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## Experimental Testing of a Moving Force Identification Bridge Weigh-in-Motion Algorithm

C.W. Rowley, E.J. OBrien, A. Gonzalez, A.Žnidarič

### Abstract

Bridge weigh-in-motion systems are based on the measurement of strain on a bridge and the use of the measurements to estimate the static weights of passing traffic loads. Traditionally, commercial systems employ a static algorithm and use the bridge influence line to infer static axle weights. This paper describes the experimental testing of an algorithm based on moving force identification theory. In this approach the bridge is dynamically modeled using the finite element method and an eigenvalue reduction technique is employed to reduce the dimension of the system. The inverse problem of finding the applied forces from measured responses is then formulated as a least squares problem with Tikhonov regularization. The optimal regularization parameter is solved using the L-curve method. Finally, the static axle loads, impact factors and truck frequencies are obtained from a complete time history of the identified moving forces.

**Keywords:** Bridge, Weigh-in-motion, Force identification, Regularization, Dynamic programming, Traffic loads

### Moving Force Identification Bridge Weigh-in-Motion Algorithm

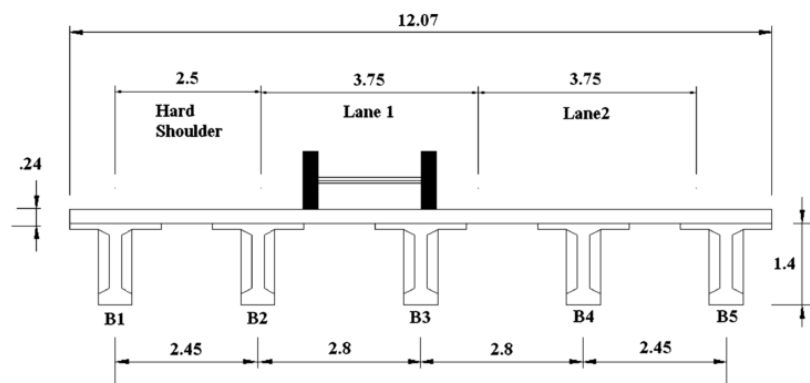
The traditional static bridge weigh-in-motion (B-WIM) algorithm provides static axle weights from minimizing the sum of squares of differences between measured total bridge strain and theoretical static strain [1]. Although the deviations with respect to the static response that vehicle and bridge dynamics introduce in the measured response tend to be averaged out during the minimization process, they remain a significant source of inaccuracy [2]. This paper proposes an alternative B-WIM algorithm that calculates the time history of moving forces as they cross the bridge, based on moving force identification (MFI) theory. The MFI algorithm implemented here was developed by the authors [3–5], and is an extension of the one-dimensional algorithm by Law and Fang [6] to two dimensions. The mathematics behind general inverse theory is available in the literature [7–11]. The MFI algorithm requires a finite element (FE) mathematical model that accurately represents the static and dynamic behavior of the bridge structure. The method of dynamic programming requires that the equilibrium equation of motion be converted to a discrete time integration scheme. In this case the equilibrium equation of motion is reduced to an equation in modal coordinates defined by:

$$\begin{aligned} & [I]_{n_z \times n_z} \{z\}_{n_z \times 1} + 2\zeta [\Omega]_{n_z \times n_z} \{z\}_{n_z \times 1} + [\Omega]_{n_z \times n_z} \{z\}_{n_z \times 1} \\ & = \Phi^T [L(t)]_{n_{\text{dof}} \times n_g} \{g(t)\}_{n_g \times 1} \end{aligned} \quad (1)$$

where  $[\Phi]$  is the modal matrix of normalized eigenvectors and  $n_z$  is the number of modes to be used in the inverse analysis.  $[\Omega]$  is a diagonal matrix containing the natural frequencies and  $\zeta$  is the percentage damping.  $[L(t)]$  is a time varying location matrix relating the  $n_g$  applied vehicle forces of the vector  $g(t)$  to the degrees of freedom,  $n_{dof}$ , of the original FE model. Tikhonov regularization [12] is applied to provide a bound to the error and ‘smoother’ solutions to the ill-conditioning nature of the MFI problem [13, 14]. The final part of the solution lies in the calculation of the optimal regularization parameter; the L-curve has been chosen to obtain the optimum [15–18]. The basic idea is to plot the discrete smoothing norm of the regularized solution versus the residual norm on a log–log scale, which will always have a ‘corner’ where the optimal regularization parameter is located.

### Bridge and Truck

The testing was carried out on the Vranksko Bridge in Slovenia as part of the research carried out within the European 6<sup>th</sup> Framework Project ARCHES (Assessment and Rehabilitation of Central European Highway Structures). The bridge is 24.8 m between the center line of the bearings with a total span of 26 m. The bridge has no skew and is of beam and slab construction. It has five concrete longitudinal beams (Fig. 1) and two diaphragm beams in the transverse direction over the supports. The bridge was instrumented with strain transducers on the beams at mid-span to record the input for the MFI algorithm, and at quarter points for axle detection. The data acquisition was carried out using SiWIM (B-WIM system developed by ZAG and the Cestel Company [19]). The scanning frequency was 512 Hz per channel. The first natural frequencies identified from measurements were 5.11, 6.50, 9.89, 16.12 and 20.55 Hz. Bridge damping was found to be approximately 1% for the first natural frequency. A pre-weighted rigid three-axle truck with steel suspension was used during testing. The truck had static axle weights of 75.04, 102.51 and 101.04 kN. The first and second axle spacings were 3.56 and 1.38 m respectively.



**Fig. 1** Cross section of Vranksko Bridge

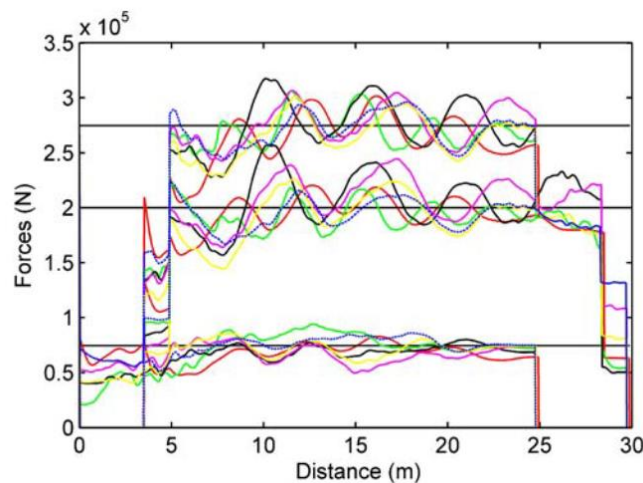
### FE Model of the Bridge

The bridge deck is modeled using a  $C_1$  conforming orthotropic rectangular plate element [20]. Both the longitudinal and transverse beam members are modeled using two-noded beam elements with three degrees of freedom per node, one translation and two rotations [21, 22]. The bridge is

discretized into 800 plate elements and 236 beam elements with a total of 861 nodes and 3,444 degrees of freedom. Poisson's ratio of 0.15 and density of  $2,400 \text{ kg/m}^3$  were assumed. It was found that Young's moduli,  $E_x = 41 \times 10^9 \text{ N/m}^2$  and  $E_y = 30 \times 10^9 \text{ N/m}^2$  for the deck and  $E_b = 35 \times 10^9 \text{ N/m}^2$  for the longitudinal and diaphragm beams, gave good agreement with the frequencies identified from measurements.

## Results

Figure 2 shows the identified forces for some of the nine single vehicle events with the test vehicle traveling in lane 1. For each event, the predicted variation of the forces exerted by the front axle, the rear tandem (axle 2 + axle 3), and the total vehicle force (axle 1 + axle 2 + axle 3) is represented. It is clear that the identified forces generally oscillate about their corresponding static loads represented by horizontal lines at 75.04 (axle 1), 203.55 kN (axle 2 + axle 3) and 278.59 kN (GVW). However, it should be noted that for the first axle of the test vehicle, the identified forces at the start of the bridge are always less than the static axle load. It is only over the period when all three axles are on the bridge that the identified forces for the first axle oscillate about the static axle value. It is difficult to determine the accuracy of the MFI algorithm in the field unless the truck is instrumented. However, some checks are possible. The axle weights and the gross vehicle weights can be obtained by summing the identified forces in the middle 60% [23] of the time history for each axle and averaging the result. These can be compared to the known static values for the test vehicle. The main truck frequencies can also be derived from the time history of the identified forces and the predicted frequencies compared over the range of test runs.



**Fig. 2** Identified axle forces versus distance

Table 1 shows the percentage errors for the predicted static weights for each of the runs. In general the percentage error in the static axle weights and the GVW is reasonable, with a maximum error in GVW of 3.8% and a maximum error for axles 2 and 3 of 6.9%. However, there is one event for the vehicle traveling at 11.34 m/s where the error for the first axle is 17%, whilst the error in the gross vehicle is only 0.3%. This equates to an under estimation of the front axle and an over-estimation of the rear axles, this could be due to an error in the calculated vehicle velocity. In each of the test runs, the predicted maximum impact factors (ratio of the maximum applied dynamic force to the

static force) were 1.35, 1.28 and 1.27 for the front axle, rear tandem and total vehicle vertical forces, respectively. The impact factors for the front axle range between 1.10 and 1.35, and for the rear tandem range between 1.08 and 1.28. The average frequencies associated to the identified force histories for the front axle and rear tandem are 3.2 and 3.12 Hz respectively. These are thought to be reasonable frequencies for the body bounce of a rigid three-axle truck with an old steel suspension [24]. The differences between runs are very small which gives confidence in the accuracy of the results. The axle hop frequencies have not been identified, perhaps because they are at a frequency too high to be detected by the algorithm.

**Table 1** Percentage error in the static axle weights, impact factors and frequencies for the identified forces

| Axle 1                            |               |                | Axles 2+3                         |               |                | GVW                               |               | Velocity (m/s) |
|-----------------------------------|---------------|----------------|-----------------------------------|---------------|----------------|-----------------------------------|---------------|----------------|
| Percentage error in static weight | Impact factor | Frequency (Hz) | Percentage error in static weight | Impact factor | Frequency (Hz) | Percentage error in static weight | Impact factor |                |
| 7.9                               | 1.13          | 2.9            | -1.5                              | 1.12          | 3.1            | 1.0                               | 1.09          | 11.97          |
| -2.0                              | 1.30          | 3.0            | 5.0                               | 1.08          | 3.3            | 3.0                               | 1.07          | 11.63          |
| -17.4                             | 1.34          | 3.1            | 6.9                               | 1.08          | 3.0            | 0.3                               | 1.12          | 11.31          |
| -3.9                              | 1.26          | 3.3            | 3.4                               | 1.09          | 3.0            | 1.4                               | 1.12          | 10.55          |
| 10.8                              | 1.03          | 3.0            | -2.7                              | 1.14          | 3.0            | 0.9                               | 1.09          | 13.93          |
| 5.0                               | 1.08          | 3.0            | -0.5                              | 1.28          | 2.7            | 0.9                               | 1.16          | 14.84          |
| 9.5                               | 1.09          | 3.0            | -4.0                              | 1.22          | 3.0            | -0.3                              | 1.27          | 18.45          |
| 3.2                               | 1.15          | 3.7            | 4.0                               | 1.13          | 3.4            | 3.8                               | 1.10          | 19.14          |
| -1.7                              | 1.15          | 3.8            | 2.8                               | 1.12          | 3.7            | 1.5                               | 1.08          | 19.50          |

## Conclusions

MFI has been developed in recent years as a theoretical means of finding the complete time history of applied axle forces during the passage of a truck over a bridge. It is tested here using measured field strain data from a B-WIM system. While the method is computationally demanding, it is shown to be feasible for on-site application and the time history of forces can be used to collect information not only on the static axle weights but also on the frequency and amplitude of the dynamic forces.

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