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Sedimentation speed of a free dendrite growing in an undercooled melt

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Abstract

Free equiaxed dendrites in solidifying alloy melts are subjected to hydrodynamic effects as a result of gravity. The sedimentation of dendrites is one such effect and believed to be a cause of macro segregation in partitioning alloys. A novel computational model is proposed to estimate the settling speed of free dendrites at moderate Reynolds numbers. Growth of the dendrite, momentum changes, internal solid fraction evolution within a spherical dendrite envelope of changing diameter, and surface morphology of the dendrite while settling are taken into account in the development of the model. Comparison with results from a series of equiaxed dendrite settling experiments, on solidifying transparent alloy analogues to metals, shows good agreement between predicted and experimental settling speeds. The correlation between surface morphology of the dendrite which affects drag force and the physical parameters of the settling dendrite is studied. The feasibility of applying the proposed model to metallic systems is also explored and the outlook is positive.

Keywords : Casting; Solidification; Segregation; Crystal settling; Equiaxed dendrites

1. Introduction

The presence of free dendrites in the liquid melt during alloy solidification is a frequent phenomenon. These free dendrites can be initiated as heterogeneously nucleated equiaxed dendrites or as fragmented dendrite arms, perhaps from a region of columnar growth. Whatever their origin, the dendrite particles are subjected to hydrodynamic forces in the liquid metal due to the effects of gravity, such as natural convection and sedimentation - unless solidification is performed in

microgravity conditions [1]. Floating or sedimentation of these free dendritic crystals can contribute to segregation profiles in alloy castings. A cone-shaped negative segregation pattern close to the bottom of large alloy casting moulds is a typical observation. It is believed that this is due to the sedimentation of initially solidified (solute poor) solid phase [2]. Macrosegregation due to dendrite settling in undercooled alloys has been observed by numerous authors (e.g. ref. [3,4]). Also, attempts have been made to estimate the drag force and the speed of equiaxed dendrites during their settling motion in undercooled melts [5-7] using mathematical models. Ahuja et. al.[5] and De Groh et al. [7] considered the equiaxed dendrite as a porous object to calculate the drag coefficient of settling speed. But these previous models of dendrite settling [5-7] did not give consideration to the growth of the dendrites as they settled in the undercooled melt. Additionally, it was assumed that a free dendrite reaches its terminal velocity almost instantaneously or at least in a time that is negligible compared to the average suspension time in any realistic casting system [5-7]. However it has recently been shown, through experimental evidence of Badillo et al. [8] on settling of SCN-acetone dendrites in undercooled melts that sedimenting equiaxed dendrites may not reach terminal velocity. The authors of ref.[8] observed and measured continuous growth of dendrite arms during the settling; the dimensions, mass and velocity of each falling dendrite were clearly changing with time.

The proposed model presented in this contribution considers dendrite growth and changes in mass and speed during settling and is validated using the experimental results on settling of SCN-acetone dendrites from the literature [8]. The potential of the applicability of the proposed model to metallic alloys is also explored.

2. The Model

A new model is hereby proposed to estimate the settling speed of an isolated dendrite in undercooled liquid, assuming moderate Reynolds number (i.e. 1 - 100) scenarios. The Olson formula [9] was used in development of the mathematical model, which also considers the growth of the dendrite while settling.

The force balance (vertical direction) for settling of a particle can be written as,

$$\frac{d(mv)}{dt} = W - B - D \quad (1)$$

where m , v , W , B , D and t are mass, velocity, weight, buoyancy, drag force on the particle, and time, respectively. According to this basic equation, the particle considered need not move essentially at its terminal velocity but can be at any moment of its settling motion. Here mass m is also not considered as a constant.

We can write, the drag force D on a spherical moving object [10] as,

$$|D| = \frac{1}{2} C_d \rho A v^2 \quad (2)$$

where ρ , A and C_d are density, projected cross sectional area of the moving object perpendicular to the direction of movement, and drag coefficient, respectively.

The case under consideration here is associated with moderate Reynolds numbers (ie. between 1 and 100) and therefore according to Olson's formula [9], which addresses this range of Reynolds numbers, the drag coefficient C_d is given by

$$C_d = \frac{24}{Re} \left(1 + \frac{3}{16} Re \right)^{\frac{1}{2}} \quad (3)$$

where Re is the Reynolds number.

However, in our case here we are considering non-spherical dendrites and therefore we need to account for drag effects arising from the departure from sphericity [11, 12]. Therefore, we introduce a factor C_s to account for the effect of shape on drag forces and need to modify equation (2) by updating the drag coefficient so that

$$C_d = C_s \left[\frac{24}{Re} \left(1 + \frac{3}{16} Re \right)^{\frac{1}{2}} \right] \quad (4)$$

Figure 1.

As mentioned above, the dendrite is not a solid sphere but we can consider an equiaxed dendrite in a hypothetical spherical envelope [13-15] of radius r_1 equal to its primary dendrite arm length (assuming symmetry in all six directions) and with internal solid fraction of g_s . The total solid (shaded) in the equiaxed envelope depicted as figure 1(a) can convert to the equivalent fully solid sphere of radius r as shown (shaded) in figure 1(b). Here the solid volume ratio is equivalent to the internal solid fraction g_s and hence for the 3-dimensional case,

$$g_s = \left(\frac{r}{r_1} \right)^3 \quad (5)$$

The shape of the dendrite has an influence on its settling velocity. According to ref. [16] the settling velocity u_{ns} of a non-spherical (n.s.) particle can be calculated using the ratio of total surface area of spherical and non-spherical objects. Therefore, in this work ‘sphericity’, ψ is defined as,

$$\psi = \frac{A_{ns}}{A_s} \quad (6)$$

where A_{ns} is surface area of the non-spherical particle and A_s is surface area of the spherical particle of equal volume.

Calculation of the sphericity ψ of a dendrite may be a difficult task due to its very complex shape. The minimum sphericity is equal to a value of 1.0, for a perfect sphere. Calculation of surface area of a real alloy dendrite with very complex branching initially requires some basic simplification of the dendrite shape. Hence, for a three dimensional solid dendrite, the six primary arms were simplified in this work to conical shapes and then the external surface areas of the six cones were calculated in order to find ψ , as described in Appendix 1. Other researchers have employed different simplifications – e.g. orthogonal square pyramids have been used [17] to represent the dendrite arms. This merely illustrates the universality of the approach - the actual dendrite surface area is likely to be even higher due to its complex shape, and the effect of additional drag due to side arm branching from the identical conical (in this case) primary arms must be accounted for. So, the total influence on drag due to the non-sphericity of a dendrite can be incorporated into the relevant coefficient as

$$C_s = B\psi \quad (7)$$

Here, B incorporates the additional drag effects of real dendrite shapes which deviate from the six conical shape simplifications, and the factor which bridges shape and the effective drag coefficient.

Therefore, using C_d as defined, the projected area can be written using the volume equivalent solid sphere and we can calculate the magnitude of the drag force, D for the non spherical dendrite falling at speed v as follows, using equation (2) and taking substitutions from equations (4) and (5). Here, the length scale ($2 \times r_l$), which equals the dendrite diameter, is used to define Re as it is the actual dimension of the object moving through the liquid.

$$|D| = \frac{1}{2} \frac{24}{\left(\frac{\rho v \pi r_l^2}{\mu} \right)} \left(1 + \frac{3}{16} Re \right)^{\frac{1}{2}} C_s \rho \left\{ \pi \left(g_s^{\frac{1}{3}} r_l \right)^2 \right\} v^2 \quad (8)$$

where μ is dynamic viscosity of the liquid, and equation (8) reduces to

$$|D| = 6\pi \mu C_s \left(1 + \frac{3}{16} Re \right)^{\frac{1}{2}} g_s^{\frac{2}{3}} r_l v \quad (9)$$

Equation (1) converts to the following considering the derivative of the LHS and using equation (9) to expand the RHS,

$$v \frac{dm}{dt} + m \frac{dv}{dt} = V\rho_s g - V\rho_l g - 6\pi \mu C_s \left(1 + \frac{3}{16} Re \right)^{\frac{1}{2}} g_s^{\frac{2}{3}} r_l v \quad (10)$$

where ρ_s , ρ_l , g and V are density of solid, density of liquid, gravitational acceleration and solid volume of dendrite respectively. Here, assuming negligible Reynolds number change in one very small time step, Δt , it is possible to write equation (10) in discretised format as,

$$v^0 \frac{(r^1 - m^0)}{\Delta t} + m^0 \frac{(r^1 - v^0)}{\Delta t} = \frac{4}{3} \pi (r^1)^3 g_s^0 (\rho_s - \rho_l) g - 6 \pi \mu C_s \left(1 + \frac{3}{16} Re \right)^{\frac{1}{2}} g_s^{0\frac{2}{3}} r_1^1 v^1 \quad (11)$$

Here the superscript “1” denotes a new time-step value and the superscript “0” denotes the previous time-step value. In this case, the volume of the dendrite is written by considering its primary dendrite arm length r_l and internal solid fraction g_s . Re-arranging equation (11) to obtain the temporally updated speed,

$$v^1 = \frac{v^0 + \frac{1}{m^0} \left[(r^0 - m^1) v^0 + \frac{4}{3} \pi (r^1)^3 g_s^0 (\rho_s - \rho_l) g \Delta t \right]}{1 + \frac{\Delta t}{m^0} 6 \pi \mu C_s \left(1 + \frac{3}{16} Re \right)^{\frac{1}{2}} g_s^{0\frac{2}{3}} r_1^1} \quad (12)$$

where r_l at the new time step can be obtained from,

$$r_1 = r_0 + z \Delta t \quad (13)$$

r_0 being the dendrite arm length at the previous time step and z , the dendrite tip velocity, can be determined experimentally or by using one of a number of suitable dendrite tip kinetics models [e.g. 18-20]. However, the presence of the fluid flow relative to the moving dendrite should be noted here and suitable modifications are appropriate [21]. The internal solid fraction g_s^0 of the dendrite envelope can be estimated via the Scheil or lever rule approximations, or via other methods such as a solute/heat transport and balance model [14]. In the current work, g_s^0 values computed [17] from the experimental observations were used. In fact Badillo et al. [17] used two different approaches to calculate internal solid fraction for their experiments. The first involved a force balance calculation on the settling crystal. In order to avoid a circular argument (as we are also using force balances to

predict settling speeds in the current contribution), and because the authors estimated [17] an uncertainty of about 15% in such calculations, we have used their second approach to g_s determination. The latter approach was based on a model of heat and solute balances for each equiaxed crystal. In this way we have independent assessments of internal fraction solid for the experiments [8].

The mass m of the dendrite can be written as,

$$m = g_s \cdot \frac{4}{3} \pi r_1^3 \rho_s \quad (14)$$

3. Simulation Parameters

The proposed model was used to simulate the dendrite settling experiments of Badillo et al.[8].

These experiments were performed using dilute SCN – acetone alloys of varying composition. In the experimental set up, a 40 cm tall cylindrical glass column was filled the SCN–acetone alloy and inserted into a large rectangular bath with transparent walls. Small seeds, from which the equiaxed crystals were grown, were produced in-situ using a so-called ‘crystal generator’ located at the top of the 40-cm-long glass column. A small restriction near the bottom end of the crystal generator tube allowed only one crystal to enter the column inside the tank and start to sediment; all other crystals were effectively blocked. During the experiments, the location, size, shape, and orientation of the equiaxed crystals were measured as a function of time using two cameras.

Numerical implementation of the current model was performed using a computer program coded in Fortran90. The computational results predict the settling location and speed as a function of time of all eight dendrite settling experiments published in ref. [8]. Table 1 shows experimental details and SCN- acetone properties used in the computations.

Table 1.

In table 1 – C_0 , ΔT , r_1^{ini} , v^{ini} , z and T_0 are composition, undercooling, initial primary dendrite arm length, initial settling velocity, average dendrite tip velocity and temperature (in Celsius), respectively. In addition to the values above, the internal solid fraction values were obtained from ref.[17] (which, as stated, were calculated using a mathematical model of solid fraction correlated to thermal and solutal convection) for the same set of experiments. In ref.[17], the definition of dendrite envelope was geometrically different to the definition used in the current work, hence appropriate geometrical calculations are used for conversion, as outlined in Appendix 2.

The value for the constant B in each experiment was chosen to calibrate the model calculations with the settling speeds reported in ref.[8]. The resulting values of B for each experiment are given in table 2.

Table 2.

4. Results

Comparison of the predicted and the experimental variation of settling speed and location with time, for each falling dendrite, is presented in figure 2 and figure 3, respectively. According to Badillo et al. [8] location was the primary experimentally measured parameter, from which speed was subsequently calculated.

Figure 2.

Figure 3.

5. Discussion

The salient features of the proposed model include consideration of the growth of a dendrite while it is settling, computation of the varying settling speed without assuming the dendrite travels at terminal velocity, and correlation of the settling speed variation with the dendrite surface morphology. Initially, the complex shape of the dendrite was reduced to a relatively simple shape consisting of six cones. Later by adjusting a parameter B the model was calibrated to mimic the effect of the real dendrite shape and surface area. As the current model treats the solid area of the dendrite directly there is no need to consider mushy envelope phenomena - unlike most previous dendrite settling models [5,7] which treated dendrites as a porous mush. Such an approach applied to the current model would lead to double counting of surface drag effects.

For the model predictions, dendrite tip growth tip velocity z is an experimentally determined average value [8] for each of the eight experiments considered. This avoids the uncertainty of calculating z through dendrite kinetic laws, which could still be inaccurate in the presence of fluid flow. As presented in the Results section, fairly reasonable agreement was found between the model predictions and the experimental results, with some deviations. However, this discrepancy may require an examination of the physical meaning of the constant B which was used in the predictions. The meaning of sphericity ψ in this work is the surface area ratio between the simplified cone-based model dendrite and a volume-equivalent sphere. The effective drag coefficient C_d was then defined using ψ and B . In this proposed model, basically B represents two requirements,

- (i) to account for the real and projected surface area of the dendrite
- (ii) to bridge the ratio (relationship) between ψ and C_s

Pettyjohnne and Christiansen [16] and De Groh et al. [7] had proposed two different relationships between ψ and C_d using logarithmic relations for nearly similar non-spherical situations. However

in both these cases, the given relationships were related to non complex surfaces and calculation of exact and also constant surface area was straightforward. But, in the case of real dendrites, the surface is very complex in nature and the relationships given in refs.[7,16] are not sufficient. Hence the constant B was introduced to cover both requirements (i) and (ii) above. Although B for each experiment was a selected as a constant, in reality it could be varied over the settling time. Therefore, performing calculations with B as invariant with time may influence the differences between predicted behaviour and experimental observations. Also, it should be noted that any error in internal solid fraction value causes erroneous calculation of settling velocity. It is worth noting that the authors of ref. [8] mention the observation of dendrite rotation while settling in some experiments. The rotations are due to the uneven growth of dendrite arms and in such conditions the concept of a spherical dendrite envelope could be an over-simplification, and some form of front-tracking [22] or alternative approach would be needed to simulate non-symmetric growth of equiaxed envelopes. Another interpretation, however, could be that such rotations even out any asymmetry temporally.

The values used for B are indicative of relative average surface area of each dendrite. This means that the dendrite in experiment 7 should have highest surface area (largest value of B in Table 2), maybe with highly complex side branching, while the dendrites in experiments 2 and 3 should have relatively low surface area. Therefore, ideally B could be a non-constant parameter which can vary with the evolution of the real dendrite surface area. For example, the initial rapid changes in predicted settling speed (Fig. 2) are caused by the mismatch between the B value used and the initial experimental settling speed v^{ini} . The value of B would depend on many physical factors present such as solute concentration (C_0), the solid fraction, temperature and settling speed (v) etc.. Here, we consider the dependence of B on two parameters which would be expected to affect the dendrite surface morphology. Figure 4 shows the variation of B with the value of C_0/v ,

Figure 4.

A reasonably good linear correlation between B and C_0/v can be observed from figure 4. This interesting relation can be analysed in the context of solute build-up at the solid/liquid interface and interface perturbation formation to yield dendrites in alloys. From the interface stability theory of Mullins and Sekerka [23], the distance between two protrusions (wavelength) λ far from the limit of constitutional undercooling is given by [24]

where D_l is diffusivity of solute in the liquid, Γ is the Gibbs-Thompson coefficient, and ΔT_0 is defined as [24],

where k is the partition coefficient and m_{liq} is the liquidus slope. When taken together, equations (15) and (16) imply that higher solute concentration C_0 causes a lower distance between protrusions. Such small dendrite arm spacing infers more protrusions in the solid/liquid interface, thus increasing surface area of the dendrite. Therefore it suggests a positive link between C_0 and B .

According to hydrodynamic principles, a liquid boundary layer of thickness δ surrounding an object moving through liquid is related to the velocity of the object v according to the relationship [10],

$$\delta \propto \left(\frac{\mu x}{v} \right)^{\frac{1}{2}} \quad (17)$$

where x is the length of the interface in the direction of the velocity vector (simply, the length of the moving object). According to this inverse proportionality between boundary layer thickness and velocity, it is obvious that at lower sedimentation rates the boundary layer is relatively thick and lot of solute rejected (from the growing dendrite) is retained around the moving dendrite without being dissipated to the undercooled liquid. So the liquid boundary layer can move with the dendrite. Therefore, the lower the falling velocity, the higher the level of solute retained around the dendrite, increasing the solute level at the solid/liquid interface. If we again refer to equations (15) and (16) and the previous paragraph: higher solute concentration at the solid liquid interface (related to C_0) causes increased protrusions and higher surface area. So, lower settling speeds can lead to a relatively higher dendrite surface area for cases we discussed, suggesting inverse proportionality between settling speed v and the factor B . In summary, equations (15)-(17) point towards the relationship predicted in Fig. 4 by the model simulations.

Experimental determination of dendrite surface area is a very challenging task for metals. However, a recently reported technique [25], combining real time X-ray radiography and topography suggests a promising way to determine exact dendrite surface areas. For calculations of buoyancy in the current simulations, the properties of the liquid alloy between dendrite arms (interdendritic liquid) were assumed equal to those of the liquid alloy far away from the dendrite. Apart from that, if rejected solute is denser than the solidified dendrite (which has less of the relatively denser solute – e.g. certain Al-Cu alloys) then dendrites may even rise in the undercooled liquid. In such a case drag force acts in a downward direction, opposing the motion. Similar situations were observed in *in-situ* X-ray radiography experiment with Al-30%Cu alloy [26, 27] However, the X-ray radiography samples were very thin in comparison to the size of the dendrites observed. In such

conditions it is easy to have quick build-up of solute. But, this kind of build-up is also quite possible when a large number of dendrites are solidifying simultaneously and/or a significant portion of the solidifying domain is occupied by solid dendrite(s). For the numerical computation in this contribution, we did not consider any such overall concentration changes in liquid with time. As in the set of experiments taken into account for the validation, dendrite volume is very small compared to the liquid volume. Thus, we believe it is reasonable to assume that there was no noteworthy increase of the solute level in the bulk undercooled liquid in the experiments.

Table 3.

Although validation of the model is carried out using transparent analogues, the assumptions and all other considerations remain valid for metallic alloy systems. As calculated and shown in table 3, Reynolds numbers of certain metallic systems of interest are not significantly different from the scenario with the SCN- acetone (SCN-ACE) binary alloy. In all cases we assume settling velocity of 4mm/s and dendrite diameter of 3mm (typical from table 1). Reynolds numbers for such scenarios with metallic systems remain in the validity range of the Olson formula which was used to calculate drag coefficient C_d in equation (3). Therefore, it seems to be quite reasonable to apply the current model to the scenario of metallic alloy solidification in future. For example, it is feasible to integrate the dendrite settling model proposed in this contribution with the authors' recent volume averaging model of equiaxed solidification [28], to enhance the predictive capabilities under the effects of gravity of equiaxed solidification.

6. Conclusion

A novel model of equiaxed dendrite settling is presented. The model assumes moderate Reynolds numbers for dendrites moving due to buoyancy forces in an undercooled melt. Dendrite growth and internal solid fraction evolution during settling are taken into account and the surface area of the

dendrite is initially defined by using six cone shapes with a cubic base in a spherical envelope.

When this shape was refined for dendrite surface area with protrusions, a reasonable agreement was found between predicted and experimental settling speeds of several experiments conducted using SCN – acetone alloys. The discrepancies between predicted speed and experimental values in some cases are supposed to be due to the varying actual surface area of the equiaxed dendrites, which cannot be easily tracked or estimated. It also found that this model is readily applicable to metallic alloy systems such as Al-Si and Al-Cu.

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References

- [1.] S. McFadden, L. Sturz, H. Jung, N. Mangelinck-Noel, H. Nguyen-Thi H., G. Zimmermann, B. Billia, D.J. Browne, D. Voss, D. Jarvis, *Journal of the Japan Society of Microgravity Application*, 25:3, 2008, pp. 489-494
- [2.] M.C. Flemings, *Solidification Processing*, McGraw-Hill Inc, 1974
- [3.] T.E. Strangman, T.Z. Kattamis T.Z., *Metall. Trans.*, 14,1973, pp.2219-2221
- [4.] H.C. De Groh III, V. Laxmanan, *Metall. Trans. A*, 19, 1988, pp.2651-2658
- [5.] S. Ahuja, C. Beckermann, R. Zakhem, P.D. Weidman, H.C. De Groh III, *Micro/Macro Scale Phenomena in Solidification*, eds. C. Beckermann et al., ASME, New York, HTD-Vol. 218, 1992, pp.85-91
- [6.] R. Zakhem, P.D. Weidman, H.C. De Groh III, *Metall. Trans. A*, 23, 1992, pp.2169-2181
- [7.] H.C. De Groh III, P.D. Weidman, R. Zakhem, S. Ahuja, S., C. Beckermann, *Metall. Trans. B*, 24, 1993, pp.749-753
- [8.] A. Badillo, D.L. Ceynar, C. Beckermann, *J. Crystal Growth*, 309, 2007, pp.197-215
- [9.] R.B. Olson, S.J. Wright, *Essentials of engineering fluid mechanics (5th Edi.)*, Harper and Row publishers, NY,1990
- [10.] G.K. Batchelor, *An introduction to fluid dynamics*, Cambridge Uni. Press, 1967
- [11.] L. Prandtl, O.G. Tietjens, *Applied Hydro and Aeromechanics*, Ch. V. Dover Pub. Inc., New York, 1957
- [12.] W.H. Graf, *Hydraulics of sediment transport*, Water Resources publ. USA, 1998
- [13.] J.D. Hunt, *Mater. Sci. Eng.*, 65, 1984, pp.73-83
- [14.] M. Rappaz, Ph. Thevoz., *Acta Metall.*, 35, 1987, pp.1487-1497
- [15.] Ch.-A. Gandin., S. Mosbah., Th. Volkman, D.M. Herlach, *Acta Mater*, 56, 2008, pp.3023-3035
- [16.] E.S. Pettyjohn, E.B. Christiansen, *Chem. Eng. Progress.*, 44:2, 1948, pp.157-172

- [17.] A. Badillo, D.L. Ceynar, C. Beckermann, *J. Crystal Growth*, 309, 2007, pp.216-224
- [18.] M.H. Burden, J.D. Hunt, *J. Cryst. Growth*, 22, 1974, pp.109-116
- [19.] J. Lipton, M.E. Glicksman, W. Kurz, *Mater Sci. Eng.*, 65, 1984, pp.57-63
- [20.] W. Kurz, B. Gioranola, R. Trivedi, *Acta Metall.*, 34, 1986, pp.823-829
- [21.] Q. Li, C. Beckermann, *J. Crystal Growth*, 236, 2002, pp.482-498
- [22.] D.J. Browne, J.D. Hunt, *Numer. Heat Trans. B*, 45, 2004, pp.395-419.
- [23.] W.W. Mullins, R.F. Sekerka, *J. Applied Physics*, 35:2, 1964, pp.444-451
- [24.] W. Kurz, D.J. Fisher, *Fundamentals of Solidification*, Trans Tech Publications, 1992
- [25.] B. Billia, H. Nguyen-Thi, N. Mangelink-Noel, N. Bergeon, A. Buffet, G. Reinhart, T. Schenk, J. Baruchel, H. Jung, J. Hartwig, *Proceedings of TMS annual conference 2009*, p.102
- [26.] R.H. Mathiesen, L. Arnberg, L. P. Bleuet, A. Somogyi, *Metall. Trans. A.*, 37, 2006, pp.2515-2524
- [27.] D. Ruvalcaba, R.H. Mathiesen, D.G. Eskin, L. Arnberg, L. Katgerman, *Acta. Mater.*, 55, 2007, pp.4287-4292
- [28.] W.U. Mirihanage, D.J. Browne, *Comput. Mater. Sci.*, 46, 2009, pp.777-784
- [29.] P. Tin, H.C. De Groh III, *Inter. Jour. of Thermophysics*, 25:4, 2004, pp.1143-1153

Appendix 1

A 2-dimensional view of a dendrite is rendered in Figure A1 for viewing and to obtain essential dimensions for calculations.

Figure A1.

The complex shape of the dendrite is simplified to a basic non-spherical 3D geometry consisting of six cones, each placed on one of the six surfaces of a centred cube of side length $2l$. The distance from the centre of the cube to the tip of a cone is equal to the primary dendrite arm length, r_l . A cross section cut through the central axis along the centre of four of the cones is shown in figure A1.

Using the dimensions, r_l and l it is possible to calculate the total volume of the dendrite and that is equal to the solid volume within the spherical dendrite envelope of Fig. 1(b). So,

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This takes the form of a cubic equation; a function of the unknown variable l , the solution to which is required to calculate the exact surface area of the model dendrite.

-

A root of this cubic equation provides the value for the variable l . Finding roots to this equation analytically is cumbersome and therefore we use Newton's method to solve it by through numerical iteration,

Sphericity, ψ is the ratio between the solid conical surface area (of 6 cones + edges of cubic base still exposed after covering from circular base of the cone on all 6 sides) and the surface area of dendrite volume equivalent sphere of radius, r .

$$\psi = \frac{6\pi lL + 6(4l^2 - \pi l^2)}{4\pi r^2} \quad (A4)$$

where,

$$L = \sqrt{l^2 + \left(\frac{4}{3}l\right)^2} \quad (A5)$$

Appendix 2

In ref.[17] internal solid fraction was calculated for the dendrite envelope by considering six square-based pyramids, each placed on one of the six surfaces of a centred cube. The instantaneous exact shape of the envelope is defined using geometrical parameters primary dendrite arm length r_1 and half angle α . In our current work a spherical envelope for the equiaxed dendrite with primary arm length r_1 is used for the definition of internal solid fraction. Therefore, a conversion ratio of internal solid fraction between these two geometrically dissimilar dendrite envelopes, using primitive geometrical parameters, is required, in order to have a consistent analysis.

Figure A2.

-

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where, $x + y = r_l$ and $x = y \tan \alpha$ (A9)

which gives,

Therefore,

-

This gives the ratio to conversion between two different envelopes,

— —————

Values of pyramid half angle, α used for computations are listed in table A1.

Table A1.

Figure and Table captions

Figures

Figure 1. (a) Equiaxed grain with hypothetical spherical envelope (b) Equivalent solid sphere in hypothetical spherical envelope

Figure 2. Experimental and predicted settling speeds

Figure 3. Experimental and predicted dendrite location

Figure 4. Correlation, B vs. C_0/v

Figure A1. Cross section cut through the central axis along six cones model dendrite

Figure A2. Cross section cut through one central axis of pyramid envelope

Tables

Table 1. Experimental data [8] and properties [29] used in calculations

Table 2. Value B , for each simulation

Table 3. Typical Reynolds numbers for dendrites settling in different systems

Table A1. Experimental values for angle α from ref. [17]