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Metamodel-based metaheuristics in optimal responsive adaptation and recovery of traffic networks

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ABSTRACT

Different emerging threats highlighted the relevance of recovery and adaptation modelling in the functioning of societal systems. However, as modelling of systems becomes more complex, its effort increases challenging the practicality of the engineering analyses required for efficient recovery and adaptation. In the present work, metamodels are researched as a tool to enable these analyses in traffic networks. One of the main advantages of metamodeling is their synergy with the short decision times required in recovery and adaptation. A sequential global metamodeling technique is proposed and applied to three macroscopic day-to-day user-equilibrium models. Two reference contexts of application are researched: optimal recovery to a perturbation (with response times reduced by 98% with loss of accuracy lower than 1%) and adaptation under uncertainty with perturbation-dependent optimality. Results show that metamodeling-based metaheuristics enable fast resource-intensive engineering analyses of traffic recovery and adaptation, which may change the paradigm of decision-making in this field

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Resilience; System recovery; System adaptation; Metamodeling; Decision-making; System analysis

1. Introduction

Traffic networks are complex systems composed of multiple elements that are expected to perform in different operational conditions. A traffic network can be composed of multiple links, which connect origin-destination (OD) pairs that respond to a demand imposed by the network users. One of the common forms to study the operation of traffic is to use travel time or cost, which characterizes the distribution of users through the network and its efficiency. This measure can be obtained by an optimization procedure that, under ideal conditions, will distribute the users of the network through the (real or perceived) shortest or least expensive paths to travel their intended OD pair, attending to the capacities allowed in the network.

Travel time or cost is a measure of high relevance in the analysis of traffic networks. In conditions of adaptation and recovery, it has been an important indicator to measure the performance of a network and has been recurrently applied to measure its resilience (Bocchini & Frangopol, 2012; Nogal et al., 2016; Sun et al., 2020; Twumasi-Boakye & Sobanjo, 2021; Vugrin et al., 2014; Wang et al., 2016). It is therefore intrinsically related to the network performance and plays a major role in decision-making and operation of traffic networks.

As calculating the cost of traveling in a network involves optimization procedures, it can demand multiple onerous calculations; and these only increase in complexity as the analyses become more detailed (e.g., more routes are included). As a result, analyses that demand a large number of evaluations are hindered by the amount of effort that is required to complete a full evaluation of all the scenarios of network operation. Network management optimization schemes are examples of these.

One of the most effective techniques to solve limitations coming from the evaluation of expensive performance functions in traffic network is to apply metamodels to replace the traffic network analysis function. Pisano (2010); Osorio and Chong (2015) have formulated a metamodel-based approach to surrogate the simulation-based optimization of microscopic traffic networks. Ciuffo et al. (2013) applied a Kriging model in the sensitivity analysis of a mesoscopic model. Chen et al. (2014) studied the application of different metamodels, and with sequential learning, in order to surrogate the traffic cost in a mesoscopic traffic modelling problem, and prove that these can perform as accurate predictors (in the case to study road-pricing). Pereira et al. (2014) used metamodels to set bounds of error for traffic predictions, with definition on observed data. Gu et al. (2019)

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further elaborate on a tolling problem using also a Kriging model in an optimization scheme with the Efficient Global Optimization (EGO) of Jones et al. (1998). Teixeira et al., (2021a) applied metamodeling in reliability analysis of a user-equilibrium model with dependence on input random variables, showing that, despite the relatively low time for a single evaluation of a network in operation (in the order of seconds), a reliability analysis could extend to hours when thousands of network operational points are required to set a probabilistic understanding of its performance. Despite this spectrum of works and considering their large potential to surrogate complex problems (Zhao & Dong, 2021), application of metamodels in traffic network analysis has been limited.

In the particular case of recovery or adaptation, response times for decision-making play an important role. Metamodels have shown before synergy with the need to reduce response times in engineering analyses that depend on it (Kamiński, 2015). If the decision-making depends on effort-consuming modelling techniques, metamodeling becomes even more relevant. There is, therefore, an important role to be filled by metamodels in the response and adaptation of engineering systems, including traffic networks. Significant research is being performed to increase the fidelity of engineering models in order to enable progressively more accurate predictions of operation (e.g., enabling digital twinning). Higher fidelity models recurrently mean more modelling efforts and analysis time. If a change in steady conditions occurs in an engineering system, running recovery or adaptation decision-making schemes supported by such models will have slow response times (for optimal decisions, an optimization is needed which demands several evaluations of the model). However, having a metamodel paired to such model allows virtually zero-response times. In the case of traffic networks and the need for effective response times in adaptation or recovery, this feature is of interest. Additionally, metamodels allow the creation of digital twins for the network *a priori* to any event to improve decision-making responsiveness. The possibility of evaluating scenarios *a priori* in the network has been identified of relevance to inform post-disaster scenarios Liu et al. (2020).

In the present work, metamodeling is studied in a context of enabling recovery and adaptation analyses that become resource-consuming or even unfeasible in a context that limits efficient and responsive optimal decision-making processes. A sequential algorithm focused on practice and applicability is proposed to create accurate surrogates of the traffic network performance. Metamodeling is then applied to reduce the cost of optimal resource allocation in recovery, and an innovative adaptation decision-making approach is introduced in order to set optimal decisions in uncertain scenarios with roots in the metamodeling capability to perform virtually cost-free traffic performance evaluations. A Kriging model is applied in the present implementation. It is noted that the framework presented can be implemented to any type of metamodel (e.g., Neural Networks, Polynomial Chaos Expansions, or Support Vector Machines). The assumptions behind the choice of Kriging are discussed; however, it should be highlighted that depending on the shape of the function to be approximated, other models are also feasible and are of interest.

To achieve the proposed goal of discussing the application of metamodeling in adaptation and recovery, Section 2 introduces the rationale of traffic assignment, which is then applied in the reference study, Section 3 introduces the topic of metamodeling and discusses the sequential technique applied in the present work, Section 4 presents and discusses results of applications of metamodeling for adaptation and recovery of traffic networks, and in Section 5, the main conclusions of the developed work are presented.

2. Traffic assignment models

Traffic assignment models are used to estimate the traffic flows in a network. These models estimate how users select their routes depending on the traffic network conditions. Inputs of the model are usually the network topology, the link performance functions, which relates the travel time and the traffic volume on each link of the network, and the OD matrix. Based on these, the traffic model will calculate traffic volumes and the travel times for the network. There are different types of traffic assignment models. Models can be classified as static or dynamic traffic assignment models, STA and DTA, respectively, and also as deterministic or stochastic traffic assignment models. Nogal et al. (2019), for reference, compare these four models when evaluating traffic resilience. For a more detail explanation of these models, the interested reader is directed to (Sheffi, 1985) for static models and (Martinez-Pastor, 2018) for static and dynamic models.

When using traffic assignment models, a classification depending on the behavioral assumption governing route choice can be done, and two main groups can be identified (a) when users as individual elements try to minimize their own travel times and (b) when the objective is to minimize the total travel time of the network as a whole. These two approaches are usually known as (a) user equilibrium (UE) and (b) system optimum (SO).

The first approach, UE, is based on the Wardrop's first principle (Wardrop, 1952) and can be defined as 'for each OD pair, at a user equilibrium, the travel time on all used paths is equal and (also) less than the travel time that would be experienced by a single vehicle on any unused path' (Sheffi, 1985). The second approach, SO, is based on the Wardrop's second principle, which states that the average (or total) travel times should be minimized. In the context of adaptation and recovery, dynamic approaches are of relevance (Nogal et al., 2016). Dynamic equilibrium models introduce the time dimension and are based on the generalisation of the notion of Wardropian equilibrium in time: the network must be 'in equilibrium' at all moments during the design period (Dehoux & Toint, 1991). Day-to-Day (D2D) models are highlighted in the present implementation. These types of dynamic models are of interest in the context studied because they can deal with disequilibrium states that have an evolution over the time.

3. Metamodeling

Metamodels are black-box functions that allow relating a set of input variables to one or more output values using a closed form mathematical function. They appear in different forms, and their balanced accuracy-efficiency has captivated significant research interest during the last decade. Their main application resides in metamodeling of expensive-to-evaluate functions, enabling practical analyses that would be unfeasible through explicitly running simulations (e.g., optimization for problems that depend on high-fidelity, such as finite element models).

Different types of metamodels can be identified, each with its own assumptions. These assumptions define their metamodeling capability and their main limitations in performing as surrogates (Teixeira et al., 2021b). Among the different alternatives for metamodeling, Kriging models have gained particular relevance due to their capability to perform as interpolation models for highly complex performance functions. Kriging models are also relevant due to their capability to enclose uncertainty in the surrogate approximation. A surrogate of $g(x)$ built with a Kriging model, using its most fundamental form, is characterized by

$$G(x) = f(a;x) + Z(x) \text{with} \left(\begin{array}{c} f(a;x) = a_1 f_1(x) + \ldots + a_p f_p(x) \\ Z(x) = N(0, C(x)) \end{array} \right. \quad (1)$$

where $f(a;x)$ is a polynomial regression in its standard form with p ($p \in \mathrm{IN}^+$) basis trend functions $f_p(x)$ and p regression coefficients a to be defined. It is noted that other trend functions can be implemented. $Z(x)$ is a Gaussian stochastic process with zero mean, defined with basis on a covariance matrix (C) that relates generic x points by using a constant process variance (σ^2) and a correlation function $R(x;\theta)$. A Matérn correlation is applied in the present work. Any prediction for $g(u)$ in a random point u has expected value $G_\mu(u)$ and a standard deviation $G_\sigma(u)$. Like other metamodels, to define $G(x)$, a set of support points is necessary, the so-called experimental design (ED). Kriging models are also called kernel-based metamodels, where the correlation function characterizes how predictions relate to the points in the ED and that depends on a set of hyperparemeters θ to be trained. One of the particularities of being a kernel-based metamodel that uses hyperparamters (one for each dimension in the most common form) is that it allows the metamodel to perform as an approximation of both locally and globally complex functions. It is noted that this capability comes at the additional cost of tuning multiple parameters. Nevertheless, when compared with complex performance functions, after the definition of the surrogate, new predictions of the performance function are virtually effortless.

In the present implementation Kriging models are applied due to their capability to act as a robust surrogate without any prior assumption on the function to surrogate (e.g., performing even in highly non-linear functions (Teixeira et al., 2020)). Moreover, their intrinsic uncertainty characteristics are used to sequentially filter new points to enrich the ED.

It is noted that if the goal is to build a metamodel using a traffic network performance measure, it is possible to use a spectra of models. In some applications, a simpler model may suffice the analysis, and applying a kernel-based method will not add any benefit, instead it will increase the effort and time required to perform the analysis. When addressing metamodeling for traffic networks, there is a need to understand the form of the network's performance response. It, alongside the range of operability, will define the best metamodeling approach. With regard to the travel cost (and setting a surrogate of it), networks are commonly described by a highly non-linear behaviour where capacity reduction leads to progressively larger increases in travel cost. This indicates that kernel-based methods, which are characterized by their capability to approximate highly non-linear functions, are of interest. In the definition of any metamodel, the ED will have the most influence, and an effective ED approach will result in an accurate prediction of the traffic network performance.

3.1. Metamodeling and sequential experimental design

One of the techniques that has proven to be more effective for the development of accurate metamodels is that of introducing sequential ED. Different methods have been developed recently with great efficiency to set sequential ED for metamodeling problems (Kleijnen, 2017; Teixeira et al., 2021b). The idea of sequential ED is that of an iterative enrichment of a pre-established ED in accordance with a measure of accuracy or improvement, which frequently evaluates the metamodel capability to surrogate the problem in-hand. Sequential enrichment is efficient because it allows the set up of an ED sample of a size strictly necessary according to a notion of improvement to have an accurate model.

If the goal is to set accurate mappers of the network performance ($f(x)$) based on criteria of adaptation and recovery, a global description of the network performance (for the measures studied) is required, and the ED needs to be selected in order to enable this global description.

In the context of generating ED, different sampling techniques can be applied, with the most fundamental example being the application of Monte Carlo sampling (MCS). Despite being global and of simple implementation, MCS has limitations in the creation of efficient ED samples. As a result, several techniques have emerged as an alternative to MCS in the creation of efficient global representative samples for ED, such as latin hypercube sampling (LHS) or low-discrepancy samples. Low-discrepancy samples are sequences that have global properties in accordance with the joint density of the ED. As a result, these have synergy and are well-suited for the idea of creating a sequential ED. Low-discrepancy samples not only are exploratory in the ED space but also capture the performance function locally. They have been shown before to perform as global experimental designs in different contexts of application (Tsvetkova & Ouarda, 2019). LHS can also be also implemented as a sequence with refinement of the variable density partitions at additional complexity in their implementation (Blatman & Sudret, 2010). It is noted that other sampling techniques have been created in the context of efficiently using ED points, and optimal ED have been investigated before in the context of sensitivity (Fajraoui et al., 2017). Nonetheless, low-discrepancy samples are expected to offer a balance between efficiency and complexity in achieving a sequential ED. These appear in different forms, such as Dalton or Sobol sequences. The latter is applied in the present research due to its robust capability to produce global balanced samples for the ED. Further discussion of these is presented in Burhenne et al., (2011); Sobol (1998); and Sobol et al. (2011).

The sequential implementation proposed uses a low-discrepancy sequence to define the ED for a Kriging surrogate of the performance function for the traffic network, $f(x)$. $S = [s_1, s_2, , s_N] \in \mathbb{R}^d$ being a low-discrepancy sequence of size N in a d-dimensional space (representing the number of support variables), with N being the ED budget and with $S_h \subseteq S$ being also a low-discrepancy sequence $S_h = [s_1, s_2, , s_h]$ of size $h \leq N$. Then, an ED sequence based on $X_h = [X_m, , X_N] \in \mathbb{R}^d$ can be defined where each sample X_{h_i}indexedtoaniteration$i = [1, , n(h)]$ and $h = [m, , N]$ is a low-discrepancy sequence $S_{h_i} \subseteq S$. With $\mathbb{M}_i(x)$ being the metamodel built on the ED sample $\hat{X}_{h_i} \subset X_{h_i}$, then any sequence $h = [m, , N]$ can be used to search for the minimum ED in S that holds an accurate $\mathbb{M}(x)$, accordingly to a defined criterion. Approximation accuracy is evaluated on the capacity of $\mathbb{M}(x)$ to approximate $f(x)$.

If a $\mathbb{L}$ loss function is defined in a complementary point $x_k \notin X_{h_i}$ i given by

$$\mathbb{L}_k = \mathbb{M}(x_k) - f(x_k) \tag{2}$$

then an estimation of accuracy can be built with dependence on this loss function. If a large enough K number of x_k points is applied, and x_k this loss has relatively small values or does not exceed a defined accuracy limit, then $\mathbb{M}(x)$ can be assumed to approximate well $f(x)$. This is the principle of cross-validation.

Therefore, any ED $\hat{X}_{h_i} \subset X_{h_i}$ in the sequence $h = [m, , N]$, with respective $\mathbb{M}_i$ can be evaluated such that,

$$\text{cv}_{a_i} = \max\left(\frac{|\mathbb{L}_k|}{f(x_k)}\right) k = 1, , K \cup \text{cv}_{s_i} = \frac{1}{K}\sum_{k=1}^{K} \mathbb{L}_k^2 \tag{3}$$

where both the cross-validation errors ($[\text{cv}_{a_i}, \text{cv}_{s_i}] \in \text{cv}_i$) measure the accuracy of the $\mathbb{M}_i$ surrogate.

If $\text{cv}_{1,,j} = [\text{cv}_{i-j+1}, , \text{cv}_i]$ are j cross-validation errors in successive i iterations (in the form of cv_a or cv_s) built on the ED subset sequences supported by the samples $X_{h_{i-j+1,,i}}$, then if

$$\text{cv}_{1,,j} \leq \varepsilon \quad [\text{cv}_{i-j+1}, , \text{cv}_i] \tag{4}$$

is true, where ε is an evaluator of accuracy, then $\mathbb{M}_i$ is assumed to be an accurate metamodel of $f(x)$. The sequence h sets the support sample increment to define the ED which occurs in units, or larger increments of support points, and $[i - j + 1, , i]$ evaluates the stability of cv_i in the last j iterations. ε is a measure of error. cv_a evaluates maximum relative error. cv_s relates to the squared error. It is noted that other measures of accuracy can be defined using this same principle, e.g., corresponding to a mean value of its response in $f(x)$

and/or squared errors. The synergy of this approach with Kriging, using a quantifier of relative error, is that it allows a filtered selection $\hat{X}_{h_i} \subset X_{h_i}$ of the points used to enrich the metamodel based on the intrinsic measures of uncertainty provided by the metamodel. Hence, only the points in X_{h_i} which prediction in $\mathbb{M}_{i-1}$ fulfills

$$\frac{Z\mathbb{M}_{i-1}^{\sigma}}{\mathbb{M}_{i-1}^{\mu}} \geq \varepsilon \tag{5}$$

are included in $\hat{X}_{h_i} \subset X_{h_i}$. Z relates to the confidence and uncertainty in the present prediction as defined by the standard normal distribution, taken as 3.291 for a 99.9% confidence, and that tunes the sensitivity to enclose new points in the $\hat{X}_h$.

If cv_a is applied with $\varepsilon = 0.05, Z = 3.291$, and $j = 2$, then convergence is achieved when the maximum loss in cross-validation is below 5% in relative error, where enriching $\hat{X}_{h_i}$ only considers points in X_{h_i} that have less than 99.9% confidence in fulfilling this condition, and considering the stability of the prediction by using the last two successive values of h with reference to i. As the intent is to use large samples to create the ED, then a large confidence level should be used in order to ensure that all necessary points are added to the ED, such as 99.9%. Pursuing very large accuracy, in particular, in high-dimensional spaces, should consider the fact that each new dimension adds complexity of space exploration. In most circumstances and considering computational efficiency, $j = 1$ is of interest. Larger values of j can be used in order to ensure that robustness in the cv error, hence, in the ED size, has been achieved.

It is noted that cv_a is a constrained measure; and such characteristic should be accounted for in the implementation, in particular, when addressing high dimensional problems and when the performance function of the network becomes complex. The cv is expected to converge to a threshold in respect to a pair metamodel-function, which is useful to select the convergence parameters and select the most appropriate metamodels (Teixeira et al., 2021a; Viana et al., 2009). Furthermore, as Kriging is a kernel based interpolation metamodel, it is well suited to approximate complex functions, and it is expected to be functional with the constrained measure cv_a. For costly traffic models, the leave-one-out error (Teixeira et al., 2021a) or k-fold cross-validation (Xiao et al., 2018) can be applied instead of using an additional sample x_k.

The following sequence describes the procedure to create the metamodel used to map the traffic network:

(1) Use a low-discrepancy sequence of size N to set the support sample to create the ED X_{h_i}, with $h = [m, , N]$, which define the ED size in each iteration. The computational budget, N, should be set to a large number. Additionally, set a sample $x_k \notin X_N$ (a minimum size of LHS sample of $3n(x)$ is applied and assumed to provide a balanced space-filling coverage of the output space (Iman & Helton, 1988; Manache & Melching, 2008; Teixeira et al., 2019).
(2) If $i = 1$ the algorithm is started with $h_i = m$ and $f(x)$ is evaluated on the X_m sequence of points, at $i = 1$ the ED is $[\hat{X}_{h_1} = X_m; f(X_m)]$. If $i|i > 1$ iteration, predict X_{h_i} with $\mathbb{M}_{i-1}$ and only perform true function evaluations $f(X_{h_i})$ for the X_{h_i} points that fulfill Equation (5). Enrich the ED $\hat{X}_{h_{i-1}}$ with these to obtain $\hat{X}_{h_i}$.
(3) Fit the metamodel (Couckuyt et al., 2014) to the ED defined $\hat{X}_{h_i}$ and respective true function evaluations and predict the cross-validation sample. Estimate the maximum $\mathbb{L}$ accordingly to cv_a, or cv_s in case it is alternatively applied. All the variables should be transformed into the standard normal space for the metamodel to be fit, which was identified to allow accurate modeling results. Lower and upper limits for the standard normal space should be defined for any finite variance variables.
(4) While there is computational budget available, evaluate the accuracy criterion using ε. While $i < n(h)$; if in j values of i, the convergence criterion is fulfilled then the surrogate is assumed to be accurate and the sequential enrichment is completed; otherwise, return to Step 2 and proceed with $i = i + 1$.

The relevance of using a metamodel in the present implementation is highlighted in the following section in four representative examples.

4. Application in adaptation and recovery of traffic networks

The previous section discussed the relevance of metamodels in order to enable the analysis of resource demanding problems, and in particular traffic networks, using relatively simple iterative schemes. In the present section, it is discussed how these can be implemented to facilitate time-consuming analyses. In the case of adaptation and recovery, response-times play a significant role in efficient decision-making. The present implementation is inspired by works that enable the decision-making schemes through the usage of optimization schemes, such as (Bocchini & Frangopol, 2012; Liu et al., 2020; Vugrin et al., 2014); however with particular focus on how metamodels facilitate the application of these methodologies.

In order to solve the traffic assignment problem in the reference examples, an equilibrium model is applied. For the mathematical description of the equilibrium model, let us consider a connected traffic network with set of nodes N and set of links A. For certain OD pairs of nodes, $pq \in D$, where D is a subset of N x N, connected by a set of routes R_{pq}, there are positive demands d_{pq} which give rise to a link flow pattern $v = (v_a)_a \in A$, and a route flow pattern $\mathbf{h} = (h_{pqr})_{r \in R_{pq}, pq \in D}$, when distributed through the network. Furthermore, for each link a, there is a positive and strictly increasing travel cost function c_a.

Mathematically, this equilibrium state can be expressed as an optimization problem for each time interval t, that is

$$\underset{\mathbf{h},v,\rho_{\mathbf{r}}}{\text{Minimize}} \sum_{a \in A} C_a(v_a(t)) \tag{6}$$

subject to

$$\sum_{r \in R_{pq}} h_{pqr}(t) = d_{pq}(t) \qquad pq \in D \tag{7}$$

$$\sum_{pq \in D} \sum_{r \in R_{pq}} \delta_{apqr} h_{pqr}(t) = v_a(t) \qquad a \in A \tag{8}$$

$$h_{pqr}(t) = \rho_r(t) h_{pqr}(t - \Delta t) \quad r \in R_{pq}, \quad pq \in D \tag{9}$$

$$|\rho_r(t) - 1| \leq \alpha \qquad r \in R_{pq} \tag{10}$$

$$h_{pqr}(t) \geq 0 \qquad r \in R_{pq}, \quad pq \in D \tag{11}$$

with

$$\delta_{apqr} = \begin{cases} 1, \text{if route } r \text{ from node } p \text{ to node } q \text{ contains arc } a; \\ 0, \text{otherwise,} \end{cases}$$

and

$$t = 0, , 1, T$$

where $C_a(\cdot)$ is the integral of the travel cost function. α limits the user daily adaptation capability. Further details on this model are discussed in Nogal et al. (2016).

In the present work, three reference networks are applied to validate the results of the proposed methodology. The Nguyen-Dupuis traffic network is applied to discuss implementations on a simple state-of-art network that has been widely used in research; then the analysis is extended to the Sioux-Falls and Cuenca networks in order to validate the usage of metamodels in the analysis of more complex traffic models. The Bureau of Public Roads (BPR) cost function is applied to model the relationship between link service capacity, the demand and travel time in each link, with BPR parameters $\alpha = 0.263$ and $\beta = 6.869$ (Mtoi & Moses, 2014) in The Nguyen and Sioux-Falls networks, and $\alpha = 1$ and $\beta = 3$ Martinez-Pastor (2018) in the Cuenca network, determining the shape of this function in the examples studied in the present work. Constant values of free-flow velocity are considered in all networks (80 km/h in the Nguyen-Dupuis, 120 km/h in the Sioux-Falls and 40 km/h in the Cuenca network).

4.1. Nguyen-Dupuis and Sioux-Falls networks recovery with temporal perturbation scenarios

For the present representative example, it is assumed that an external perturbation damages the set of links $A_d \subseteq A$ in the traffic network in a percentage d_a of their capacity C_a, during T consecutive days. Then, a number n_{tm} of teams with an individual capability to restore a percentage r_a of C_a per day are deployed in order to restore the affected links' capacity and mitigate the effects of the perturbation. This problem setting can be solved using a bi-level optimization (Sinha et al., 2017) where the minimum daily traffic disruption is aimed, at using as decision variables $\delta_a \in \mathbb{N}_0^+$ that sets the number of teams allocated at each link a in time t. The optimization problem is formalised as a bi-level optimization in t that uses as lower-level the optimization previously described and is denoted as follows

$$\underset{\delta_a,\rho_r,\mathbf{h},v}{\text{Minimize}} \sum_{a \in A} C_a(\delta_a(t), \rho_r, \mathbf{h}, \mathbf{v}) \tag{12}$$

s.t.:

$$C_{\text{a}}^*(t) + C_{\text{a}} r_a \delta_a(t) \leq C_{\text{a}} \qquad a \in A_d \tag{13}$$

$$\sum_{a \in A_d} \delta_a(t) = n_{tm}, \qquad t \in [1, , T] \tag{14}$$

$$\rho_r, \mathbf{h}, v \in \Psi(\delta_a) \tag{15}$$

and

$$\delta_a(t) \in [0, \ldots, n_{tm}] \tag{16}$$

$$C_{\text{a}}^*(t) = (1 - d_a)C_{\text{a}}, \text{if} t = 1, \text{otherwise}, C_{\text{a}}^*(t) = (1 - d_a)C_{\text{a}} + \sum_{t^*=1}^{t-1} C_{\text{a}} r_a \delta_a(t^*) \tag{17}$$

with $\Psi(\delta_a)$ being a set-valued mapping in t

$$\Psi(\delta_a) \in \underset{\mathbf{h},v,\rho_{\mathbf{r}}}{\text{argmin}} \sum_{a \in A} C_a(v_a(t), \delta_a) \tag{18}$$

subject to the constraints (7)–(11).

In order to solve this problem, an optimization scheme is applied enclosing an integer Genetic Algorithm (GA) that searches for the solution D_R that jointly with the lower-level optimization problem of Equation (18) (solved using a continuous non-linear optimization) minimizes the travel time. It is noted that operation management has been one of the main areas of application of GA (Katoch et al., 2020). GA is a meta-heuristic optimization method that is known to provide *near*-optimum solution to highly complex optimization problems within reasonable computational time. In this case, the choice of this selection of meta-heuristics is mainly justified because of the discrete nature of the decision variable and the computational complexity that the nested optimization problem involves. Hence, this procedure is costly to run and as a recovery or adaptation decision-making scheme is well suited for a near-optimal heuristic solution (practicality of the results is relevant). It is noted that the nature of metamodeling also facilitates a parametric evaluation of the optimization. Examples of such synergy in GA with a similar application problems are discussed in (Bocchini & Frangopol, 2012; Kammouh et al., 2021).

It is noted that other optimization techniques can be applied to solve this problem, and a comparison of these is not the aim of the present work. Similarly, the optimization proposed can be easily transformed in a single step allocation of $\delta_a(t)$ t, however, in the present implementation, it was identified that partitioning it in smaller optimization problems that merge the upper and lower level optimization in t resulted in a more efficient implementation. In the implementation, Matlab®'s *ga* function is applied, when not stated otherwise, using default parameters.

4.1.1. Example I: Nguyen-Dupuis recovery decision-making in perturbation scenario

In the first reference example, applied to the Nguyen-Dupuis network, it is assumed that seven links are damaged in $d_a = 50\%$ of their capacity, during 4 days, $T = 4$. Then, $n_{tm} = 4$ teams with a recovery capacity per team $r_a = 12.5\%$ of link capacity per day are deployed during T in order to restore and mitigate the effects of this perturbation in the traffic network. It is noted that this might not guarantee the total recovery; nevertheless, it implicitly considers a typical budget restriction.

The Nguyen-Dupuis traffic network used in the present example consists of 13 nodes, 38 links, 66 routes, and 34 OD pairs, see Figure 1(a) for a graphical description with the link numbers included adjacent to each link. C_a is assumed to be the same for all links and has the value of 80 users/hour. In the present example, users are assumed to have ideal adaptive capability (no bounds in ρ_r). Table 1 presents the OD pairs considered in the present example, with respect to routes and their demands.

The location of the damaged links is presented in Figure 1(a) – red arrow links with link number highlighted in red. Figure 1(b) presents the results for definition of a surrogate of the total travel time (C) in the Nguyen-Dupuis with dependence on the damaged links.

The convergence of the sequential metamodel is presented for a value of $j = 10$. A LHS sample of 50 points is applied to evaluate cv_a, which uses a value of $\varepsilon = 0.05$. It is possible to infer that cv_a is stable below the value of ε, even when relatively low computational time is used, i.e., 313 s (and where most of the time is spent evaluating the network function). The built metamodel has a maximum absolute error in a cross-validation of the LHS sample of approximately 4.3%, and only demands 35 $f(x)$ evaluations to be built. After it is defined, the metamodel is expected to predict accurately any scenario of damage in these seven links, individually and combined, of up to 50% in their initial capacity. It is noted that the metamodel is defined in a region of interest for prediction,

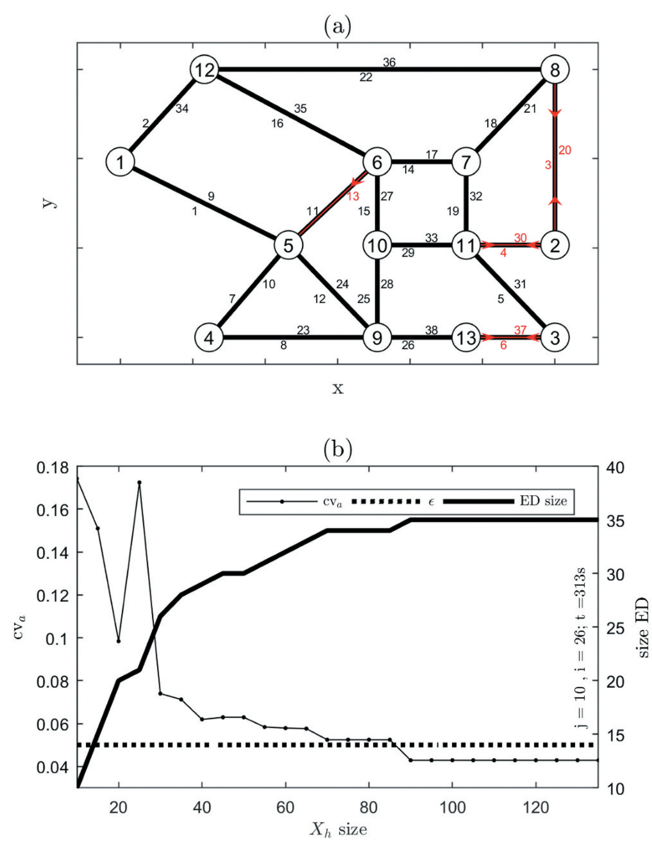

Figure 1. Results of convergence (b) for the creation of a Kriging surrogate of the travel cost with an ED that uses seven links accordingly to the case specified in (a). A LHS sample of 50 points is applied. Note: The size of the ED should be read on the right y axis and cv_a on the left y axis.

Table 1. OD pairs and routes used in the reference example of the Nguyen-Dupuis network. Demand in the OD pair is measured in average users per hour during daily hours, denoted here in daily average hourly traffic (DAHT).

OD (nodes)	Route links	Demand (DAHT)
1 → 2	1–11-14-18-20; 2–35-14-18-20; 2–36-20	75
1 → 3	1–11-14-19-31; 1–11-15-29-31; 1–12-25-29-31; 1–12-26-37; 2–35-14-19-31; 2–35-15-29-31	150
1 → 8	1–11-14-18; 2–35-14-18; 2–36	75
1 → 5	1	25
1 → 12	2	25
2 → 1	3–21-17-13-9; 3–21-17-16-34; 3–22-34	75
2 → 4	3–21-17-13-10; 3–21-19-33-28-23; 4–33-28-23	75
2 → 8	3	25
2 → 12	3–21-17-16; 3–22	50
3 → 1	5–32-17-13-9; 5–32-17-16-34; 5–33-17-16-34; 5–33-27-13-9; 5–33-28-24-9; 6–38-24-9	150
3 → 4	5–33-28-23; 6–38-23	50
3 → 11	5	25
3 → 12	5–32-17-16; 5–33-27-16	50
4 → 2	7–11-14-18-20; 8–25-29-30; 8–25-29-32-18-20	75
4 → 3	8–25-29-31; 8–26-37	50
4 → 8	7–11-14-18; 8–25-29-32-18	50
8 → 1	21–17-13-9; 21–17-16-34; 22–34	75
8 → 4	21–17-13-10; 21–19-33-28-23	50
8 → 12	21–17-16; 22	50
9 → 1	24–9	25
9 → 13	26	25
10 → 5	28–24	25
10 → 11	29	25
11 → 3	31	25
11 → 6	33–27	25
11 → 7	32	25
11 → 10	33	25
12 → 1	34	25
12 → 2	35–14-18-20; 36–20	50
12 → 3	35–14-19-31; 35–15-29-31	50
12 → 6	35	25
12 → 8	35–14-18; 36	50
13 → 3	37	25
13 → 9	38	25

nonetheless, in practice, it can be defined to predict even larger ranges of operation while maintaining the ε threshold of accuracy in the fitting.

In order to compare the efficiency of proposed approach, average results for the optimization using a GA to set the recovery strategy for ten different seeds are presented in Figure 2. The perturbation introduced in the links previously referenced (Figure 1(a)) increases the total travel time in the network by approximately 35% for 4 days. Using an efficient allocation of the recovery teams, it is possible to reduce the average increase in travel time for these 4 days to approximately 10% (with a loss of 0.5% in the average metamodel GA prediction, and 1.04% in maximum loss of daily prediction, in day 4, for the best known solution). In a practical context, there are significant benefits in reducing by 25% its increment due to the perturbation event. In the present case, the GA was able to converge to a consistent solution within a small number of random seeds. The results for the meta-heuristic solution are similar for the optimization that uses the metamodel and network evaluations, however, using the Kriging model the GA is solved in 0.02% of the time. With the 313 s used to establish an accurate metamodel this value increases to approximately 1.8% in total time. It is possible to infer that the true prediction from the metamodel solution (see the last bar of Figure 2(a)), evaluated in the model and obtained using GA and the metamodel, is very approximate to the full GA optimization solution. The Kriging capability to accurately map the performance function is key to the obtained results.

Figure 3 presents the results for best known team distribution for both the meta-GA and GA. It is possible to infer that in Day 1 and Day 2, the meta-GA provides the same solution as the full GA assessment that uses the equilibrium model. In the last two days, the allocation of teams diverges. Analysis of the daily evolution shows that most of the loss in travel time is recovered in the first 2 days, whereas in the last 2 days, interventions are mainly concerned with recovering less than 5% in travel time, a region where the metamodel as defined is more

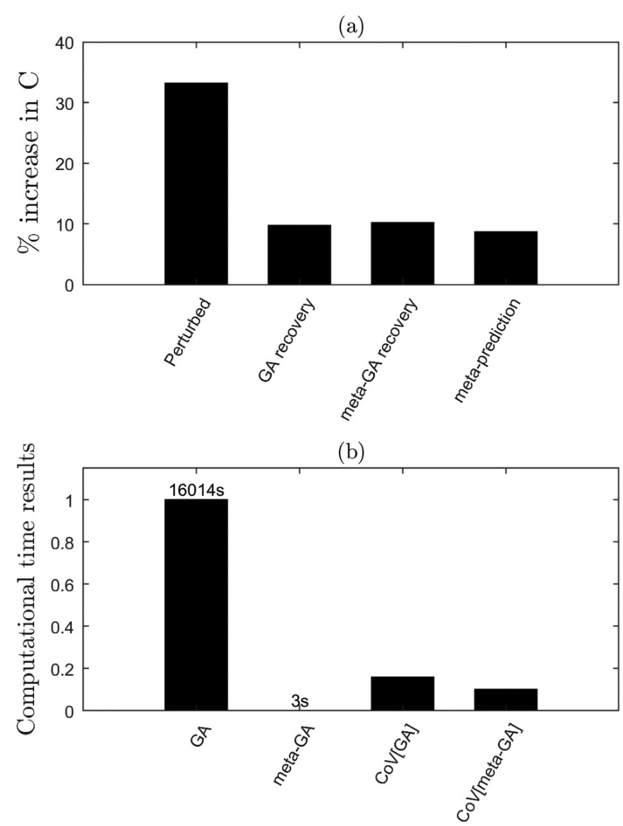

Figure 2. Average implementation results for recovery using 10 random seeds (with the same seed being used to solve the GA) and the proposed approach. (a) Average value of loss in travel time during the perturbation. (b) Comparison of computational time for the GA implementation. CoV – Coefficient of variation. Performance of the network is evaluated as the increase in percentage in relation to the non-perturbed travel time C. The GA is halted when 20 generations show no improvement in the penalty function, using a population size of 100.

likely to provide different results. Nonetheless, in this region, losses of travel time are relatively negligible when compared with the impact of recovery decisions in the days 2 and 3, with a maximum of 1.04% in day 4.

Links 3 and 20 are highly important for the network as defined. While 3 initiates all routes from node 2. Link 20 is part of multiple routes that have limited redundancy in two OD pairs. It is noted that in the surrogate definition a more or less conservative criterion can be used depending on the accuracy pursued and complexity of the problem in-hand by adjusting the accuracy evaluation parameters.

It was highlighted that the interest of metamodeling is particularly relevant in applications where user response has significant importance (Kamiński, 2015), which is the case of decision-making for recovery. Metamodeling also allows the mitigation of the influence of inherent limitations imposed by some stochastic optimization algorithms, such as GA. It allows a fast analysis of the results at limited cost, being capable of defaulting its stochastic characteristics. This aspect of the implementation is particularly relevant since the performance of optimization algorithms is parametric, see with reference to GA, the discussion of Beasley et al. (1993).

4.1.2. Example II: Sioux-Falls recovery decision-making with restricted user adaptation

A similar approach is implemented in the Sioux-Falls network, graphically represented in Figure 4.

The Sioux-Falls is representative of an implementation in a network with larger complexity. It includes 76 links and 24 nodes. In the present example, 14 OD pairs are considered in the modeling, each comprising 4 routes, see Table 2. The capacity of each link is set

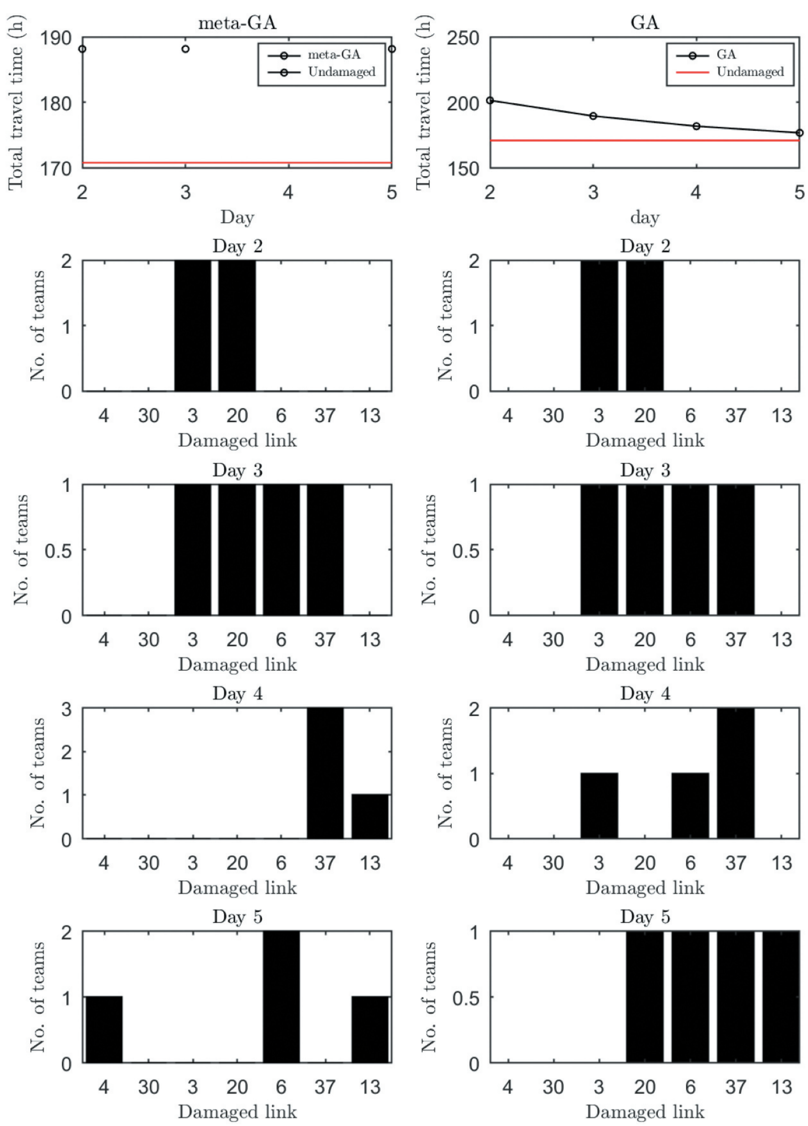

Figure 3. Daily changes in travel time for best known prediction and the results for the number (No.) of teams allocated to each damaged link in order to set the repair strategy.

initially to be 50% larger than the OD pair demand, with a value of 300 users/hour. User capability to adapt their travel choices in day t is accounted using an $\alpha = 0.8$, and compared with the case of no restrictions in adapting capability. The implementation using this network is a reference of the methodological application to a more involved example.

In order to study recovery, eight teams have been considered, $n_{tm} = 8$, with a recovery capacity per team of $r_a = 12.5\%$. The damage level is $d_a = 50\%$ of the link capacity starting at day 2 and lasting 7 days with 16 damaged links. The links damaged in the present example are 1 to 12, 14, 15, 31 and 35; see Figure 4. This simulates a loss of capacity in the upper region of the network.

Cross-validation of a random LHS sample of 100 points simulating scenarios of damage is presented in Figure 5. The final metamodel is fitted using the travel time of the perturbed network with consideration of user adaptation restriction from steady conditions. It fulfills the halting criterion with $\varepsilon = 0.05$ at an ED of 790 points and with an average absolute relative error of approximately 1.21% and a maximum of 4.96% in approximating this LHS sample. Results for the decision-making scheme of team allocation for recovery are presented in Figure 6. It is assumed that in day 2 ($t = 2$) the perturbation is introduced. Due to users' limited adaptivity a significant increase in the total traveling time in the network is experienced. If no action is taken this value will progressively decrease considering the user's capability to adapt

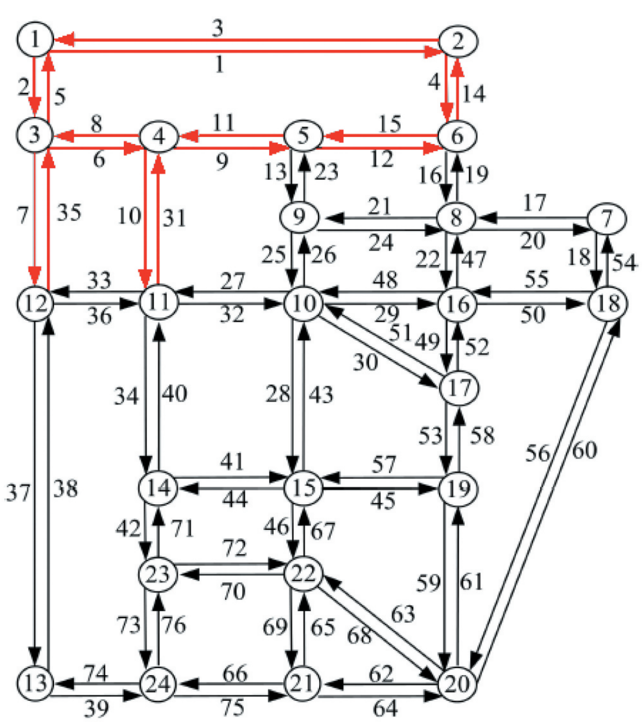

Figure 4. Sioux-Falls network, represented by nodes and links.

Table 2. OD pairs and demands used in the reference example of the Sioux-Falls network. Demand in the OD pair is measured in average users per hour during daily hours denoted here in daily average hourly traffic (DAHT).

OD pairs (nodes)	OD routes links	Demand (DAHT)
OD 1–18	2-6-10-32-29-50; 1-4-16-20-18; 2-6-9-13-24-22-50; 2-7-36-34-42-72-68-60	200
OD 1–18	2-6-10-34-42-72-68; 1-4-16-22-49-53-59; 2-7-37-39-75-64; 2-6-9-13-25-29-50-56	200
OD 2–13	4–15-11-8-7-37; 4–16-22-49-53-59-62-66-74; 3-2-6-10-34-42-73-74; 4–16-21-25-27-33-37	200
OD 3–21	7–37-39-75; 6–10-34-42-72-69; 6-9-13-25-28-46-68-62; 5-1-4-16-22-49-53-59-62	200
OD 6–12	15–11-8-7; 16–22-48-27-33; 14-3-2-7; 16–22-48-27-33	200
OD 6–24	16–22-49-53-59-62-66; 15–13-25-28-46-70-73; 14-3-2-7-37-39; 15–11-10-34-42-73	200
OD 10–19	30–53; 28–45; 29–49-53; 27–34-41-45	200
OD 13–4	39–76-71-40-31; 38–35-6; 39–75-65-67-43-26-23-11; 38–36-31	200
OD 13–8	39–76-71-41-43-26-24; 38–36-32-29-47; 39–75-64-60-54-17; 38–35-6-9-12-16	200
OD 13–18	39–75-64-60; 38–36-32-29-50; 39–76-71-41-43-26-24-20-18; 38–35-6-9-12-16-22-50	200
OD 18–3	55–48-27-33-35; 54–17-19-15-11-8; 55–47-19-14-3-2; 56–63-67-44-40-31-8	200
OD 18–23	55–48-28-46-70; 56–62-66-70; 54–17-21-25-27-34-42; 55–49-53-57-44-42	200
OD 18–23	65–67-44-40-31; 64–61-58-51-26-23-11; 66–74-38-35-6; 65–67-43-27-31	200
OD 21–5	63–67-43-26-23; 61–58-52-47-19-15; 62–65-70-71-40-31-9; 60–54-17-21-23	200

until a steady state is reached. Alternatively, if a recovery strategy is set, it is possible to infer that in 4 days the travel time is recovered to less than an 1% increase from the reference travel time.

The metamodel fitted is able to predict accurately a recovery strategy for the problem in-hand that has a performance similar to the application of GA combined with evaluation of the network performance function. Losses in all predictions in t are relatively small when compared with the GA that uses the true function. Moreover, this example shows that adequate recovery decision-making is highly dependent on the users capability to adapt. This capacity is restricted by users' lack of knowledge about the traffic conditions and other user behavior. In the present example, if decisions are based on steady conditions – i.e. all users are able to fully adapt, the recovery is less efficient (curve of GA-steady). This lower efficiency is more evident in the first days of recovery from when the perturbation impacts the network, where only a limited cumulative number of users will be able to set new route choices.

Figure 7 presents team allocation results for the most efficient recovery strategies found with both restricted user adaptation and assuming that users have full adaptive capabilities (i.e. if a decision-making procedure would use the meta-GA results). A major difference that was identified in the recovery solutions is the

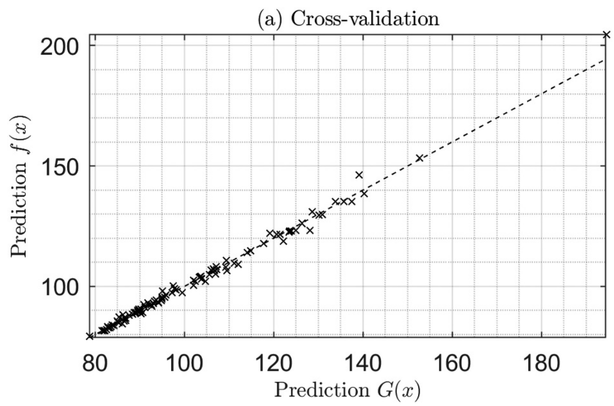

Figure 5. Cross-validation sample prediction provided by the fitted metamodel.

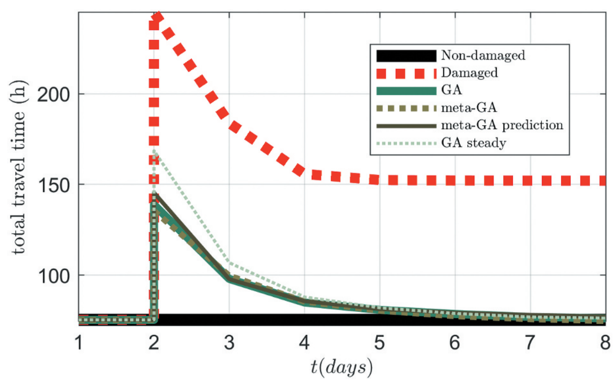

Figure 6. Recovery results from decision-making in the Sioux-Falls network with 16 damaged links at d_a = 50%, considering the damage scenario described. The GA is halted when 50 generations show no improvement in best penalty function value. Twenty random population sizes between 30 and 120 individuals are applied.

change in allocation from link number 2 to 31. Link 31 has, in steady conditions, more users than link 2 (approximately 35% more). However, when full adaptation is possible, most of these users are able to select alternative routes that are not damaged and are still competitive. Whereas, link 2 is located in an area of high damage (i.e. adjacent links are also damaged) and due to this, when a perturbation is in place, users remain in link 2, as there are no competitive alternatives. When adaptation is restricted, the steady behaviour of link 31 cannot be replicated, and only few users leave it. This exacerbates its importance in recovery, and as a result there is interest in allocating teams to recover this link in day 1. This change of allocation in day 1 results in a reduction of 37 hours in total travel cost for this day. The fact that users have limited capability to adapt changes the dynamics of the recovery procedure, and the metamodel can be trained to capture this aspect of the performance.

One of the possibilities that such mapping strategy opens in the selection of a recovery strategy is that of including uncertainty in the approximation, which facilitates the analysis of *ad-hoc* decision-making to a particular perturbation. This is one of the main motivations of metamodeling. Furthermore, Liu et al. (2020) showed before that the possibility of working with scenarios of disruption *a priori* can provide relevant information on the functioning of a network and this can be easily explored at virtually no cost with the an accurate metamodel. Moreover, it is also possible, due to the metamodel characteristics, to further increase the mapping with relevant stochastic variables of interest for an accurate uncertainty analysis.

In the present work it was also assumed that links are always accessible due to the number of teams and partial damages, however this may not be always the case. Accessibility has large relevance in the context of recovery Zhao and Zhang (2020). While, using a metamodel allows to perform decision-making on a subset of damaged links that can be considered accessible or not, it would be of interest to further investigate on alternatives to combine accessibility and metamodeling within the context of optimal recovery.

4.2. Nguyen-Dupuis and Cuenca with double-loop GA adaptation, limited budget and uncertain scenarios of perturbation

The second representative example of application uses both the Nguyen-Dupuis and Cuenca network. It consists in implementing adaptation measures in order to improve the network performance in future uncertain perturbation scenarios. An adaptation budget B is set to enhance the traffic network performance during perturbation events; in the form of additional unitary capacity increments ΔC_a. A decision vector $D_A = [\delta_a]$ sets the decision-making scheme for budget distribution in the $A_d \subseteq A$ links. In order to introduce and discuss the innovative character of the present implementation, in the first example researched steady conditions are assumed, and in the second $\alpha = 0.8$. In this case, the optimization of adaptation can be also set as a bi-level optimization problem (Sinha et al., 2017) that follows:

$$\underset{D_A,\rho_r,\mathbf{h},v}{\text{Minimize}} \sum_{a\in A} C_a(D_A,\mathbf{h},\mathbf{v},\rho_r) \tag{19}$$

s.t.:

$$\sum_{a\in A_d} \delta_a \Delta C_a = B \tag{20}$$

$$\rho_r, \mathbf{h}, v \in \Psi(D_A) \tag{21}$$

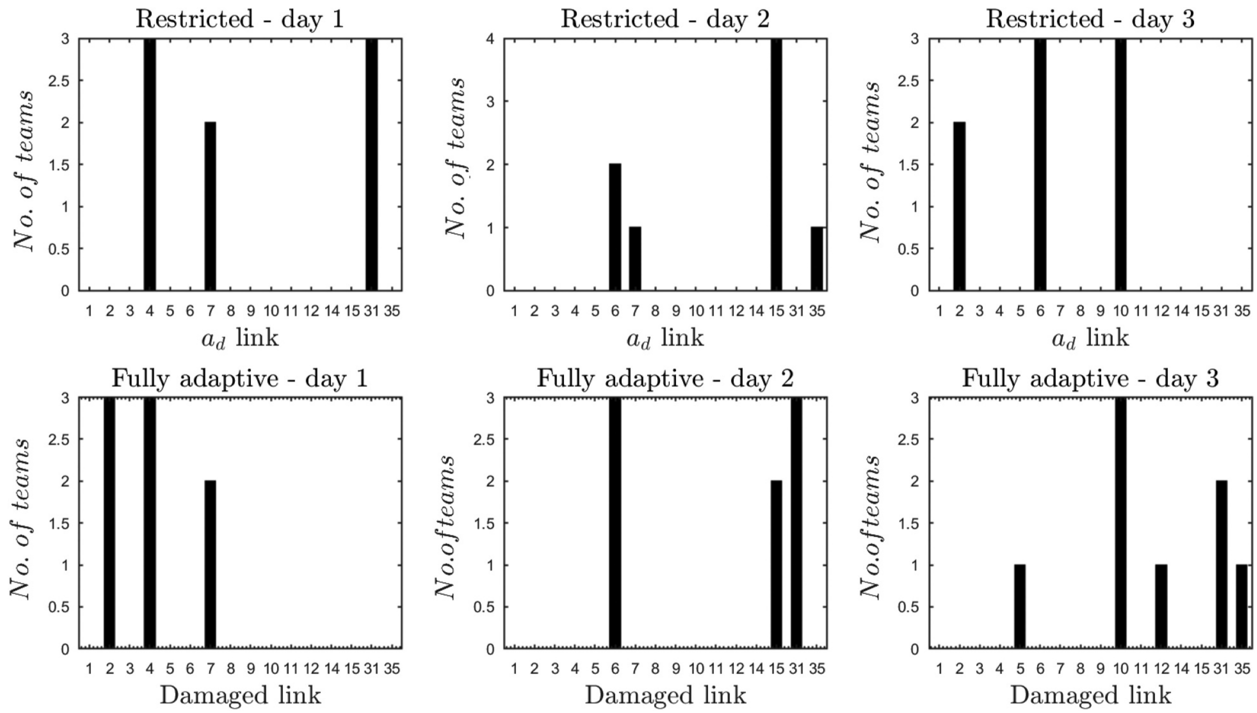

Figure 7. Team allocation per link affected in the first 3 days of recovery after the network is impacted by the perturbation, considering restricted and unrestricted user adaptation.

and $\Psi(D_A)$ represents the lower level optimization set-valued mapping,

$$\Psi(D_A) \in \underset{\rho_r, \mathbf{h}, v}{\operatorname{argmin}} \sum_{a \in A} C_a(v_a(t), D_A)$$

subject to the constraints of Equations (7)–(11). This procedure can then be applied in undamaged and damaged scenarios, where different levels of d_a are considered for a random subset $\hat{a}_d \subset A_d$ of links with $n(\hat{a}_d)$ being the number of links, or cardinality of, $\hat{a}_d$. Similarly to the previous example a GA is applied in order to distribute the budget in a black-box optimization of the simulation model.

4.2.1. Example I: Nguyen-Dupuis adaptation with uncertain scenarios of damage

An hazardous event is simulated affecting A_d links given in Table 3. This perturbation is expected to vary in a probabilistic range; where there is a 5% probability of $n(\hat{a}_d) = 1$, a 7.5% probability of $n(\hat{a}_d) = 2$, 12.5% probability of $n(\hat{a}_d) = 1$, a 25% probability of $n(\hat{a}_d) = 4$, a 20% probability of $n(\hat{a}_d) = 5$, a 20% probability of $n(\hat{a}_d) = 6$, and a 10% probability of $n(\hat{a}_d) \geq 7$. In all the cases d_a follows a random uniform distribution with bounds [40%,60%].

Figure 8(a) presents the representative histogram for $n(\hat{a}_d)$. In this context, any combination $\hat{a}_d \subset A_d$ may be damaged at a time. Figure 8(b) presents the type of damage that a link experiences when damaged.

Therefore, if $n(\hat{a}_d)$ links are damaged, the damage of each link will follow a uniform distribution within the bounds [40%,60%], resulting in a large combination of damage scenarios for the subset of links considered in the present analysis.

A metamodel is fitted in order to accurately predict the network operation in a range of damage up to 60% of capacity, and an increase of 50% in C_a. The final model is fitted with a 0.99% average absolute relative error in prediction of a LHS sample of 100 points; and with cv_a of 4.67%. The sequential implementation is halted after 590 network function evaluations. The Nguyen-Dupuis is applied as defined in Example 1 (demands, OD pairs and capacities). Table 3 presents the subset of potentially damaged links considered in the present example to define A_d.

A $B = 50\%$ of link capacity is considered, that can be divided in 50 independent increases of $\Delta C_a = 1\%$ (it is noted that C_a is equal for all links).

The idea of this implementation is that of discussing how the proposed application can be used to extend the present state-of-art in adaptation of civil engineering systems. Because of the possibility of optimizing the adaptation decision in regard to the existing B at virtually no cost, it is possible to define the meta-heuristic best adaptation measure for each scenario of perturbation, which then defines a meta-heuristic optimal distribution of adaptation. This curve characterizes the response of the adapted system if the damage scenario could be exactly predicted before its occurrence and a scenario-specific (*ad-hoc*) adaptation can be applied. In practice, however, having *ad-hoc* adaptation to multiple scenarios may not be possible in most circumstances; and the most common alternative is to have a single adaptation decision that is expected to maintain the highest network performance in most cases. It is noted that if scenario-dependent adaptation can implemented, such as highlighted in Gilrein et al. (2021), this curve has significant relevance and would not be obtained on a feasible basis without the support of a metamodel (2500 GA evaluations of the optimization described are used to define it).

Since in most cases *ad-hoc* adaption is not possible, any decision-maker is faced with the challenging decision of setting the most suitable distribution of the budget that allows a robust performance to different scenarios. A straightforward decision of adaptation for this effect is to optimize the resources available in the network in order to increase its efficiency in recurrent operational scenarios. This would involve the application of meta-heuristic optimal D_A for the undamaged network (D_A if $a \in A_d, d_a = 0$). Alternatively, it would be intuitive to pursue a damage-based approach to optimize the network adaptation for an expected value of damage in the links, i.e. all the links equally at $d_a = 50\%$ (D_A if $a \in A_d, d_a = \mu_{d_a}$); or to adapt the network to so-called worst case scenario, i.e., where all the links are exposed to $d_a = 60\%$ of initial capacity (D_A if $a \in A_d, d_a = d_{lb}$). It is understandable that defining a scenario to optimize the adaptation of resources that will hold the best results in the overall response of the system to a perturbation is not simple; and comparison of different scenarios is time-consuming. Moreover, with uncertain scenarios of damage, it is difficult to know and find *a priori* what adaptation strategy and

Table 3. Reference links used to generate $\hat{a}_d$, totaling an ED of 12 variables. The description of nodes follows Figure 1(a).

Potentially damaged links	Nodes connecting
1; 9	1–5; 5–1
3; 20	2–8; 8–2
4; 30	2–11; 11–2
5; 31	3–11; 11–3
6; 37	3–13; 13–3
18; 21	7–8; 8–7

scenarios should be used to compute an overall efficient strategy. Even if critical links are identified, quantifying the optimum distribution of budget is not a simple task as will be shown in the following example.

In this particular context, the possibility of defining a meta-heuristic optimal distribution and having a virtually free emulator of the network adaptation enables the possibility of searching for the measure that will better approximate it. This is possible using a double-loop optimization that can be set with an additional optimization step, where the distribution for meta-heuristic adaptation to random scenarios is defined and then a second optimization is performed in order to define the adaptation decision for division of capacity that produces a distribution that is closer to this curve. In the present example this is achieved by minimizing the squared difference between the optimal and adapted scenarios at $x_{\hat{a}_d}$,

$$\Delta_{opt} = \underset{D_A}{Minimize} \sum_{x_{\hat{a}_d}} \left(C_{A_{opm}}(x_{\hat{a}_d}) - C_{D_A}(x_{\hat{a}_d})\right)^2 \quad (22)$$

where each $(x_{\hat{a}_d})$ is a scenario of damage, and Δ_{opt} is the difference between the total travel cost of adaptation D_A, C_{D_A}, to the meta-heuristic optimal adaptation cost obtained for *ad-hoc* response, $C_{A_{opm}}$. This analysis encloses all the $n(x_{\hat{a}_d})$ random scenarios of damage considered. Constraints related to $\mathcal{B}$ are maintained accordingly to the formulation introduced previously. In the present example, the second optimization is also performed using a GA; i.e. one to find the heuristic optimal curve and the other to search for the D_A that minimizes the squared difference to this curve.

Results from the implementation of different adaptation decisions under a limited budget of capacity, in the context defined, are presented in Figure 9, where the response probability density functions for different adaptation decisions are compared.

As expected, the meta-heuristic optimal distribution (black continuous curve) is on the left of the remaining curves. The distribution that represents the mean damage scenario in the considered links provides the most fitted solution to the problem in-hand that uses no selection measures other than a 'rule of thumb' (red curve). D_A for the network normal operation (using an undamaged network) provides a reasonable response to the damage scenarios simulated (yellow curve). It is noted that this solution has a larger variability, which indicates that it performs well in conditions that are approximate to the state of $a \in A_d, d_a = 0$, but less so in other scenarios. The worst case scenario curve provides the adaptation solution that has worse performance. It represents an unlikely reference scenario for adaptation. The results show that the solution obtained

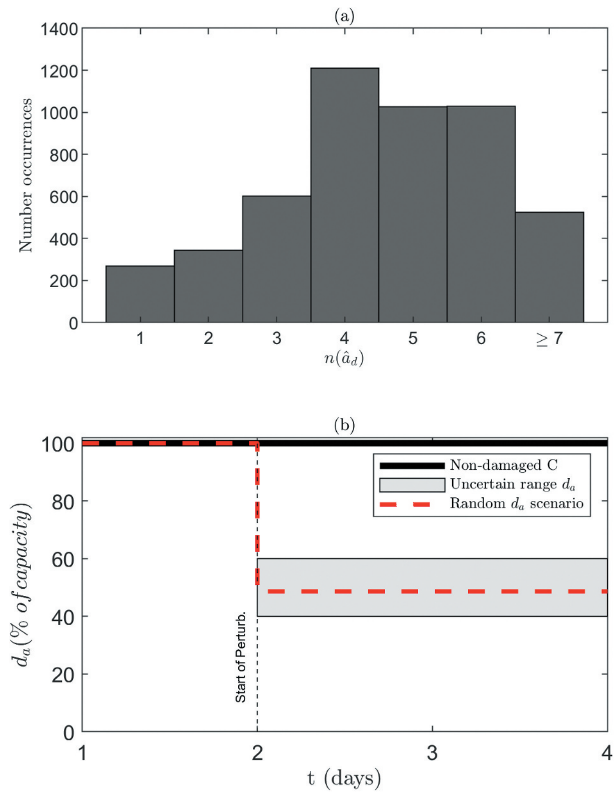

Figure 8. (a) Histogram of the number of damaged links in 5000 hazard occurrences. Combinations of damaged links from the subset considered are randomly generated. (b) Damage uncertainty interval in an affected link, representative of independent link damage.

with a double-loop optimization (green curve), which searches for the single adaptation decision that minimizes the difference between optimum distribution and the adapted response, surpasses the remaining alternatives and results in the performance probability density function that is closer to the meta-heuristic optimum curve. While the metamodel simplifies performing a parametric search for the best adaptation, in the face of uncertainty and limited *a priori* information on the scenarios of damage, it is expected to provide a systematic efficient solution with no prior assumptions on the network.

4.2.2. Example II: Cuenca network adaptation with uncertain scenarios of damage, dimensional reduction, and user restricted movement

In the present case, it is assumed that uncertainty in relation to the perturbation is even larger; with two potential global scenarios of perturbation. There is a 60% probability of scenario I, which consists in a 25% probability of $n(\hat{a}_d) \leq 4$, a 35% probability of $n(\hat{a}_d) = 5$, a 20% probability of $n(\hat{a}_d) = 6$, a 15% probability of $n(\hat{a}_d) = 7$, and 5% probability of $n(\hat{a}_d) \geq 8$. In all the cases, d_a is considered to be $d_1 = 50\%$ of C_a. Then, scenario II has a 40% probability of occurring and in it there is a 30% probability of $(n\hat{a}_d) \leq 6$, a 20% probability of $n(\hat{a}_d) = 7$, a 20% probability of $n(\hat{a}_d) = 8$, a 15% probability of $n(\hat{a}_d) = 9$, and 15% probability of $n(\hat{a}_d) \geq 10$. In all the cases of scenario II, d_a is considered to be $d_2 = 60\%$ of C_a. Summarizing, two scenarios with different intensities are considered, resulting in more or less damaging links, and that may generate losses of $d_1 = 50\%$ or $d_2 = 60\%$ in C_a. Adaptation in this context is studied such as in the previous example; where scenarios of, no damage, damage to all links at 50% and 60% of C_a, no adaptation, and double GA adaptation are considered. A $\Delta C_a = 1\%$ with $B = 100\%$ of C_a is applied in order to study adaptation. Increments of capacity in C_a are limited to a maximum increment of 50% C_a, or $\frac{1}{2}B$. The Cuenca network is applied as a reference of a more complex example. This network is composed of 232 nodes and 209 routes that pass through 368 links. Damages are simulated in the fifteen most critical links as defined by a centrality metric Sun et al. (2020), the weighted betweeness.[1] This metric is particularly interesting in this context as it is based on the network topology as defined by its routes, and is a simulation-free technique to reduce the ED size. It allows effortless reduction of the dimensional complexity of metamodeling, which is of interest to research metamodeling in complex networks. Figure 10 highlights the most critical links used to build the ED.

The final metamodel is fitted with a sample size of 2291 points and maximum cv_a of 4.96%. The final average absolute error in cross-validation of a LHS sample of 100 points was 1.47%. Box-plots for the best-known response of adaptation to a 2500 random scenario of damage are presented in Figure 11.

When compared with the cases of $a \in A_d, d_a = d_1$ and $a \in A_d, d_a = d_2$, the benefit of double GA adaptation is at maximum marginal in median and extreme values. Nonetheless, an interesting outcome of the double GA adaptation in the present application is the reduction of the response variance. It is possible to infer that in the 75% quantile there is a reduction of 101, 82, and 145 hours in the travel time when compared with the other alternatives for adaptation. Moreover, the tail of the Double GA response increases slower, which can be identified in the (blue) dots above the 75% quantile. These are plotted at increments of 5% in quantile. Adaptation at $d_a = d_2$ and $d_a = 0$ provide the most efficient results for extreme occurrences (at the higher and lower order statistics, respectively). $d_a = 0$ provides a competitive alternative for adaptation. This is related to the consideration of a subset of important links. As expected, the event-dependent response outperforms all the alternatives, and in the worst case scenario guarantees a performance equal to $d_a = d_2$. As previously mentioned, this response can be of relevance if means of event-dependent adaptation are available (e.g., adapting capacity in link by managing lanes). When looking at the best-known solution results for each case, Figure 12, it is possible to infer that the double GA combines information from the perturbation-based scenarios, while providing a more balanced distribution of the adaptation budget in the links considered. This balance benefits the performance in responses at higher order statistics of travel time, hence, damage, such as identified. It is interesting to infer that even in a subset of most relevant links, there is an even smaller subset of links that enclose most of the influence in the perturbation-based travel time.

To conclude, it is noted that creating a sequential metamodel was identified to be effective in moderate losses of capacity, or large losses when the capacities in the network are large. As the loss of capacity in the network increases, an exponential growth of the travel cost occurs, and despite the metamodel capability to surrogate this trend, in scenarios of large damage it may fail to capture the exact exponential shape in the function if the density of points in the exponential region is not adequate. Nonetheless, this can be tackled by a division of the space of metamodeling as discussed in Teixeira et al., 2021b) or applied in the piecewise implementation of Marelli et al. (2021). Future considerations about the modeling space are expected to accelerate convergence (also considering a discussion on the travel function and modeling limits of applicability, Mtoi and Moses (2014)). In the present case, it was seen that for most of the dimensional space convergence in respect to ε was fast and used few points. Mainly highly nonlinear regions were identified to delay convergence and increase uncertainty, resulting in more points being added to the ED. In practice, the Kriging has the capability to approximate highly complex functions, which is a point of interest to study adaptation and recovery decision-making, in systems, which has zero-response times. In such case, the ED approach to the Kriging will have an important role. It is a key feature in improving the efficiency of implementing surrogates and decision processes in the analysis of recovery and adaptation of complex systems, including traffic networks. In practice, a metamodel can be built such that it replicates the full operational conditions of the traffic network, including disruptions, which can be prepared *a priori* to any event.

5. Conclusions

Applications of metamodeling in adaptation and recovery of traffic networks were studied in the present work. Metamodels were applied to surrogate the

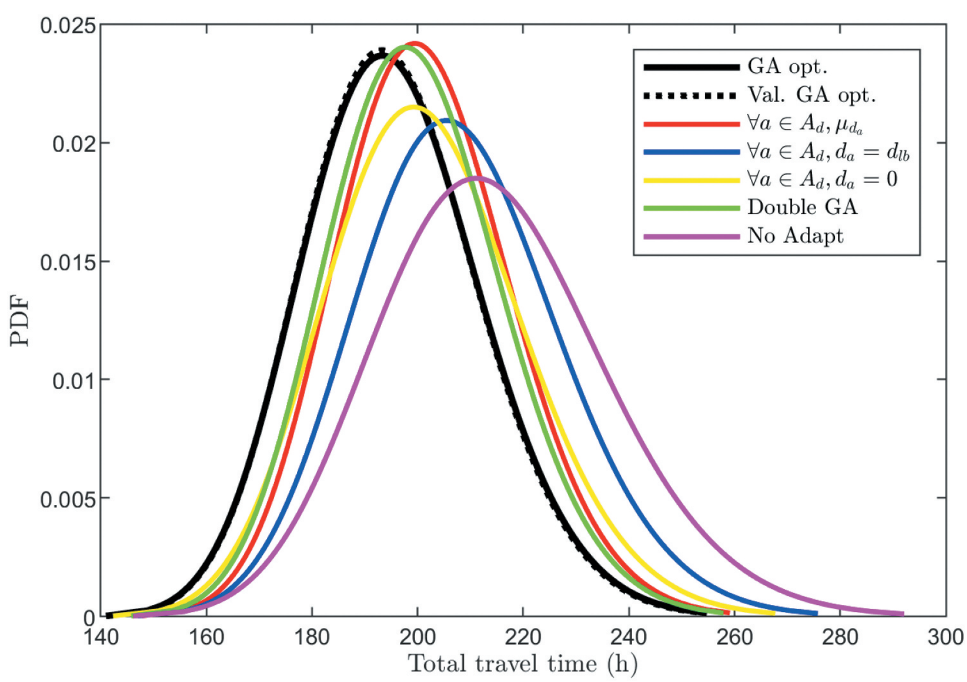

Figure 9. Probability density function for the travel time in the network, considering implementation of different adaptation decisions and 2500 random scenarios of perturbation. Results presented use an average of random seeds [1, , 10], and a GA population of 100 individuals, with the GA being halted at 50 consecutive generations with no improvement in best penalty function value.

performance function that defines traffic networks characterized by a day-to-day equilibrium. An iterative metamodel with sequential experimental design was proposed and implemented, efficiently creating accurate global surrogates of the network performance. The global surrogates were then applied in adaptation and recovery of traffic networks. Three networks with distinct levels of complexity were used, the Nguyen-Dupuis, the Sioux-Falls and the Cuenca traffic networks. First, metamodeling was implemented to accelerate an optimization of decision-making for recovery, producing accurate results (relative error of less than 1%) and using only a tiny fraction of the effort required to perform a full analysis that uses the network performance function. The influence of restricted user adaptation was also successfully investigated. Then, allocation of an adaptation budget for a network under random scenarios of perturbation was researched. By using metamodeling, it was possible to optimize the best adaptation decision for every uncertain perturbation, and define perturbation-dependent optimal adaption. This is of interest to find the adaptation decision that minimizes the difference between optimal and actual responses. This decision was identified to outperform other probable optimal decisions based on deterministic scenarios of perturbation or rules of thumb.

One of the most interesting features of metamodeling in the context researched is related to the possibility of being implemented *a priori* as digital twins for decision-making processes that depends on time-consuming support tools. This enables zero-response times for relevant problems, such as adaptation and recovery.

A significant remark is related to the need for particular considerations regarding the dimensional space. In the case of traffic networks, due to their complexity, the experimental design may become large. In such circumstances, the cost of metamodeling increases and becomes significant with no relevant analysis gains. In respect to this aspect of implementation, it is noted that traffic networks have synergy with the idea of dimensional reduction (usually a subset of links encloses most of the influence on travel time), being well-suited to combine it with metamodeling. This was successfully tested in the Cuenca network adaptation example, where in a subset of the most important links, a smaller subset enclosed most of the influence.

In terms of future works, the role of metamodeling was also highlighted in the context of its potential to enable fully adaptive systems, which are concurrent with the idea of complex adaptive systems, and can be exploited to enable zero-response times in problems of transport. These have the flexibility to be combined with other approaches that enhance the effectiveness of

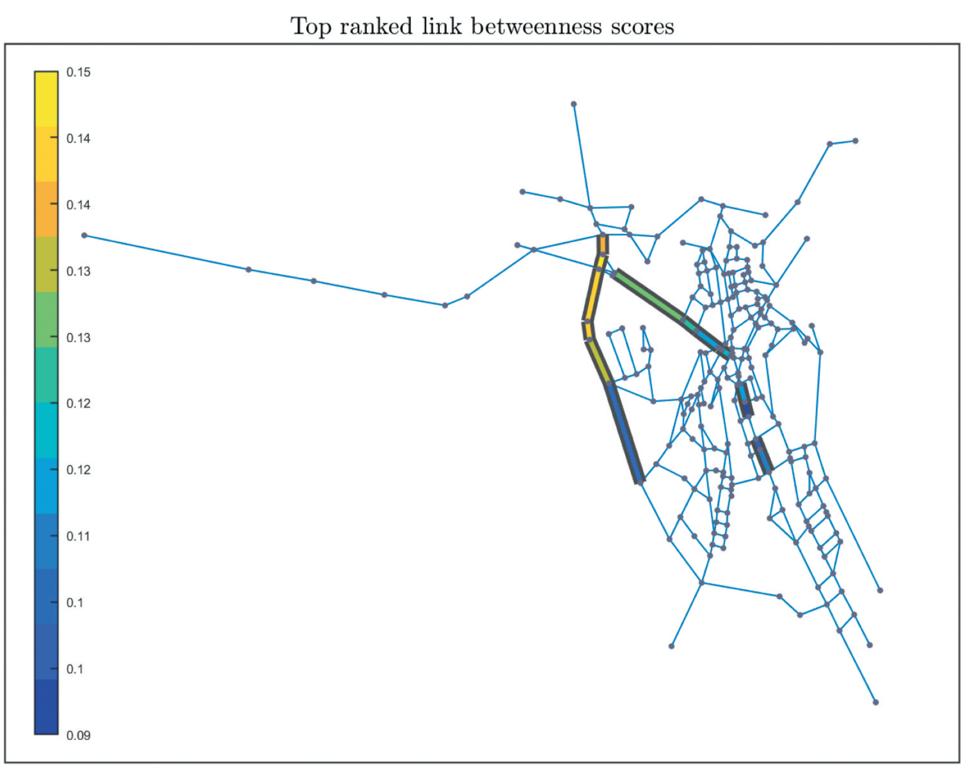

Figure 10. Cuenca newtork links used to generate the metamodel ED as ranked by link-betweeness.

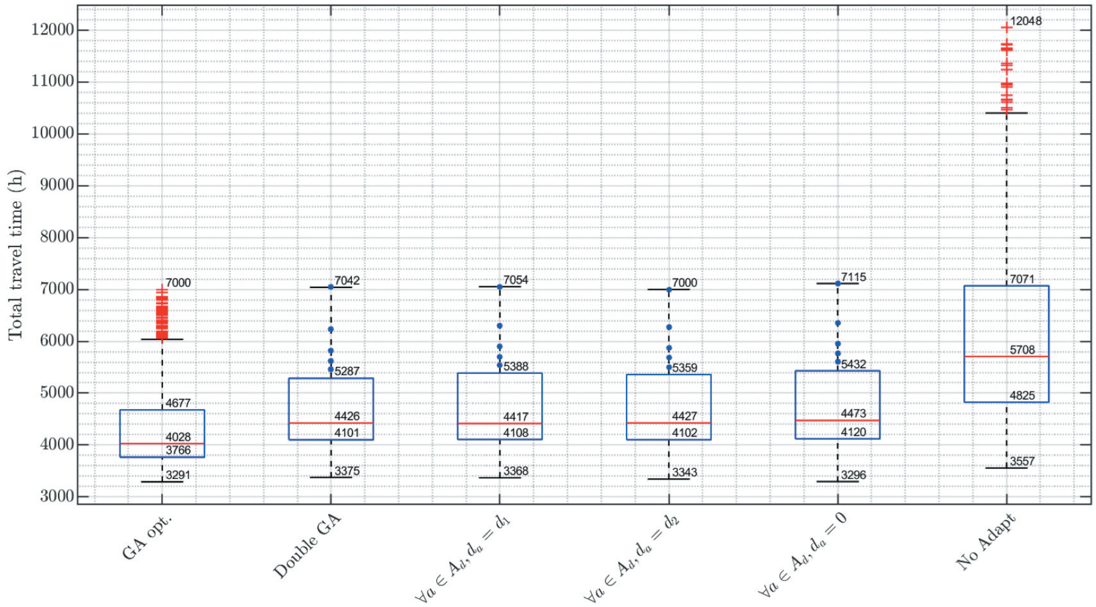

Figure 11. Box-plots for best known adaptation decision based using 2500 scenarios of damage.

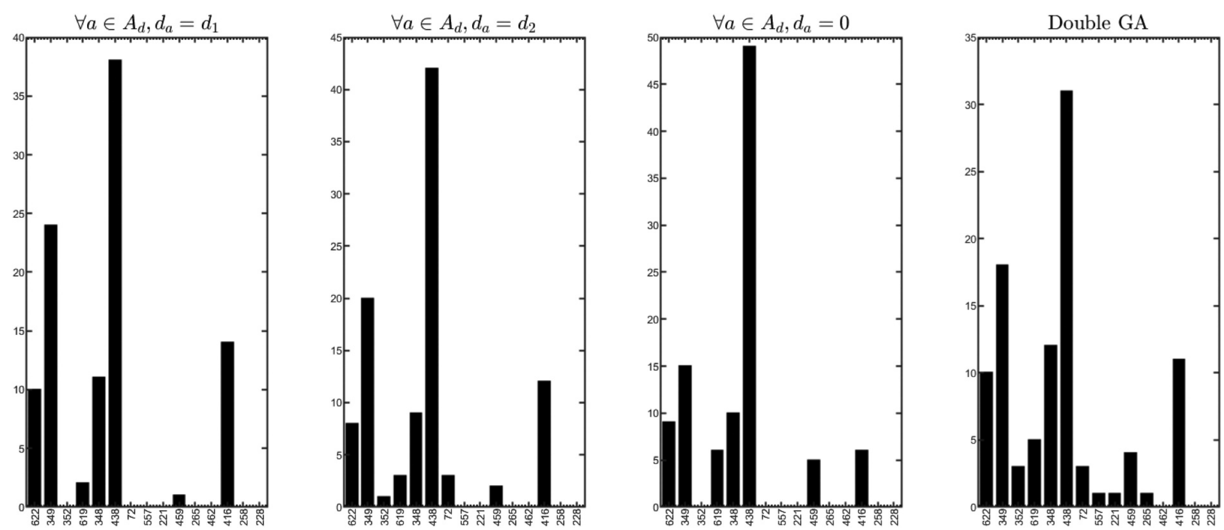

Figure 12. Best-known solution for allocation of adaptation budget.

optimal recovery and adaptation decision-making, such as updates to traffic models using a Bayesian approaches Castillo et al. (2008), or agent-based analyses, such as LS/ATN Dorer and Calisti (2005); Neagu et al. (2006). The present implementation is representative of the enabling potential of metamodeling in one of the most relevant topics of the present, recovery, and adaptation in the context of engineering systems.

Note

1. It quantifies the number of times a link is used along the shortest path between two ODs.

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References

Beasley, D., Bull, D. R., & Martin, R. R. (1993). An overview of genetic algorithms: part 1, fundamentals. *University Computing*, *15*(2), 56–69.

Blatman, G., & Sudret, B. (2010). An adaptive algorithm to build up sparse polynomial chaos expansions for stochastic finite element analysis. *Probabilistic Engineering Mechanics*, *25*(2), 183–197. doi:10.1016/j.probengmech.2009.10.003

Bocchini, P., & Frangopol, D. M. (2012). Restoration of bridge networks after an earthquake: multicriteria intervention optimization. *Earthquake Spectra*, *28*(2), 426–455. doi:10.1193/1.4000019

Burhenne, S., Jacob, D., & Henze, G. P. (2011). Sampling based on sobol'sequences for monte carlo techniques applied to building simulations. In *Proc. Int. Conf. Build. Simulation* (pp. 1816–1823). Sydney, Australia.

Castillo, E., Menéndez, J. M., & Sánchez-Cambronero, S. (2008). Predicting traffic flow using bayesian networks. *Transportation Research Part B: Methodological*, *42*(5), 482–509. doi:10.1016/j.trb.2007.10.003

Chen, X., Zhang, L., He, X., Xiong, C., & Li, Z. (2014). Surrogate-based optimization of expensive-to-evaluate objective for optimal highway toll charges in transportation network. *Computer-Aided Civil and Infrastructure Engineering*, *29*(5), 359–381. doi:10.1111/mice.12058

Ciuffo, B., Casas, J., Montanino, M., Perarnau, J., & Punzo, V. (2013). Gaussian process metamodels for sensitivity analysis of traffic simulation models: case study of aimsun mesoscopic model. *Transportation Research Record*, *2390*(1), 87–98. doi:10.3141/2390-10

Couckuyt, I., Dhaene, T., & Demeester, P. (2014). Oodace toolbox: A flexible object-oriented Kriging implementation. *Journal of Machine Learning Research*, *15*, 3183–3186.

Dehoux, P., & Toint, P. L. (1991). Some comments on dynamic traffic modelling in the presence of advanced driver information systems. In *Drive Conference (1991: Brussels, Belgium)* (Vol. 2). Brussels, Belgium.

Dorer, K., & Calisti, M. (2005). An adaptive solution to dynamic transport optimization. In *Proceedings of the fourth international joint conference on Autonomous agents and multiagent systems* (pp. 45–51). The Netherlands.

Fajraoui, N., Marelli, S., & Sudret, B. (2017). Sequential design of experiment for sparse polynomial chaos expansions. *SIAM/ASA Journal on Uncertainty Quantification*, *5*(1), 1061–1085. doi:10.1137/16M1103488

Gilrein, E. J., Carvalhaes, T. M., Markolf, S. A., Chester, M. V., Allenby, B. R., & Garcia, M. (2021). Concepts and practices for transforming infrastructure from rigid to adaptable. *Sustainable and Resilient Infrastructure*, *6*(3–4), 213–234.

Gu, Z., Waller, S. T., & Saberi, M. (2019). Surrogate-based toll optimization in a large-scale heterogeneously congested network. *Computer-Aided Civil and Infrastructure Engineering*, *34*(8), 638–653. doi:10.1111/mice.12444

Iman, R. L., & Helton, J. C. (1988). An investigation of uncertainty and sensitivity analysis techniques for computer models. *Risk Analysis*, *8*(1), 71–90. doi:10.1111/j.1539-6924.1988.tb01155.x

Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. *Journal of Global Optimization*, *13*(4), 455–492. doi:10.1023/A:1008306431147

Kamiński, B. (2015). A method for the updating of stochastic Kriging metamodels. *European Journal of Operational Research*, *247*(3), 859–866. doi:10.1016/j.ejor.2015.06.070

Kammouh, O., Nogal, M., Binnekamp, R., & Wolfert, A. R. (2021). Multi-system intervention optimization for interdependent infrastructure. *Automation in Construction*, *127*, 103698. doi:10.1016/j.autcon.2021.103698

Katoch, S., Chauhan, S. S., & Kumar, V. (2020). A review on genetic algorithm: Past, present, and future. *Multimedia Tools and Applications*, 1–36. doi:10.1007/s11042-020-10139-6

Kleijnen, J. P. (2017). Regression and Kriging metamodels with their experimental designs in simulation: A review. *European Journal of Operational Research*, *256*(1), 1–16. doi:10.1016/j.ejor.2016.06.041

Liu, Y., McNeil, S., Hackl, J., & Adey, B. T. (2020). Prioritizing transportation network recovery using a resilience measure. *Sustainable and Resilient Infrastructure*, 1–12. doi:10.1080/23789689.2019.1708180

Manache, G., & Melching, C. S. (2008). Identification of reliable regression-and correlation-based sensitivity measures for importance ranking of water-quality model parameters. *Environmental Modelling and Software*, *23*(5), 549–562. doi:10.1016/j.envsoft.2007.08.001

Marelli, S., Wagner, P.-R., Lataniotis, C., & Sudret, B. (2021). Stochastic spectral embedding. *International Journal for Uncertainty Quantification*, *11*(2). doi:10.1615/Int.J.UncertaintyQuantification.2020034395

Martinez-Pastor, B. (2018). *Resilience of traffic networks to extreme weather events: analysis and assessment*. Ph. D. thesis, Trinity College Dublin.

Mtoi, E. T., & Moses, R. (2014). Calibration and evaluation of link congestion functions: Applying intrinsic sensitivity of link speed as a practical consideration to heterogeneous facility types within urban network. *Journal of Transportation Technologies*, (2). doi:10.4236/jtts.2014.42014

Neagu, N., Dorer, K., Greenwood, D., & Calisti, M. (2006). Ls/atn: reporting on a successful agent-based solution for transport logistics optimization. In *IEEE Workshop on Distributed Intelligent Systems: Collective Intelligence and Its Applications (DIS'06)* (pp. 213–218). Prague, Czech Republic: IEEE.

Nogal, M., Honfi, D., & O'Connor, A. (2019). Resilience of road transport systems considering the stochastic response of travellers. In *13th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP13 Seoul*, South Korea, May 26-30, 2019.

Nogal, M., O'Connor, A., Caulfield, B., & Martinez-Pastor, B. (2016). Resilience of traffic networks: from perturbation to recovery via a dynamic restricted equilibrium model. *Reliability Engineering & System Safety*, *156*, 84–96. doi:10.1016/j.ress.2016.07.020

Osorio, C., & Chong, L. (2015). A computationally efficient simulation-based optimization algorithm for large-scale urban transportation problems. *Transportation Science*, *49* (3), 623–636. doi:10.1287/trsc.2014.0550

Pereira, F. C., Antoniou, C., Fargas, J. A., & Ben-Akiva, M. (2014). A metamodel for estimating error bounds in real-time traffic prediction systems. *IEEE Transactions on Intelligent Transportation Systems*, *15*(3), 1310–1322. doi:10.1109/TITS.2014.2300103

Pisano, C. O. (2010). *Mitigating network congestion: Analytical models, optimization methods and their applications*. Ph. D. thesis, Citeseer.

Sheffi, Y. (1985). *Urban transportation networks* (Vol. 6). Englewood Cliffs, NJ: Prentice-Hall.

Sinha, A., Malo, P., & Deb, K. (2017). A review on bilevel optimization: From classical to evolutionary approaches and applications. *IEEE Transactions on Evolutionary Computation*, *22*(2), 276–295. doi:10.1109/TEVC.2017.2712906

Sobol, I. M. (1998). On quasi-monte carlo integrations. *Mathematics and Computers in Simulation*, *47*(2–5), 103–112. doi:10.1016/S0378-4754(98)00096-2

Sobol', I. M., Asotsky, D., Kreinin, A., & Kucherenko, S. (2011). Construction and comparison of high-dimensional sobol'generators. *Wilmott*, *2011*(56), 64–79. doi:10.1002/wilm.10056

Sun, W., Bocchini, P., & Davison, B. D. (2020). Resilience metrics and measurement methods for transportation infrastructure: The state of the art. *Sustainable and Resilient Infrastructure*, *5* (3), 168–199. doi:10.1080/23789689.2018.1448663

Teixeira, R., Martinez-Pastor, B., Nogal, M., & O'Connor, A. (2021a). Reliability analysis using a multi-metamodel complement-basis approach. *Reliability Engineering & System Safety*, *205*, 107248. doi:10.1016/j.ress.2020.107248

Teixeira, R., Nogal, M., O'Connor, A., & Martinez-Pastor, B. (2020). Reliability assessment with density scanned adaptive Kriging. *Reliability Engineering & System Safety*, *199*, 106908. doi:10.1016/j.ress.2020.106908

Teixeira, R., Nogal, M., & O'Connor, A. (2021b). Adaptive approaches in metamodel-based reliability analysis: A review. *Structural Safety*, *89*, 102019. doi:10.1016/j.strusafe.2020.102019

Teixeira, R., O'Connor, A., & Nogal, M. (2019). Probabilistic sensitivity analysis of offshore wind turbines using a transformed kullback-leibler divergence. *Structural Safety*, *81*, 101860. doi:10.1016/j.strusafe.2019.03.007

Tsvetkova, O., & Ouarda, T. B. (2019). Quasi-monte carlo technique in global sensitivity analysis of wind resource assessment with a study on UAE. *Journal of Renewable and Sustainable Energy*, *11*(5), 053303. doi:10.1063/1.5120035

Twumasi-Boakye, R., & Sobanjo, J. O. (2021). A computational approach for evaluating post-disaster transportation network resilience. *Sustainable and Resilient Infrastructure*, *6*(3–4),235–251.

Viana, F. A., Haftka, R. T., & Steffen, V. (2009). Multiple surrogates: How cross-validation errors can help us to obtain the best predictor. *Structural and Multidisciplinary Optimization*, *39*(4), 439–457. doi:10.1007/s00158-008-0338-0

Vugrin, E. D., Turnquist, M. A., & Brown, N. J. (2014). Optimal recovery sequencing for enhanced resilience and service restoration in transportation networks. *International Journal of Critical Infrastructures*, *10*(3/4), 218–246. doi:10.1504/IJCIS.2014.066356

Wang, D. Z. W., Liu, H., Szeto, W. Y., & Chow, A. H. F. (2016). Identification of critical combination of vulnerable links in transportation networks – A global optimisation approach. *Transportmetrica A: Transport Science*, *12*(4), 346–365. doi:10.1080/23249935.2015.1137373

Wardrop, J. G. (1952). Road paper. some theoretical aspects of road traffic research. *Proceedings of the institution of civil engineers*, *1*, 325–362. 3.

Xiao, N.-C., Zuo, M., & Zhou, C. (2018). A new adaptive sequential sampling method to construct surrogate models for efficient reliability analysis. *Reliability Engineering & System Safety*, *169*, 330–338. doi:10.1016/j.ress.2017.09.008

Zhao, T., & Zhang, Y. (2020). Transportation infrastructure restoration optimization considering mobility and accessibility in resilience measures. *Transportation Research Part C: Emerging Technologies*, *117*, 102700.

Zhao, Y., & Dong, S. (2021). Probabilistic fatigue surrogate model of bimodal tension process for a semi-submersible platform. *Ocean Engineering*, *220*, 108501. doi:10.1016/j.oceaneng.2020.108501