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<b>Title</b>	Estimation of lifetime maximum distributions of bridge traffic load effects
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<b>Publication date</b>	2012-07-08
<b>Publication information</b>	O'Brien, Eugene J., Donya Hajializadeh, Emma Sheils, and Bernard Enright. "Estimation of Lifetime Maximum Distributions of Bridge Traffic Load Effects." CRC Press, July 8, 2012.
<b>Conference details</b>	The Sixth International IABMAS Conference on Bridge Maintenance, Safety, Management, Resilience & Sustainability, Stresa, Lake Maggiore, Italy, 8-12 July 2012
<b>Publisher</b>	CRC Press
<b>Item record/more information</b>	<a href="http://hdl.handle.net/10197/4119">http://hdl.handle.net/10197/4119</a>

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# Estimation of Lifetime Maximum Distributions of Bridge Traffic Load Effects

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**ABSTRACT:** This paper considers the problem of assessing traffic loading on road bridges. A database of European WIM data is used to determine accurate annual maximum distributions of load effect. These in turn are used to find the probability of failure for a number of load effects. Using the probability of failure as the benchmark, traditional measures of safety – factor of safety and reliability index – are reviewed. Both are found to give inconsistent results, i.e., a given factor of safety or reliability index actually corresponds to a range of different probabilities of failure.

## 1 INTRODUCTION

Repairing or replacing deteriorated bridges is expensive due to the cost of the repair itself but also due to the disruption to traffic and the resulting delays. Large savings are therefore possible by proving that bridges are safe without intervention. To assess the safety of an existing bridge, the traffic loads to which it may be subject in its lifetime need to be accurately quantified. Statistical or probabilistic approaches are commonly adopted and various methods used to infer the bridge safety based on its loading and its resistance to that load.

A great deal of work has been done on the resistance of bridges to load but considerably less on traffic loading itself. This paper focuses on the load side of the load/resistance inequality.

A number of alternative ways of measuring safety are considered. The probability of failure – Figure 1 – is clearly a benchmark measure of safety. For ease of use, other concepts are used in practice. The appropriateness of reliability theory is considered in this paper and the consistency of the relationship between the reliability index and probability of failure. The standard load and resistance factor design (LRFD) approach is also examined.

### 1.1 Assessing Traffic Load

In a bridge assessment, the cost of intervention justifies a more thorough examination of the true safety than would be the case for a new bridge. For accurate assessments, it may be possible to get site-specific information on traffic load in the form of Weigh-in-Motion (WIM) measurements.

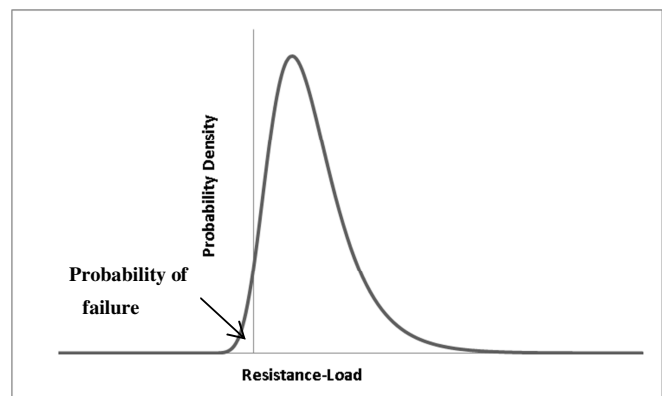


Figure 1. Probability density function of difference between resistance and load effect. Probability of failure is area under the graph left of the Y-axis.

Weigh-In-Motion was initially developed in the 1960s and 1970s. Since the early 1990s, a series of WIM related European research projects have been carried out, supported by the European Commission through the COST Transport and Framework Programmes: COST323(Jacob et al. 2002), WAVE (Jacob & O'Brien 1999), Top Trial [1], (Opitz & Kuhne 2005) and REMOVE (Rooke et al. 2005). COST323, WAVE and Top Trial focused more on the accuracy and reliability of the WIM sensor systems while REMOVE developed strategies to address the problems of overloading and overload enforcement. The application of WIM data to the bridge safety assessment problem has been considered in the 6<sup>th</sup> Framework ARCHES [2] and 7<sup>th</sup> Framework ASSET [3] projects.

The concept of using WIM data to assess the level of loading on a bridge is well established in the scientific literature. A common approach (Nowak 1993, Vrouwenvelder & Waarts 1993, Miao & Chan 2002) is:

- (i) To measure traffic data for some weeks;
- (ii) To calculate the load effects for the measured traffic scenarios;
- (iii) To identify the maximum-per-day load effects;
- (iv) To fit an extreme value probability distribution to the daily maximum load effects and
- (v) To use this distribution to extrapolate to the characteristic maximum effects.

This estimation may require a considerable degree of subjective judgment (Gindy & Nassif 2006, Kulicki et al. 2007). Furthermore, there is considerable variability in the results, suggesting low levels of accuracy.

An alternative approach is developed in the ARCHES and ASSET projects (O'Brien & Enright 2011):

- (i) Traffic data is again measured for some weeks or months;
- (ii) Probability distributions are fitted to characteristics of the traffic (weights, axle configurations, gaps, etc.);
- (iii) Monte Carlo simulation is used to simulate loading scenarios which are validated by comparison with data directly measured;
- (iv) Load effects are calculated for the simulated traffic scenarios;
- (v) The simulation is continued for thousands of years of traffic to determine the characteristic maximum value.

The principal advantage of this 'long run' simulation approach is that the results are far more reliable. Provided the simulation is valid, the characteristic load effect is found by interpolation rather than extrapolation and is much more accurate. An added bonus is that 'typical' characteristic maximum loading scenarios can be examined to confirm the validity of the calculations.

### 1.2 Load and Resistance Factor Design

The standard design philosophy in most countries is the limit state approach where, for the limit state of failure, the factored effect of load is required to be less than the factored resistance to that load (Figure 2). In the United States, this is referred to as Load and Resistance Factor Design (LRFD).

The load factor exceeds unity and the resistance factor is less than unity with each factor reflecting the uncertainty associated with the load or resistance. The loads and resistances are already characteristic values so the factored loads and resistances are approximations of other, more extreme, characteristic values. An alternative, simpler approach is to calculate a single factor of safety as the ratio of characteristic resistance to characteristic load.

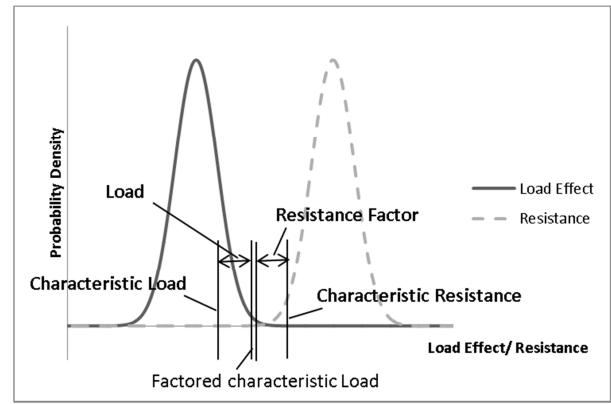


Figure 2. Load and Resistance Factor Design Philosophy

LRFD has clear advantages in terms of simplicity but there are inevitable inconsistencies in the levels of safety implicit in the design or assessment.

### 1.3 Reliability Theory

Reliability theory has gained in popularity in recent years. The reliability of system can be defined as the system's ability to fulfill its design functions for a specified time. This ability is commonly measured using probabilities. Reliability is therefore the probability that the structure will not fail, resulting in (Melchers 1999, Ayyub et al. 2002):

$$\text{Reliability} = 1 - \text{Failure Probability} \quad (1)$$

Based on this definition, reliability is one of the components of risk. Safety can be defined as the acceptability of a risk for the system, making it a component of risk management. Reliability and risk measures can be considered as performance measures specified as target reliability levels or target reliability indices. The selected reliability level of a particular structural element reflects the probability of failure of that element (Ayyub et al. 2002).

It should be noted that the reliability based design procedure requires the definition of performance functions that correspond to limit states for significant failure modes. The reliability of each element is achieved when the resistance (capacity) is greater than the load (demand). A generalized form for the performance function of a structural component is given by (Melchers 1999):

$$g = R - L \quad (2)$$

where  $g$  = performance function;  $R$  = resistance; and  $L$  = load effect on the structural element. Due to the variability in both resistance and load, there is always a probability of failure,  $P_f$ , that can be defined as (Melchers 1999, Kenshal 2009):

$$P_f = P(g < 0) = P(R < L) \quad (3)$$

In general, the probability of failure, for all possible stresses, is given by:

$$P_f = \int_{-\infty}^{\infty} f_S(x) F_R(x) dx \quad (R, S \text{ independent}) \quad (4)$$

where  $x$  is the underlying variable(s);  $f_S(x)$  is the probability density function for load effect  $L$ ; and  $F_R(x)$  is the cumulative distribution function for resistance  $R$ . The solution to this integral can be accurately found using numerical techniques such as the fast probability integration technique and the sampling technique. The fast probability integration technique can be grouped into two types, namely, first-order reliability method (FORM) and second-order reliability method (SORM).

(Cornell 1969) proposed the use of only first and second moments to characterize the entire set of random variables, and linearization by means of the Taylor series expansion of the limit state function  $g(x)$  at some appropriate checking point. The measure of reliability is given by the reliability or safety index:

$$\beta_c = \frac{m_g}{\sigma_g} \quad (5)$$

where  $m_g$  is the mean value and  $\sigma_g$  is the standard deviation of  $g(X)$ . This approximation technique simplifies the original complex probability problem. Figure 3 shows the relation between reliability index and probability of failure which is given by,

$$\beta_c = \Phi^{-1}(P_f) \quad (6)$$

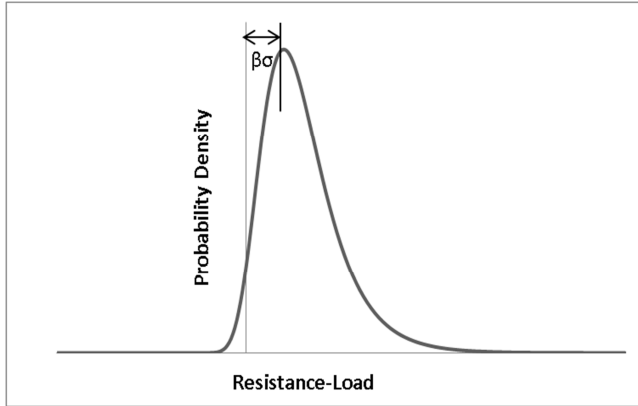


Figure 3. Probability density function of difference between resistance and load effect

It should be noted that this relationship assumes all the random variables in the limit state equation to have Normal probability distribution and the performance function to be linear. (Lee & Hwang (2008); Melchers 1999, Ayyub et al. 2002, Ayyub & McCuen 2003, kenshal 2009) However, in practice, it is common to deal with nonlinear performance functions with a relatively small level of linearity. Some authors (Ayyub et al. 2002, Ayyub & McCuen 2003, kenshal 2009) assume, if this is the case, then the error in estimating the probability of failure is small, and that, for all practical purposes, this equation can be used to evaluate  $P_f$  with sufficient accuracy.

(Hasofer & Lind 1974) propose to linearize about a point which lies on the failure surface and which

corresponds to the maximum likelihood of failure occurrence (HL method). The reliability is measured through the Hasofer-Lind safety index and it is defined as the minimum distance from the origin to the failure surface as illustrated in Figure 4. (Rackwitz & Flessler 1978) extend the HL method to include random variable distribution information, which is denoted as the HL-RF method. The HL-RF method requires the least amount of storage and computation in each step. For most situations this method not only converges, but also converges more quickly.

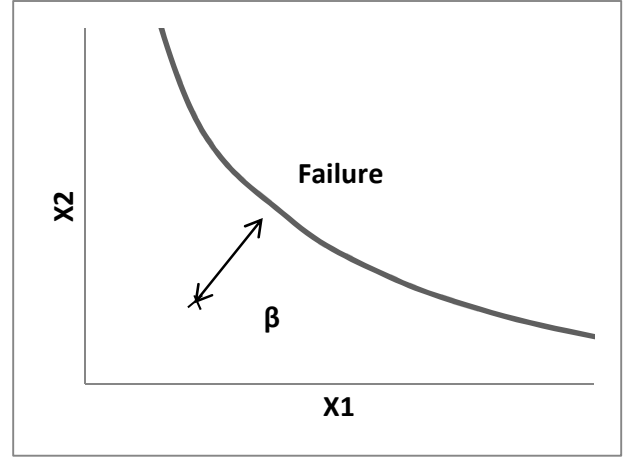


Figure 4. Failure surface in design space, where  $x_1$  and  $x_2$  are design parameters (properties of the bridge)

The SORM approach is first explored by (Fiessler et al. 1979) using various quadratic approximations. A simple closed-form solution for the probability computation using a second-order approximation,  $P_f$ , is given by (Breitung 1984) using the theory of asymptotic approximations as,

$$P_f = \Phi(-\beta) \prod_{i=1}^{n-1} (1 + \beta k_i)^{-1/2} \quad (7)$$

where  $k_i$  denotes the principal curvatures of the limit state at the minimum distance point, and  $\beta$  is the reliability index using the FORM.

(Tvedt 1990) develops alternative SORM formulations. Tvedt's method uses a parabolic and a general second-order approximation to the limit state, and it does not use asymptotic approximations. The SORM approach can obtain the wrong curvature due to the numerical noise and the computation time is increased when the number of random variables increases.

## 2 LOAD EFFECT CALCULATIONS

A large database of WIM measurements was collected for trucks weighing in excess of 3.5 t at five European highway sites between 2005 and 2008. One of these sites, in Slovakia, has bidirectional traffic with average daily truck traffic of 1100 in each direction. A database of 750,000 trucks from the Slovakian site, where the legal gross weight limit is

40 t, is used in this study. Above this limit, trucks would be expected to have special permits, but it is not possible to identify from the WIM data whether an extremely heavy vehicle has a special permit or is illegally overloaded. The recorded data were cleaned to remove unreliable observations which amounted to about 1.5% of the total.

Some extremely heavy vehicles were recorded, with the maximum gross weight being 117 t, and it is these extremely heavy vehicles that govern the maximum loading likely to occur in the lifetime of a bridge.

The traffic modelled here is bidirectional, with one lane in each direction, and independent streams of traffic are generated for each direction. In simulation, many millions of loading events are analysed, and for efficiency of computation it is necessary to use a reasonably simple model for transverse load distribution on two-lane bridges. For bending moment the maximum stress is assumed to occur along the centre line of the bridge, with equal lateral distribution from both lanes. In the case of shear stress at the supports of a simply supported bridge, the maximum occurs when each truck is close to the support, and the lateral distribution is very much less than for mid-span bending moment. In this case a reduction factor of 0.45 (Enright 2010) applied to the axle weights in one lane. This factor is based on finite element analyses performed for different types of bridge. The load effects and bridge lengths examined in the simulation runs are shown in Table 1.

Table 1. Load effects and bridge lengths

Load Effect	Bridge Lengths Considered (m)
LE1 Mid-span bending moment, simply supported	15, 35
LE2 Support shear at entrance to a simply supported bridge	15, 35
LE3 Central support hogging moment, 2-span continuous	35

Using the long run simulation philosophy, the traffic is simulated for 5000 years, assuming 250 working days per year. The output consists of yearly maximum load effects, and these can be used to calculate a very accurate characteristic values. Yearly maxima can be plotted on Gumbel probability paper, which is a re-scaled cumulative distribution function.

Sample results are shown in Figure 5 for the 5000-year simulation run. Two load effects are shown – shear (LE2) on a simply supported 15 m bridge, and hogging moment (LE3) over the central support of a two-span bridge of total length 35 m. Due to the randomness inherent in the process, there

is some variability between the results of multiple simulation runs, and this variability is most pronounced in the upper tail region. Weibull fits to the upper tail are used to smooth this variability (as shown in the Figure 5), and these are used to estimate the characteristic values.

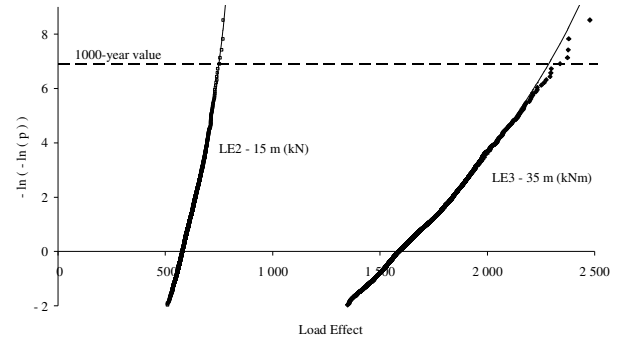


Figure 5. Yearly maximum load effects – 5000 years of simulation

### 3 COMPARISONS

The long run simulations were used to determine the yearly maximum distributions of load effect for the five load effects. This study considers only load effect. To avoid any complication arising from assumptions regarding resistance, the load effect and resistance distributions are assumed to be mirror images of each other (Figure 6), i.e., a mirror image of the load effect distribution is assumed for the distribution of resistance. By varying the line of symmetry (shown in the figure), different probabilities of failure can be considered and the corresponding safety index values and factors of safety calculated.

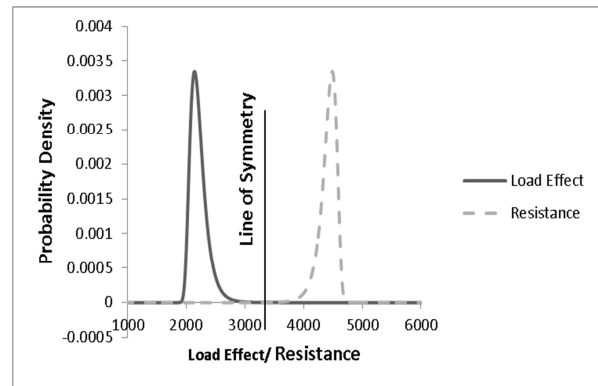


Figure 6. Load Effect and Mirrored Resistance Distribution

The factors of safety (ratio of factored resistance to factored load) are illustrated in Figure 7 for a range of probabilities of failure. It can be seen that, while greater factors of safety correspond to lower probabilities of failure, the relationship is highly non-linear. Further, for a probability of failure of  $10^{-6}$ , the factor of safety varies with load effect from 0.94 to 1.25. Taking the converse of this, for a factor of safety of 1.2, the probability of failure varies by

several orders of magnitude – from  $2.4 \times 10^{-13}$  to  $2.0 \times 10^{-6}$ .

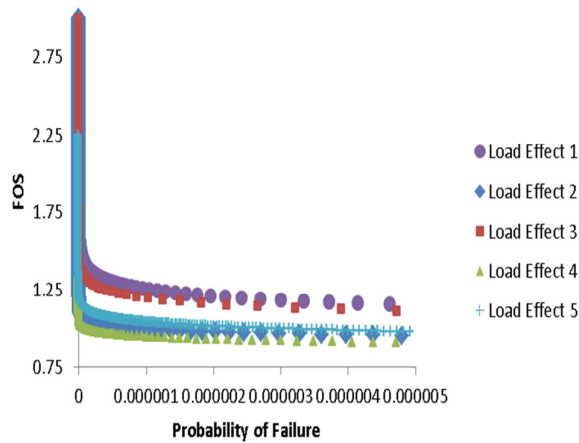


Figure 7. Probability of Failure vs Factor of Safety for different Load Effects

The safety index might be expected to be more consistent with probability of failure. In its simplest form, for a given load effect, the safety index is the inverse Normal distribution function of the probability of failure. Applying this relationship gives a safety index value of 4.2649 for a probability of failure of  $10^{-6}$ . However, as can be seen in Figure 8, the safety index at this probability varies from 2.0 to 3.3, depending on load effect. It is not surprising that the relationship between safety index and probability of failure is different from that commonly assumed though the extent of the difference is great. What is perhaps more of concern is the great variability in the safety index values for a given probability of failure.

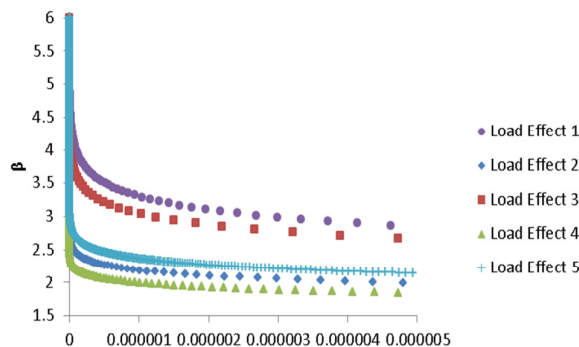


Figure 8. Probability of Failure vs Safety index for different Load Effects

Looking at the relationship conversely, for a safety index value of 3.0, the probability of failure varies from  $6.0 \times 10^{-13}$  to  $2.7 \times 10^{-6}$ . Clearly using the safety index results in quite inconsistent estimates of the probability of failure.

The relationship between safety index and probability of failure can be seen better when the latter is plotted on a double log (Gumbel) scale. This is illustrated in Figure 9. On this scale a probability of failure of  $10^{-6}$  corresponds to a point on the X-axis of -

2.63 and, as before, it can be seen that the safety index for this probability varies from 1.98 to 3.29.

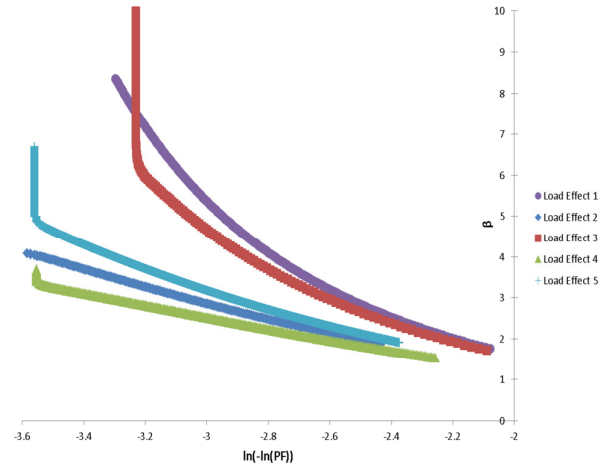


Figure 9. Probability of Failure(Gumbel scale) vs Safety index for different Load Effects

## 4 CONCLUSIONS

This paper uses Weigh-in-Motion data and elaborate Monte Carlo simulations of traffic loading scenarios to establish yearly maximum probability distributions for five different load effects. The distribution of resistance in each case is assumed to be a mirror image of the distribution of load effect. By varying the axis of symmetry, a range of levels of safety are considered in each case. A single factor of safety is, unsurprisingly, shown to correspond to a wide range of probabilities of failure. Reliability theory is also considered and beta index is shown to be little better in terms of its correlation with probability of failure.

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