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# Tools for Analytical and Numerical Analysis of Electrostatic Vibration Energy Harvesters: Application to a Continuous Mode Conditioning Circuit

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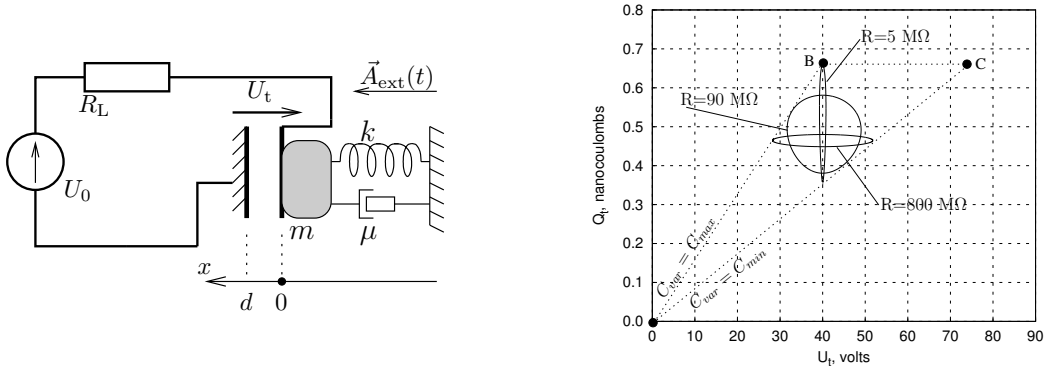
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**Abstract.** This paper reports the application of different analytical tools to a basic continuous conditioning (CC) circuit for electrostatic vibration energy harvesters (e-VEHs). We address the fundamental issues of this conditioning circuit and give design advice that enhances the performance of e-VEHs employing this circuit. This circuit is widely used for harvesters with or without an electret layer. Despite its wide use, its fundamental problems have been weakly addressed even for simple configurations of e-VEHs since it is impossible to solve the corresponding equations in closed form. As a consequence, appropriate semi-analytical methods that provide an insight into the physics of the system are required.

## 1. Introduction

Vibration energy harvesting is a promising technique for the generation of electricity from ambient vibrations for the supply of autonomous microsystems. Capacitive (electrostatic) vibration energy harvesters appear to be the best candidates for miniaturization. They are not as bulky as electromagnetic harvesters, and they do not suffer from fatigue or depolarization like piezoelectric devices. A large number of studies have investigated electrostatic vibration energy harvesters, and many have focused on the design of a transducer device including a mechanical resonator and a capacitive transducer. However, the main challenge of e-VEH implementation is related to the design of conditioning and interface electronics. Currently, there are several families of circuits for the conditioning of capacitive energy converters. We can group them into three categories. (i) Constant-charge or constant-voltage operation mode [1], (ii) Circuits employing a charge pump [2, 3] and (iii) “Basic” conditioning circuit obtained by connecting a capacitive transducer to a source of voltage in series with a load resistor [4].

The first two families of circuits require complex control electronics in order to correctly sequence different phases of energy conversion. They are considered to be an encouraging prospect for the implementation of smart adaptive e-VEH systems, able to react to a variation in the input vibrations. On the other hand, the operation of a basic conditioning circuit does not require any smart electronics. Its cost is low, however its efficiency is also lower in comparison with the other families. The basic conditioning circuit shown in fig. 1a has been extensively used in two contexts. Firstly, when the main research or design work is focused on a transducer or a resonator, this circuit is used for the testing of the device [4]. Secondly, this circuit is often used in e-VEH employing an electret layer for transducer biasing [5]. In this case, the voltage source is incorporated into the model of a variable capacitor with a charged electret layer.



**Figure 1.** Schematic view of the continuous conditioning circuit (left) and charge-voltage diagrams for different values of load resistance at  $U_0=40 \text{ V}$  (right).  $X=12 \mu\text{m}$ ,  $f = 150 \text{ Hz}$ .

In spite of its extreme simplicity and wide use, there are no established or formalised methods that will allow a designer to choose optimal parameters of the circuit (the optimal resistance and bias voltage) or analytically predict the circuit performance (the converted power). This paper presents a study of the basic conditioning circuit and its operation in conjunction with a mechanical resonator. Three methods are employed for the analysis: charge-voltage diagram method, mechanical impedance method and multiple-scale method.

## 2. Mathematical Model

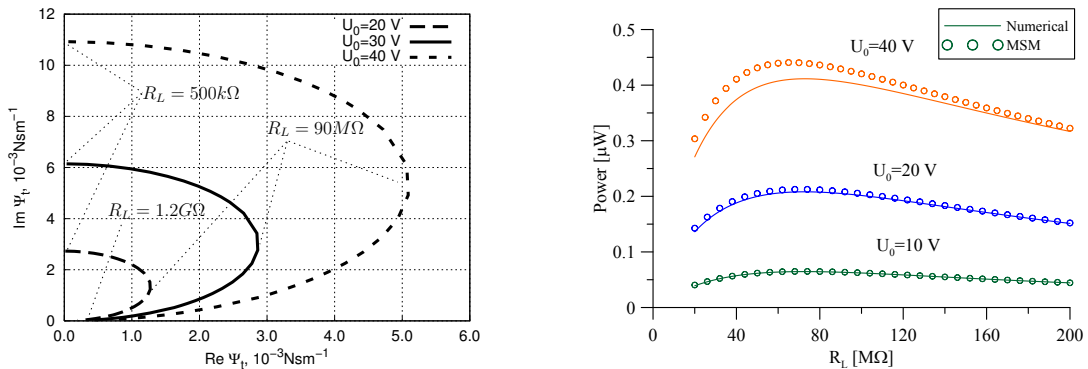
The system is described by two equations. One models the displacement of a resonator, and the other models the circuit itself:

$$\begin{cases} \dot{v} = A_{\text{ext}}(t) - k/m \cdot x - \mu/m \cdot v + 1/m \cdot F_t(x) \\ \dot{x} = v \\ \dot{q} = (U_0 - q/C_{\text{var}}(x))/R_L \end{cases} \quad \text{where } F_t(x) = \frac{1}{2} \frac{q^2}{C_{\text{var}}^2} \frac{\partial C_{\text{var}}}{\partial x} \quad (1)$$

In these equations,  $q$  is the instantaneous charge of the transducer capacitance,  $x$  and  $v$  are the instantaneous displacement and velocity of the mobile mass respectively,  $A_{\text{ext}}$  is the instantaneous acceleration of the external vibrations,  $m$ ,  $k$  and  $\mu$  are the mass, stiffness and damping factor of the resonator. The function  $C_{\text{var}}(x)$  gives the capacitance of the transducer as a function of the displacement  $x$ :  $C_{\text{var}} = \epsilon_0 S/(d - x)$ , where  $d$  is the initial gap between the plates,  $S$  is the area of overlap between the plates. The parameters of the device are as follows:  $m = 66 \cdot 10^{-6} \text{ kg}$ ,  $\mu = 7.9 \cdot 10^{-3} \text{ Nsm}^{-1}$ ,  $k = 68 \text{ Nm}^{-1}$ ,  $d = 43.5 \cdot 10^{-6} \text{ m}$ , the natural frequency is  $f_0 = \omega_0/(2\pi) = \sqrt{k/m}/(2\pi) = 161.5 \text{ Hz}$ , the rest capacitance is  $C_0 = 1.08 \cdot 10^{-11} \text{ F}$ .

## 3. Analysis in the Electrical domain: Method of Charge-Voltage Diagrams

Even in the absence of mechanical coupling, i.e., when the mobile mass vibration amplitude does not depend on the electrical processes, the analysis of the “basic” conditioning circuit is very difficult. The latter is described by a linear ordinary differential equation with time-variable coefficients, which cannot be integrated in closed form even for a simple  $C_{\text{var}}(x)$  function. Charge-voltage diagrams are used to give a graphical representation of the state evolution of a variable capacitor in the  $(U, Q)$  plane. For the study of an energy harvester, charge-voltage diagrams are plotted in the steady-state mode when the mobile electrode of a transducer oscillates with a given sinusoidal law:  $x(t) = X \cos(\omega t)$  and  $C_{\text{var}}(t) = \epsilon_0 S/(d - X \cos(\omega t))$ . In this way, the transducer capacity varies between the two known values,  $C_{\text{max}}$  and  $C_{\text{min}}$ . The charge-voltage diagram for different values of the load resistance  $R_L$  is shown in fig. 1. A typical QV diagram is a closed cycle. The area of the cycle is equal to the energy converted by the transducer during the cycle – this energy is dissipated on the  $R_L$ . It can be seen that for small  $R_L$ , the transducer voltage is equal to  $U_0$ , and the QV cycle is degenerated to a vertical line. For very large  $R_L$ ,



**Figure 2.** Impedance of gap-closing transducer plotted for  $R_L$  in the range ( $0.5 \text{ M}\Omega$ ,  $1.2 \text{ G}\Omega$ ), for different  $U_0$  (left). The converted power as a function of  $R_L$  for three different  $U_0$  values (right). The solid lines show the converted power obtained from numerical simulations of the system (1) while the circles show the the converted power calculated using the multiple scales method.

the charge on the transducer is virtually constant, and the cycle is degenerated to a horizontal line. There is an optimal  $R_L$  for which the converted power is maximal. It can be seen, that the energy converted in the optimal cycle is much lower than the energy converted with a constant charge conditioning circuit (the triangle OBC) [1], having the same bias voltage and the same amplitude of the transducer capacitance variation.

#### 4. Mechanical Impedance Method

The mechanical impedance method for analysis of a vibration energy harvester was proposed in [6]. The method assumes that a high-Q resonator responds only to the first harmonic of the external actuation. In the mechanical impedance method, the mobile mass is supposed to move following the sinusoidal law (see above). All forces applied to the mass are represented by mechanical impedances  $\Psi$  equal to the ratio between minus the first (fundamental) harmonic of the force expressed by its complex amplitude ( $\dot{F}^\omega$ ) and the complex velocity of the mass motion ( $\dot{V}$ ). Therefore, the impedance and the power of mechanical energy conversion, related to a force with impedance  $\Psi$ , is given by:

$$\Psi = -\dot{F}^\omega / \dot{V}, \quad P = |\dot{V}| \text{Re } \Psi. \quad (2)$$

The power  $P$  is positive if the energy leaves the mechanical domain. The imaginary part of the impedance provides important information about the resonance frequency shift due to the nonlinearity of the resonator/transducer.

More generally, the mechanical impedance of the transducer  $\Psi_t$  is calculated by analysing the system composed of a transducer whose mobile electrode moves following the sinusoidal law. As explained in [6],  $\Psi_t$  depends on the motion amplitude  $X$ , since the considered system is usually nonlinear. Knowing the  $\Psi_t(X)$  function, it is possible to find the amplitude of mobile mass vibration in the coupled electromechanical mode. It is important to understand that although the transducer impedance relates two mechanical quantities, it is obtained by analyzing the electrical side of the system.

Figure 2 presents an example of loci of the transducer impedance for three values of bias voltages. The load resistance  $R_L$  varies from  $0.5 \text{ M}\Omega$  to  $1.2 \text{ G}\Omega$ . As expected, the real part of the impedance is positive (the transducer removes energy from the mechanical domain), and it increases with the bias voltage. There is an optimal value of the resistance  $R_L$  at which  $\text{Re } \Psi_t$  is maximal. This value corresponds to the maximal power which can be converted by the conditioning circuit at the given mobile mass vibration amplitude. It can be seen that the imaginary part of the impedance varies with the load resistance as well, from a maximal value

towards a value close to zero. The imaginary part of the impedance is positive: it highlights the shift of the resonance frequency toward the low values. This highlights a very important phenomena which has never been studied: the value of the load resistance has an impact on the resonance frequency shift of the vibration energy harvester.

### 5. Perturbation Technique: Multiple Scales Methods

The method of multiple scales (MSM) is a type of perturbation technique that is often applied for the analysis of weakly nonlinear oscillators [7]. Its approach is based on presenting oscillations in a quasi-harmonic form and find adjustments to oscillation characteristics, such as amplitude and phase, that result from the nonlinearity. In order to apply MSM, we introduce different time scales  $T_k = \varepsilon^k t$  that reflect the shift in the frequency and present the solution in the form of a series  $x(t) = \sum_{k=0}^N \varepsilon^k x_k(t)$  that accounts for the change in the amplitude of oscillations due to nonlinearity. This approach allows one to analyse the system simultaneously in the mechanical and electrical domains. As a result, we are able to find both the displacement of the resonator and the instantaneous charge on the transducer as functions of time. In addition, MSM allows one to take into account various additional nonlinearities such as, for instance, mechanical nonlinearity.

The method is applied to dimensionless equations and we rewrite system (1) using normalised parameters and variables [8]:

$$y'' + 2\beta y' + y = \alpha \cos \Omega \tau + \nu Q^2(y), \quad Q' = 1 - \rho(1 - y)Q \quad (3)$$

where we introduced the following normalised variables: time  $\tau = \omega_0 t$ , dissipation  $\beta = \mu/(2m\omega_0)$ , normalised external vibration frequency  $\Omega = \omega_{\text{ext}}/\omega_0$ , normalised external acceleration amplitude is  $\alpha = A_{\text{ext}}/(m\omega_0^2 d)$ , the coefficients  $\rho = (C_0\omega_0 R_L)^{-1}$  and  $\nu = U_0^2/(2mC_0\omega_0^4 d^2 R_L^2)$ , dimensionless displacement  $y = x/d$  and dimensionless charge  $Q = q/(U_0\omega_0 R_L)$ . The solution of equations (3) yields the displacement of the resonator and the charge:

$$y(\tau) = Y_0 + Y \cos(\Omega \tau + \psi_0) \quad (4)$$

where the amplitude  $Y$  of oscillations is found from the equation

$$\frac{\alpha^2}{4} = \left( \beta f_0(Y) + \frac{\nu h_1(Y)}{2} \right)^2 + \left( Y \sigma + \frac{\nu g_1(Y)}{2} \right)^2 \quad (5)$$

and  $f_0$ ,  $g_1$  and  $h_1$  are the coefficients of the Fourier series for the transducer force  $F_t = \nu Q^2$ . The charge  $Q$  is found from the equation:

$$Q(\tau) = \left[ \frac{a_0}{\rho} + \sum_{n=1}^{\infty} a_n \cos n\tau - \sum_{n=1}^{\infty} b_n \sin n\tau \right] \times \left[ \frac{a_0}{\rho} + \sum_{n=1}^{\infty} \frac{a_n (\rho \cos n\tau + n \sin n\tau)}{n^2 + \rho^2} + \sum_{n=1}^{\infty} \frac{b_n (\rho \sin n\tau - n \cos n\tau)}{n^2 + \rho^2} \right] \quad (6)$$

Here,  $a_0$ ,  $a_i$  and  $b_i$  are the coefficients of the Fourier series of the function  $\exp[-\beta a \sin t]$ , and they are the functions of the amplitude of oscillations  $Y$ . The first few coefficient are given below:

$$a_0 = I_0(\rho Y), \quad a_1 = 0, \quad b_1 = -2I_1(\rho Y) \quad a_2 = -2I_2(\rho Y), \quad b_2 = 0 \quad (7)$$

where  $I_n(z)$  is the modified Bessel function of the first kind. The coefficients  $a_i$  and  $b_i$  decay as the index  $i$  increases. The series (6) must be truncated when the corresponding coefficients are small enough. The dynamics of the system strongly depend on the product  $\rho \cdot Y$  where  $Y$  is the amplitude of oscillations in the resonator. For instance, when  $\rho \cdot Y \approx 1$ , it is enough to

account only few first terms in (6), and in this case  $Q(\tau)$  represents almost harmonic oscillations. In addition, we note that system (1) can display stiffness, and it is difficult to solve this system numerically. The stiffness of the system is also determined by this product (it is stiff if  $\rho \cdot Y \gg 1$ ).

The power dissipated at the resistor is given by the formula

$$P = \frac{1}{T} \cdot \frac{U_0^2}{R_L} \int_{T_0}^{T_0+T} (\dot{Q})^2 d\tau \quad (8)$$

where  $T = 2\pi/\Omega$  is the dimensionless period of oscillations and  $T_0$  is the time required to reach a steady state. The comparison of the converted power obtained by numerical solution of system (1) and the converted power yielding from the application of MSM is given in fig. 2. A very good correspondence is seen, in particular, for the plot obtained at  $U_0 = 10$  and  $U_0 = 20$  V. In all cases, there is an optimal value of the load resistance  $R_L$  at which the converted energy is maximal. This value is  $R_L = 73$  M $\Omega$  and it does not depend on the applied constant voltage  $U_0$ . It is interesting to mention, that at the peak of the converted power in fig. 2,  $\rho \cdot Y \approx 1$  for all values of  $A_{\text{ext}}$  and  $U_0$ .

## 6. Conclusions

We have presented three different techniques for the analysis of a basic continuous conditioning circuit for electrostatic vibration energy harvesters. This circuit is widely used for harvesters with or without an electret layer. Despite its wide use, its fundamental problems have been weakly addressed even for simple configurations of e-VEHs as it is impossible to solve the corresponding equations in closed form.

The method of charge-voltage diagrams is used for the graphic representation of the state evolution of a variable capacitor in the  $(U, Q)$  plane. Though it is very simple, it provides the qualitative understanding of the operation of the system. However, there is no closed form expression providing the value of the energy converted in one cycle or the optimal value of the resistance. The mechanical impedance method starts from an electrical analysis of the system, and provides the information about how the conditioned transducer impacts on the mechanical dynamics of the resonator (power conversion, resonance frequency shift). The multiple scale method, although advanced, allows one to solve the governing equation simultaneously in the mechanical and electrical domain and obtain the displacement of the resonator and the charge on the transducer as function of time. Based on the obtained solutions, we can calculate the converted power and find the optimal value of the load resistance  $R_L$  at which the converted power is maximised.

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