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Extreme Measures of Agricultural Financial Risk

By

John Cotter*, Kevin Dowd* and Wyn Morgan♦

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* Centre for Financial Markets, Smurfit School of Business, University College Dublin, Carysfort Avenue, Blackrock, Co. Dublin, Ireland. Email: john.cotter@ucd.ie. Cotter acknowledges the support of Science Foundation Ireland under Grant Number 08/SRC/FM1389.

* Pensions Institute, Cass Business School, 106 Bunhill Row London EC1Y 8TZ.

♦ School of Economics, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom. Email: wyn.morgan@nottingham.ac.uk.. (Corresponding author)

Extreme Measures of Agricultural Financial Risk

The agricultural marketing environment is inherently risky. Having accurate measures of risk helps farmers policy makers and financial institutions make better informed decisions about how to deal with this risk. This paper examines three tail quantile-based risk measures applied to the estimation of extreme agricultural financial risk for corn and soybean production in the US: Value at Risk (VaR), Expected Shortfall (ES) and Spectral Risk Measures (SRMs). We use Extreme Value Theory (EVT) to model the tail returns and present results for these three different risk measures using agricultural futures market returns data. We compare estimated risk measures in terms of size and precision, and find that they are all considerably higher than Normal estimates. The estimated risk measures are also quite imprecise, and become more so as the risks involved become more extreme.

Keywords: Agricultural financial risk, Spectral risk measures, Expected Shortfall, Value at Risk, Extreme Value Theory.

JEL Classification: E17, G19, N52

1. INTRODUCTION

The inherent variability in agricultural production (weather, pests, animal illness and so forth) alongside demand variations (food scares, fads, etc.) and policy reform that exposes farmers to greater market influences (e.g. Agenda 2000) make for a marketing environment for farmers that is characterised by significant levels of financial risk (Moschini and Hennessy (2001), Chern and Rickettsen (2003) and Carter and Smith (2007)). In an environment characterised by risk, two key questions arise: how big is the risk being faced and how do agents try to deal with it? It is the former which forms the focus of this paper, but by providing an answer, it implicitly deals with the latter.

Being able to quantify the risks faced by farmers is of great benefit to those who design and provide risk management tools for the agricultural sector, and indeed, design policies that shape the agricultural sector. Equally, accurate risk measurement is especially acute for those farmers seeking loans and other financial instruments to support and develop their businesses. Lending institutions would have greater confidence in their own risk if they had more accurate measures of the risk the borrowers faced, and in times when lending is scrutinised more heavily, improving lending efficiency through more accurate measures of risk is a highly desirable aim. This becomes even more pertinent when the possibility of extreme risks exists. Generally, the literature focuses heavily on risk measurement matters although as Chambers and Quiggin (2004) argue, the coverage has not always been comprehensive and, key for present purposes, lacks a multi-variate portfolio analysis approach.

This paper thus seeks to address the key question of what measures can be used to assess the levels of risk more accurately allowing for multi-variate portfolio analysis and by doing so, draw on the recent developments in the field.¹ As a backdrop, the last

¹ A reviewer has quite rightly pointed out that risk measures are typically calculated for a firm with a portfolio of commodities, or just the market risk associated with a portfolio of commodities. However calculation of risk measures is important for a large spectrum of economic agents associated with agricultural activity. The calculation of risk measures use market prices and agents face some function of these. For example, a corn farmer would be interested in analysing market price decreases in traded corn to ensure they have sufficient income and thus would examine risk measures for a long position of a distribution of returns. In contrast, an agricultural producer who uses corn as in input and needs to buy it for processing would be worried about market price increases and thus focus on risk for a short position for a distribution of returns. Both parties would be interested in both the magnitude of price

decade and a half have witnessed an explosion of research on different measures of financial risk, and especially on one particular measure, the Value-at-Risk (VaR). VaR models were first used by financial institutions for their own risk management purposes, but have since been adopted by many non-financial companies as well (Nocera, 2009). Amongst their many uses, VaR models can be used to determine capital and reserving requirements, establish position limits and inform hedging strategies. They can also be used to manage cashflow, liquidity and credit risks as well as the market risks for which they were first developed. Estimation methods have improved considerably over the years, and the properties – and especially the limitations – of the VaR itself have become better understood. Various new measures of financial risk have also been proposed and these include, most notably, the coherent risk measures proposed by Artzner *et al* (1999). These risk measures have the highly desirable property of sub-additivity, which the VaR lacks. The lack of sub-additivity may impair the use of VaR in a multi-variate setting due to potentially unfavourable diversification results. Perhaps most notable amongst these risk measures are the Expected Shortfall (ES), which tell us what we can expect to lose in the event of a loss exceeding the VaR, and Spectral Risk Measures (SRMs), which take account of the user's degree of risk aversion. Thus, not only have VaR estimation methods improved over time, but there have also been significant developments in the literature on financial risk measures themselves.

In this paper we are also interested in a second, related, issue: the issue of extremes, or the risk associated with the prospect of low probability, high impact losses. In an agricultural setting this is highly pertinent, as such extreme losses can occur as crops are wiped out by drought or pests and indeed livestock can be devastated by illness and disease. From a risk management perspective, the importance of extremes is self-evident: these are the prospective events that, albeit improbable, are also most damaging. The literature on extremes tells us that these should be modelled separately from the rest of the distribution using the distributions implied by Extreme Value (EV) theory, and should *not* be modelled by fitting full distributions to the data in an ad hoc way (e.g., such

increases and decreases as they show the extent to which prices (and consequently their income) can vary.

as Normal and Student distributions).² This is very useful because it means that EV theory tells us what distributions we should (and should not) fit to the data. Perhaps the most useful version of EVT is Peaks-Over-Threshold (POT) approach in which we model the exceedances over a high threshold using a Generalised Pareto Distribution (GPD; see, e.g., Embrechts *et al* (1997)).³ The application of the GPD can be justified by theory that tells us that the tail observations should follow a GPD in the asymptotic limit as the threshold gets bigger. Once the GPD curve is fitted to the data, it can then be extrapolated to give estimates of any extreme quantiles or tail probabilities we choose. It is important to note that EVT is an unconditional approach that gives average risk measures over the period of analysis. In our case we use weekly data for Corn and Soybean futures between January 1979 and December 2006 so give average weekly risk measures for this timeframe. We could also examine the conditional distribution where we would determine if the risk measures varied across the sample or we could split the unconditional distribution for different sub-samples, for example two equal sub-samples between 1997 and 2006. We do not pursue this latter approach as EVT is affected by sample size, and the smaller the number of cohort tail observations, the more unstable the tail risk measures.⁴

In this paper, we combine these two themes – measures of financial risk and extremes – in an agricultural context: more particularly, we estimate VaRs, ESs and SRMs using EV/POT methods applied to US corn and soybean returns data. Bearing in mind that the usefulness of any estimates of financial risk measures also depends crucially on their precision – and that EV estimates are always highly imprecise due to the sparsity of extremes data – we also examine alternative methods of estimating their precision. Given the heavy reliance of A Gaussian distribution in the literature, we also

² For more on EVT, see, e.g., Embrechts *et al.*, 1997, or Beirlant *et al.*, 2004. Note that tail risk measures are underestimated using a Gaussian distribution and this estimation bias deteriorates as one moves further out into the tail (Cotter, 2007).

³ The alternative is Generalised EVT in which we model the distribution of extreme returns directly. However, the POT approach (typically) involves one less parameter and fits more easily with the likelihood that extreme losses occur in clusters.

⁴ The most promising approach to examine the conditional distribution is to use alternative approaches that successfully model the time-varying dynamics of the distribution of asset prices. An example would be fitting a GARCH model and these are heavily used to model various financial time-series. This would be a comprehensive undertaking and is quite separate from the unconditional EVT approach followed here. However, the authors are currently undertaking this in a separate study (Cotter *et al*, 2009)

produce estimates of risk measures using a Gaussian distribution for comparison, and these results highlight the dangers of assuming a Gaussian distribution in a risk management context.

This paper is organised as follows. Section 2 reviews the literature on financial risk measurement in the agricultural context. Section 3 examines the risk measures and estimation methods used in this study. Section 4 explains the data set used. Section 5 presents our results and section 6 concludes.

2. RISK AND FINANCIAL RISK MEASURES IN AGRICULTURE

There are two related strands in the literature that are germane to the current paper. The first focusses on responses to the existence of risk. Institutional responses to risk range over a number of issues including the design of crop insurance schemes (Miranda and Glauber, 1997) while Mahul (1999) and Mahul and Wright (2003) examine the optimal form of insurance policies. However, as Goodwin (2001) notes crop insurance schemes whether provided by the private or public sector, are not always effective in dealing with risk.

Alternative routes for risk reduction are discussed; Skees (1999) argues that risk-sharing instruments can be devised to cover catastrophic risks, partly via weather derivatives. The choice is between using contract insurance or futures markets with the former being used if risks are independent or the latter if highly correlated. Pennings and Leuthold (2000) employ a confirmatory factor analytical model to examine futures market usage.

These areas of enquiry implicitly assume that the scale of the risk is known and understood by agents. This is not necessarily the case and this brings forward the second strand of research that explores the measurement of risk. The scale of the risk has been examined in two ways. The first is the *perceived* risk and research in to attitudes to risk has often been based on survey work (Lagerkvist, 2005; Lence, 2000, Hueth and Ligon, 1999) but while estimating indications of attitudes they do not show the *actual* risks farmers face nor their scale, which is the second aspect and the focus of the current paper.

The literature on agricultural financial risk measures is summarised in Table 1. These papers typically address very different specific applications (see column 2) and use a variety of estimation methods (column 4). We note the following points:

Insert Table 1 here

Most of these studies use multivariate parametric approaches to estimate VaR based on the assumption that underlying risks factors are distributed as multivariate Normal. We would note here that any form of Normal distribution – univariate or multivariate – is inconsistent with EV theory and inappropriate for extremes. More importantly, for a Normal distribution, VaRs are not sub-additive. Thus they can result in an overestimation of portfolio risk due to negative diversification effect when assets are combined together. Some studies also use historical simulation methods to estimate the VaR, and only one study (Zhang *et al.* (2007)) uses Monte Carlo methods.

Two studies include results based on Extreme-Value Theory (EVT). Of these, Siaplay *et al* (2005) report EV estimates of VaR in a single table obtained using the EV function in Palisade Corporation's '@Risk' package, but provide no EV analysis as such. We also note there that this function only allows the user to model a Gumbel EV distribution, and this distribution is not compatible with the extremes of heavy-tailed returns. Modelling the market risk in financial data such as agricultural commodity prices with the heavy-tailed Frechet EV distribution is the standard approach to EV modelling. Odening and Hinrichs (2002) provide an analysis based on Generalised EV theory, but they report rather unstable estimates of the tail index parameter – a common problem in this area - and this makes their results unreliable.

It is also noteworthy that all but one of these studies focuses exclusively on the VaR risk measure. The one exception (Zhang *et al.*, 2007) looks at lower partial moment measures based on the downside risk literature (e.g., Fishburn, 1977) rather than the coherent risk measures that have been much discussed in the mainstream financial risk literature. However, since the VaR and the ES can be regarded as special cases of the lower partial moment measures if the lower partial moment parameter takes the values 0

or 1 respectively (see Dowd, 2005, p. 26), then this study can also be regarded as reporting the ES as well as the VaR.

To our knowledge, there are no studies so far of coherent risk measures applied to agricultural risk problems and it is this shortfall that we address in the rest of this paper.

3. RISK MEASURES AND ESTIMATION METHODS

Suppose X is a realised random loss variable – a variable that assigns loss outcomes a positive sign and profit outcomes a negative one - for a commodity over a given horizon. If the confidence level is α , the VaR at this confidence level is:

$$VaR_{\alpha} = q_{\alpha} \quad (1)$$

where the term q_{α} is the α -quantile of the loss distribution. For any given horizon, the VaR is defined in terms of its conditioning parameter, the confidence level, which is arbitrarily specified by the user. Viewed as a function of the quantiles of the loss distribution, the VaR places all its weight on a single quantile that corresponds to the chosen confidence level and places no weight on any others. This implies that the user only ‘cares’ about a single loss quantile, and is not concerned about higher losses – a rather strange form of negative risk aversion – and it is this property that causes the VaR risk measure to be non-subadditive (Acerbi, 2004) ⁵. Sub-additivity is required for standard diversification of risks, that is the risk of a portfolio is less than or equal to the sum of the components of the portfolio. The lack of sub-additivity implies that risk diversification does not work and this is strange from a risk management perspective as well as invalidating the main plain platform of portfolio theory. More formally, suppose

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⁵ Specifically, suppose we let X and Y represent any two portfolios and let $\rho(\cdot)$ be a measure of risk over a given forecast horizon. The risk measure $\rho(\cdot)$ is subadditive if it always satisfies the condition $\rho(X+Y) \leq \rho(X) + \rho(Y)$. Subadditivity reflects the idea that risks should not increase, and should typically decrease, when we put them together, i.e., it reflects the notion that risks should diversify. The coherent risk measures are always sub-additive by construction, because sub-additivity is one of the axioms of coherence, but the VaR is not coherent and the failure of VaR to be sub-additive leads to the VaR having some strange and undesirable properties as a risk measure. See Artzner *et al.* (1999, p. 217, Dowd (2005, pp. 31-32)

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The second measure, the ES, is loosely speaking the average of the ‘tail losses’ or losses exceeding the VaR:

$$ES_{\alpha} = \frac{1}{1-\alpha} \int_{\alpha}^1 q_{\alpha} dp \quad (2)$$

The ES is superior to the VaR in a number of respects (e.g., it is subadditive and coherent and because it tells us something about the expected magnitude of tail losses of which the VaR tells us nothing). However, the ES is specified in terms of the same conditioning parameter as the VaR and, as with the VaR, there is generally little to tell us what value this parameter should take. Moreover, the ES has the undesirable property of implying that the user is risk-neutral, and this sits uncomfortably with the use of such measures by risk-averse agents.

Our third measure is the Spectral Risk Measure (SRM). Following Acerbi (2002), we consider the following exponential Spectral Risk Measure:

$$M_{\varphi} = \frac{R e^{-R}}{1 - e^{-R}} \int_0^1 e^{Rp} q_p dp \quad (3)$$

where $R > 0$ is the coefficient of absolute risk aversion.⁶ This weighting/risk-aversion attaches higher weights to larger losses, and, moreover, the weights rise more rapidly as the user becomes more risk-averse. The distinctive feature of an SRM is, thus, that it specifically incorporates a user's degree⁷ of risk aversion. Once a user chooses the value of R that reflects their attitude to risk, it can then obtain an 'optimal' risk measure that directly reflects its degree of risk aversion. So, whereas the VaR or ES are contingent on the choice of an arbitrary parameter, the confidence level, whose 'best' value cannot easily be determined, a spectral risk measure is contingent on a parameter whose 'best' value can be selected by the agent that uses it. Another useful property of SRMs is that SRMs are a subset of the family of coherent risk measures, so they have the attractions of coherent risk measures as well.

Turning now to estimation methods, we model the distributions governing the agricultural returns using a Peaks over Threshold (POT) approach which focuses on the realisations of a random variable X over a high tail threshold u . More particularly, if X has the distribution function $F(x)$, we are interested in the distribution function $F_u(x)$ of exceedances of X over a high tail threshold u (the relationship between exceedances and different tail thresholds is shown later in Figure 1):

$$F_u(x) = P\{X - u \leq x | X > u\} = \frac{F(x+u) - F(u)}{1 - F(u)} \quad (4)$$

As u gets large, the distribution of exceedances tends to a Generalized Pareto Distribution (GPD) (Embrechts *et al.*, 1997):

$$G_{\xi, \beta}(x) = \begin{cases} 1 - (1 + \xi x / \beta)^{-1/\xi} & \text{if } \xi \geq 0 \\ 1 - \exp(-x / \beta) & \text{if } \xi < 0 \end{cases} \quad (6)$$

⁶ The exponential SRM is thus based on the assumption that the coefficient of absolute risk aversion is constant and is generated by exponential risk-aversion or utility function. An alternative SRM is a power SRM based on the assumption that the coefficient of relative risk aversion is constant. The properties of these various SRMs are examined in more detail by Dowd *et al.* (2008).

⁷ The measurement of risk aversion has figured significantly in the agricultural literature with papers including *inter alia* Lagerkvist (2005) and Lence (2000).

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where

$$x \in \begin{cases} [0, \infty) & \text{if } \xi \geq 0 \\ [0, -\beta/\xi] & \text{if } \xi < 0 \end{cases}$$

and the shape ξ and scale $\beta > 0$ parameters are estimated conditional on the threshold u (Balkema and de Haan, 1974; Embrechts *et al.*, 1997, pp. 162-164). Note that the shape parameter ξ sometimes appears in GPD discussions couched in terms of its inverse, a tail index parameter α given by $\alpha = 1/\xi$.

The behavior of the GPD tail depends on the values of these parameters, and the shape parameter is especially important. A negative ξ is associated with very thin-tailed distributions that are rarely of relevance to financial data, and a zero ξ is associated with thin tailed distributions such as the Normal, but the most relevant for our purposes are heavy-tailed distributions associated with $\xi > 0$. The tails of such distributions decay slowly and follow a heavy tailed ‘power law’ function. Moreover the number of finite moments is determined by the value of ξ (or α): if $\xi \leq 0.5$ (or, equivalently, $\alpha \geq 2$), we have infinite second and higher moments; if $\xi \leq 0.25$ (or $\alpha \geq 4$), we have infinite fourth and higher moments, and so forth. α therefore indicates the number of finite moments. Evidence generally suggests that the second moment is probably finite, but the fourth moment is more problematic [as it may not be defined](#) (see, e.g., Loretan and Phillips, 1994).

The values of the GPD parameters can be estimated by Maximum Likelihood (ML) methods using suitable (e.g., numerical optimization) methods. The log-likelihood function of the GPD is:

$$l(\xi, \beta) = -n(\ln(\beta)) - (1 + 1/\xi) \sum_{i=1}^n \ln(1 + \xi x_i / \beta) \quad \text{for } \xi \neq 0 \quad (7)$$

$$l(\beta) = -n(\ln(\beta)) - \beta^{-1} \sum_{i=1}^n x_i \quad \text{for } \xi = 0 \quad (8)$$

where in both cases x_i satisfies the constraints specified above for x .

Assuming that u is sufficiently high, the distribution function for exceedances is given by:

$$F_u(x) = 1 - \frac{N_u}{n} \left(1 + \xi \frac{x-u}{\beta} \right)^{-\frac{1}{\xi}} \quad (9)$$

where n is the sample size and N_u is the number of observations in excess of the threshold (Embrechts *et al.*, 1997, p. 354). The p^{th} quantile of the return distribution - which is also the VaR at the (high) confidence level p - can then be obtained by inverting the distribution function, viz.:

$$q_p = \text{VaR}_p = u + \frac{\beta}{\xi} \left\{ \left(\frac{n}{N_u} p \right)^{-\xi} - 1 \right\} \quad (10)$$

The ES is then given by:

$$ES_p = \frac{q_p}{1-\xi} + \frac{\beta - \xi u}{1-\xi} \quad (11)$$

and the exponential SRM is given by:

$$SRM = \int_0^1 \frac{e^{-(1-p)/R}}{R(1-e^{-1/R})} \left[u + \frac{\beta}{\xi} \left\{ \left(\frac{n}{N_u} p \right)^{-\xi} - 1 \right\} \right] dp \quad (12)$$

Having obtained the risk-measure formulas, estimates of the risk measures themselves are then obtained by estimating/choosing the relevant parameters and plugging these into the appropriate formula (i.e., (10) for the VaR, (11) for the ES, and (12) for the SRM). This is straightforward for the VaR and the ES; however, for spectral risk measures, we need to use a suitable numerical integration method (e.g., a trapezoidal

rule, Simpson's rule, etc. - for further details see Miranda and Fackler, 2002, or Cotter and Dowd, 2006).

Finally, in this section, we note that the estimates of standard errors and confidence intervals reported in this paper were obtained using a semi-parametric bootstrap set out by Cotter and Dowd (2006). To implement this procedure, we begin by taking 5000 bootstrap resamples, each of which consists of n uniform random variables, where n is our sample size. Each resample is then sorted into ascending order so that its relative frequencies can be considered 'as if' they were a set of resampled cumulative probabilities. For example, for the j^{th} resample, these relative frequencies are as $p_1^j, p_2^j, \dots, p_n^j$, where $p_i^j \leq p_{i+1}^j$. We then use the fitted GPD (i.e., (10)) to obtain each element of the j^{th} resample set of losses. Thus, if p_i^j is the i^{th} cumulative probability in the j^{th} resample, then q_i^j , the i^{th} highest loss in the j^{th} resample, can be obtained from

$$q_i^j = \hat{u} + \frac{\hat{\beta}}{\hat{\xi}} \left\{ \left(\frac{n}{\hat{N}_u} p_i^j \right)^{-\hat{\xi}} - 1 \right\} \quad (13)$$

where (13) is a version of (10) in which the '^' refer to the sample-based estimates of the GPD parameters. Since the VaRs are quantiles, (13) gives us direct resample estimates of the VaRs. Resample estimates of the ES and SRM are then obtained using (11) and (12) respectively (with q_p replaced by q_i^j and parameters replaced by their '^' estimates). For each resample, the standard errors and confidence interval were obtained from the set of resample estimates of the appropriate risk measures.

4. DATA

Our data set consists of weekly logarithmic price changes for Corn and Soybean futures contracts traded on the CBOT between January 1979 and December 2006 totalling 1461 observations. Our period of analysis is a one week time horizon⁸ and we take the nearby

⁸ We choose the period of a week for illustrative purposes and to simplify the analysis although a week would also be appropriate for some financial users of these risk measures. However, we accept that different users would have different forecast horizons, but try to deal with all these possibilities

futures contract in our analysis. The use of nearby contracts for rolling over the data is the standard approach that ensures high levels of trading volume associated with the futures. We examine the tails of both long and short positions for each series, thus giving us a total of 4 cases in total. We choose these particular crops for their importance in the US agricultural sector. Corn is the most widely produced feed grain in the US and accounts for 90% of the total value of feed grains produced. Approximately 32 million hectares are planted to corn with most being in the heartland states, with the number of large farms (greater than 200 hectares) growing in recent years. Soybeans are also selected as the US is the world's largest producer and exporter of them and approximately 2.5 billion bushels were produced in 2007 from around 30 million hectares. Of this area, 72% of farms grew less than 100 hectares but contributed less than 26% of the total crop⁹. Thus, we believe our choice of crops captures significant agricultural activity and thus could be viewed as representative of arable production in the US.

5. RESULTS

As a preliminary, we note that the mean returns for both contracts are near zero, and the corresponding standard deviations suggest weekly volatilities in excess of 3%. The corn contract has a slight positive skew (0.005) whereas the soybeans contract has a strong negative skew (-0.444). Their kurtoses are 6.857 and 6.359 respectively. QQ-plots and Jarque-Bera test results indicate that a Gaussian distribution is always rejected. These results suggest that we are dealing with distributions that are heavy tailed and (at least in the soybean case) probably skewed as well.

We now fit the distributions of exceedances and ML estimates of the GPD parameters are given in Table 2 for both long and short trading positions. [We show exceedances for different thresholds, u, in Figure 1. We choose from the range of stable tail parameter estimates as they feed into our risk measures. In contrast, we see for very](#)

would induce considerable complications to our analysis, e.g. sample frequency would not match forecast horizons, so forecasts would be multiples of the scaling frequency. We thank an anonymous referee for highlighting this point.

⁹ Data are drawn from the National Agricultural Statistics Service (NASS) of the USDA website at <http://www.nass.usda.gov/>

[small number of exceedences, that tail estimates become instable. This is understandable as we are using a very small sub-sample to describe the tail parameter.](#) ~~The~~ Table 2 gives the assumed thresholds u , the associated numbers of exceedences (N_i) and the observed probabilities of being less than the threshold ($prob$). Also included and of most interest for the risk measures are the tail indices, ξ , and the scale parameter, β . The tail indices are generally positive (though not statistically significant) for the spot and futures contracts, and the scale parameters vary around 2. The numbers and probabilities of exceedences vary somewhat.

Insert Table 2 here

[Insert Figure 1 here](#)

Turning now to the risk measures themselves, Table 3 reports the estimated risk measures based on the counterfactual and heavily used assumption that returns are Normal. We will comment on these shortly when presenting the POT estimates of these risk measures.

Insert Table 3

GPD estimates of VaR and ES are given in Table 4 for confidence levels of 99%, 99.5% and 99.9%: To illustrate, the VaR of 8.338 at the 99% level implies that there is a 1% chance of having losses greater than 8.338% of the value of the corn futures contract for a long trading position. These show, as we might expect, that estimated risk measures rise with the confidence level, and that the estimated VaRs are notably small than the estimated ESs. There are no great differences between the different contracts, but the short and long results can be somewhat different from each other. It is also noteworthy that the estimated risk measures are usually much higher than the Normal-based estimates in Table 3 and the divergence increases as one moves to more extreme probability levels. This suggests that extreme risks are large, and that assuming a Gaussian distribution in these circumstances can lead to very considerable under-estimates of our risk measures.

The Table also reports the bootstrapped standard errors of the estimated risk measures, and these rise considerably with the confidence level: this indicates that estimated risk measures become considerably less precise as the confidence level rises.

This is a well-known phenomenon, and reflects the fact that as the confidence level rises, we are dealing with an increasingly extreme tail measured with fewer and fewer observations.

Insert Table 4 here

Table 5 shows bootstrapped estimates of the standardized 90% confidence intervals for the VaR and ES: these are estimates of the 90% confidence intervals divided by the estimated mean risk measure, and are easier to interpret than conventional confidence intervals. So, for example, the first two results in the first row of Table 6 tell us that the 90% confidence interval for the corn VaR varies from 88.6% to 112.7% of the mean VaR, and so forth. Two features of these results stand out. First, the standardized confidence intervals for the ES are generally a little narrower than those for the VaR: this confirms that in relative terms, estimates of the ES are more precise than estimates of the VaR. Second, the confidence intervals are fairly symmetric for the risk measures predicated on the 99% confidence level, but become asymmetric as the confidence level rises and, in particular, we see that the right bound is further from the mean risk measure than the left bound. This finding is also to be expected and reflects the fact that as we move further out into the extreme tail, observations become increasingly dispersed.

Insert Table 5 here

We now turn to estimates of spectral risk measures. As we have discussed already, these risk measures make use of the coefficient of absolute risk aversion R rather than the confidence level as their conditioning parameter. The value of this coefficient depends on the user's attitude to risk, and can in principle be any positive number (assuming that the user is in fact risk-averse). However, in the present EV context it only makes sense to work with fairly high values of R : the higher is R , the more we are concerned about very high (i.e., extreme) losses relative to more moderate

ones. A concern with extremes therefore suggests a high value of R . Accordingly, we consider here values of R equal to 20, 100 and 200.¹⁰

Once a value of R has been chosen, we can estimate the value of the integral (12) using numerical integration. The idea behind this is to discretize the continuous variable p into a large number N of discrete ‘slices’, where the discrete approximation gets better as N gets larger. We then choose a suitable numerical integration method, and the ones we considered were the trapezoidal rule, Simpson’s rule, and numerical integration procedures using quasi-Monte Carlo methods based on Niederreiter and Weyl algorithms respectively.¹¹

However, we first need to evaluate the accuracy of these methods. To help us do so, Table 6 gives estimates of the approximation errors generated by these alternative numerical integration methods based on alternative values of N and a plausible set of benchmark parameters. These results indicate that all methods have a negative bias for relatively small values of N , but they also indicate that the bias disappears as N gets large. In addition, they suggest that for high N , the trapezoidal method is at least as accurate as any of the others.

Insert Table 6 here

For the remaining estimations, we selected a benchmark method consisting of the trapezoidal rule calibrated with $N=1$ million.¹²

Estimates of SRMs and their bootstrapped standard errors and standardised 90% confidence bounds are given in Table 7. Broadly speaking these results are comparable to those obtained earlier for the VaR and ES, but with R playing the same role as the earlier confidence level. In particular, we see that:

¹⁰ Obviously risk aversion is user specific. For those concerned by smaller losses the conditioning parameter would take on lower risk aversion values.

¹¹ The choice of numerical integration method was also influenced by the need to have fast integration algorithms for use in our bootstrap algorithms. We used the Miranda-Fackler (2002) CompEcon functions, which are very fast indeed.

¹² There is of course a trade-off between calculation time and accuracy, but the choice of $N=1$ million gives us results that are accurate to within half a percentage point in the illustrative case examined in Table 7, and this is accurate enough for our purposes.

- Estimated SRMs are considerably higher than the normal estimates in Table 3.
- Estimated SRMs rise notably as R increases.
- Estimated SEs and the widths of confidence intervals rise as R increases; we also see some asymmetry in the confidence intervals for very high values of R again with the right bound being a little further away from the mean than the left bound.
- Differences across contract types are fairly small, and the only noteworthy difference between the corn and soybean results is that the latter have more pronounced differences between long and short positions.

Insert Table 7 here

6. CONCLUSIONS

Accurate risk measurement in agriculture is important to farmers, policy makers and financial institutions, not least as policy reform in many developed market economies agricultural sectors potentially exposes producers to greater market risk. The importance of such measures is central to this paper which applies the Peaks-Over-Threshold version of Extreme Value Theory to estimate the extreme financial risk measures for a selection of agricultural contracts. The risk measures considered were the Value at Risk (VaR), the Expected Shortfall (ES), and the Spectral Risk Measure (SRM) based on an exponential risk-aversion function for a given coefficient of absolute risk aversion. We examine the properties of these risk measures and suggest that SRM is to be preferred to the ES, which in turn is to be preferred to the VaR. We then estimate both the risk measures themselves and some precision metrics obtained using a parametric bootstrap procedure. Our empirical results suggest three main conclusions, and this is the case for all three risk measures. First, we find that the estimated risk measures are all considerably higher than the estimates we would have obtained under a Gaussian distribution. This suggests that a Gaussian distribution can lead to major under-estimates of extreme risks. Second, we find that estimated risk measures are quite imprecise, as judged by the estimated standard errors and confidence intervals. This is to be expected, as EV problems almost by definition involve small numbers of extreme observations. Third, we find that the degree

of uncertainty associated with our estimated risk measures increases as we go further out into the tail. This finding also makes intuitive sense: the further we go into the tail, the more sparse our observations become, and the more uncertain any estimates will be. In a nutshell, extreme risk measures are large, but also uncertain.¹³

¹³ A major limitation of our work, however, is that it is restricted only to the univariate case. A natural but nontrivial extension is therefore to apply multivariate EV theory to the estimation of agricultural financial risks. This is technically demanding as it would require the estimation of an appropriate copula function – that is a copula function consistent with EV theory. For more details on how that might be done, see, e.g., McNeil et al (2005, pp. 311-326).

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