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Sensitivity to damage of the forced frequencies of a simply supported beam subjected to a moving quarter-car

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Abstract. The vibration of bridges under operational conditions can be measured via accelerometers to extract their dynamic features. These features can then be monitored in time, although only a reduced number of cause-effect scenarios can be verified on the field. Therefore, theoretical models of the bridge are often employed for covering a wider range of scenarios. For instance, a variety of damage conditions can be introduced in a calibrated bridge model to obtain the associated frequencies, which can be subsequently compared to frequencies measured onsite for assessing the bridge condition. It must be noted that these frequencies may be influenced by factors other than damage, i.e., environmental effects due to temperature changes and operational effects due to traffic. During the forced vibration of a bridge caused by a moving vehicle, the frequencies governing the bridge response depend on the mass and stiffness ratios of the vehicle to the bridge. Therefore, records in free vibration are usually preferred or alternatively, the influence of operational loads is removed from forced vibration records before assessing whether damage has occurred or not. This paper shows that forced vibration stores relevant information about damage beyond the frequency changes derived from free vibration. Eigenvalue analysis is employed to investigate how forced frequencies change with the positions of a crossing vehicle and damage. The vehicle is modelled using a quarter-car and the bridge as a simply supported finite element beam, where damage is introduced via localized stiffness losses.

Keywords: Vehicle-Bridge Interaction, Damage Detection, Forced Vibration.

1 Introduction

Structural Health Monitoring (SHM) is becoming increasingly important in several research disciplines, such as civil engineering, aeronautics, and mechanical engineering [1]. The condition of bridges, aircrafts, and machines can be monitored by changes to the dynamic properties of a structural system, i.e., natural frequencies, mode shapes and damping ratios. However, it must be noted that these changes may or may not be related to damage, i.e., frequencies will also change due to environmental and operational effects. SHM methods based on mode shapes include mode shape curvature [2], modal strain energy [3], changes in dynamic flexibility [4] and others, which are typically able to locate and quantify damage at the cost of installing a large number of sensors on the structure. On the other hand, methods based on monitoring frequencies are not so demanding in the number of sensors or energy consumption, but their capabilities are commonly limited to detecting the presence of damage, without locating or quantifying it. In the case of a bridge, the installation of only one sensor (i.e., an accelerometer) can be sufficient to capture the frequency information being sought. Frequencies can be derived from an acceleration signal by various signal processing techniques, including the Fast Fourier Transform (FFT), the Wavelet Transform (WT) and the Hilbert-Huang Transform (HHT) amongst others [5].

Compared to other structures, bridges are characterized by an operational traffic load. The latter consists of a vehicle or fleet of vehicles applying spatial- and time-varying forces to the bridge that they are crossing. Bridge and vehicle are two structural systems that, when treated in isolation from each other, have their own masses, stiffness and associated frequencies. However, when the vehicle is on the bridge, the two isolated systems become one unique system, encompassing all masses and stiffness. It is true that there is usually a disproportion between the magnitude of mass and stiffness of vehicles and bridge, which leads to very close frequencies of the bridge in free and forced vibration. Nonetheless, when heavy traffic is involved or the values of bridge and vehicle frequencies are close, the differences in dynamic behaviour between forced and free vibration of the bridge may not be negligible. The importance of these differences has been numerically investigated by [6-10], and experimentally confirmed by [11-12].

This paper aims to carry out a preliminary assessment of how significant the impact of a vehicle is on the forced frequencies of a bridge compared to damage. Forced frequencies are calculated varying both the position of a localized stiffness loss and the position of a vehicle on the bridge. For this purpose, eigenvalue analysis is applied to the coupled system composed of a simply supported finite element beam and a quarter-car model. Section 2 describes the numerical model employed to obtain the bridge frequencies for healthy and damaged scenarios with (i.e., forced vibration) and without (i.e., free vibration) the presence of a vehicle. Section 3 shows how eigenvalues vary with the position of the quarter-car on the beam model and with the position of damage. Section 4 compares the relative changes in frequency between the different damaged scenarios and the healthy condition for free vibration and for forced vibration considering each position of the vehicle on the bridge. Finally, conclusions and recommendations for further research are presented in Section 5.

2 Bridge and vehicle models

A 20 m long simply supported beam discretized into 200 finite beam elements (0.1 m long) serves as a simplified planar model of a short/medium span road bridge. The bridge is assumed to have a uniform solid rectangular cross-section with a depth of 1 m, a width of 15 m, a density of 2500 kg/m^3 and Young's modulus of 35 GPa. Table 1 provides other properties associated with this model. The values of natural frequencies are rounded to two decimal places.

Table 1. Bridge properties.

Parameter	Symbol	Value	Unit
2nd moment of area	I	1.25	m^4
Mass per unit length	m	37500	kg/m
1st natural frequency	$f_{b,1}$	4.24	Hz
2nd natural frequency	$f_{b,2}$	16.97	Hz
3rd natural frequency	$f_{b,3}$	38.17	Hz

The vehicle is modelled as a quarter-car with two degrees of freedom related to the vertical movement of the two masses composing the model. The largest and smallest masses represent the body and axle masses respectively, and they are connected through a spring-dashpot system, which simulates the suspension stiffness and damping. The connection between vehicle and bridge models is made through a second spring, which simulates the tire stiffness. Two different configurations of the vehicle are tested, namely Vehicles A and B, whose properties are listed in Table 2.

Table 2. Vehicle properties.

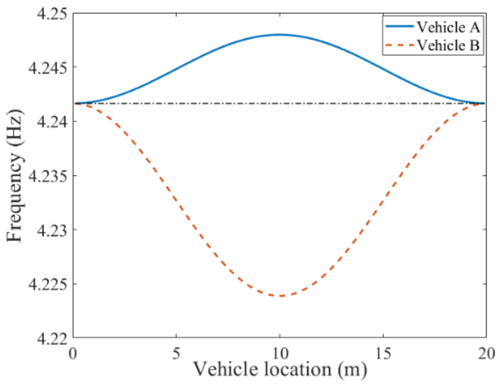
Parameter	Symbol	Value		Unit
		Vehicle A	Vehicle B	
Body mass	m_{body}	4250	7000	kg
Axle mass	m_{axle}	750	3000	kg
Suspension stiffness	k_s	1.8×10^6	6×10^5	N/m
Suspension damping	c_s	5000	2000	Ns/m
Tire stiffness	k_t	10^6	4×10^6	N/m
1st frequency	$f_{v,1}$	1.89	1.37	Hz
2nd frequency	$f_{v,2}$	10.09	6.26	Hz

Vehicles A and B use typical values for steel and air suspensions respectively. Gross vehicle weights of 5 tonnes for Vehicle A and 10 tonnes for Vehicle B are adopted. It must be noted that Vehicle B has the 2nd frequency closer to the 1st frequency of the bridge than the frequencies of Vehicle A. This fact is of significance given that previous research [7-8] has concluded that when analyzing the effect of a vehicle on the forced frequencies of a bridge, not only the vehicle to bridge mass ratio is important, but also the proximity of the vehicle and bridge frequencies.

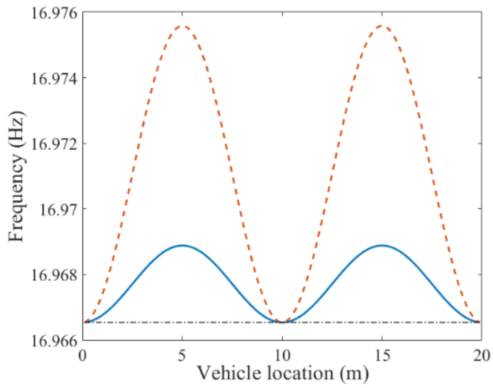
3 Eigenvalue analysis of coupled bridge and vehicle models

As previously mentioned, forced vibration of the bridge caused by the presence of vehicles occurs at frequencies that may differ from those in free vibration significantly. Figure 1 shows the variation of the first three bridge frequencies for different locations of Vehicles A and B on the bridge. The quarter-car is placed on one of the nodes of the finite element beam model, the stiffness and mass matrices of the coupled system are computed, and the eigenvalue problem is solved. This calculation is repeated to obtain the eigenvalues for every position of the quarter-car on the beam. These eigenvalues provide the varying forced frequencies of the combined system as the vehicle moves along the bridge.

Bridge frequencies in free vibration given in Table 1 are represented by horizontal dashed-dotted lines in Figure 1. For the models under investigation, the forced frequency is equal or larger than the frequency in free vibration associated with a specific mode of vibration, except for the 1st mode and Vehicle B. For the three modes of vibration, maximum or minimum forced frequencies occur for locations of the vehicle exactly matching points of maximum amplitude of the mode shape. A maximum change in forced frequency of –0.42% with respect to free vibration corresponding to the 1st mode can be noticed in Figure 1(a) for Vehicle B placed at mid-span. Smaller maximum relative changes in the forced frequency of 0.05% and 0.01% are found for the 2nd and 3rd modes respectively and Vehicle B. Maximum relative changes in 1st, 2nd and 3rd forced frequencies due to Vehicle A are 0.15%, 0.01% and 0.002% respectively. Hence, Vehicle A has a significantly lesser impact on forced frequencies than Vehicle B.



(a)



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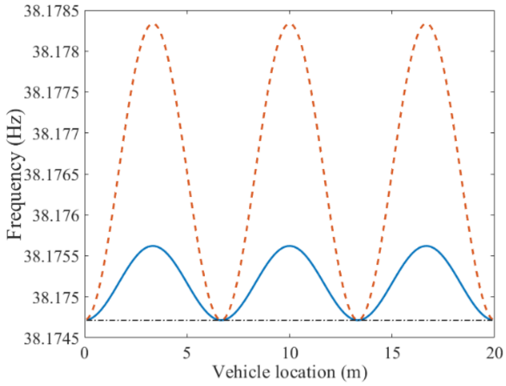
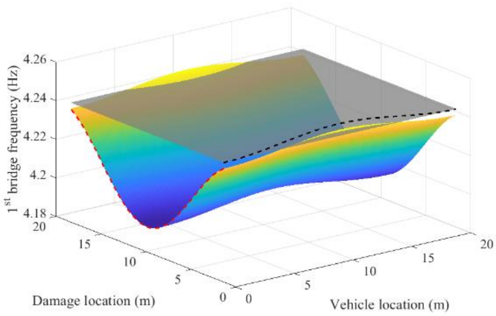


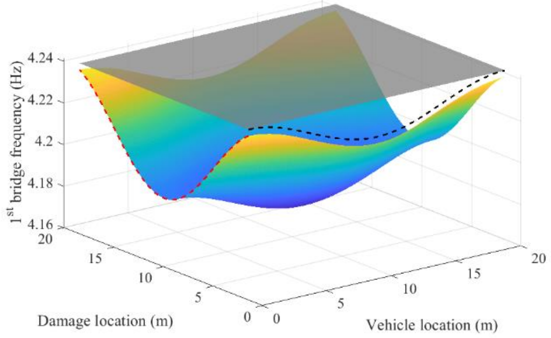
(c)

Fig. 1. Bridge forced frequencies due to the vehicle: (a) 1st frequency, (b) 2nd frequency and (c) 3rd frequency.

To place the significance of these variations in frequency due to vehicle position in context, they are compared to variations caused by damage, whose location and characterization is the main goal for SHM systems. Damage is modelled here following Sinha's approach [13]: a crack of a given depth causes a loss of stiffness that varies linearly from the location of the crack to zero over a length equal to 1.5 times the depth of the bridge. Therefore, for the 1 m deep bridge under investigation, the loss of stiffness extends over 3 m. A crack depth of 5 cm is considered here (i.e., crack to beam depth ratio of 0.05), which for a rectangular cross-section, it is equivalent to a 14.26% stiffness loss at the location of the crack. Figures 2, 3 and 4 show the combined effect of damage and vehicle for all possible locations on the bridge, on the 1st, 2nd and 3rd bridge frequencies, respectively. The original bridge frequencies in free vibration given in Table 1 are represented by horizontal planes. The damage location closest to the supports starts at 1.5 m from the support to allow for the 1.5 m affected by damage to be fully located inside the bridge. In these figures, the dashed lines represent the isolated effects of vehicle and damage, whereas the continuous surfaces represent the combined effect of both on the first three frequencies of vibration of the bridge.

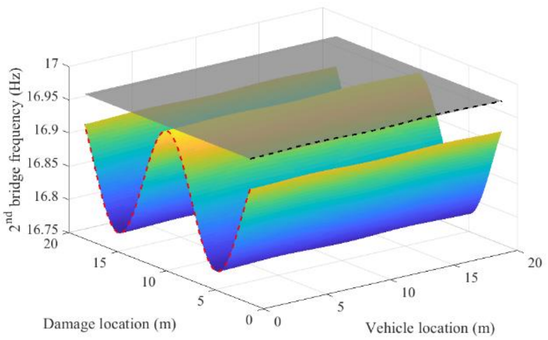


(a)

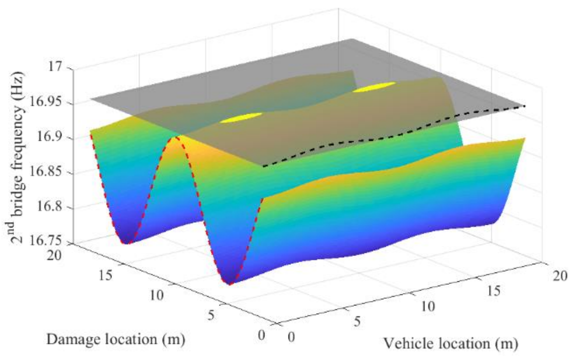


(b)

Fig. 2. 1st frequency of vibration of the bridge for different damage and vehicle locations: (a) Vehicle A and (b) Vehicle B.

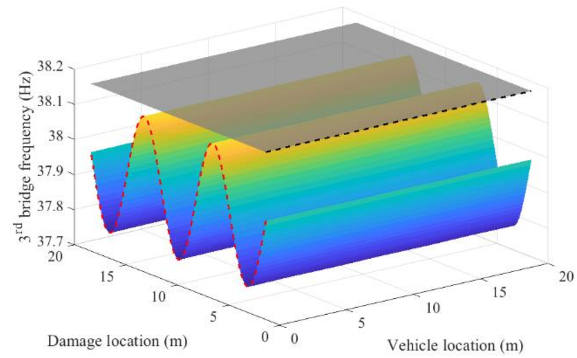


(a)

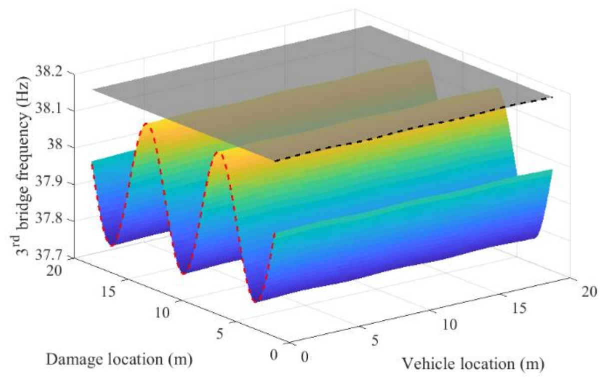


(b)

Fig. 3. 2nd frequency of vibration of the bridge for different damage and vehicle locations: (a) Vehicle A and (b) Vehicle B.



(a)



(b)

Fig. 4. 3rd frequency of vibration of the bridge for different damage and vehicle locations: (a) Vehicle A and (b) Vehicle B.

It is noted that for Vehicle A and the 1st frequency of the bridge (Figure 2(a)), the frequency variation due to the vehicle itself is higher than that of damage when the damage is very close to the supports. Overall, the vehicle tends to stiffen the bridge (i.e., bridge frequencies increase), while damage makes the bridge more flexible (i.e., bridge frequencies decrease). Hence, in Figure 2(a), Vehicle A causes an increase in the 1st bridge frequency that outweighs the decrease in 1st frequency caused by damage in some scenarios. Therefore, damage may be concealed by the vehicle effect. The same is true for Vehicle B and the 2nd frequency (Figure 3(b)), when damage is located near mid-span (minimal effect of damage on 2nd frequency) and the vehicle is positioned near ¼ or ¾ of the span (maximal effect of the vehicle position on forced frequency). Changes in frequency due to damage that are hindered by those due to the position of the vehicle can be visualized by the surface rising above the horizontal plane. In contrast, Figure 2(b) shows how Vehicle B has the same effect on the 1st bridge frequency as damage, i.e. it causes the frequency to decrease.

In the case of the 3rd frequency (Figure 4), the surface never surpasses the horizontal plane. This means that the impact of the vehicle on the frequency of the bridge is never larger than that of damage. It could be argued that this is partially due to a lack of vehicle frequencies in the proximity of the 3rd bridge frequency. Finally, given that the sensitivity to damage by any mode of vibration differs depending on the location of the damage, the use of three frequencies would minimize the risk of blind spots associated with a specific location and mode. For example, the 3rd frequency appears to exhibit relatively larger variations for damages closer to the supports than the 1st and 2nd frequencies.

4 Comparison of relative changes in frequency

The previous section has established that for a given set of vehicle parameters and damage severity, the variation of bridge frequencies depends largely on the positions of damage and vehicle. This section analyses the relative changes in frequency for the first three modes of vibration of the bridge. Relative changes in free and forced frequency are defined by Equations (1) and (2) respectively.

$$\Delta f_{free,i} = 100 \cdot \frac{f_{d,i} - f_{h,i}}{f_{h,i}} \tag{1}$$

$$\Delta f_{forced,i} = 100 \cdot \frac{f_{d+v,i} - f_{h+v,i}}{f_{h+v,i}} \tag{2}$$

where i = 1,2,3 indicates the mode under consideration; $f_{d,i}$ represents the frequency in free vibration of the damaged bridge, $f_{h,i}$ is the frequency in free vibration of the healthy bridge, and $f_{d+v,i}$ and $f_{h+v,i}$ are the forced frequencies of the damaged and healthy bridges respectively when the vehicle is on the bridge. While $\Delta f_{free,i}$ yields a unique value for a given damage location, $\Delta f_{forced,i}$ takes a different value for each position of the vehicle, even for a fixed damage location.

Figure 5 shows the relative change in the 1st bridge frequency for every location of the vehicle. Each sub-figure corresponds to a different damage location, separated by intervals of 1 m in the first half of the bridge (from damage at 2 m to damage at 10 m). The variation due to damage in the second half of the bridge is symmetric with respect to mid-span and it is omitted to avoid duplication. Dashed horizontal lines represent the variation in free vibration ($\Delta f_{free,1}$) and curved lines represent the variation in forced vibration ($\Delta f_{forced,1}$). The variation due to Vehicle A is plotted using a dashed-dotted line, and the variation due to Vehicle B is shown with a solid line. It can be seen how the slope of the $\Delta f_{forced,1}$ curve is zero or close to zero at a point located close to the location of the damage. This is more clearly visible for vehicle B than for vehicle A and for sub-figure 5(c) than sub-figure 5(a), i.e. for locations of damage close to mid-span, where the amplitude of the 1st mode is maximum. When locating damage, it must be noted that there are a few points of the $\Delta f_{forced,1}$ curve meeting the condition of zero slope and that this uncertainty can be reduced by considering more than one mode simultaneously.

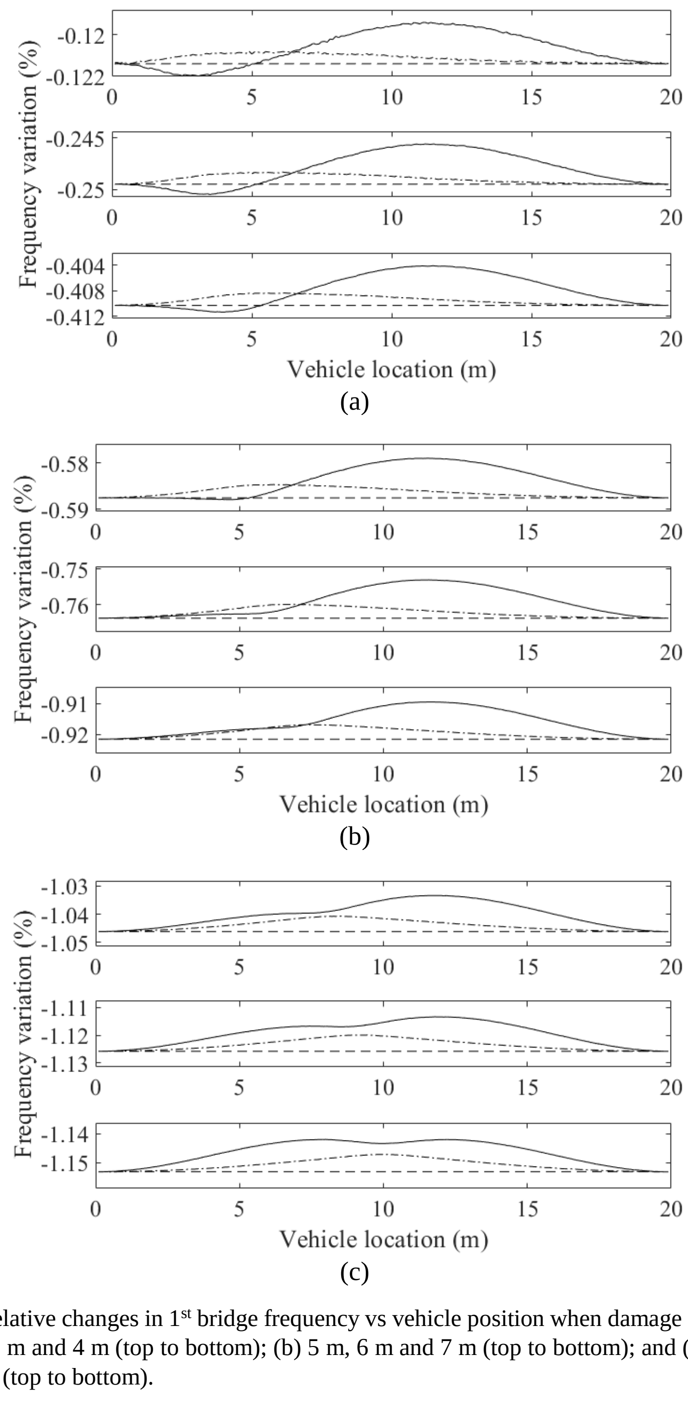
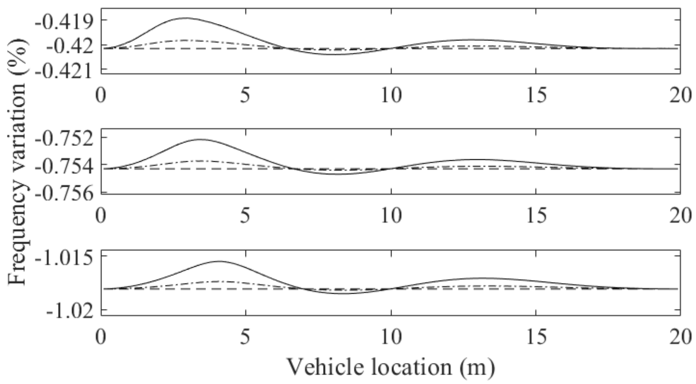


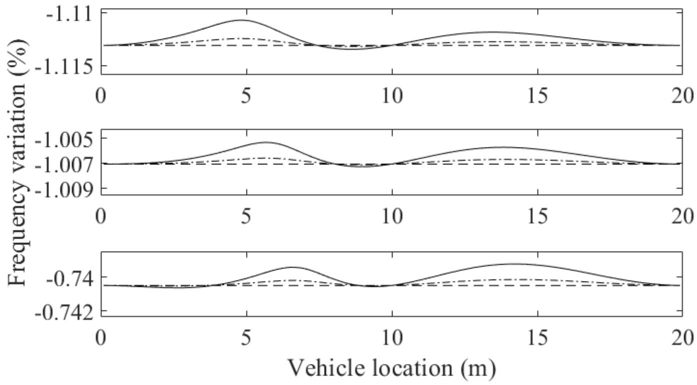
Fig. 5. Relative changes in 1st bridge frequency vs vehicle position when damage is located at (a) 2 m, 3 m and 4 m (top to bottom); (b) 5 m, 6 m and 7 m (top to bottom); and (c) 8 m, 9 m and 10 m (top to bottom).

In terms of the magnitude of the change, for damage located at 2 m from the left support (Figure 5(a)), the relative changes in frequency, both for free and forced vibration, are in the range of –0.12% to –0.122%. Compared to those values, damage located at mid-span (bottom Figure 5(c)) leads to relative changes in frequency that are around ten times higher, with values in the range of –1.14% to –1.15%. It is also noticeable how the variation in forced frequency by Vehicle A is generally closer to free vibration than the variation by Vehicle B. The curve corresponding to Vehicle A is flatter than the curve of Vehicle B, hence rendering the location of damage more challenging for Vehicle A than for Vehicle B.

Figure 6 shows relative changes in frequency for the 2nd mode and Figure 7 does the same for the 3rd mode. In agreement with Figure 5, the variation of relative change in forced frequency due to Vehicle A remains closer to $\Delta f_{free,i}$, in comparison to Vehicle B, which appears to be more ideally suited to locating damage. In the case of $\Delta f_{forced,2}$ (Figure 6), the relative change in frequency at mid-span is always equal or almost equal to the value in free vibration, while noticeable peaks are found for positions of the vehicle near ¼ or ¾ of the span. The magnitude of the highest relative changes in the 2nd frequency is in the order of –1.11% (top Figure 6(b)), similar to those found for the 1st frequency (bottom Figure 5(c)), but the slope of the curve becomes zero at a different damage location, i.e., at around 5 m from the left support (top Figure 6(b)). It becomes clear that when using the 2nd frequency, the damage is more accurately located near the points of maximum amplitude of the 2nd mode of vibration. For the 3rd mode, maximum changes of the $\Delta f_{forced,3}$ curves correspond to damages located at 3 m (middle Figure 7(a)) and 10 m (bottom Figure 7(c)) from the left support, with similar magnitudes (–1.04% to –-1.06%) as for the first two frequencies. In summary, frequencies are most sensitive to damage for locations near the modal points of maximum amplitude, and the location of damage can be characterized by a zero slope of the $\Delta f_{forced,i}$ curve.



(a)



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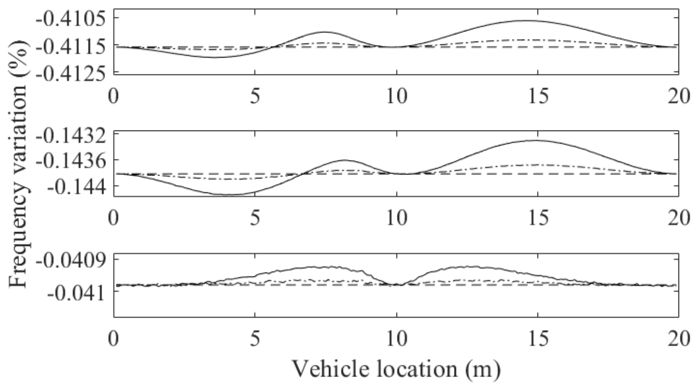
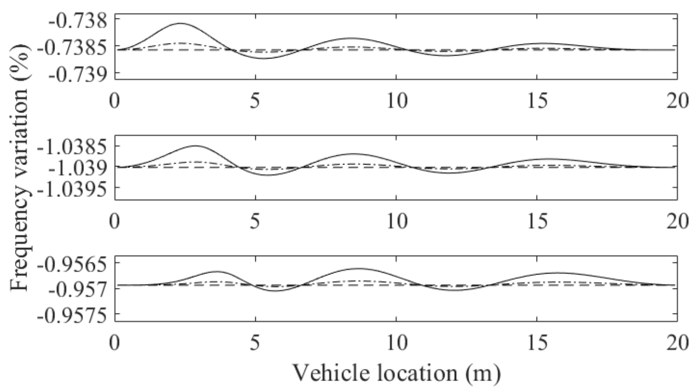
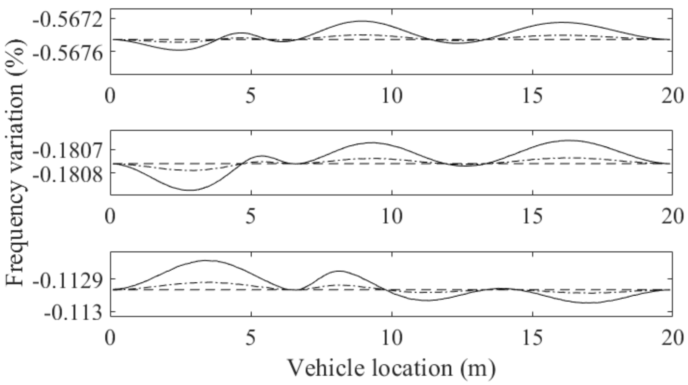


(c)

Fig. 6. Relative changes in 2nd bridge frequency vs vehicle position when damage is located at (a) 2 m, 3 m and 4 m (top to bottom); (b) 5 m, 6 m and 7 m (top to bottom); and (c) 8 m, 9 m and 10 m (top to bottom).



(a)



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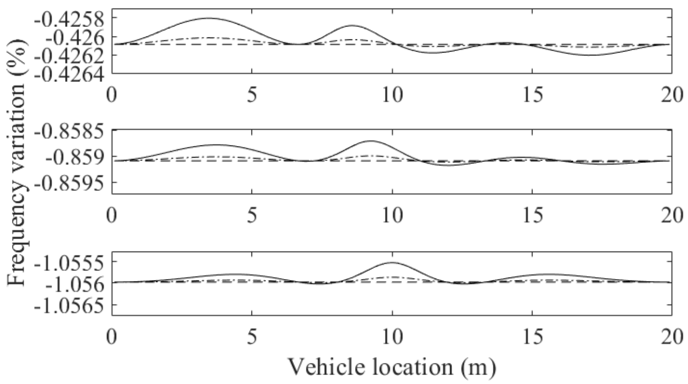

(c)

Fig. 7. Relative changes in 3rd bridge frequency vs vehicle position when damage is located at (a) 2 m, 3 m and 4 m (top to bottom); (b) 5 m, 6 m and 7 m (top to bottom); and (c) 8 m, 9 m and 10 m (top to bottom).

5 Conclusions

This paper has built on previous research about forced frequencies of a bridge in healthy condition due to a moving vehicle to assess how they are influenced by damage. While free vibration records provide one single value of frequency for each mode of vibration, frequencies obtained from forced vibration vary with each position of the vehicle on the bridge. A change of frequency in free vibration indicates a difference in mechanical properties or boundary conditions of the bridge between two points in time, but it gives no indication of the possible location of damage. The simplified bridge and vehicle models employed here have served to illustrate how the variation of bridge frequencies with the position of the vehicle can be used to detect and to locate damage. This variation has been more pronounced when using the vehicle with higher mass and frequencies closer to the bridge frequency. A mode of vibration can be more sensitive to damage than others depending on the location of damage. The relative change in frequency has been larger for damage locations closer to the points of maximum amplitude of the mode being considered. The slope of the curve of relative changes in the forced frequency versus vehicle position has become zero when the vehicle was in the proximity of damage. The results have shown potential for development of new damage detection algorithms that aim to exploit the damage features contained in the forced vibration signal, for application to traditional SHM systems when few or null records in free vibration are available, or to modern SHM technologies based on energy-harvesting sensors or drones with a limited flying duration, i.e., drones charging bridge sensors and downloading vibration data from them for a short period of time to optimize energy efficiency. Nevertheless, this research is at an early stage, and the promising results achieved by eigenvalue analysis need to be tested in a real-life situation by capturing small changes of forced frequency with sufficient accuracy in the time-frequency domain.

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