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Assessing the Effect of Network Order on Epistemic Uncertainty Quantification for Reduced-order Grey-box Energy Models

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Abstract

Grey-box building energy models are becoming extremely popular for modeling building thermal energy performance and subsequently evaluating base case energy consumption, establishing efficiency scenarios, implementing model predictive control and forecasting building thermal behavior. Energy simulation inputs and model parameters in such models introduce uncertainty and hence, highly affect the accuracy and reliability of energy simulation results. Furthermore, increasing the reduced-order model complexity eventually increases the epistemic uncertainty (lack of knowledge) in energy simulation results due to an associated increase in number of model parameters. Existing studies often provide disintegrated analysis of model complexity, accuracy and uncertainty when implementing reduced-order grey-box models. This study proposes a framework to create reduced-order grey-box energy models and henceforth, quantify and analyze the effect of epistemic uncertainties through variation of network order. The devised framework further enables the identification of a balance between network complexity, accuracy and model uncertainty. A strong relationship exists between network order and model parameter uncertainty. Increasing the model complexity has no significant effect on model accuracy (CVRMSE reduces from 3.65% to 2.55%). The epistemic spread of uncertainties increases by a significant amount ($\sim 10\%$).

Key Innovations

- Integrated workflows for reduced-order model development and uncertainty analysis.
- Thermal network models for building energy performance simulation.
- Probabilistic outputs would facilitate informed design decisions.

Practical Implications

Simplified networks identified using the devised approach are crucial in the integration of thermal networks with other energy vectors, for instance, electricity. Energy modelers could integrate uncertainty

with complexity and accuracy when identifying grey-box building energy models.

Introduction

Buildings and buildings' construction sectors combined account for over one-third of global final energy consumption and nearly 40% of total direct and indirect CO₂ emissions (Ürge-Vorsatz et al., 2015). The associated energy demand continues to rise, mainly driven by improved access to energy in developing countries, greater ownership and use of energy-consuming devices, and rapid growth in building floor area. Global energy efficiency improvements are experiencing a decline over the past few years. In 2018, the primary energy intensity (strong indicator of energy use by the global economy) improved only by 1.2%, which represents the slowest rate since 2010. This rate corresponds to a level well below the average 3% improvement in line with IEA's Efficient World Strategy (Abergel and Delmastro, 2020). Despite the technologies and processes becoming more efficient, factors such as transformation of transport modes and more building floor area per person are dampening the technical efficiency gains of energy demand and thereby, slowing the global energy intensity improvements.

Building stakeholders usually rely on building energy performance simulation (BEPS) strategies to retrofit existing energy-intensive buildings. The development of detailed building models (using BEPS tools) often requires detailed geometric and non-geometric data and therefore, is not a cost-effective solution for evaluation of energy-efficient retrofits (Harb et al., 2016). As each individual building differs in terms of structural parameters and nature of operation, developing a detailed model for each building would be impractical. Furthermore, owing to the existing complex and inter-dependant relationships in the system network, detailed building models often turn out to be computationally intensive (Coakley et al., 2014). Therefore, it is crucial to devise modeling approaches that utilise sparse data and represent the actual thermal behaviour of the building under study. Surrogate modeling (grey-box models) is one such approach,

which has been used on a wide scale (Lin et al., 2012). While grey-box building energy models have been widely implemented, the applicability of these models has often been specific to particular applications and stakeholders.

Grey-box building energy models employ numerous parameters, the details of which are often not available to modelers (Tian et al., 2018). These uncertainties are seldomly quantified creating a false sense of validity and engineering rigor (Wang et al., 2012). Although it is quite crucial to model each uncertainty category in a different manner, uncertainty classification is often overlooked while performing uncertainty analysis. In terms of the inherent variability, uncertainty categories typically consist of aleatory and epistemic uncertainties (Helton et al., 2010). Aleatory uncertainty (type A uncertainty or variability) arises due to the inherent or natural variation of the system under consideration whereas epistemic uncertainty (type B uncertainty or uncertainty) arises mainly due to lack of knowledge (Helton et al., 2010). As different uncertainty types employ different techniques for mitigation, differentiating between the type of uncertainty is important. When implementing reduced-order grey-box models, it is quite crucial to identify an optimal balance between model complexity and model epistemic uncertainty. Increasing the model complexity (model parameters) eventually increases the epistemic uncertainty in the formulated model where model order/model complexity refers to the number of capacitances in a grey-box model.

A significant number of studies in literature identified the importance of implementing grey-box models in behaviour prediction and control of a building or a group of buildings. The devised models broadly differ on the basis of identified network order and grey-box model parameters. Bacher and Madsen (2011) performed a series of experiments with grey-box building energy models and concluded that grey-box model performance does not improve significantly for model orders beyond three. Reynders et al. (2014) formulated various grey-box networks ranging from the first to the fifth order. While low-order models (≤ 2) do not effectively represent the building dynamics, higher order models, namely, fourth and fifth order models, effectively represent the underlying dynamics in terms of system complexity. Higher order models, however, require further heat flux measurements in addition to the temperature measurement. Fonti et al. (2017) devised experimental scenarios to evaluate the accuracy of grey-box models for short-term building energy forecasts. The authors concluded that second-order models estimate the thermal behaviour within acceptable limits (ASHRAE Guideline 14) while maintaining the required complexity. Existing studies still fail to provide a generalised structure to the model identification procedure, which differs significantly from one study to the other.

When performing uncertainty analysis, surrogate modeling techniques (data-driven or reduced-order) are gaining immense popularity in order to further reduce the computational costs (Kim, 2016). For instance, Wate et al. (2020) formulated an emulation based uncertainty and sensitivity analysis framework to account for uncertainties in design variables and occupant behavior. Heo et al. (2015) employed normative energy models calibrated using a Bayesian approach to quantify the parametric uncertainties in the energy simulation model. These studies provide useful insights into uncertainty analysis of surrogate models in terms of model complexity and model accuracy. Thébault and Bouchié (2018) proposed a combined implementation of RC network models and uncertainty analysis. However, a comprehensive overview of the effect of model complexity on model uncertainty still remains absent. Increasing the reduced-order model complexity would eventually increase the epistemic uncertainty (lack of knowledge) in energy simulation results due to an associated increase in number of model parameters. Therefore, a systematic framework to create and analyze reduced-order building energy models could provide confidence in model accuracy.

This study lays out a framework to create reduced-order grey-box models. Furthermore, the framework introduces a process to quantify epistemic uncertainties and assess the effect of variation of network order on these uncertainties. This research introduces novelty through the generalized and integrated use of model identification and quantification procedures. The identification process generalizes, simplifies and automates the generation of grey-box models; uncertainties can then be quantified in these models through the quantification procedure. Furthermore, the implemented 2D Monte Carlo approach segregates the quantification and propagation of different uncertainty types, which is crucial as different uncertainty types can trigger different responses and the possible amalgamation of these might lead to erroneous inferences. The devised framework further enables the identification of a balance between network order, complexity and accuracy.

The paper comprises following sections: Section 2 provides detailed description of the methodology for epistemic uncertainty quantification of reduced-order models. Section 3 discusses the administrative building case study to devise grey-box model structures and quantify the associated uncertainties. Section 4 presents the conclusions and future work.

Methodology

The devised methodology facilitates a generalized formulation of reduced-order grey-box models with an integrated implementation of uncertainty analysis of the developed models to account for the effect of epistemic uncertainties. In the context of reduced-order

grey-box models, as majority of the surrogate modeling studies in literature focus either on model complexity or model uncertainty, the proposed technique considers both complexity and uncertainty to identify an optimal balance.

The reduced-order model development and uncertainty analysis follows four crucial processes, namely, data collection, reduced-order model development, reduced-order uncertainty analysis and epistemic uncertainty analysis (Figure 1). The data collection process identifies and collects the required data for grey-box model development. The reduced-order model development process involves data analysis, order identification, parameter estimation and model validation to formulate and identify reduced-order grey-box building energy models. The reduced-order uncertainty analysis process quantifies uncertainties in energy simulations with a special focus on model parameter uncertainties of the developed reduced-order models. The epistemic uncertainty analysis process uses the identified model complexity and model uncertainty to identify whether an increase in the model complexity bears a significant increase in the model epistemic uncertainty.

Data Collection

Data collection refers to a standardized procedure for collecting data pertinent to building stock modeling. The devised approach categorises the variables into two sets, namely, mandatory and optional variables. The mandatory variables comprise weather variables, building site information, building physical parameters and building operation variables. The optional variables include building renovation history and HVAC system information. While mandatory variables decide the initial order (complexity) of the grey-box network, the optional variables enrich the

dynamics of the formulated grey-box structure.

Weather Variables mainly constitute ambient temperature, global solar radiation and wind speed. Weather variables mainly act as inputs to the grey-box model and do not play any role in grey-box model identification. Weather information is easily available for any major location in the world in the form of IWEC (National Renewable Energy Laboratory (NREL), 2020). Building site information mainly includes the location, number of buildings in the neighborhood and building type. Although these variables do not affect the model formulation process, these variables help in the identification of the required complexity in representing individual buildings or a cluster of buildings. For instance, when buildings are connected via a district heating network, similar types of buildings could be represented through a grey-box network with similar complexities. Building physical parameters consist of gross interior floor area, window area, number of floors and number of zones. These physical parameters directly affect the order of the grey-box network. Building operation variables form the most crucial variable set and constitute building heat demand patterns, internal temperature profiles and building space usage. Building heat demand and indoor temperature profiles, measured at the building level, usually reflect the ongoing activities inside any building. These profiles then relate to the complexities required in representing the dynamics. Building space usage variable specifies the individual proportions of spaces used for various functions (for instance, offices, storage and toilets). Renovation history lists all the past retrofits applied to the building and relates to the increased heterogeneity in the building envelope. As any fabric related renovation will alter the building dynamics, this information could identify any required modifications in the

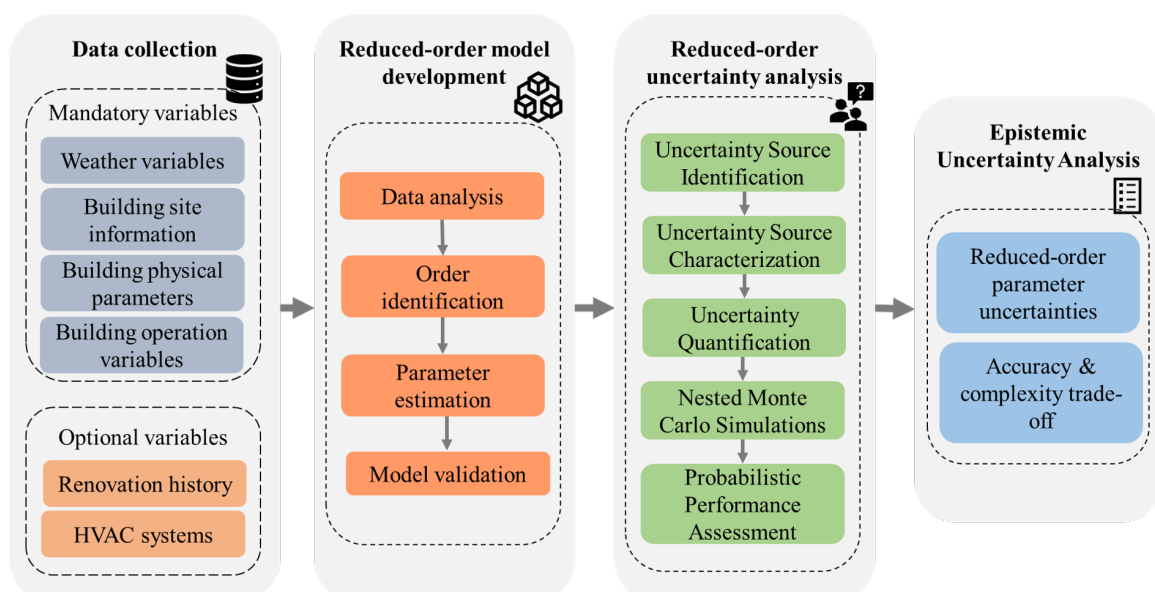


Figure 1: Methodology for reduced-order model development and uncertainty analysis to identify the effect of model complexity on epistemic uncertainties.

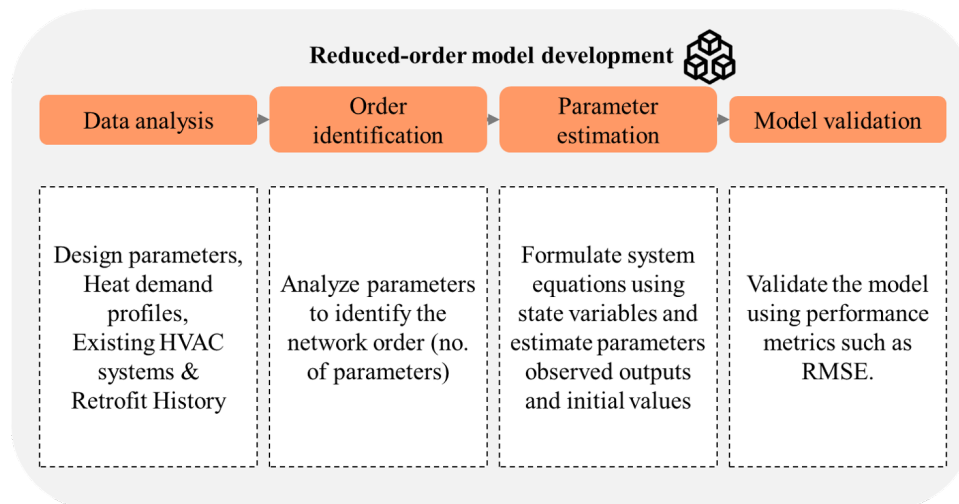


Figure 2: Process workflow to formulate generalized reduced-order grey-box building energy models.

structure of the formulated grey-box model. HVAC system information mainly constitutes of information regarding the HVAC system efficiency, existing ventilation strategies and the presence of embedded heating or cooling systems in the building fabric. These factors alter the building dynamics, thus, increasing the complexity associated with the grey-box model.

Reduced-order Model Development

This process identifies the structure of grey-box models (RC networks). A typical resistance capacitance (RC) network represents the dynamics of the building assuming a steady-state heat transfer across the building envelope. The generalized development of reduced-order models to estimate the building heat demand involves data analysis, order identification, parameter estimation and model validation (Figure 2). These processes offer an automated and a generalized workflow to formulate grey-box networks using easily identifiable building parameters.

The data analysis process involves an initial statistical analysis of building heat demand patterns to describe any existing patterns in the respective profiles. This step is crucial in the generation of generalized reduced-order grey-box models and represents a novel process in the proposed methodology. As the demand profiles only give an indication of the network order, the profile variations need to be established using tests of statistical significance such as ANalysis Of Variance (ANOVA), which checks the impact of one or more factors by comparing the means of different samples. ANOVA is used to analyze the fluctuations in heat demand, the samples being the hourly demand profiles. This test establishes the presence of peaks in the profiles that can be termed as statistically significant.

The order identification process outlines the underlying structure of the grey-box network. This process considers thermal zoning of spaces inside the building where each thermal zone possesses similar characteristics and can be represented using a similar RC

network. The order identification procedure uses the results of ANOVA analysis from the data analysis process. This process could be further enriched with available data regarding the past retrofits and existing HVAC systems. For instance, radiant heating or cooling systems alter the heat capacity of the floor, which might require another RC branch for appropriate representation of the building dynamics. Similarly, fabric renovation of only one side of the building might require a separate RC branch to represent the associated wall dynamics.

The parameter estimation (or model calibration) process involves the estimation of network parameters, namely, thermal resistance and thermal capacitance and the associated time constants. Although there exist several parameter estimation procedures, this study implements the parameter estimation procedure using Continuous Time Stochastic Modeling in R (CTSM-R) that uses Maximum Likelihood Estimation (MLE) and automates the estimation procedure (Juhl et al., 2016). By using a continuous time formulation of the dynamics and discrete time measurements, the tool bridges the gap between physical and statistical modeling.

The model validation process acts as a measure of the goodness of fit for the model. Error KPIs, for instance, Root Mean Square Error (RMSE) give an indication of the suitability of the model and aid in understanding the effects not properly described by the model. This study further uses Mean Absolute Percentage Error (MAPE) and R^2 values as other validation metrics.

Reduced-order Uncertainty Analysis

Reduced-order uncertainty analysis involves uncertainty source identification, uncertainty source characterization, uncertainty quantification, nested (2D fuzzy) Monte Carlo (MC) simulations and probabilistic performance assessment (Figure 3). These processes examine the impact of uncertainties, thereby enhancing the simulation quality and the robustness

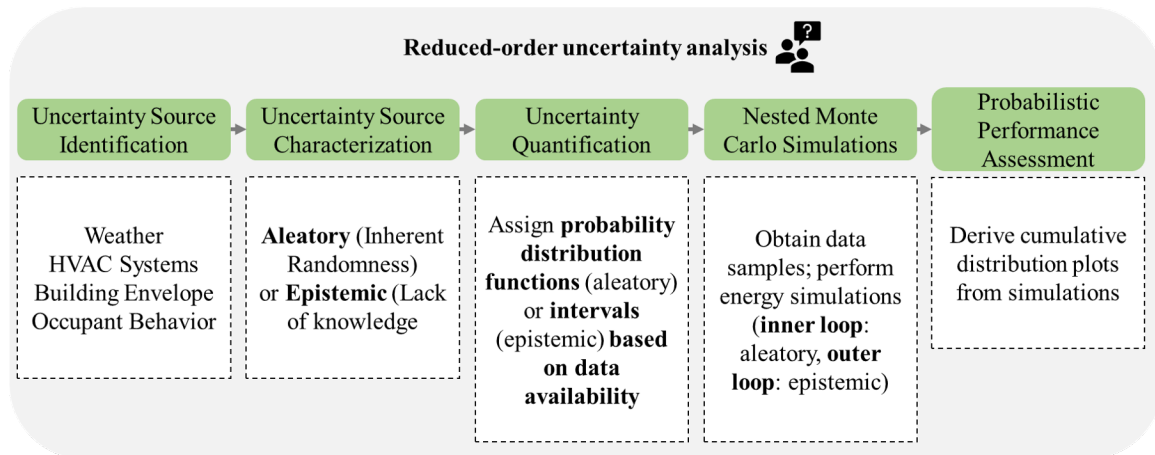


Figure 3: Process workflow to quantify uncertainties in reduced-order grey-box models.

of results.

Uncertainty source identification involves the identification of the most prominent sources of uncertainty existing in the system under study. For instance, in a typical building simulation, uncertainties can arise due to weather data, building envelope, HVAC systems or occupant behavior. Uncertainty characterization involves characterization of identified uncertainties into epistemic and aleatory categories. All uncertainties in the simulation need to be characterized before the implementation of uncertainty quantification techniques. For instance, weather data are put into the category of aleatory uncertainty based on inherent randomness while grey-box parameter values are put into the category of epistemic uncertainty based on limited access to knowledge. Uncertainty quantification involves the quantification process (identifying probability density functions or fuzzy intervals) to quantify different uncertainty types using suitable methods. This study models epistemic uncertainties using fuzzy sets and second-order probability technique and aleatory uncertainties using the probability distribution framework. Nested MC simulations involve input sampling and subsequent energy simulations to identify the model outputs. The nested MC approach is an extension of MC simulation and employs two loops allowing variability and uncertainty to be modelled separately (Figure 4). The variability is modelled in the inner loop and the uncertainty in the outer loop. Nested MC analysis can produce bounds on the output of a particular model at any credible level and takes into account the parameter uncertainty for each random quantity in the model. Performance probabilistic assessment concludes the uncertainty analysis workflow and involves the presentation of probabilistic model predictions using numerical indicators, such as mean, median, standard deviation etc. or using typical graphical methods, for instance, histograms, box plots and density plots.

The two-dimensional MC approach models the joint uncertainties in building energy performance simulations that result from variations in weather variables,

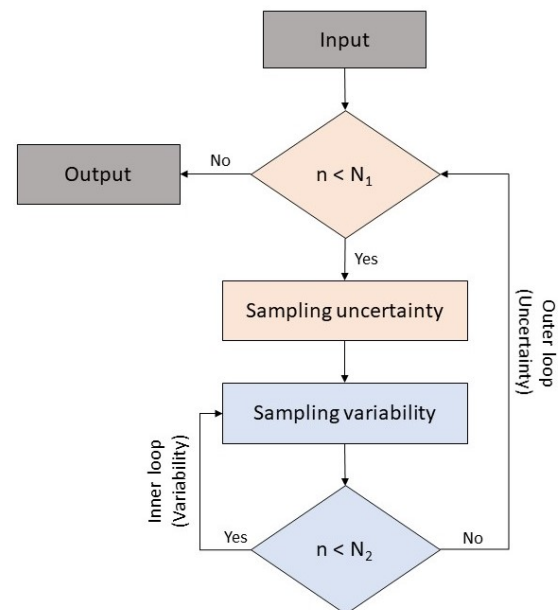


Figure 4: Implemented nested MC approach to quantify the uncertainty in heat demand. N_1 and N_2 represent the number of outer and inner loop simulations.

model parameters and other building specific inputs (for example, infiltration rate). The fuzzy set approach is appropriate to deal with sources of uncertainties when the historical data record is normally sparse and human-based judgment is dominant. As the model input data for grey-box modeling is often unavailable, the fuzzy set approach would ideally associate intervals to the information gathered using previous literature. The nested simulations segregate the uncertainties and produce a family of probability plots for the identified second as well as third-order grey-box models. Each individual plot represents aleatory uncertainty and the spread of plots represents epistemic uncertainty.

Epistemic Uncertainty Analysis

This process combines the results of model development and uncertainty analysis workflows and involves the analysis of reduced-order parameter uncertain-

ties and reduced-order model complexity/accuracy. With a classic building simulation, design decisions are mostly binary (either a yes or a no). On the contrary, these decisions can be answered using probabilities with uncertainty analysis, which aids in answering the design questions with probability distributions rather than single point values as outputs.

Case Study

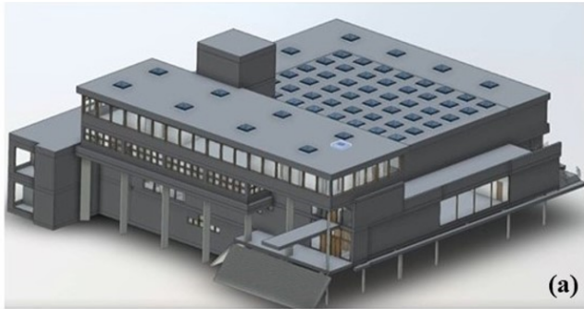


Figure 5: Revit model of the chosen administrative building as the demonstration case study.

The main objective of this paper is to identify the effect of increasing model complexity on reduced-order grey-box model uncertainty. The proposed methodology integrates the generalized reduced-order model development approach with uncertainty analysis to predict the building heat demand with varied levels of confidence. To demonstrate the application of the devised framework, an administrative building connected to the district heating network at University College Dublin's (UCD) campus is analyzed for uncertainty in heat demand predictions (Figure 5). The data collection process extracts the required variables from the Revit model and available building design drawings. The building occupies an area of 3390 m². The outer walls of the building consist of brick blocks insulated according to Building Regulations 2000 (Office, 2000). The building rests on an insulated floor slab consisting of 30 mm insulation and 150 mm of cast concrete. Windows constitute of approximately 42% of the external wall area. The building interiors consist of a large office area with small cubicles on the ground floor while the first and second floors consist of individual offices and meeting rooms. The roof is flat and insulated according to Building Regulations

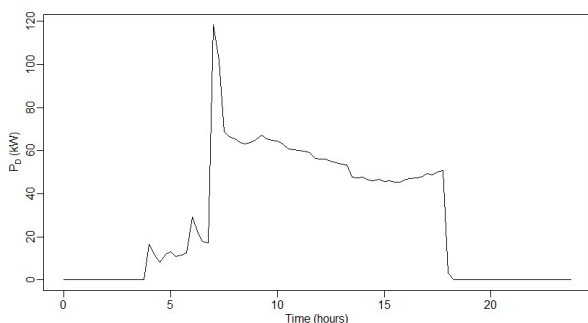


Figure 6: Hourly heat demand for the administrative building.

2000. The occupancy pattern follows a typical 9AM - 5PM office schedule. Measured energy consumption on 15 minute basis is available for this building. The office building is modelled using Dymola, which employs the Modelica language in the back end for equation-based computations.

The grey-box model development process uses these variables to identify the grey-box network for the administrative building. When implementing the grey-box model development workflow, the data analysis process outlines the ANOVA test results. Corresponding to a significance level of 0.10, the variations (Figure 6) are found to be non-existent in the heat demand profile of the chosen building (p -value = 0.323). The order identification process suggests a lumped parameter model would be sufficient to represent the building dynamics. Corresponding to a floor area of 3390 m², normalizing the heat demand with respect to the floor area yields high demand levels per m² of space use. Henceforth, separate RC network branches are assigned to exterior walls and interior mass. As the demand fluctuations are deemed insignificant by the ANOVA test, a second order grey-box model would be sufficient to represent the building dynamics. Furthermore, the building does not have any embedded heating system and has a homogeneous fabric installed all throughout the building envelope. The parameter estimation process uses CTSM-R to parameterise the identified second order grey-box model and predict the internal temperature profiles. The model validation process uses CVRMSE, MAPE and R² performance metrics to evaluate the developed models. For the second-order model, the corresponding values of CVRMSE, MAPE and R² are 3.65%, 2.83% and 0.94 respectively. The parameter estimation process is repeated with a third-order grey-box model, which then predicts the internal temperature profile of the building. The values of CVRMSE, MAPE and are found to be 2.55%, 1.72% and 0.96 respectively (Kim et al., 2014).

The uncertainty analysis workflow follows the reduced-order model development workflow. The source identification process identifies weather variables, internal gains, infiltration rate and grey-box model parameters as the uncertain sources. There can be other sources of uncertainties such as building ageing etc. but these are considered to stay constant as these will require historical building data which is not available for the building. Moreover, while calibrating the grey-box network, these uncertainties are indirectly accounted for in the analysis. The source characterization process identifies weather variables as aleatory and internal gains, infiltration rate and grey-box model parameters as epistemic. The uncertainty quantification process assigns probability density functions to weather variables and triangular distributions to internal gains, infiltration rate and grey-box model parameters. While the grey-box model

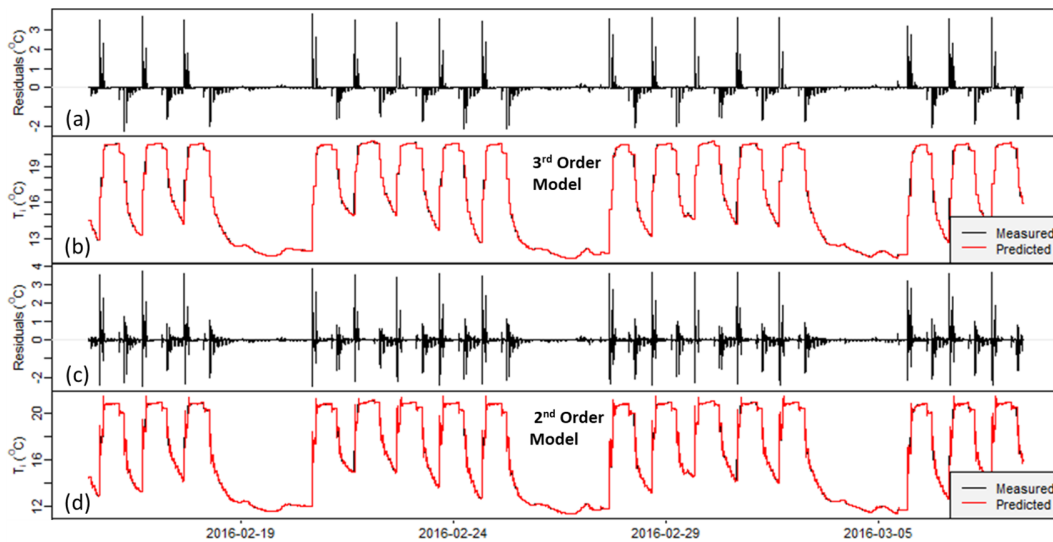


Figure 7: Residuals and estimated and measured internal temperature profile comparison for (a) and (b) third order model, (c) and (d) second order model.

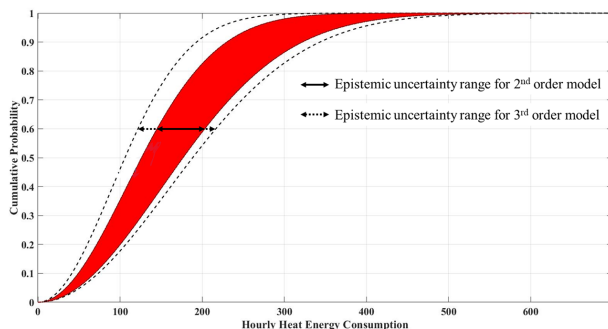


Figure 8: Epistemic uncertainty variations for the second order and third order model.

parameters are estimated using CTSM-R, the interval bounds for internal gains and infiltration rate are identified using previous literature. The values of equipment and lighting heat gains lie in the range of 8 - 12 W/m² and 6 - 12 W/m². The considered interval limits for air infiltration rate are 0.1 - 0.25 ACH.

The nested MC simulations uses the quantified uncertain variables to identify the cumulative distribution functions of building heat demand. Based on these plots, aleatory and epistemic uncertainties share an equal dominance in daily heat energy consumption (Figure 8). Improved information in all fields will enhance the estimate of outputs. It is worthwhile to note that reduction in the spread of the plot (epistemic uncertainty) is possible through acquiring more knowledge of the building in consideration.

When analyzing model accuracy and model complexity trade-off, it can be seen that increasing the model complexity (third order network) has no significant effect on model accuracy (Figure 7); only a slight reduction is seen in CVRMSE values (3.65% to 2.55%) although both models comply with the ASHRAE Guideline 14 (Landsberg et al., 2014). Furthermore, when analyzing both second and third-order models for epistemic uncertainties (increase in number of

uncertain parameters), the epistemic spread of uncertainties increases by a significant amount (Figure 8). This clearly indicates that increasing the grey-box model complexity does not necessarily indicate a good model conformity in terms of model uncertainty.

Conclusion

This study proposes an integrated reduced-order model development and uncertainty analysis methodology to building confidence in building energy simulations. Grey-box modeling could provide additional insights and information for designers considering novel alternative design approaches, where prior information may not be readily available. The task of modeling energy in buildings becomes more and more challenging as the associated scale (city/district/urban) changes. It is often difficult to gather geometric and non-geometric data associated with each and every building at an urban scale. The devised reduced-order model development and uncertainty analysis workflows produce simplified and reliable thermal networks, which would facilitate integration with other energy vectors such as electricity. The proposed methodology implements reduced-order models that contain the knowledge of building dynamics often considered essential in the identification of cost effective energy retrofits. The optimal balance between complexity and accuracy makes such models a powerful tool to inform policymakers of the energy use and thereby, identify investments in energy-conversion measures for the commercial building stock. A probabilistic framework employing advanced techniques would allow stakeholders to identify influential inputs by considering the factors behind the risks in a given family of distributions. Moreover, energy modellers could implement the uncertainty metric in the identification of grey-box models along with other metrics, namely, complexity and

accuracy as increasing the number of model parameters (model order) increases the uncertainties in the output energy simulations. Policymakers could use the framework to devise future probabilistic scenarios related to changes in climate and subsequent changes in the heating or cooling consumption patterns.

In the event the uncertainty analysis involves numerous parameters, sensitivity analysis would help in the identification of influential factors and would be a valuable addition to the workflows. As occupant behavior has a significant influence on building energy performance, implicit occupant models might be integrated to simulate the variations of occupant behavior.

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