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A bespoke signal processing algorithm for operational modal testing of post-tensioned steel and concrete beams

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ABSTRACT

The extraction of modal properties, specifically natural frequency, damping ratio and mode shape is a difficult task, especially when output-only data is measured. The accuracy of the estimation these modal properties is compromised by noisy signals, and signal filtering is required to suppress unwanted frequency content. Care is required however to avoid over-filtering of the output data, which can eliminate valid structural frequency content if required care is not exercised. This paper describes the development of a bespoke signal processing algorithm to extract the modal properties of both simply supported post-tensioned steel and concrete sections. Dynamic impact testing was conducted on a series of different post-tensioned steel rectangular hollow sections, and 9 different post-tensioned concrete beams, each with differing straight-profiled post-tensioning strand eccentricities. Acceleration time-history data was recorded for each of the steel and concrete beams via an accelerometer. This data was subsequently processed, first centring the acceleration-time history using a moving average filter, and subsequently removing any zero drift in the accelerometer via a second order low pass Butterworth filter. Electrical noise was then removed via a notch filter. The accelerometer data was then smoothed in the time domain. The Fast Fourier Transform (FFT) was applied to the signal to convert into the frequency domain and finally a bespoke peak-picking algorithm was invoked to extract the natural frequencies of the beams. A comparison is subsequently made between the accuracy of the estimation of the modal properties of the steel and concrete beams for filtered and unfiltered data, and a sensitivity analysis of the filtering and peak picking parameters is conducted to determine the effect that this has on the accuracy of the estimation of the modal parameters. The results show the effectiveness of the bespoke signal processing algorithm in increasing the accuracy of the estimation of the modal properties as opposed to the raw unprocessed signals.

Keywords: Operational Modal Analysis, Fast Fourier Transform, Filtering, Signal Processing.

INTRODUCTION

Experimental modal analysis has been conducted on a series of steel and concrete beams in the lab [1], [2]. The steel and concrete beams have been post-tensioned by treading a post-tensioning strand through their cross section and jacking the beam on a live end. A series of impact hammer strikes were applied to the tested structures, and the dynamic response of the structure was measured using an accelerometer. The output signal only was measured, no measurement signal was taken of the dynamic input signals therefore the experiments conducted are considered as output-only dynamic tests. These tests are becoming increasingly common, especially in the field of Operational Modal Analysis (OMA), in which the modal parameters of a structure are determined from the structures response to ambient vibration conditions. The modal properties are thus extracted from the Fourier Transform representations of the system output signals.

The testing was conducted in order to determine the existing relationship between post-tensioning force magnitude and modal properties of post-tensioned steel and concrete specimens, as there is significant disagreement between researchers in this field [3]–[13].

This paper outlines a bespoke signal processing algorithm, and subsequent bespoke peak-picking methodology applied to a series of impact response signals obtained from the dynamic testing described in full [1], [2]. The paper is organized as follows; the fundamental theory behind digital signal processing is first introduced. The bespoke signal processing algorithm followed to filter the obtained signals is described in the following section. A detailed description of each of the steps of the signal processing algorithm and the effect of each signal filter on the raw data signal is subsequently described in the following sections. A curve smoothing function used to smooth both time and frequency domain representations is then described. The following section describes the peak-picking algorithm used to identify the natural frequencies of the structural system(s) tested. The cumulative effect of the filtering on the raw data is then outlined. The Inverse Fourier Transform is subsequently applied to the smoothed, normalized, filtered, frequency domain representation. Finally, conclusions based on the described filtering regime are drawn.

SIGNAL PROCESSING

The main assumption underlying all forms of modal analysis is that any dynamic response signal may be represented, in the time domain, as an infinite series of sinusoids, each with their own unique amplitude and frequency. The amplitudes are considered as the modal contribution of each mode of vibration to the overall dynamic signal, whereas the frequencies are defined as the natural vibration frequencies of the structural system, for the corresponding mode shape of vibration. All other seemingly random portions of the vibration response are considered as a noise components of the system, and as such, asystematic.

Noise

Noise is any unwanted or seemingly random portions of the measured vibration response of the system. Quality assurance of the measured dynamic data is of utmost importance. The signals obtained should be of sufficient strength and clarity and free of excessive noise [14]. This unwanted frequency content is attenuated by appropriate use of signal processing and signal filtering that is outlined in detail in this paper.

Aliasing

Aliasing is an error associated with digital signal processing (DSP) in which the existence of very high frequency components in the signal are missed as the sampling rate is not large enough. It is not possible to determine any frequency greater than the Nyquist frequency, ω_{Nyq}. The solution to this is to use an anti-aliasing filter. The anti-aliasing filter is a low pass, sharp cut-off filter. The type of filter implemented is usually a high order low pass Butterworth filter [17]. Anti-aliasing filters are not automatically built-in to signal analysers and data collectors, and thus the signal must be post-processed.

Leakage

Leakage is a signal processing problem related to discrete signals and the definition of the Fourier Transform. The Fourier Transform is defined as an integral over the range of $(-\infty, +\infty)$. Leakage occurs due to the need to take only a finite length of time-history, coupled with the assumption of periodicity [14]. When the periodicity assumption is not valid and there is a discontinuity at the beginning or the end of the signal in the time domain, the resulting frequency spectrum does not indicate the original frequency. This frequency is actually not represented. Energy has `leaked' into a number of the spectral lines close to the true frequency and the spectrum is spread over several lines [14]. The effects of leakage were mitigated against throughout the course of this study by making use of *'zero padding'* (adding zeros before and after signal) which artificially removed the periodicity requirement. However, impact signals, experience minimal distortion due to leakage provided the signal is analysed over the appropriate region.

Windowing

Windowing involves the modification of the signal in the time domain in order to suppress the effects of unwanted frequency content and minimize the effects of leakage. Avitabile [15] warns that all windows distort data, as they distort the peak amplitude and appear to indicate more damping than actually exists. Avitabile [15] suggests in order to avoid the use of windows to continuously sample a periodic repetition of the data, or to completely observe the signal in one data sample. The latter is obtained by choosing impulse/impact excitation as the type of dynamic excitation, which was chosen as the form of input excitation chosen for this study.

SIGNAL PROCESSING REGIME APPLIED TO OBTAINED SIGNALS

The following procedure was applied to the raw impact response signals obtained from the steel and concrete testing and presented in [1], [2];

1. The initial acceleration vs. time data obtained from the accelerometer attached to the steel or concrete beam following impact hammer strike was read in.
2. Initial *'zero padding'*, *'windowing'* and offset correction was applied to the obtained accelerometer data.
3. The initial data analysis was performed.
   - The Fast Fourier Transform (FFT) of the unfiltered initial acceleration-time data, as outlined above, was computed.
   - The peaks in the initial spectrum were computed using a bespoke peak picking algorithm – the peaks are the computed natural frequencies of the system.
4. The initial data was subsequently filtered. Following initial data analysis, the natural frequencies were unintelligible, thus the signal required to be filtered to remove excessive noise/frequency content corrupting the identification of the natural frequency. The *'zero padded'*, *'windowed'* and offset initial data was subsequently filtered as outlined below.
5. The following filtering was performed;

- The *'zero-drift'* was removed by applying a low pass Butterworth filter to the data.
- The signal was smoothed, first by removing electrical noise frequency content of 50Hz and all harmonics, and then by subsequently applying a smoothing function to remove excessive high-frequency noise in the signal.
- A high pass filter was applied to remove all low frequency noise components corrupting the signal.
- An anti-aliasing filter was applied to the data.

6. The data was then subsequently windowed again.
7. The FFT of the filtered acceleration-time data was then computed to convert the acceleration-time signal into the frequency domain.
8. The main peaks were then extracted from the frequency domain curve to determine the natural frequencies of the structural system.

Figure 1 outlines a flow chart describing the bespoke signal processing algorithm followed for the concrete beam specimens as outlined previously [1], [2].

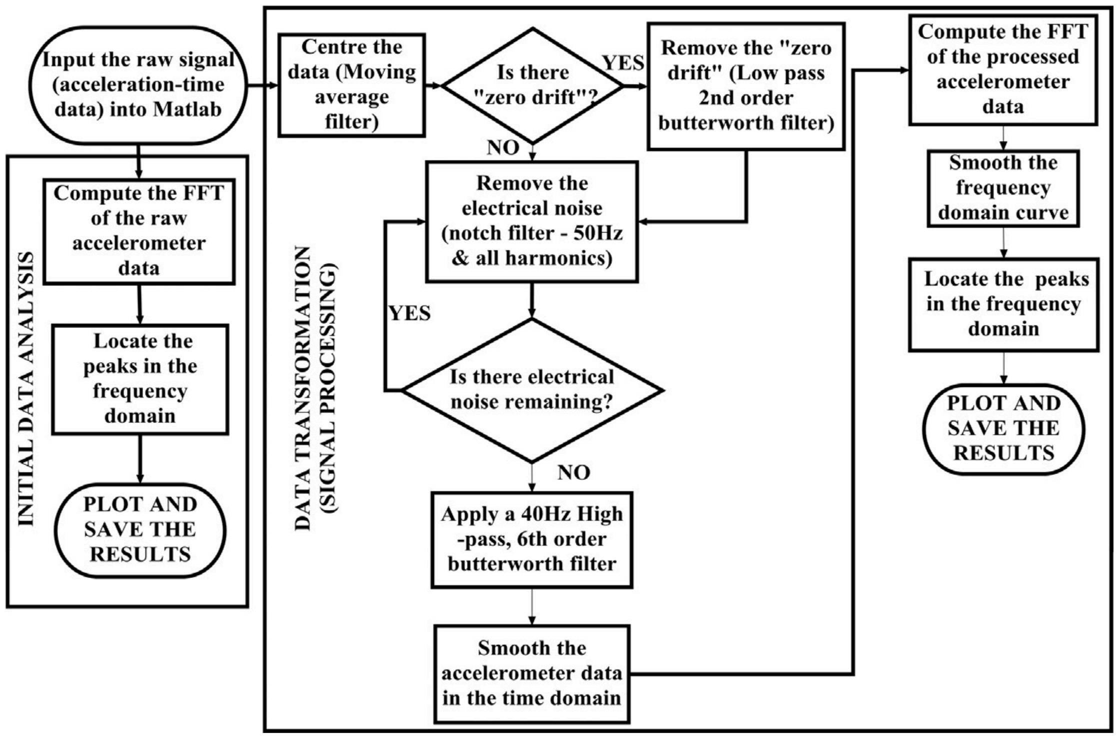


Figure 1. Signal processing flow chart as applied to the concrete beam tested (reproduced from [2]*)*

In certain cases, the fundamental frequency may not be identifiable from the raw data signal, as evident for the signal represented in the time and frequency domains in Figure 2 and Figure 3 respectively, and as outlined previously [2]. This paper identifies the need, in some instances for signal filtering and the removal of low and high frequency noise for the correct estimation of the fundamental frequencies of vibration.

Initial 'zero padding' and 'windowing'

The initial data signal is characterized by the blue signal in Figure 2. The signal has an initial offset, as shown in Figure 2. The signal is initially processed by cutting an initial portion of the data in the time domain. The position of the maximum acceleration in the raw signal was identified, and the first 1/3 of the signal, from the initial acceleration to the maximum acceleration was removed, and thus the signal was 'windowed' in order to ignore/remove any frequency content in this portion of the signal. The remaining signal was then forced to begin at 0. The initial offset was removed by subtracting the initial offset from all parts of the signal. The red signal in Figure 2 is the result of preprocessing the blue signal by 'zero padding' and removing the initial offset.

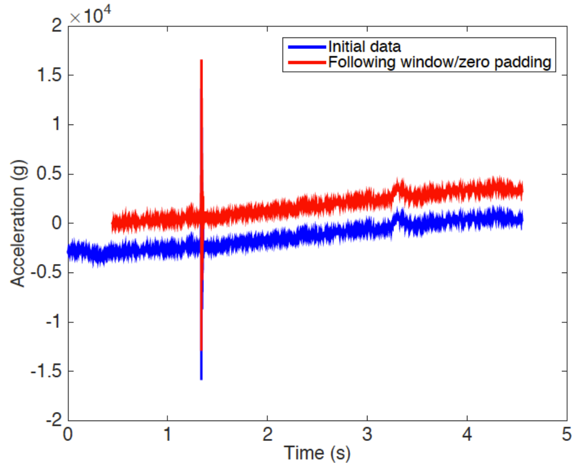
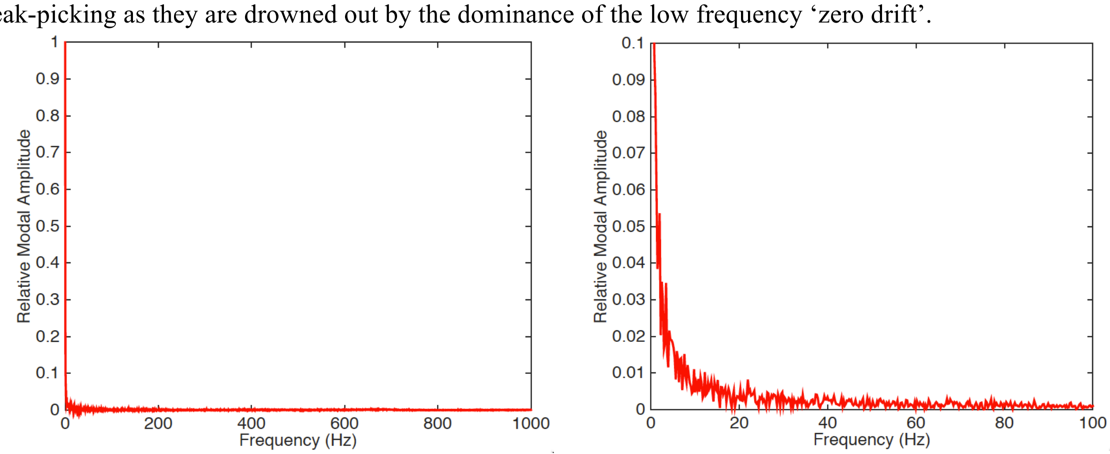


Figure 2. Raw output data collected (blue) and following windowing/zero padding (red) in the time domain

Initial data analysis

Figure 3 shows the initial frequency domain representation of the windowed data following initial processing (Figure 2). As shown in Figure 2, there is a very low frequency component in the signal, where the overall trend in the data is rising slightly from left to right. This is known as 'zero drift' and is an instrumentation error in the accelerometer. When the Fourier Transform is performed on the data, it assumes that all parts of the data/trends in the data are periodic, and that therefore the 'zero drift' is also a periodic component of the signal. As a result, the very low frequency content dominates the frequency domain representation of the signal, and thus the fundamental structural modes of vibration are not readily identifiable through peak-picking as they are drowned out by the dominance of the low frequency 'zero drift'.



(a) *(b)*

Figure 3. Frequency domain representation of the initial (raw) time domain data

Moving average filter

A moving average filter was applied to the initial data in order to remove the effects of some high frequency noise on the data signal. Figure 4 shows both the time and frequency domain of the raw data signal following the implementation of a moving average filter. The blue data in both the time and frequency domain represents the unfiltered, raw initial data, while the red represents the data following the application of the moving average filter. The moving average filter is a standard technique in signal processing whereby the signal is smoothed by removing high-frequency, unwanted, line noise in order to uncover the fundamental structural frequencies. A moving average filter, of length N takes the average of every N consecutive samples in the signal, where N is the sample size. The filter itself has a delay of (N-1)/2, and this must be adjusted for in the filter. As outlined in Figure 4, the effect of the filter is to smooth the signal, removing unwanted high frequency noise.

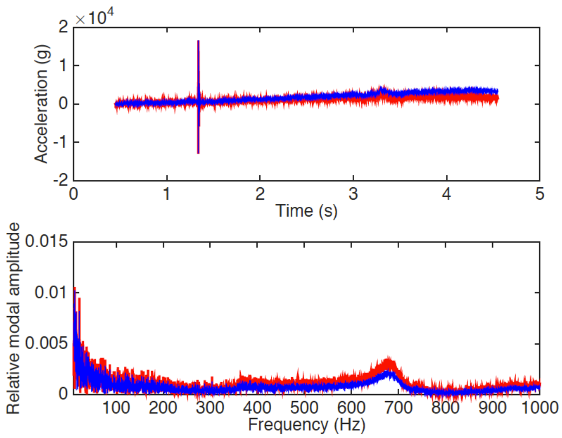


Figure 4. Time and frequency domain representation of raw data signal following application of moving average filter.

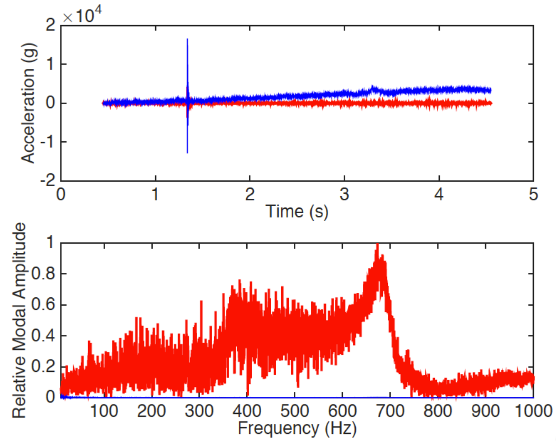


Figure 5. Time and frequency domain representations of raw data signal following removal of low frequency 'zero drift'.

Remove 'zero drift'

As outlined in **Initial data analysis**, 'zero drift' is a low frequency noise component in the accelerometer signal due to an instrumentation error in the accelerometer, meaning that the signal oscillates about a non-zero or drifting datum. As outlined previously since the Fourier Transform assumes periodicity, this low frequency noise dominates the signal, thus making the structural vibration frequencies unintelligible. A low (2^{nd}) order, high-pass Butterworth filter, with a cutoff frequency of 0.1Hz was thus applied to the signal. This meant that low frequency components of the signal (<0.1Hz) were suppressed/attenuated, thus removing the very low frequency 'zero drift' from the data, and thus forcing the data to oscillate about the zero datum, as shown in **Figure 5**.

Smooth signal – remove electrical noise

The main component of noise in the data is electrical noise. The sources of accelerometer noise can be broken down into the electrical noise from the circuitry that converts the mechanical vibration into a voltage signal that is thus correlated against a given acceleration, and the mechanical noise from the accelerometer itself. Ambient vibration of the system will also contaminate the pure structural signal. The A/C mains by which the accelerometer is powered has an alternating current with a frequency of 50Hz. This 50Hz signal, including all of its harmonics, is represented in the output data. This 50Hz electrical noise and its harmonics are filtered out by means of a 'notch' filter. A notch filter is a high order bandpass filter that removes specifically defined frequencies, while passing all other frequencies. Figure 6 shows the result of applying a series of notch filters of 50Hz and its multiples to the data. The blue signal in Figure 6 represents the raw signal, whereas the red signal represents the raw signal following removal of the electrical noise.

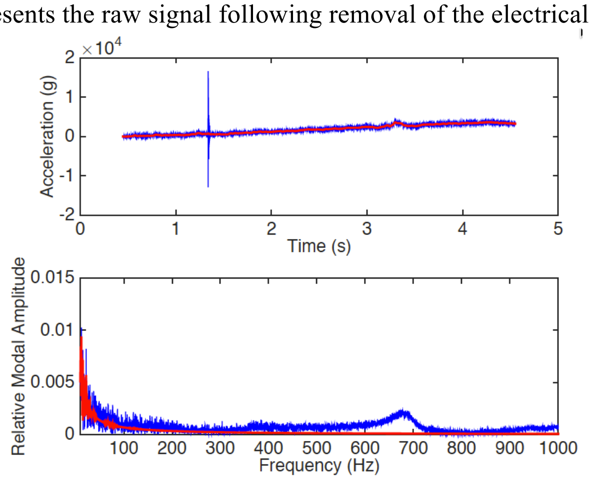


Figure 6. Time and frequency domain representation of raw signal before and after removal of electrical noise.

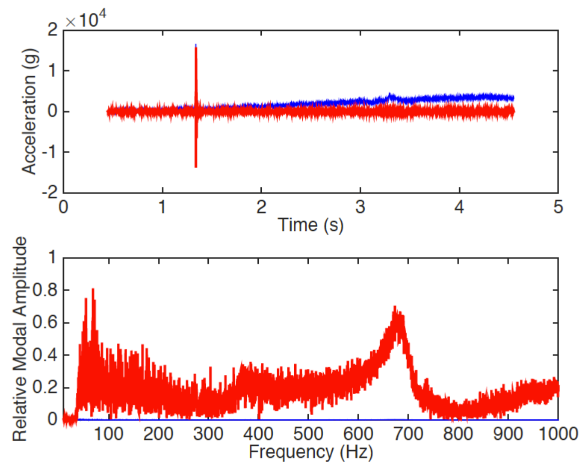


Figure 7. Time and frequency domain representation of raw data signal following application of 40Hz high pass filter.

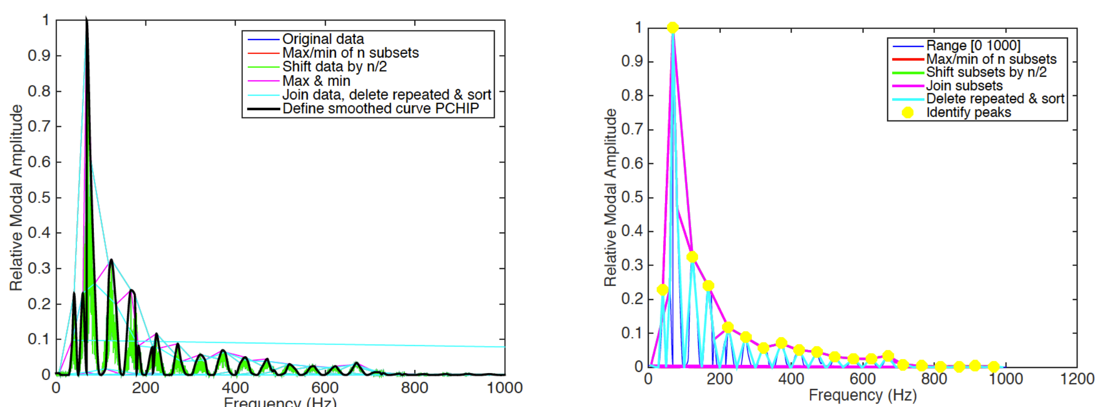
High pass filter

The fundamental bending frequency of the post-tensioned concrete beam sections tested was estimated to be 78Hz [2]. This estimation does not account for the effect of the mass of the jacks situated either end of the beam, tensioning the post-tensioning strand. The effect of the jacks should be to reduced the predicted frequency. The fundamental bending frequency, and subsequent harmonics were of interest in the testing conducted, thus a high pass filter was applied to the data in order to suppress the unwanted low frequency noise and to determine the fundamental bending frequencies via peak picking. Figure 7 shows the time and frequency domain representation of the raw data signal following the application of the 40Hz high pass filter. The blue data represents the raw data signal, whereas the red signal represents the signal following application of the high order (8^{th} order) 40Hz high-pass filter. The frequency domain representation indicates the suppression of the frequency content of the signal <40Hz.

Curve smoothing function

Figure 1 shows a flow chart outlining the series of different filters applied to the raw data. Following the application of each of the filters described previously, a frequency domain representation was obtained by which the fundamental structural vibration frequency could be obtained. In order to determine an estimation of the fundamental frequency, the frequency domain curve required smoothing. A smoothing algorithm was applied to the frequency domain data by identifying the most relevant points. With this aim, the data set is split into a series of *n* subsets. The smaller the value of *n*, the smother the final curve is. Then, the maximum and minimum values of the frequency domain representation of the data in each subset are calculated. In order to remove the effect of lag on the data, the data set is shifted by a value of *n/2*, and the new maximum and minimum values of the shifted subsets, calculated. In that way, the points identified during both rounds, which are up to *4n* positions, represent the most relevant points of the frequency domain curve. Finally, a continuous curve is defined using a piecewise cubic Hermitic polynomial by interpolation within the identified points.

Figure 8(a) shows the performed steps of the curve smoothing function of the frequency domain representation of a response signal. The blue curve shows the frequency domain representation of the filtered impact response signal in the range of interest of 0 to 1,000Hz. The red curve shows the definition of the maximum and minimum values in each of the defined subsets. The green curve shows the filtered impact signal shifted by n/2. The magenta curve shows the maximum and minimum values of the shifted subsets. The cyan curve shows the sorted data. Finally, the black curve shows the representation of the cubic interpolation function joining the maximum and minimum points in the subsets.



(a) *(b)*

Figure 8. (a) Curve smoothing function of frequency domain representation of signal using piecewise cubic interpolation. (b) Peak-picking function of the smoothed frequency domain representation

Find the peaks in the frequency domain

The peaks of the frequency domain representation of the signal, are, by definition, the natural frequencies of vibration of the structural system. The peaks are identified via a peak-picking algorithm. The peaks are identified using the *'findpeaks'* function in MATLAB [18]. The function returns a vector with the local maxima and the corresponding index of the input signal vector. This index is then used to identify the corresponding frequency from the frequency vector. A local peak is a data point that is larger than its two neighboring points. The unsmoothed frequency domain representation produces too many local peaks to identify a fundamental frequency, therefore the frequency domain representation required smoothing. Figure 1 shows the steps involved in identifying the fundamental frequencies of the signal.

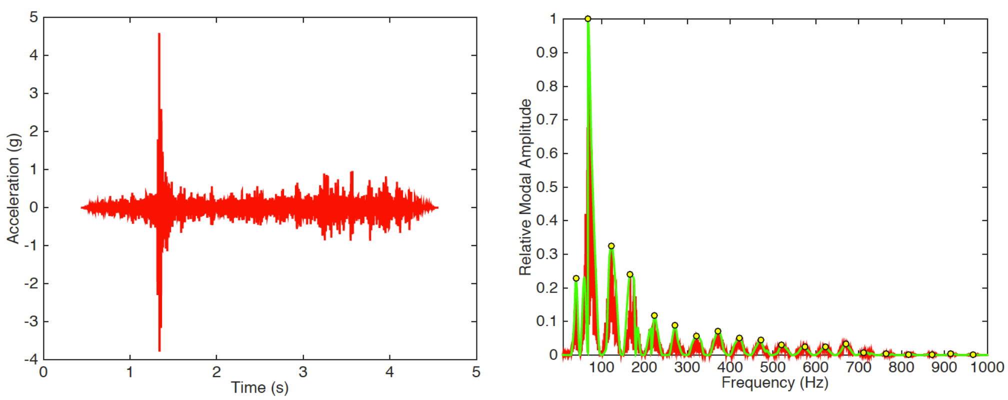
1. The smoothed frequency domain data was organized into column vectors and plotted (blue curve).

2. The data was filtered into the range of interest (0-1000Hz).
3. The filtered and smoothed frequency content was divided into a series of n subsets.
4. The maximum and minimum values of each subset was subsequently calculated (red curve).
5. The subset data was shifted by a value of n/2 (green curve).
6. The maxima and minima values of the shifted subsets were thus calculated and the values were joined (magenta curve).
7. The repeated values were deleted and the values were sorted (cyan curve).
8. The MATLAB [18] *'findpeaks'* function was thus used to identify the peaks in the smoothed frequency domain (yellow data points).

These peaks are identified as the fundamental frequencies of vibration. Figure 8(b) shows the steps of the function in graphical format. The fundamental frequency of vibration for the post-tensioned concrete beams tested was estimated to be 74Hz [2]. The fundamental frequency identified from peak-picking for the given signal was 69Hz. This represents a 7% error in estimation of the predicted fundamental frequency, however, the structural system tested included the mass of the jacks on either end of the beam [2], and this is expected to reduce the expected frequency.

Cumulative effect of filtering

The subsequent sections describe the filters applied to the data in depth, and outline the effect of each individual filter on the raw data signal. Figure 9 presents the time and frequency domain representations of the filtered signal following the cumulative effects of the signal filtering as described in Figure 1 and subsequent sections.

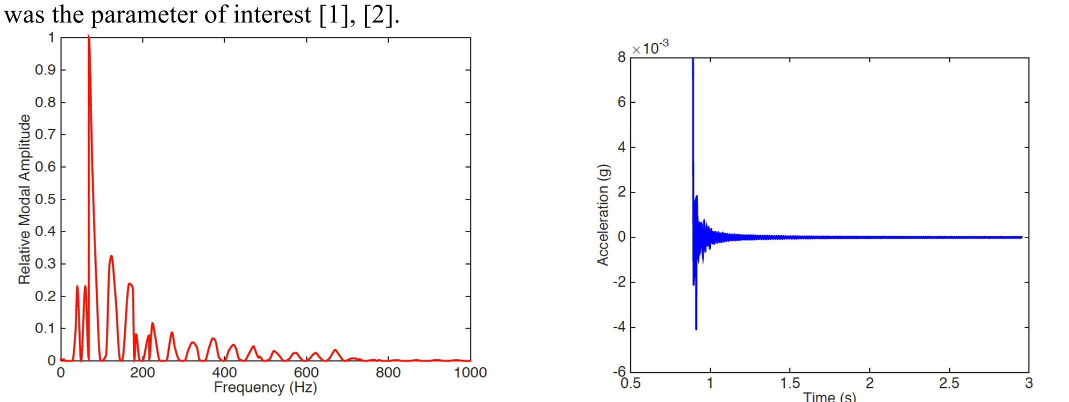


(a) *(b)*

Figure 9. Time (a) and frequency (b) domain representation of filtered response signal

Smoothed frequency domain curve

Figure 10(a) shows the smoothed frequency domain representation of the output response signal. The Inverse Fourier Transform (IFT) was subsequently performed on the smoothed, normalized, frequency domain representation. The IFT transforms the signal from the frequency domain back into the time domain. Since the frequency domain representation has been normalized, the IFT does not represent the correct amplitude of vibration. This is not of interest however, as the frequency was the parameter of interest [1], [2].



(a) *(b)*

Figure 10. (a) Smoothed frequency domain representation of output response signal; (b) Inverse Fourier Transform (IFFT) of frequency domain representation of output response signal.

CONCLUSIONS

This paper outlined the importance and effect of signal filtering on raw acceleration response data for steel and concrete beams in order to identify the fundamental frequencies of vibration [1], [2]. The paper outlines the difficulty in identifying the appropriate fundamental frequency using unfiltered accelerometer data. The effect of noise components is such as to distort the structural vibration signal, meaning that the main structural vibration frequencies may be unintelligible. The noise components may be removed through careful filtering. The filters applied to the accelerometer data, and described throughout the paper are; moving average filter, zero-drift filter, notch filter removing electrical noise, 40Hz high pass filter, and an anti-aliasing filter. The most important filters in terms of identification of the natural frequencies are the zero drift and 40Hz high-pass filters, removing the dominant low-frequency noise components. The second most important filter in terms of structural frequency identification is the electrical notch filter, removing the electrical noise from the A/C circuitry in the accelerometer.

Care must be taken in signal filtering in order to avoid over-filtering. Over filtering of the signal leads to confirmation bias, furthermore, if care is not exercised, valid structural frequency content may be removed. The described signal processing regime allowed for the correct identification of the predicted fundamental frequencies of vibration for both steel and concrete sections as outlined previously [1], [2].

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