumers.

7.2. Larger Systems Based on the 10-unit system

Additional case studies have been generated by replicating the base 10-unit system and scaling the system load accordingly to show the scalability of the proposed solution methodology and to show the effects of increasing the system dimensions on the Computing Time (CT). The required data for the base 10-unit system can be found in subsection 7.1.

Firstly, 10 different scenarios are considered to model the state space of the problem in all studies. Table 4 shows the results of these cases obtained under OCM and PSM mechanisms. The CT increases drastically as the system size grows under both mechanisms. The CT for 20- and 40-unit systems is very high under PSM mechanism. In a separate study for each case under PSM mechanism, a time limit of 2 hours was set to stop CPLEX in order to find a possible near optimal solution. The results of such studies were surprisingly the same as those reported in Table 4 suggesting the use of such time limit or another stopping criterion such as a carefully tuned threshold on payment. It is interesting that in all studies the increase in Offer Cost (OC) caused by applying PSM mechanism is lower than the increase in Payment to Suppliers (PS) under OCM mechanism.

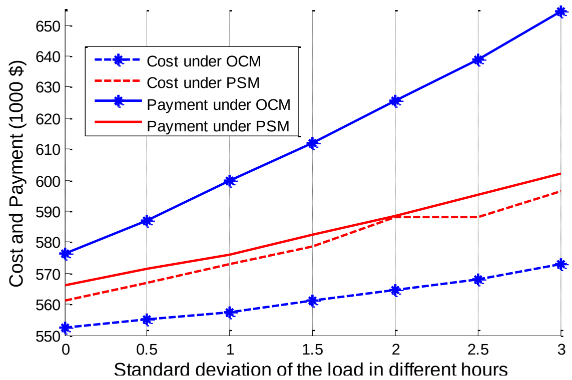

Fig. 6. *System cost and payment for different load standard deviations.*

Table 4 Total Payment (1000$), Total Cost (1000$) and Computing Time (s) for Case 7.2

	Ns=10 PSM	Ns=10 OCM	Ns=20 PSM	Ns=20 OCM
10-unit, PS	627.80	658.63	629.71	662.12
10-unit, OC	598.32	572.65	601.84	576.02
10-unit, CT	2775.2	9.8	8524.5	93.1
20-unit, PS	1245.84	1331.17	1249.51	1338.53
20-unit, OC	1183.27	1139.98	1189.27	1145.17
20-unit, CT	8119.6	160.1	20568.3	557.7
40-unit, PS	2473.46	2630.91	2479.03	2639.80
40-unit, OC	2332.63	2277.85	2337.63	2281.12
40-unit, CT	19084.7	763.2	36000.0*	2119.6

*10-hour limit is set for 40-unit system with 20 scenarios.

Table 5 Location, Type, and Power Output (MW) of System Units, subsection 7.3

Bus-No	Type	P_g^{ocm}	P_g^{pcm}	Bus-No	Type	P_g^{ocm}	P_g^{pcm}
1-1	U20	-	-	15-15	U12	-	-
1-2	U20	15.99	-	15-16	U12	-	-
1-3	U76	76.00	76.00	15-17	U12	-	-
1-4	U76	76.00	76.00	15-18	U12	-	-
2-5	U20	-	-	15-19	U12	-	-
2-6	U20	-	-	15-20	U155	155.0	155.0
2-7	U76	76.00	76.00	16-21	U155	155.0	155.0
2-8	U76	76.00	76.00	18-22	U400	400.0	354.4
7-9	U100	-	-	21-23	U400	400.0	354.4
7-10	U100	-	-	23-24	U155	155.0	155.0
7-11	U100	-	-	23-25	U155	155.0	155.0
13-12	U197	-	68.99	23-26	U350	344.8	245.0
13-13	U197	-	68.99				
13-14	U197	-	68.99				

The number of scenarios is next increased to 20. Table 4 shows the results. The effect of duplicating the number of scenarios on CT under OCM is lower than the effect of duplicating the number of units while the situation is reversed under PSM mechanism. The reason is the high number of complementary slackness conditions caused by duplicating the number of scenarios and units and the higher number of Lagrange optimality conditions caused by duplicating the number of scenarios. In a separate study, a 2-hour time limit was set for all cases with 20 scenarios. The higher increase in PS with respect to the optimal values presented in Table 4 was about 2.3 % corresponding to the largest test system. For a 4-hour time limit the same results as Table 4 were obtained.

7.3. IEEE-RTS Without Uncertainty

The main target of this study is to extract the results of applying the proposed framework on a larger test system with network constraints. In this study, the uncertainties are neglected. The results will be compared with those of the next sub-section in which the uncertainties are considered to analyse the effects of system uncertainties.

Here, the uncertainties are neglected. The units' data and other useful information can be found in [25]. The capacity of lines 11–13, 15–16, and 15–24 have been reduced to 175, 60, and 175 MVA, respectively. The load profile corresponds to 12 noon of a Saturday of a winter week with total load of 2084.8 MW [25]. Table 5 shows the UC results. Under PSM mechanism more units have been committed to reduce the LMPs. This increases the system start-up cost. However, the total payment is still lower.

The payments to suppliers for OCM and PSM and total system costs for OCM and PSM are 66749, 66492, 60587 and 67111 ($), respectively. Each mechanism leads to the minimum value of the regarding objective function. The CT is about 19 and less than 1 second under PSM and OCM mechanisms, respectively. No network congestion is observed and the LMP is the same for all network buses (DC-OPF is used). The LMP is 13.50 and 14.50 $/MWh under PSM and OCM mechanisms, respectively.

7.4. IEEE-RTS Considering Uncertainties

The main target of this case study is to analyse the effects of including the reserve problem and uncertainties on the payments, costs, ELMPs and RLMPs, by comparing the results with those obtained in subsection 7.3. The relationship between energy and reserve LMPs are also analyzed in order to check if the proposed LMP definition accounts for the lost opportunity cost for selling the convertible products.

The UC data and Reserve offers can be found in [11], and [25], respectively. All the components are assumed to be fully reliable except unit 23-U350, unit 21-U400 and line 14-16. Only the outage of these components leads to DC-LF constraints activation [11]. Standard deviation of 2% is considered for the bus loads.

The UC results are given in Table 6. The CT is about 318 and 12 seconds under PSM and OCM mechanisms, respectively. More units have been committed to reduce the total payment under PSM mechanism. This leads to relatively lower production and therefore, lower incremental cost for each unit. The LMPs and then total payment are lower in this way. However, this increases the system start-up cost. Table 7 shows the system costs and payments. The PSM mechanism has increased the total cost by 10% while the increase in total payment under OCM mechanism with respect to its optimum value is more than 20%. The ELPMs are depicted in Fig. 7 for the most probable scenario. The ELMPs and their variations are increased comparing to those obtained in the previous section. Inclusion of the reserve problem has increased the energy cost and payment with respect to those of Table 5. From Table 7 and the previous observations in subsection 7.1, it seems that PSM mechanism increases the reserve payment and in fact, commits more reserve compared to OCM mechanism. The reason was discussed in 7.1.

Table 6 Location, power output (MW) and committed reserve (MW) of system units, subsection 7.4

Bus-No.	OCM P_g	PSM P_g	OCM R_g^{up}	PSM R_g^{up}	OCM R_g^{dn}	PSM R_g^{dn}
1-1	-	-	-	-	-	-
1-2	-	-	-	-	-	-
1-3	76.00	76.00	-	-	-	-
1-4	76.00	76.00	-	-	-	-
2-5	-	-	-	-	-	-
2-6	-	-	-	-	-	-
2-7	76.00	76.00	-	-	-	-
2-8	76.00	76.00	-	-	-	-
7-9	68.51	25.00	-	-	-	-
7-10	68.51	25.00	1.63	3.15	-	-
7-12	63.82	25.00	31.35	75.00	-	-
13-13	-	69.00	-	-	-	-
13-13	-	69.00	-	-	-	-
13-14	-	69.00	-	-	-	-
15-15	-	2.00	-	-	-	-
15-16	-	2.00	-	-	-	-
15-17	-	2.00	-	-	-	-
15-18	-	2.00	-	9.03	-	-
15-19	-	2.00	-	-	-	-
15-20	155.00	155.00	-	-	-	-
16-21	153.17	155.00	-	-	7.75	-
18-22	388.25	400.00	11.75	-	52.14	136.10
21-23	223.49	226.18	25.91	16.42	0.00	-
23-24	155.00	155.00	-	-	42.46	-
23-25	155.00	155.00	-	-	-	-
23-26	350.00	242.60	-	-	210	102.60

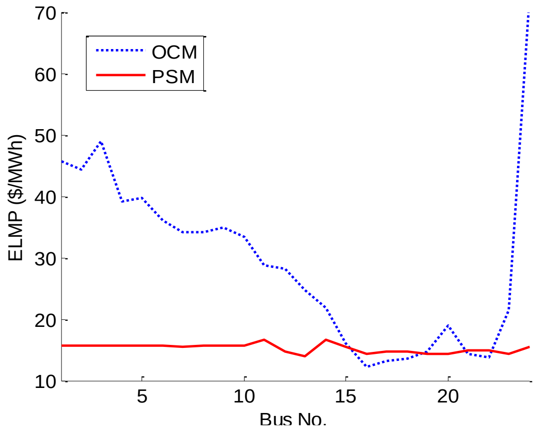

Fig. 7. *ELMP for scenario* 1, *OCM and PSM mechanisms.*

Table 7 Costs and Payments ($) Under OCM and PSM.

	System cost	Energy cost	Reserve cost	Total payment	Energy payment	Reserve payment
OCM	66426	66166	260	79890	79585	305
PSM	73109	72553	555	65963	65214	749

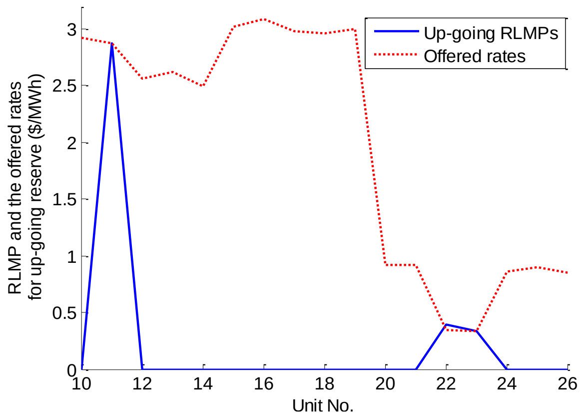

Fig. 8. *RLMP and offered rates for up-going reserves, OCM.*

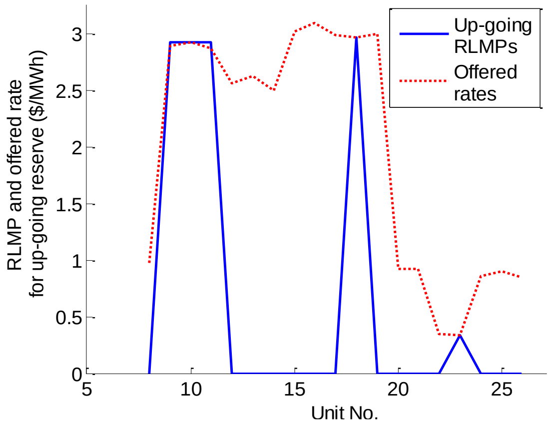

Fig. 9. *RLMP and offered rates for up-going reserves, PSM.*

The committed reserves are also given in Table 6. Fig. 8 shows the offered rates and up-going RLMPs under OCM mechanism. The RLMPs of 2 accepted offers (out of 3) are equal to their offered rates. For unit 22, the RLMP is higher than the offered rate to compensate for the lost opportunity of selling more energy. Based on this observation, it seems that the upper limit constraint has been activated for unit 22 (confirmed by Table 6). Fig. 9 shows the up-going reserve RLMP and the offered rates under PSM mechanism. The same phenomenon is observed for units 9 and 11. The offered rates of up-going reserve for units 9, 10, 11 are 2.89, 2.92, and 2.87 $/MWh, respectively, while the energy offers are the same. Since all the units are contributed in up-going reserve provision, the RLMP is the same for these units and is equal to the higher accepted reserve offer at this bus (2.92 $/MWh). The upper limit constraints are activated for units 9 and 11, but not for unit 10 (Table 6 confirms this). The increase in RLMPs when the generation constraints are activated is not quite visible in the figures. However, as the load grows, this phenomenon can well be seen. Fig. 10 shows the RLMPs and offered rates for the down-going reserves. The same phenomenon is observed for unit 26 under both mechanisms due to activation of the minimum generation constraint.

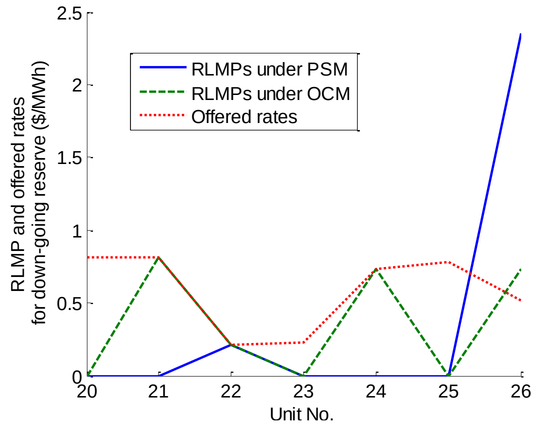

Fig. 10. *RLMP and down-going reserve offered rates, PSM.*

8. Conclusions

A tri-level optimization framework was proposed to incorporate the uncertainties associated with load fluctuations and components' availability in PSM UC. The proposed tri-level framework makes it possible to implement the proper definition of LMPs in the PSM stochastic market clearing. The proposed reserve marginal pricing best suits the proposed formulation of security constrained stochastic payment minimization, considers the free market requirements and accounts for lost opportunity of selling the convertible products (energy and reserve). It was shown that the system uncertainties should be considered in market clearing to commit the proper reserves to save the system in different scenarios (feasibility) and to find the UC results which lead to the minimum stochastic payment (optimality).

It is possible to model the uncertainties associated with renewable energy resources using the proposed approach. However, inclusion of these resources in electricity markets requires further research to deal with different aspects of pricing of their energy and settlement of the regarding trades. Inclusion of the demand side contributions in flexibility reserve provision is also proposed for the future studies, since by contributing in flexibility provision, consumers can further reduce their electricity cost in an electricity market under payment minimization mechanism.

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10. References

[1] Borenstein, S., Bushnell, J.B., Wolak, F.A.: 'Measuring market inefficiencies in california's restructured wholesale electricity market', The American Economic Review, 2002, 92, (5), pp. 1376–1405

[2] Fernandez.Blanco, R., Arroyo, J.M., Alguacil, N., Guan, X.: 'Incorporating priceresponsive demand in energy scheduling based on consumer payment minimization', IEEE Trans. Smart Grid, 2016, 7, (2), pp. 817–826

[3] Jamalzadeh, R., Ardehali, M.M., Hosseinian, S.H., Choopani, M.: 'Payment cost minimisation auction for deregulated electricity market using mixed-integer linear programming approach', IET Gener. Transm. Distrib., 2013, 7, (8), pp. 907–918

[4] Chen, Y., Luh, P.B., Yan, J.H., Stern, G.A., Blankson, W.E. 'Payment cost minimization for simultaneous auctions in energy and spinning reserve markets', In: 2006 IEEE Power Engineering Society General Meeting

[5] Zhao, F., Luh, P.B., Yan, J.H., Stern, G.A., Chang, S.: 'Payment cost minimization auction for deregulated electricity markets with transmission capacity constraints', IEEE Trans. Power Syst., 2008, 23, (2), pp. 532–544

[6] Li, C., Hedman, K.W., Zhang, M.: 'Market pricing with single-generator-failure security constraints', IET Gener. Transm. Distrib., 2017, 11 , (7), pp. 1777–1785

[7] Fernandez.Blanco, R., Arroyo, J.M., Alguacil, N.: 'A unified bilevel programming framework for price-based market clearing under marginal pricing', IEEE Trans. Power Syst., 2012, 27, (1), pp. 517–525

[8] Han, X., Luh, P.B., Bragin, M.A., Yan, J.H., Yu, N., Stern, G.A. 'Solving payment cost co-optimization problems', In: 2012 IEEE Power and Energy Society General Meeting

[9] Carvalho, C., Cuervo, P.: 'Payment minimization in a combined energy market through a bilevel linear model', 17th Power Syst. Computation Conf., Stockholm, Sweden, 2011

[10] Nouri, A., Hosseini, S.H.: 'Comparison of lmps' sensitivity under payment cost minimization and offer cost minimization mechanisms', IEEE Syst. J., 2015, 9, (4), pp. 1507–1518

[11] Nouri, A., Hosseini, S.H.: 'Payment minimisation auction with security constraints', IET Gener. Transm. Distrib., 2017, 11, (6), pp. 1370–1380

[12] Wu, L., Shahidehpour, M., Li, Z.: 'Comparison of scenario-based and interval optimization approaches to stochastic scuc', IEEE Trans. Power Syst., 2012, 27, (2), pp. 913–921

[13] Xu, Y., Hu, Q., Li, F.: 'Probabilistic model of payment cost minimization considering wind power and its uncertainty', IEEE Trans. Sustainable Energy, 2013, 4, (3), pp. 716–724

[14] Hamian, M., Darvishan, A., Hosseinzadeh, M., Lariche, M.J., Ghadimi, N., Nouri, A.: 'A framework to expedite joint energy-reserve payment cost minimization using a custom-designed method based on mixed integer genetic algorithm', Eng. Appl. Artif Intel., 2018, 72, pp. 203 – 212

[15] Bizhaniaram, B., Nouri, A. 'Stochastic payment cost minimization in energy markets with high penetration of renewables'. 2018 IEEE International Conference on Environment and Electrical Engineering and IEEE Industrial & Commercial Power Systems Europe.

[16] Nouri, A., Hosseini, S.H.: 'Value-based reserve market settlement', IET Gener. Transm. Distrib., 2016, 10, (9), pp. 2216–2224

[17] FernÃ¸ andez-Blanco, R., Dvorkin, Y., Ortega-Vazquez, M.A.: 'Probabilistic security- constrained unit commitment with generation and transmission contingencies', IEEE Trans. Power Syst., 2017, 32, (1), pp. 228–239

[18] Zhao, F., Luh, P.B., Yan, J.H., Stern, G.A., Chang, S.: 'Payment cost minimization auction for deregulated electricity markets with transmission capacity constraints', IEEE Trans. Power Syst., 2008, 23, (2), pp. 532–544

[19] Morales, J.M., Conejo, A.J., Perez.Ruiz, J.: 'Economic valuation of reserves in power systems with high penetration of wind power', IEEE Trans. Power Syst., 2009, 24, (2), pp. 900–910

[20] Han, X., Luh, P.B., Yan, J.H., Stern, G.A. 'Payment cost minimization with transmission capacity constraints and losses using the objective switching method'. In: IEEE PES General Meeting, 2010

[21] Raj, V.M., Chanana, S. 'Analysis of unit commitment problem through lagrange relaxation and priority listing method'. In: 2014 6th IEEE Power India International Conference (PIICON), 2014. pp. 1–6

[22] Xu, D., Chen, Z., Yang, L.: 'Scenario tree generation approaches using k-means and lp moment matching methods', J. Comput. Appl. Math, 2012, 236, (17), pp. 4561 – 4579

[23] Uzuncan, E., Hesamzadeh, M.R., Balkwill, A.: 'Optimal transmission access for generators in wind-integrated power systems: stochastic and robust programming approaches', IET Gener. Transm. Distrib., 2017, 11 , (6), pp. 1345–1359

[24] Samadi Gazijahani, F., Salehi, J.: 'Stochastic multi-objective framework for optimal dynamic planning of interconnected microgrids', IET Renew. Power Gen., 2017, 11, (14), pp. 1749–1759

[25] Reliability Test System Task Force.: 'The ieee reliability test system-1996', IEEE Trans. Power Syst., 1999, 14, (3), pp. 1010–1020

11. Appendix

For the optimization problem (A1), the Lagrange optimality, primal feasibility, dual feasibility and complementary slackness conditions are given in (A2), (A3), (A4) and (A5), respectively. *EC* and *IC* are the sets of equality and inequality constraints.

$$\begin{aligned} &\underset{x}{Min}\ \ f(x) \\ &h_i(x)=0 \quad (\mu_i) \quad \forall i \in EC \\ &g_j(x) \le 0 \quad (\gamma_j) \quad \forall j \in IC \end{aligned} \tag{A1}$$

$$\nabla f(x) + \sum_{i \in EC} \mu_i \nabla h_i(x) + \sum_{i \in IC} \gamma_i \nabla g_j(x) = 0 \tag{A2}$$

$$\begin{aligned} &h_i(x)=0 \quad (\mu_i) \quad \forall i \in EC \\ &g_j(x) \le 0 \quad (\gamma_j) \quad \forall j \in IC \end{aligned} \tag{A3}$$

$$\gamma_j \ge 0 \quad \forall j \in IC \tag{A4}$$

$$\gamma_j g_j(x) = 0 \qquad \forall j \in IC \tag{A5}$$