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Detection of Defects in a Plate Using Rayleigh-Lamb Waves

Nondestructive testing of materials is essential wherever they are used in safety critical applications. Conventional ultrasonic techniques, such as C-scanning, generally use waves propagating normal to the plane surface. While accurate, such methods are slow for structures with large plane areas. The use of longitudinal waves offers the possibility of quicker testing of such structures.

This paper considers the reflection of low frequency Rayleigh-Lamb waves by a two dimensional centrally embedded vertical defect in aluminium plate. A numerical method is used, based on results from dimensionless analytical work. It is shown that such longitudinal ultrasonic waves can successfully detect structural defects.

1. Introduction

Conventional ultrasonic non-destructive evaluation (NDE) of plates typically relies on the through thickness propagation of waves. This is generally quite time consuming for condition monitoring of large engineering structures. An alternative way to test structures, particularly those being thin such as plates and pipes, is to propagate ultrasonic waves along the major plane of the component. This can result in significant time savings since the transducer is only passed along the edge of the structure, rather than passing over its entire area. In plates, such longitudinal waves are referred to as Rayleigh-Lamb waves.

2. Mathematical Analysis

The basic mathematics of Rayleigh-Lamb waves have been extensively studied [1, 2, 3]. For the present analysis, the equations were expressed in dimensionless form, after [4]. In this way, the fundamental characteristic of any mode of vibration becomes its dimensionless frequency:

$$\Omega = \frac{\omega h}{c_t} \quad (1)$$

where ω is frequency ($\text{rad} \cdot \text{s}^{-1}$), h is the plate half thickness (m), and c_t is the speed of transverse waves ($\text{m} \cdot \text{s}^{-1}$) and is a function of material properties. At low frequencies ($\Omega < 1.5$), there are only two possible modes of vibration: one symmetric (S_o), and one anti-symmetric (A_o). In addition, at low frequencies, the S_o wave is virtually non-dispersive, while the A_o wave is highly dispersive. This leads us to concentrate on the S_o waves, since non-dispersive behaviour gives more coherent wave trains.

3. Computational Modelling

To study the reflection of Rayleigh Lamb waves by the defect, a finite element model was created [5]. The modelled aluminium plate has a length of $100 \times$ plate half thickness. The plate length was chosen to be significantly greater than the wavelength of the waves to be studied. The interest in the present work is in symmetric modes, and for this reason only half the thickness of the plate was modelled, with a symmetry boundary condition imposed on the bottom surface of the model to represent the plate centre line.

The plate was meshed with quadrilateral plane strain isoparametric elements, with vertex nodes and 3×3 integration points. A mesh size in the direction of propagation of $0.25 \times$ plate half thickness was chosen, giving 400 elements along the plate, and at least 20 elements per wavelength (based on expected wavelengths of frequencies to be tested). 20 elements were used to mesh the half thickness of the plate. 10 elements were used to mesh the crack face, and 10 to mesh the intact part of the plate above the crack. This gave adequate mesh density on the face of the defect. The ratio of crack length to plate thickness was varied by changing the positions of the nodes. 20 'infinite' elements were used along the non-cracked end of the plate, to provide a quiet boundary to the finite element model.

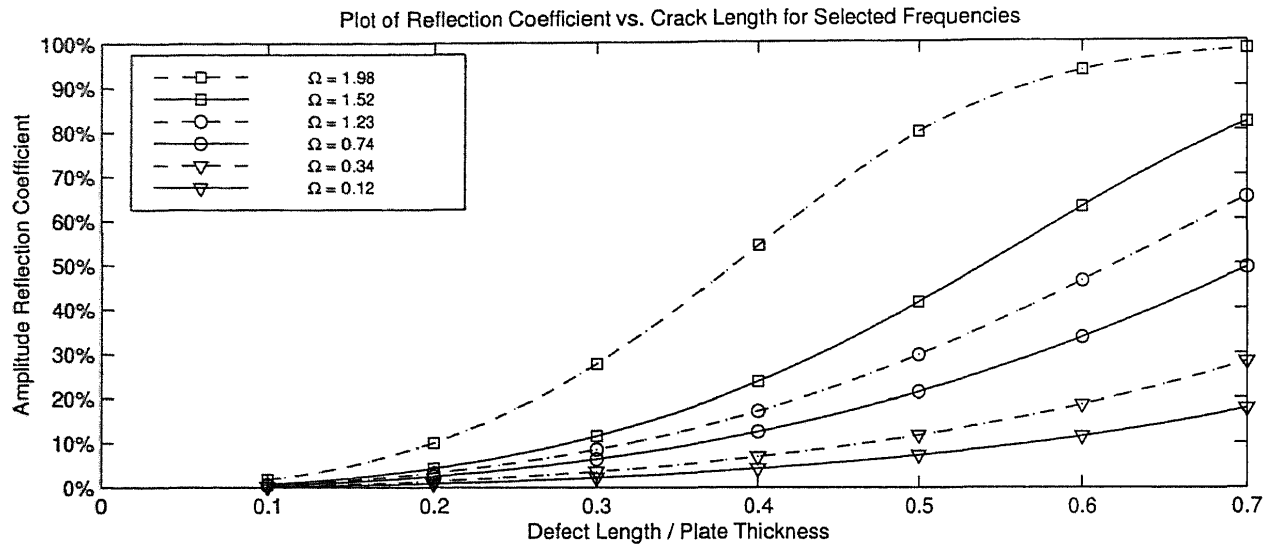
Since the face of the crack represents a free surface, it is necessary for it to be stress free. However, the incident wave will excite stresses on the surface of the crack. In order to keep the surface free of stress, the reflected wave must vibrate the plate such that it cancels out these stresses on the free surface with equal but opposite stresses.

In this model, the stress equation for the S_0 wave was programmed into a user subroutine which then applied the required stress to the free surface of the crack. The intact portion of the plate, above the crack, was fixed in the x -direction by means of a plane of symmetry.

The stresses were applied for frequencies up to $\Omega = 2$, and for a selection of crack lengths between 0.1 and 0.7 of the plate thickness. The amplitudes of the displacements excited in the plate were then compared to those of the displacements which are excited by the corresponding incident wave. The ratio of these two amplitudes was used to define a reflection coefficient.

4. Results and Discussion

Selected results obtained from the model are shown on the chart below. A total of 84 analyses were run, comprising of 7 different crack lengths and 12 frequencies. Six representative frequencies are plotted on the figure below. It is seen that broader cracks produce a larger reflection coefficient. Also, higher frequencies produce larger reflection coefficients. This is expected from analytic work, since at higher Ω , more of the longitudinal motion of the wave becomes concentrated at the centre line of the plate. The defects studied here are symmetric about, and located at, the plate centre line; it is natural that they would reflect higher frequency waves more strongly.



5. Conclusions

The usefulness of low frequency symmetric Rayleigh-Lamb waves for the detection of embedded symmetric vertical flaws in aluminium plate has been examined and characterised by means of a dimensionless index. It has been shown that physically measurable reflection coefficients ($> 10\%$) can be obtained for defects as small as 0.3 of plate thickness while still using the non-dispersive low frequency longitudinal S_0 wave.

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